A COMPREHENSIVE FRAMEWORK FOR GLOBAL AND DOMESTIC ASSET ALLOCATION

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List of Abbreviations

Abbreviation	*Descriptions*
ACWI	All countries world index
AIC	Akaike's information criterion
APT	Arbitrage pricing theory
AR	Auto-regressive model
ARIMA	Auto-regressive integrated moving average model
ARIMA-Reg	Auto-regressive integrated moving average-regression model
BIC	Bayesian information criterion
BGAI	Barclays global aggregate index
BL-CR	Black-Litterman under country risk based estimation
BL-Model	Black-Litterman model
BSP	Brent spot oil prices
CAPM	Capital asset pricing model
CC-VCM	Constant correlation variance covariance matrix
CPI	Consumer price index
EPU	Economic policy uncertainty index
ESR	Excess sharp ratio
EWP	Equally weighted portfolio
GICS	Global industry classification standard
GMVP	Global minimum variance portfolio
HI	Herfindahl index
Hist	Historical averages
IEER	Implied equilibrium excess return vector
JB	Jarque-bera test statistics
KSE	Karachi stock exchange (guarantee) limited
MSCI	Morgan stanley capital international
MSPE	Mean square prediction error
MVP	Efficient portfolios based on mean-variance criteria
OLS	Ordinary least square
PCA	Principal component based covariance estimation
RMSE	Root mean square error
S&P GSCI	Standard & Poor Goldman Sachs commodity index
SD	Standard deviation
Sh-VCM	Shrinkage method for variance covariance matrix
SI-VCM	Single index variance covariance matrix
S-VCM	Sample variance covariance matrix
Var	Variance
VCM	Variance covariance matrix

ABSTRACT

There are two main streams to deal with traditional asset allocation strategies i.e. theoretical approach and implementation approach. These approaches are the prime focus of this study. Portfolio optimization is based upon two fundamental ingredients i.e. estimation of return vector and covariance matrix. This study compares the 12 covariance matrix under four categories i.e conventional methods, factor models, portfolio of estimators and shrinkage approach. This study also compares the performance of 7 alternative ways for estimation of return vector. Study also develops portfolios based on mean-variance optimization, minimum variance portfolios, constraints portfolios and naïve diversification. This study first time introduces the 'country risk' as unprice risk factor in the Black-Litterman model and uses this augmented Black-Litterman formula (BL-CR) for the estimation of expected return vector. The comparison of asset allocation strategies are base upon the financial efficiency and diversification dimensions using 10 asset classes from 5 emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand, 4 asset classes from global environment and 22 asset classes from Pakistan. Study reveals that factor models as a group outperform the competing covariance estimators in all the emerging countries. From the number of positive and negative weights to asset classes, maximum and minimum value of weights, other diversification measures of the mean-variance framework, it is reveal that mean variance portfolios are concentrated, mostly counterintuitive, results more short positions and highly sensitive to the choice of input. Similarly the financial efficiency of these portfolios is also highly sensitive to the input estimates. Results of asset allocation strategies suggest that, on an average, equally weighted portfolios result a competitive strategy in Pakistan and in global environment. Therefore study also recommends that investment managers and academia should at least consider the naïve diversification as a first obvious benchmark in comparison with other asset allocation strategies. The BL-CR model outperform the original model as it has relatively less short positions, more number of positive weight, less variance, low value of Herfindahl index and high value of excess sharp ratio. Therefore BL-CR model is more appropriate on mathematical and empirical ground in asset allocation than original model to disperse country risk. This study also recommends that investment managers and academia should consider the Black-Litterman model under country risk for tactical asset allocation decisions in emerging Asian countries.

Keywords: Asset allocation, Black-Litterman model, covariance matrix, emerging markets, expected return vector, mean-variance criteria, portfolio optimization

JEL classification: C13, C51, C52, G11, G15, G17

Chapter 1

1. INTRODUCTION

Generally individuals earn and spend money. But it is hard to assume that current earning exactly match the current consumption desires. Sometime individuals have more money than they want to spend and vice versa. This imbalance either in the shape of excess current earning or excess consumption leads towards savings or borrowings. If current earning is more than consumption then people tends to save. Reason behind saving may be the trade-off between present consumption and higher level of future consumption. Individuals may give up the immediate possession of savings against some future amount of money for future consumption. But this possession happens if they expect to receive larger amount of money than they give up today.

Individuals who defer their today's consumption i.e. savings, expect to receive higher level of money than they give up. On the other hand, if current income is less than current consumption i.e. borrowing, then individual will return more than they borrowed. The rate at which current consumption and future consumption are exchanged is the rate of interest.

In fact investment decisions are how much not to consume today with the view that more can be consumed in future. Investment decisions are directly linked with consumption decisions. In this context, investments are the intended deferred consumption. Investment decisions questions the current consumptions. The consumption/investment decision is equally important for individual investor as well as manager of corporation. Individuals opt to save one dollar if expected future benefit exceeds the benefit of consuming it today. Manager of corporation decides the trade-off between present consumption (to pay dividend to the shareholders) and future consumption (retain for the future productive opportunities).

Benefit of capital market to the society requires a comparison between consumption and investment in the presence of capital market and those without well-functioning capital market. In the world with perfect capital market, if total saving and total investment is more than a world without capital market then presence of capital market benefit to the society. Moreover, this increase in total saving and total investment is established without making any individual worse-off while at least one individual getting better off with capital market. Some basic principles regarding the consumption and investment with and without capital market are developed in first half of twentieth century by American economist Irving Fisher.

Fisher (1930) shows how well functioning capital market enhance the utility of economic agents. Capital market provides cheaper means to borrow and lend. Lending through well-functioning capital market to individual borrower may result a higher return for savers while borrowers can get relatively less expensive financing due to less searching cost. Savers opt to save more in the world with capital market than the world without capital market. Similarly borrowers can borrow more and less expensive financing in the world with capital market than if forced to search for financing without capital market. Therefore total saving and total investment is more in the world with capital market than otherwise.

According to Fisher separation theorem, capital market yields a single interest rate. Borrowers and lenders can use interest rate in consumption and investment decisions. There is separation between both of the decisions i.e. investment and financing. The acceptance criteria of any investment opportunity should be the direct comparison between the rate of return from that specific project and market interest rate. If investment opportunity's rate of return is equal or greater than market interest rate then companies and individuals should accept it. For this, individuals and companies should move to capital market for financing if they cannot finance it internally. Every individual has his own consumption preferences but borrowers do not need to consider these preferences. Similarly savers do not need to search borrowers for a specific type of investment. With this separation between the investment and financing decisions, firm do not need to customize their investment decisions according to the individual preferences.

1.1.Theoretical Background of Study

Main theories which provide foundation to modern finance are utility theory, state preference theory, Markowitz portfolio theory, capital market theory and arbitrage pricing theory. Common subject of all these theories are to facilitate the individuals and societies for the allocation of their scarce resources into investment avenues. Theoretical backgrounds of these basic foundations of asset allocation are described as follow.

1.1.1. The theory of choice

Study of economics deals with the allocation of scarce resources and distribution of wealth among one another over time horizon. Individuals as a consumer have alternatives about how much to buy and consume. Consumers have to decide among these choices and select on the basis of maximizing the utility. Utility is the amount of happiness gained from a goods or services. In the language of economics, utility has some properties. Utility number may have positive and negative signs and a bigger utility number is better in term of happiness as compare to smaller utility number. Also utility numbers have ordinal properties as oppose to the cardinal properties. Consumer can compare utility numbers instead to perform any calculation on it.

The outcome related to any choice may be certain or uncertain. Consumer makes decision under certainty as well as uncertainty. In some situation consumer exactly knows the outcome and get benefit from consuming the specific goods. Conversely, sometime single decision leads toward different outcomes and occurrence of these outcomes also varies in term of probabilities. A single decision leads multiple outcomes and therefore it associates multiple utilities. In this situation consumer may consider competing models like the expected income hypothesis and expected utility hypothesis. As per the expected income hypothesis, individual calculate the expected income from each choice and select among those on the basis of maximizing expected income. Expected income is the probability weighted average of income from all choices. Mathematically, $EI = \sum P_i I_i$ where EI is the expected income, P_i is the probability of the occurrence of ith outcome and I_i is the income from ith outcome. Expected utility is the probability weighted average of utilities of consumer from all choices. Mathematically, $EU = \sum P_j U(I_j)$ where P_j denotes the probability of occurrence of ith outcome, $U(I_j)$ the utility resulting from ith outcome. Individual selects among these choices to maximize expected utility.

Individuals also decide whether to consume today or save and consume relatively larger amount in future. That is, individuals have choices whether to consume now or defer this today's consumption for the betterment of their future i.e. investment. This is utility theory of choice over time horizon. The theory of investor choice has some basic assumptions regarding the individual behavior when individual faces the task of ranking the timeless (one period) risky alternatives. Five axioms of choice under uncertainty are the minimum requirement for rational and consistent individual behavior. These are comparability, transitivity, strong independence, measurability and ranking.

1.1.2. State preference theory

In the economy of a country, an equity market plays a distinct role for the movement of funds between several parties having different interests. Broadly, it acts as intermediary among the parties involved. On one side there are investors who want to invest their surplus or sacrifice today's need for the amelioration of future. These investors are gorgeous for corporations as well as many other institutions. Equity markets facilitate both of the parties in a sophisticated mode. Companies sell their securities and get capital for investment in real assets while individual have claims on real asset of companies by investing in securities.

All the securities inherently have time dimensions. The expected consumption over some future time period determines the individual investment decisions. Investment decisions are related to future time horizon and future is generally uncertain. From individual as well company's perspective the future value of securities at some future point in time can be written as a vector of possible future payoffs. The portfolio of an individual investor is the matrix of probable payoff resulting from involved securities in that specific portfolio.

As securities have time dimension and therefore inherently involve uncertainty and risk. Any security is a set of probable payoffs and these payoffs entirely depends upon the future state of nature. Furthermore, the future states of nature are also mutually exclusive. As long as future state of nature is revealed, the payoffs associated with that specific security is determined. So security represents a claim to a vector of state-contingent payoffs. In state preference model the form of uncertainty is related to the future state of nature. Future payoffs associated with securities are driven on the basis of probable state of nature. Therefore payoffs related to each risky security are known when future state of nature is known. Individual's total end of period wealth is known by simply adding the future payoff of all the holding securities.

Theoretically speaking, the number of future state of nature can be infinite and future payoffs associated with these state of nature are also infinite. Critical properties of these infinite states of nature are that they are mutually exclusive and exhaustive. In other word, among these set of future state of nature one and only one state of nature will be realized at the end of any specific period and the summation of all the probabilities of individual state of nature is equal to one. Each security has its own end of period payoff's probability distribution and it is assumed that individual can associate these payoffs with each probable state of nature. Further, it is also assumed that the only concern from the individual's point of view is the amount of wealth from a state of nature otherwise indifferent from the occurrence of state of nature. That is individual have state independent utility function as oppose to the state dependent utility function. Individual utility function may be state dependent if it depends upon the individual's own wealth as well the wealth position of other individuals.

1.1.3. Mean - variance portfolio theory

Theory of investor's choice mainly deals with a world of uncertainty where risk-averse investor makes choices. State preference framework describes that any security is a set of probable payoffs and these payoffs entirely depends upon the future state of nature. Furthermore, the future states of nature are also mutually exclusive. But this state preference framework is a general approach and it seems very hard to identify and list down all the future state of nature along with payoffs in each mutually exclusive state of nature. State preference framework in its absolute form lacks in empirical testing. More easy measurement of object of choices is possible in the use of mean-variance object of choice. Mean and variance of any security's return are used to define the indifference curve of investors. The mean and variance approach may be less general than that of state preference framework but it mainly based on statistics. Due to the statistical nature of mean-variance approach its empirical testing is relatively easy as compare to other object of choice. To value risky assets properly, one should know the way to measure and price the financial risk. The ability to quantify the risk is considered a major milestone in literature of finance in last couple of decade. This development leads the economy towards better allocation of scarce resources. It helps the investors in allocating their savings into different type of risky investment opportunities.

Generally the term risk and uncertainty are used interchangeably in the literature of finance. A situation is said to be uncertain only if it results in two or more outcomes. Uncertainty may be positive or negative. But risk is specifically attached to the unfavorable outcome. Risk is the possibility of loss or probability of adverse outcome. Every risk is uncertainty but every uncertainty may or may not be a risk. Therefore uncertainty is necessary but not sufficient condition for risk.

In the beginning of 1960s financial analyst starts to consider the risk in a more rigorous fashion. But at that time there is no specific risk measure. The work of Markowitz (1952) provides basic principles for portfolio theory. Markowitz (1952) work is statistical in nature but basic rationale behind his framework for asset allocation can be captured with the well-known phrase “don’t put all your eggs in one basket’. Under a reasonable set of assumptions, Markowitz shows that variance of rate of return is appropriate measure for portfolio risk. He also derives a formula for computation purposes. The formula for risk of portfolio shows the importance of diversification in a way that how it reduces total risk of portfolio. There are several basic assumptions behind Markowitz (1952) model regarding the behavior of investor. Like, investor maximizes one period expected utility and utility curve is diminishing marginal utility of wealth. Variability in expected returns is a measure of estimation of risk of portfolio. Further it also assumes that investor’s decision solely based on the expected return and risk. Investor also prefer higher level of return over lower return for same level of risk and similarly investor also prefer less risk over higher risk for a specific level of return.

Allocation in one asset or portfolio of assets based upon above set of assumptions is considered efficient only if there is no single asset or portfolio of assets which offer higher expected return with given level of risk or lower risk with given level of expected return. Markowitz (1952) demonstrates that expected return of portfolio is weighted average expected return of all individual assets in the portfolio. Risk of portfolio (measured by standard deviation) is not only depends upon the risk of individual asset but also depends upon the covariance between return of all pairs in that portfolio. This covariance between the return of one asset with the return of other assets is the dominant factor for investor.

Selection of different assets in the portfolio should base upon the correlation coefficient. Rate of return of any portfolio can be maintain while reducing the risk of portfolio by selecting the assets having low positive or negative correlation. Efficient frontier is set of all assets or portfolio that offers highest expected return for each risk level or lowest risk for each specific level of return. Investor selects that portfolio which is at the point of tangency between efficient frontier and highest utility curve. This selection of portfolio varies among investors. Optimum portfolio for specific investor is one that has desirable risk-return characteristics for individuals. Further mathematical details on asset allocation and mean variance framework are presented at appendix A. The mean variance framework also faces many criticisms by various researchers. Hanoch and Levy (1969) analyze the preferences under utility theory and also compare it with mean variance criterion. Main finding shows that there exist differences in preferences in both the criteria. The idea of mean variance optimization may be a good starting point for portfolio selection. Practically, resulting portfolio from the mean variance criteria may be counterintuitive. The mean variance criteria also imply normally distributed returns.

1.1.4. Capital asset pricing model

The work of Markowitz (1952) on portfolio selection becomes relevant with the publication of capital asset pricing model. Capital market theory actually builds upon the portfolio theory. Assume that an investor evaluates all the risky assets and reaches to the efficient frontier. Then further assume that investor maximize his utility on the basis of risk and return of risky assets. Among all available portfolios of risky assets, investor selects the one that is at the point of tangency between utility map and efficient frontier. Then such type of investor is considered as Markowitz efficient investor. The effort of Markowitz on portfolio theory is further extended and resulted in a model to price all the risky assets.

The concept of risk free asset, along with the portfolio theory provides basis to the development of capital market theory. There is a zero correlation and covariance of risk free asset, with any of the asset. Due to this any combination of asset or group of asset with risk free asset result a linear risk and return function. Therefore, the combination of any risky asset with risk free asset on the efficient frontier result a set of straight line portfolio possibilities. Among these lines, the line which is tangent to the Markowitz efficient frontier is dominant and called capital market line.

Investment decisions of all investor target the capital market line but on the basis of individual risk preferences. Capital market theory suggests that there is a portfolio for investor in which only systematic risk of individual asset is important while unsystematic risk of individual asset is unimportant. Systematic risk is the responsiveness of asset towards the economy wide factors. This response varies within asset to asset. Portfolio diversification has no link with systematic risk of assets. Therefore, investors demand higher expected return (premium) on these risky assets depending on the systematic risk. This effort is attributed simultaneously to William Sharp (1964), Jack Treynor (1961), John Linter (1965) and Jan Mossin (1966). It showed that equilibrium rate of return is determined by the covariance of all risky assets with market portfolio.

Hypothetical world is assumed for the development of capital asset pricing model (CAPM). Capital market theory is a step further than portfolio theory; therefore, it is based upon some common as well as some additional assumptions. It is assume that all investors are risk averse and considers Markowitz efficient investors. All individual wants to maximize expected utility. Investors have homogenous expectation about return of asset and have joint normal distribution. A single risk free rate is used for the purpose of borrowing and lending for investors. Total investment can be divided into infinite pieces. The market are perfect and in equilibrium. Anyhow not all assumption which provide basis to capital market theory confirms the reality. But this model (CAPM) is very useful and revolutionary in the financial literature as first time, individuals are able to quantify and price the risk in capital market.

1.1.5. Arbitrage pricing theory

Basic building block of understanding the link between risk and expected return in the financial market has been provided by the Markowitz portfolio theory and capital asset pricing model. Sharp (1964) defined the systematic risk and explain the way investor can tradeoff between risk and expected return. Investors can either invest in risk free assets or in risky assets. Capital asset pricing model consider an appealing explanation for the tradeoff between risk and expected return in the capital market. But latter on it was not considered appropriate for the description of equilibrium in capital markets. Ross (1976) suggests that expected return of any specific asset is a function of the sensitivity of that asset with one or more systematic factors. Moreover, expected returns are linearly related to risk factors and this model builds on fewer assumptions than CAPM. This contribution by Ross (1976) is referred as arbitrage pricing theory.

Arbitrage pricing theory is more general than the capital asset pricing model but it actually based upon the same intuition. Arbitrage pricing theory suggests that expected rate of return of any given security is linear function of K multiple risk factors. Sensitivity of any given security with each risk factor is called factor loading. Arbitrage pricing theory assumes that capital markets are perfect and investor prefers more wealth over less wealth. Further, returns that are generated through a random process can be expressed as a linear function of set of K factors.

Unfortunately arbitrage pricing theory does not specify the name of different common risk factors. Due to this non specification of common risk factors, practically it seems difficult to define arbitrage pricing theory in theoretical rigorous fashion. This gap between theory and practice towards explaining the relationship between risk and expected return has further captured by the emergence of multifactor model. Later on financial researchers identify various risk factors for the explanation of risk-return behavior in capital market. Fama and French (1993) explore three risk factors that are market premium, size premium and value premium. Carhart (1997) further extend the work of Fama and French (1993) by adding another risk factor called momentum factor.

It can be safely said that both capital asset pricing model and arbitrage pricing theory are constantly under consideration for asset pricing in capital markets. Capital asset pricing model is easily understandable but has less power to explain the return in capital market. This is mainly due to its single factor i.e. market premium. But it is difficult to explain arbitrage pricing theory in economically and theoretically meaningful term. Therefore, still asset pricing theory is at unsatisfactory state.

1.2.Problem Statement

Investment is actually the commitment of funds over future time period in order to receive future payments that will compensates the times funds are committed, expected inflation and expected risk. Investor can allocates fund among available investment opportunities locally as well across borders. In the process of asset allocation, investor may confront varying situations. First, investor needs to identify and explore all the available investment opportunities for investment purpose. Secondly, investor needs to decide about the competing ways for estimation of inputs to portfolio optimization i.e. estimation of future return vector and variance-covariance matrix. In the process of asset allocation, investor can apply the concept of mean-variance optimization, minimum variance portfolios and constrained portfolios. Further investor may simply invest equally in all available investment opportunities i.e. naively diversified portfolios. Therefore investors have to decide in multiple facets simultaneously.

Financial researchers have identified various risk factors for the explanation of risk-return behavior in capital market. It is still possible to earn premium for taking on systematic exposures that are uncorrelated to the market but undesirable for certain investors. Therefore investor can extend the original Black-Litterman formula which primarily based upon single factor assumption to two or multiple factor model. The additional factor should generate positive returns and this premium cannot be accounted by the CAPM. The process of asset allocation starts with the identification of asset classes. The other problem which is more crucial is the identification of investment weights in each considered asset class. In other words, investor needs a comprehensive framework for asset allocation in Pakistan as well across the globe.

1.3.Broader context of the study

Asset allocation based upon quantitative model is perhaps first discussed by Markowitz (1952) in his famous article on portfolio selection. Undoubtedly Markowitz (1952) works on portfolio selection gains widespread acceptance in literature of finance. Kolm, Tutuncu and Fabozzi (2014) reports that the so influential research paper of Markowitz on ‘portfolio selection’ cite 19,016 times in Google scholar and for the phrase ‘modern portfolio theory’ there are about 590,000 hits in Google, 531 YouTube videos, 217 books on Amazon and thousands of tweets on Twitter. But still Michaud (1989) term the optimization by mean variance framework as “enigma” and try to establish that the portfolios based upon Markowitz efficient frontier suggests irrelevant and financially meaningless asset allocations. Financial researchers raises various points against mean-variance framework like missing factors, information levels mismatched, robustness, estimation error maximizer, sensitivity of optimal solution with inputs, non-uniqueness and mostly it results counterintuitive portfolios. Further Disatnik and Katz (2012) claim that large short sale positions under mean variance framework are due to the poor sample estimates of inputs to portfolio optimizer. Disatnik and Benninga (2007) argue that the classical approach end with questionable results. There are two main streams to deal with these questionable results i.e. theoretical approach and implementation approach and are the focuses of this study. Portfolio optimization is based upon two fundamental ingredients i.e. estimation of return vector and covariance matrix. Both of these fundamental ingredients need to be estimated because these are unknown.

The stickiest input to portfolio optimization is estimation of covariance matrix (Ledoit & Wolf, 2003). Typically covariance matrix is estimated by sample covariance matrix. But many researchers like Pafka et al. (2004), Michaud (1989) put criticism on the sample covariance matrix. The index models are supposed to be more intuitive for the explanation of movements among stocks and first introduce by Sharpe (1963). The industry association of stocks beyond the market index are suggests by King (1966) and statistical factors are also applies in the existing literature for the estimation of covariance matrix. Elton and Gruber (1973) suggests the average correlation matrix for the estimation of covariance matrix and reveal that it performs better even than the sample and single index covariance matrix. Previous literatures commonly agree that constant correlation base covariance estimators are more effective than single index models; and historical sample covariance matrix are the poor estimators of the covariance matrix.

Disatnik and Benninga (2007) describe that covariance estimation always have error. It may be due to estimation issues or specification issues. As per the basics of statistics theory there exist tradeoffs or optimal point between both of these two errors and it opens another avenue for discussion on estimation of covariance matrix. Recently, Ledoit and Wolf (2003, 2004) propose the Bayesian shrinkage approach for the estimation of optimal point between estimation and specification error. These shrinkage estimators are complicated in nature. Jagannathan and Ma (2003) come up with simple average of historical sample covariance and Sharpe (1963) model and challenge the most complex Ledoit and Wolf estimations. Disatnik and Benninga (2007) reveal that one cannot get any additional benefit from more complex shrinkage methods.

Recently Disatnik and Benninga (2007) analyze the New York stock exchange and reveal that one cannot get any additional benefit from more complex shrinkage methods. These findings are also consistent with Liu and Lin (2010). The literature dealing with the estimation of covariance matrix is still at confusing stage and also major chunk of literature skew towards the develop markets. With respect to the equity classification of emerging Asian countries, to our knowledge, there is no study that compares the performance of covariance estimation methods i.e conventional methods, factor models, portfolio of estimators and shrinkage approach in the emerging Asian countries.

Regarding the estimation of return vector, Black and Litterman working at Goldman Sachs try to work on an improve model than mean-variance and claim that asset allocation based upon traditional models has not attracted the global portfolio managers. The work of Black and Litterman (1992) point out that only few global portfolio managers consistently fascinate with the standard quantitative asset allocation because it give hefty short positions, behave badly, suggests corner solutions in many cases, mostly results with zero weights and dictate extra-large weights to the assets having low capitalization. Contribution made by BL is the identification of two sources of information regarding future returns. First source is computed from global version of CAPM and it should hold if markets are in equilibrium. The other is based upon the opinion of investment managers about asset classes. Black and Litterman author few papers on the presented idea that are also precise in nature. Empirical studies like Idzorek (2004), Schottle, Werner and Zagst (2010), Beach and Orlov (2007), Walters (2008), Fernandes, Ornelas and Cusicanqui (2012), Meucci (2008), Krishnan and Mains (2005), Meucci (2006), Simonian and Davis (2011) also put into practice the Black-Litterman model.

As BL model improves on the traditional model but it also makes several restrictive assumptions. The BL model assumes that risk can be completely characterize by covariance and built upon the same assumption as the capital asset pricing model (CAPM) and it is attributed simultaneously to William Sharp (1964), Jack Treynor (1961), John Linter (1965) and Jan Mossin (1966). CAPM is revolutionary in a sense that first time, individuals are able to quantify and price the risk in capital market. But Ross (1976) suggests that expected return of any specific asset is a function of the sensitivity of that asset with one or more systematic factors. Later on financial researchers also identify various risk factors for the explanation of risk-return behavior in capital market. Fama and French (1993) explore three risk factors that are market premium, size premium and value premium. Carhart (1997) further extend the work of Fama and French (1993) by adding another risk factor called momentum factor. Also researchers like Fama (1996), Harvey and Siddique (2000) reports evidence of getting the additional premium by taking different positions on different stocks. Similarly Watson and Head (2007) provides evidence against CAPM on a study in stock markets.

Therefore CAPM has less power to explain the return in capital market. This is mainly due to its single factor i.e. market premium. Safely it can be said that CAPM has theoretical justification but it has no valid empirical applications. Hence it is still possible to earn premium for taking on systematic exposures that are uncorrelated to the market but undesirable for investors. Generally expected return strictly depends upon the risk free rate and risk premium; reward against risk. Also it can be observe that emerging economies are supposed to be more risky than the develop markets in term of economic, financial and political risk factors. So the investors of emerging countries are supposed to bear additional risk called country risk. Also Harvey (2004) concludes that country risk should be additionally rewarded and finds that there is high association between the expected return and country risk in emerging markets. After globalization and rapid industrialization, the ability and willingness of an emerging country to meet its any obligation need to be priced. With this background this study also added the country risk factor into the original BL model. This factor expose to generate positive returns and this premium is not capture by the CAPM.

Existing literature on subject of asset allocation across different investment opportunities can be categorized on the basis of tools used for asset allocation. There may be mean-variance framework, safety first portfolio optimization, Black-Litterman model for asset allocation, expected utility asset allocation, behavioral portfolio theory for construction of portfolios, multi-stage and dynamics strategies for portfolio diversification, multi-objective optimization, other sophisticated strategies and rules for diversification and non-theory based diversification. Prime focus of Markowitz (1952) was on portfolio rather on single asset and this idea opened new avenues for investors. He, Grant and Fabre (2012) argued that there are practical problems in the mean-variance framework suggested by Markowtiz (1952) and researcher agreed that this choice for asset allocation was unrealistic. Michaud (1989) even called it "enigma". Given the errors and numerical instability of estimators, DeMiguel et al. (2009) conclude empirically that non theory-based diversification outperforms the more sophisticated asset allocation strategies. The financial literature applies a fundamental principle of statistics to optimize between the estimation error and specification error. Bengtsson and Holst (2002), Chan et al. (1999), Jagannathan and Ma (2003), Ledoit and Wolf (2003, 2004) and Wolf (2004) show empirically that shrinkage estimators and a portfolio of estimators are best suited to covariance estimation.

Black and Litterman (1992) proposed a model for asset allocation which was more intuitive by suggesting healthy estimates for expected return vector. Portfolio selection problem may be extended from single period problem to multiple period problems and further it may be extended from single objective problem to multiple objective problems. Shefrin and Statman (2000) developed behavioral portfolio theory. The main focus of this study is on the asset allocation framework especially on Markowitz portfolio selection, constrained optimization, naïve diversification, minimum variance portfolios, Black-Litterman framework, augmented Black-Litterman model (propose), alternative ways for the estimation of inputs (variance covariance matrix and expected return vector) to portfolio optimization and different ways for comparison of asset allocation strategies. Based upon this background, study has following research questions which are followed by the objective of the study.

1.4.Research Questions

The study has following research questions:

1. Whether relatively new ways for estimation of covariance matrix and expected return vector outperform the traditional ways for estimation of inputs to portfolio optimization?
2. Do asset allocations based on sophisticated asset allocation tools outperform the naively diversified strategy?
3. Whether the augmented Black-Litterman model (Proposed) outperforms the original Black-Litterman model?
4. What should be an appropriate strategy for asset allocation in Pakistan?
5. What should be an appropriate global strategy for asset allocation?

1.5.Research Objectives

The study has following research objectives:

1. To suggest the optimal ways for the estimation of inputs for portfolio optimization i.e. estimation of return vector and variance-covariance matrix which consistently outperformed the competing tools.
2. To compare and contrast the performance of portfolios based on mean-variance optimization, minimum variance portfolios, constrained portfolios and naively diversified portfolios.
3. To introduce the country risk as unpriced risk factor into the Black-Litterman formula and perform the asset allocation on the basis of this augmented Black-Litterman model.
4. To guide investors for devising the optimal asset allocation strategy in Pakistan.
5. To facilitate investors for devising the optimal asset allocation strategy in global perspective.
6. To provide a comprehensive framework for asset allocation to investors.

1.6.Significance of study

Asset allocation is the process in which investors distribute available wealth into various asset classes for investment purposes. Generally investor is someone who allocates wealth into different available investment opportunities. The term investor may ranges from individual investor to institutional investor, fund managers, trustees overseeing a corporation, university endowments or invested premium for insurance company. Asset allocation into investment opportunities is one of the central decisions for investors. Investors have discretion to invest funds across investment opportunities in two broader ways. That is, they can invest all available funds only into one asset class. The other way is to invest simultaneously into more than one asset classes. The latter phenomena of investment can be termed as diversification.

Diversification considered the most desirable way among financial analysts to manage the risk of investment by investing within investment alternatives. Its objective is to manage the risk by allocating resources into various assets and these assets generally react differently against same event. Rationale behind diversification is that a portfolio of asset has generally higher average return and has less risk than any single asset class. In the same time diversification does not guarantee a consistent healthy output but still financial analysts expect that it helps investor to accomplish long term financial goals. Further with the help of diversification, investors may able to neutralize the poor performance of one asset class with that of good performer.

Main concern of any investor is to identify the way by which investor can distribute his wealth among different asset classes. Asset allocation is critical component of process of portfolio management. Asset class can be defined as securities having same characteristics, behavior in the market and are subject to same laws and regulations. Most universal asset classes include stocks, bonds, cash, commodities, currencies and real estates. Return of any portfolio over time primarily depends on the selection of different asset classes for specific portfolio and proportion of investment in each security. Therefore, investors have to decide appropriate asset class mix. This study helps investor to identify and explore major investment opportunities in Pakistan as well across globe. With the help of identification and description of each asset class, it is easier for investors to allocate resources into different asset classes.   This allocation of resources into different asset classes ultimately escorts investors towards portfolio diversification.

Undoubtedly, foremost question for investors concerning diversification is how many stocks make a diversified portfolio? In this regard Evans and Archer (1968) described that ten stocks made a portfolio diversified. Later on, there is mix evidence in empirical research regarding number of stocks to be included in portfolio for diversification. But it was agreed that there is relationship between number of stocks added in the portfolio and risk of portfolio. For this, investors compare the marginal benefit with marginal cost and determine appropriate asset class mix. Investors have option to allocate whole investment into alternative asset classes proportionately and disproportionately. If investors allocate available resources with equal weight into available asset classes then such type of diversification is termed as naive diversification. Again investors have option whether to distribute whole investment on the basis of naive diversification or find certain level of diversification. Study also suggests the optimal ways for estimation of inputs for portfolio optimization i.e. estimation of return vector and variance-covariance matrix which consistently outperformed the competing ways in Pakistan and global perspective.

The study also contribute in the existing literature by providing a frame work for asset allocation on the basis of Markowitz mean-variance optimization, minimum variance portfolios and constrained portfolios. Further this study compares the performance of these comprehensive ways for allocation of resources with naive diversification. On the basis of recommendation of these frameworks for allocation of resources, this study provides weights in each asset class to the investors for asset allocation among different asset classes.

Researchers have identified various risk factors for the explanation of risk-return behavior in capital market. Investor can earn premium against some systematic exposure that has zero correlation to market but undesirable for investors. Hence study also extends the original Black-Litterman formula which primarily based upon single factor assumption to two or multiple factor model. A novel contribution of this study is the introduction of un-priced factor i.e. country risk into the Black-Litterman model. It augments the Black-Litterman formula in the framework of two-factors and considers the country risk as un-priced factor. Other novel feature of this study is the estimation of investor's views as input to Black-Litterman model. We develop a market model for investor's views as input into the Black-Litterman model. Study also contributing by suggesting a global strategy to investors for asset allocation.

This study contributes to the financial literature with following ways. First, it compares 12 different ways for the estimation of covariance matrix within 4 categories i.e. conventional methods, factor models, portfolio of estimators and shrinkage approach. These includes sample matrix, constant correlation model, single index matrix, principal component analysis based model, portfolio of sample matrix & diagonal matrix, portfolio of sample matrix & single index matrix, portfolio of sample matrix & constant correlation matrix, portfolio of sample matrix, single index matrix & constant correlation matrix, portfolio of sample matrix, single index matrix, constant correlation matrix & diagonal, shrinkage to the diagonal matrix, shrinkage to the single index model, and shrinkage to the constant correlation model. For comparison purpose this study uses root mean square error (RMSE) and risk of minimum variance portfolios (GMVP). Second, it suggests the optimal way for the estimation of inputs for portfolio optimization i.e. estimation of future return vector. Study uses diverse ways for estimation of expected return vector i.e. historical average estimation, Auto-Regressive (AR) estimation, ARIMA based estimation, ARIMA-Reg based estimation, CAPM based estimation, implied equilibrium excess return and Black-Litterman (BL) model. Study uses four different criteria to evaluate the performance consistencies of alternative future return estimation techniques i.e. paired sample t-test, correlation matrix, descriptive statistics and mean square prediction error (MSPE).

Third, study compare the performance of portfolios based on mean-variance optimization, minimum variance portfolios and constrained portfolios. These strategies for asset allocation then compared with naïve diversification on the basis of two evaluation dimensions: financial efficiency and diversification. Therefore study is also an attempt to uncover the standing of naïve diversification in asset allocation strategies.

Fourth, study first time examines the ‘country risk’ as unpriced risk factor in the Black-Litterman model and use this augmented Black-Litterman formula (BL-CR) for the estimation of expected return vector. This study uses three different datasets i.e. data related to emerging Asian countries, data related to global representative indices and data related to equity market of Pakistan. Under emerging Asian countries this study uses the global industry classification standard (GICS) develop by MSCI and Standard & Poor’s in 5 emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand. Under global representative indices this study uses the global equity, global bond, global commodity and global real estate as global asset classes. In Pakistan the focus of this study is on the sectors of equity market in Pakistan. Summarizing, this study provides a comprehensive framework to investors for asset allocation in Pakistan as well in global perspective.

1.7.Plan of study

The study is structured as follow. First, theoretical foundations as well as their empirical results are described in the literature review. Second, the data description and methodology section explain how the research is performed and which data set is used. Finally there are empirical findings and conclusion.

Chapter 2

2. REVIEW OF RELATED LITERATURE

Emergence of finance as a separate discipline from economics takes place during the 20th century. Merton (1990) argued that a generation ago finance discipline was just combination of some rule of thumb, collection of anecdotes and moreover it is all about the manipulation of accounting data. Early pioneer in this emerging field may be Lintner, Markowitz, Miller, Modigliani, Samuelson, Sharp and Tobin. The doctorate degree of Harry Markowitz was in jeopardy when a member of examination committee Milton Friedman started dissent against his doctoral thesis. Friedman argued that this work was not fall under the discipline of economics, business and neither it fall under the class of mathematics. However the Friedman did not sway rest of committee members. But the defense of doctoral thesis of Harry Markowitz in the economics department of the University of Chicago seems to be the most significant event in the emergence of finance.

In the 2nd decade of 21st century, it is hard to understand what finance was like before the portfolio theory? Now risk and return are such a fundamental concepts that it is hard to assume that these were once novelty. In today's world consumer takes several decisions like consumption-saving decisions and the portfolio selection decisions. Selection of portfolio mainly deals with selecting a portfolio of investment that fulfills the investment objectives over the investment horizon. Undoubtedly, investment objectives vary across individuals but stable and healthy payoffs are always enviable. Selection of portfolio is intricate mainly due to the following reasons. One reason might be the availability of large scale scattered investment opportunities to investors. Other may be the valuation or forecasting of the future payoffs associated nearly infinite investment opportunities. In the process of asset allocation, investor generally invests certain amount of money but future payoffs associated with this certain invested amount are uncertain. Portfolio selection can be done mainly under heuristic approach and quantitative approach. Heuristic approach deals with selection of portfolio on the basis of investor's view about future performance of investments. Quantitative approach focused on the mathematical model for allocation of investment. The characteristics of different investment opportunities are evaluated to determine which one should be added in the portfolio.

The work of Markowitz (1952) on portfolio selection considered a path breaking advances in the process of asset allocation. Framework used by Markowitz (1952) still considered in active portfolio management (Tu & Zhou 2011). Beside this framework, there are various approaches proposed by researchers for the asset allocation. The customary wealth theory by Markowitz (1952) and safety-first portfolio optimization by Roy (1952) were considerable work in the literature of portfolio optimization. Models suggested by Black-Litterman (1992), Khan and Zhou (2007) three fund models, Garlappi, Uppal and Wang's (2007) multi prior model and Mackinlay and Pastor (2000) were proven step forward in the portfolio selection. Further there are various rules used by researcher for the portfolio selection. Rules suggested by Brown (1976), Bawa and Klein (1976), Jorion (1986), Bawa, Brown and Klein (1979), Barry (1974), application of the idea of Stein (1955), James and Stein (1961) and rules imposing the short sale constrains like Jagannathan and Ma (2003) also applied in the existing literature for portfolio selection.

Non-theory based rule (1/N) for diversification i.e. naive diversification, also there along with above sophisticated rules. Brow (1976) generally considered the first study on the application of naive diversification. Tu and Zhou (2011) and Demiguel, Garlappi and Uppal (2009) examined the combination of different rules for diversification. Sharp (2007) presented that investor can allocate resources on the basis of maximizing the expected utility of return. Along with these

Chow (1995) selected the portfolio on the basis of risk, return and relative performance. A minimax portfolio selection with the help of linear programming had been introduced by Young (1998).

Work on portfolio diversification was further extended by researchers in mean variance framework. Mean absolute deviation model was applied by Kanno and Yamazaki (1991) for portfolio optimization. Feinstein and Thapa (1993) further extended the work of Kanno and Yamazaki (1991). Fishburn (1977) examined the mean target model and Samuelson (1958) suggested the importance of higher order moments in portfolio selection which provided the base to mean variance skewness model. Application of linear programming by Levy and Markowitz (1979) and its further extension in the form of goal programming by Tamiz (1996) was employed for asset allocation.

The above described models are focused only on one period optimization. Therefore asset allocation based upon these described strategies was specific only for one period. But in practice, it is quite possible that individuals may change their allocation time to time. Along with these theories for portfolio construction, sophisticated strategies and rules, non-theory based diversification; there also exists evidence of construction of portfolio in behavioral perspective. In this context Shefrin and Statman (2000) developed behavioral portfolio theory for construction of portfolios.

Existing literature on subject of asset allocation across different investment opportunities can be categorized on the basis of tools used for asset allocation. There may be mean-variance framework, safety first portfolio optimization, Black-Litterman model for asset allocation, expected utility asset allocation, behavioral portfolio theory for construction of portfolios, multistage and dynamics strategies for portfolio diversification, multi-objective optimization, other sophisticated strategies and rules for diversification and non-theory based diversification.

2.1.Mean-variance framework

Asset allocation based upon quantitative model was perhaps first discussed in Markowitz (1952) article on portfolio selections. Prime focus of Markowitz (1952) was on portfolio rather on single asset. Portfolio can be defined as a combination of two or more securities. Sharp (1970) defined portfolio as totality of decision determining the future prospectus of individuals. At the occasion of noble lecture in the City University of New York (USA) by Markowitz in 1990, it was disclosed by him that basic idea regarding the portfolio theory was came to his mind while reading John Burr Williams, the theory of investment value. William (1938) claimed that value of any stock today should be exactly equal to the present value of all future dividends associated with that stock. As future is uncertain so Markowitz proposed that value of any stock is actually the present value of expected future returns.

Further it was suggested by Markowitz that investor should not consider only the characteristics of individual assets but also its relation with other assets in that specific portfolio. Hence investor needs to consider the risk, return as well as co-movement of that security with other securities in the portfolio. Moreover Markowitz (1952) argued that investor can form a portfolio with higher expected return at a given level of risk or a portfolio with lower level of risk at a given level of expected return. Difference between risk and uncertainty was discussed by Knight (1921). Knight (1921) claimed that when individuals assign some numerical probabilities to randomness then it is risk and when individuals are not able to assign probabilities to alternative outcomes then it is uncertainty.

In addition to mean and variance of return as proposed by Markowitz (1952, 1959) researchers like Lee (1977) and Kraus and Litzenberger (1976) presented the inclusion of higher moments like skewness as alternative portfolio models. Markowitz (1952) combined higher level of return with minimum risk and identified an efficient frontier. Tobin (1958) further directed the investor in the identification of efficient portfolios. Dimson and Mussavian (1999) claimed that in 1952s and even at the start of 1960s it was nearly impossible to apply the Markowitz model for 2000 securities. It was mainly due to the requirement of estimation of about two million risk-return characteristics. This problem was then simplified by Sharp (1963). Sharp (1963) proposed that there was co-movement between securities and markets. This idea of Sharp (1963) was further extended to various asset pricing models.

It was argued by Dimson and Mussavian (1999) that Markowitz model for portfolio diversification along with the work of Sharp marked the end of beginning of modern finance. Fang (2007) applied the mean variance framework on the arbitrage portfolio which was a combination of long and short term investments in a way that net investment is zero. Similarly Jorion (1994) uncovered the sub optimality in the currency markets. With the application of mean-variance analysis it was argued that this sub optimality was due to the ignorance of relationship among the assets in the constructed portfolio. In insurance industry, Chiu and Wong (2012) checked the optimality of asset-liability management within the context of mean variance analysis. Study focused on the problem of insurance by minimizing the variance of terminal wealth with given level of expected wealth subject to liability payments. Miniaci and Pastorello (2010) analyzed the optimization of portfolio formed by household with the help of micro econometric approach actually based upon the mean-variance analysis.

Framework applied by Markowtiz (1952) considered a standard approach for asset allocation but there are various approaches for the same purpose. There are diverse extensions in the mean-variance framework. Chopra, Hensel and Turner (1993) used the mean-Variance framework with some constraints. It was concluded that optimization with adjustment in the input showed improved results in the shape of mean, variance and terminal wealth. After the inclusion of transaction cost, these described improvements further enhanced. Mean variance optimization preliminarily considered only the risky assets as both expected mean and variance of assets not equal to zero. But this portfolio optimization changed with the incorporation of risk free asset as an investment opportunity. This development of risk free asset changed the scenario and termed as separation theorem by Tobin (1958). With the addition of risk free asset along with risky assets in portfolio selection, the resultant new efficient frontier had more realistic risk-return characteristics. Mean variance optimization was much general framework for portfolio selection and it could be restricted by adding various constraints.

Undoubtedly, mean variance framework proposed by Markowitz had widespread acceptance in the literature of finance. Markowitz (1952) efficient frontier also provided basis to important fundamentals in modern finance like CAPM. But still Michaud (1989) termed the optimization by mean variance framework as “enigma”. Michaud (1989) tried to prove that the portfolio diversification based upon Markowitz efficient frontier suggested irrelevant or false and financially meaningless asset allocation. Further argued that mean-variance framework was actually “estimation error maximizer”. Michaud (1989) had raised various points against mean-variance framework like missing factors, information levels mismatched, sensitivity of optimal solution with inputs, non uniqueness etc.

Mean-variance portfolio optimization ended with concentrated portfolio (Black & Litterman, 1992). The resultant portfolios are counterintuitive (Michaud, 1989). Chopra and Ziemba (1993) described that this model was more sensitive toward inputs. Best and Grauer (1991) further examined this optimization and concluded that it was not robust. Chow (1995) analyzed the selection of portfolio on the basis of risk, return as well the relative performance. Chow (1995) claimed that many investors willfully reject the solution provided by mean variance analysis and choose the alternative one. He argued that it was mainly due to the fact that utility of investor was not based upon expected risk and return. According to Chow (1995) basic problem started from the forecasting and additionally the investors were not able to correctly specify the investment objectives. Further Chow (1995) proposed tracking error utility function along with mean-variance approach which constructed the portfolio on the basis of risk, return and performance relative to any benchmark. Arulraj, Pvc (2012) applied traditional mean-variance model and enhanced Black-Litterman model. Beach and Orlov (2007) described that it was the Black and Litterman (1992) model that tried to fill the gap between theory related to asset allocation and practice in real world.

2.2.Safety-first portfolio optimization

Modern portfolio construction originated from the imperative contribution of Markowitz (1952) and Roy (1952). Markowitz's work on portfolio selection considered so influential that Markowitz awarded Nobel Prize in economics in 1990. Chiu, Wong and Li (2012) argued that many of today's researchers and practitioners only recognized the contribution of modern portfolio construction to Markowitz. But Markowitz (1999) claimed that today's researchers often called him the father of modern portfolio theory, but equal recognition in this tribute could be named to Roy (1952). Markowitz (1952) principle for portfolio selection was termed as mean-variance portfolio optimization while Roy (1952) principle for portfolio construction termed as safety-first principle. Chiu, Wong and Li (2012) described that indisputably, safety-first principle easily considered foundation for portfolio selection. Roy (1952) proposed that portfolio manager proceeded in a fashion that they tried to minimize the probability of portfolio value below a specified level.

Safety-first principle for portfolio optimization was about minimization of probability of negative returns. Norki and Boyko (2012) further improved with healthy estimation of negative return probabilities. Dorfleitner and Utz (2012) extended the safety-first based upon financial and sustainability returns. Milevsky (1999) described that investor that devised their strategy on the basis of safety first utility, were time invariant with respect to asset allocation. Anyhow there was decreased in shortfall risk exponentially but proportions on each asset were time invariant. Haley and Whiteman (2005) based upon the preliminary work of Haley (2003) developed a method of portfolio construction on the basis of safety-first principle and it was called generalized safety first. Bawa (1978) described a generalized safety first rule of order n. Safety first principle by Roy (1952) extended by many researchers. Pyle and Turnovsky (1970) examined the relation between mean-variance and safety-first principle. Arzac and Bawa (1977) generalized the safety-first principle. Levy and Levy (2009) analyzed the safety first experimentally. It was documented that safety first explained a major role in decision making. They proposed an expected utility safety first criteria. Haley, Paarsch and Whiteman (2013) developed a portfolio construction method mainly based upon safety first rule by minimizing the probability of portfolio value below a specified level.

2.3.Black-Litterman model

The process of portfolio construction on the basis of mean-variance analysis was milestone in modern finance. But simultaneously various researchers had some reservation on this quantitative model. He, Grant and Fabre (2012) argued that there are practical problems in the men-variance framework suggested by Markowtiz (1952), and researchers agreed that this choice for asset allocation was unrealistic. Michaud (1989) even called it “enigma”. Meucci (2005) said that estimation of return in traditional model was done with inefficient method. Black and Litterman working at Goldman Sachs tried to work on an improved model than mean-variance model for asset allocation. Black and Litterman (1992) proposed a model for asset allocation which was more intuitive by suggesting healthy estimates for expected return vector. These expected return vector then used for the calculation of weights to assets in the portfolio. Black and Litterman (1992) claimed that asset allocation based upon quantitative models had not attracted the global portfolio managers. It was mainly due to the reasons that resulting portfolio behaved badly. It was argued that the classical quantitative models gave hefty short positions, suggested corner solutions in many cases, gave zero weight in many cases and dictated extra-large weights to the assets having low capitalization.

Path breaking work of Black and Litterman (1992) pointed out two main reasons regarding atrocious behavior of classical approach to portfolio optimization. First reason was the difficulty in estimation of expected returns. Standard approach requires expected return for all the assets considered for optimization. But investor had only conservative view about some markets. Investor used assumptions and historical information for expected return which Black and Litterman (1992) considered poor estimations. Other potential reasons pointed out were sensitivity of weights and the assumption used for expected return. Further, argued that due to these reasons only few global portfolio managers consistently fascinated with the standard quantitative asset allocation.

Contribution made by Black and Litterman (1992) was the identification of two sources of information regarding the future returns. They merged both source of information into one formula. This formula then applied for estimation of expected returns. One source of information was computed quantitatively and other was related to the view of managers. Black and Litterman (1992) considered three asset classes namely equity, currency and bond in global framework. First source was the computation from global version of CAPM and it should hold if market was in equilibrium. The other view based upon the opinion of investment managers about asset classes. Since managers had access to information, Black and Litterman (1992) gave opportunity to present view in relative and absolute sense. Scowcroft and sefton (2003) confirmed that view in relative and absolute sense was more practical and closer to the thinking process of investors. Investor incorporated as many view as number of asset in the portfolio.

The mathematics involved in the Black and Litterman (1992) model was little complex in nature and Black and Litterman wrote few papers on the presented idea but these were precise in nature. Due to this reason, researcher uncovered the detail involved in Black and Litterman (1992) model for portfolio optimization. In this regard Idzorek (2005) presented the step by step guide to the complex model and tried to facilitate further researchers. Idzorek (2005) described the procedure that how investors combined the view for construction of well diversified portfolio. This paper further presented a way to incorporate the user specified confidence level. In conclusion, Idzorek (2005) suggested that overall the model proposed by Black and Litterman (1992) for asset allocation triumph over the mostly mentioned weakness of Markowitz (1952) mean variance analysis. These mostly mentioned weaknesses were like error maximizer, unintuitive, concentrated portfolio, sensitivity of optimal solution with inputs, non uniqueness etc. Drobetz (2001) analyzed the Black-Litterman model practically.

Schottle, Werner and Zagst (2010) suggested that Black-Litterman model still getting more attention in the literature as it was comparatively more intuitive and neat and itwas a special case of Bayes model. Further Beach and Orlov (2007) described that this model was an attempt to fill the gap between theory and practice in real world. Their study applied the Black-Litterman strategy for the asset allocation in global perspective. GARCH model has been applied to derive the inputs as required in the Black-Litterman model. Schottle, Werner and Zagst (2010) analyzed the Bayes and Black-Litterman model and concluded that latter one was a special case of former to resolve the error maximizing nature of traditional model. He, Grant and Fabre (2012) applied the Black-Litterman model as investment strategy for asset allocation in global framework.

Traditional mean-variance model and enhanced Black-Litterman model have been applied by Arulraj, Meghana and Karthika (2012). Study estimated expected returns of stocks in Bombay stock exchange. With the help of bootstrap methodology, study included the error estimates in Black-Litterman model. Further argued that initial Black-Litterman model faced problems in practical implementations. Satchel (2000) addressed some issues related to the practical implementations of the original model. Meucci (2005) and Cheung (2009) discussed issues related to translate subjective views into return in Black-Litterman model. Drobetz (2001) discussed the way to elude the negative consequence in portfolio optimization and suggested to incorporate the Black-Litterman model into the decision of asset allocation. Drobetz (2001) recommended the use of Black-Litterman model by arguing that this model solved many problems of the traditional approach for asset allocation.

Meticulous discussions on the Black-Litterman model have been found in the study presented by Cheung (2009). Study described that model was attractive and alluring on the basis of theory as well as practical viewpoint. Cheung (2009) uncovered the economic interpretation, assumptions and implementations of the model. Lejeune (2011) applied the Black-Litterman model for forecasting the return associated with asset classes. These approximated returns used for construction of fund. In this context, Meucci (2006) presented a five step recipe to input views beyond the Black-Litterman model. Fernandes, Ornelas and Cusicanqui (2012) documented that Black-Litterman was practical approach for asset allocation. It also based upon the intuitions and estimation error in the framework of mean-variance allocation considered the reasons behind poor diversification. Due to this estimation error, Michaud (1998) presented the use of resampling and proposed that input should be the result of stochastic process as opposed to Markowitz (1952).

Combination of resampling technique by Michaud (1998) along with Black-Litterman model was proposed by Fernandes, Ornelas and Cusicanqui (2012). This combination then applied on bond and equity asset classes. The new proposed combination subsequently compared on the basis of different measures with traditional portfolio optimization. Meucci (2008) further widen the practical side of the Black-Litterman model and included the market risk factors rather return of assets. In this scenario Simonian and Davis (2011) proposed step further in Black-Litterman model. As asset returns were uncertain, Simonian and Davis (2011) incorporated this uncertainty with robust Black-Litterman in selection of portfolio. They claimed that this model had advantage over original model. Cheung (2012) presented augmented Black–Litterman model for portfolio construction. Krishnan and Mains (2005) considered the recession as a risk factor and named the new model as two factor BL model.Silva, Lee and Pornrojnangkool (2009) analyzed the application of Black-Litterman model in active portfolio management. The combination of quantitative views along with qualitative views further combined with little enhancement by Giacomettiy, Bertocchiy, Rachev and Fabozzi (2007). They enhanced the work of Black-Litterman model by providing a comparison under various distributions for return. Study also made comparisons by using variety of risk measures.

He, Grant and Fabre (2012) argued that there are practical problems in the men-variance framework suggested by Markowtiz (1952). Michaud (1989) even called it “enigma”. Meucci (2005) said that estimation of return in traditional model was done with inefficient method. Black and Litterman (1992) proposed a model for asset allocation which was more intuitive by suggesting healthy estimates for expected return vector. Scowcroft and sefton (2003) confirmed that it was more practical and closer to the thinking process of investors. Idzorek (2005) presented the step by step guide to the complex model and. Drobetz (2001) analyzed the Black-Litterman model practically. Schottle, Werner and Zagst (2010) suggested that Black-Litterman model still getting more attention in the literature. Beach and Orlov (2007) described that this model was an attempt to fill the gap between theory and practice in real world.

Schottle, Werner and Zagst (2010) analyzed the Bayes and Black-Litterman model. He, Grant and Fabre (2012) applied the Black-Litterman model as investment strategy. Arulraj, Meghana and Karthika (2012) and Satchel (2000) addressed some issues related to the practical implementations of the original model. Meucci (2005) and Cheung (2009) discussed issues related to translate subjective views into return in Black-Litterman model. Drobetz (2001), Cheung (2009), Lejeune (2011), Meucci (2006), Fernandes, Ornelas and Cusicanqui (2012), Meucci (2008), Simonian and Davis (2011), Cheung (2012) Krishnan and Mains (2005), Silva, Lee and Pornrojnangkool (2009) and Giacomettiy, Bertocchiy, Rachev and Fabozzi (2007) also made significant contribution on Black and Litterman model.

2.4.Multi-stage and dynamics strategies for portfolio diversification

Portfolio selection problem may be extended from single period problem to multiple period problems and further it may be extended from single objective problem to multiple objective problems. Pioneering work done by Markowitz (1959) was about single period portfolio optimization. Markowitz and Dijk (2003) stated that analysts tried to optimize consumption-investment decisions on the basis of multiple time periods. Generally believed that investor changes the weight of asset classes in portfolio at different time horizons. Fama (1970), Roll (1973), Sengupta (1983) suggested various multistage and dynamic models for portfolio optimization. Ng (2000) presented a way to conquer the non-separability and suggested a multi-period solution. Zhou and Li (2000) also presented a solution in continuous time. Li and Li (2012) argued that after the initiated work of Ng (2000) and Zhou and Li (2000) the dynamic portfolio construction becomes widespread in financial literature.

Continuous-time portfolio construction with regime switching was analyzed by Zhou and Yin (2003). Continuous-time portfolio selection was further studied by Bielecki, Jin, Pliska and Zhou (2005) and Zhu, Li, and Wang (2004). Wei and Ye (2007) also analyzed the traditional mean-variance approach for construction of portfolio in multi-period horizon. Costa and Araujo (2008) analyzed multi-period mean-variance optimization with Markov switching parameters. Leippold, Trojani and Vanini (2004) described different challenges in multi-period mean-variance portfolio selection. It was described that first challenge was difficulty in separation between two objectives of mean-variance optimization. Second challenge faced by multi-period optimization was the financial interpretation of results obtained by this optimization. It was further stated that these results were not matched intuitively with basic Markowitz portfolio theory. Third challenge was related to the inclusion of intertemporal constraints in the model. Osorio, Gulpinar and Rustem (2008) introduced the way to consider the risk in multistage in mean-variance analysis.

Usually real world portfolio constructions were multi-periods and investor revise the portfolio time to time. Zhang, Liu and Xu (2012) described that extension of single period to multi-period portfolio construction was quite natural. Liu, Zhang and Xu (2012) suggested that investor considered multiple criteria for multi-period portfolio constructions.

2.5.Multi-objective optimization

The Markowitz model for portfolio selection may be treated as bi-objective optimization. It was suggested by Markowitz that investors formed portfolio on the basis of maximizing the return and minimizing the risk of portfolio. Therefore it was based upon two criteria. Anagnostopoulos and Mamanis (2010) stated that due to these two criteria investors had a set of optimal portfolio rather a single portfolio. Anagnostopoulos and Mamanis (2010) proposed a model for asset allocation with three objectives. Two were the conventional objectives i.e. mean and variance of portfolio and third was minimizing the number of assets in the portfolio. To solve the multi-objective optimization the methodology of goal programming had been adopted by various researchers like Kumar (1978), Tamiz (1996). Gabriel, Kumar, Ordonez and Nasserian (2006) studied the multi objective problem in the area of project selection by using integer constrained optimizations. Gilbert, Holmes and Rosenthal (1985) applied the multi objective integer programming to allocate the land for various projects. Coello and Becerra (2009) presented a review on the uses of multi objective optimization in different emerging fields.

Criteria proposed by Markowitz (1952) for portfolio selection i.e. mean and variance are still facing criticism by financial analysts and researchers. Joro and Na (2006) and Zopounidis and

Doumpos (2002) argued that these two so called criteria for portfolio selection were not capable to detain all the necessary information required for portfolio selection. Guptaa, Mittal and Mehlawat (2013) stated that there might be some other criteria for portfolio selection. There might be chances that alternative criteria got more preferences than conventional criteria in investor's mind. Gupta, Mehlawat and Saxena (2008) described that portfolio selection based upon multiple criteria become more widespread in recent times. Steuer, Qi and Hirschberger (2005) compared the standard investor with non standard investor. Standard investors were the investors whose utility function was based only on return function while non standard investor also considered some other parameters like liquidity, dividends etc. Their study considered this problem as multi-objective stochastic programming problem.

Generally every real world problem is multi-objective in nature. Ahmed and Hegazi (2006) stated that like every real life problem portfolio selection was also multi-objective problem. Krink and Paterlini (2011) proposed a multi-objective algorithm for portfolio optimization. In this work they compared the result of differential evolution for multi-objective optimization with quadratic programming. Qi and Hirshberger (2007) suggested that the process of portfolio optimization became intricate in the presence of multi-objective optimization. Guptaa, Mittal and Mehlawat (2013) claimed that investors got more overall satisfaction when portfolio selection were based upon multiple criteria rather concentrated only on return and risk of portfolio. Xidonas and Mavrotas (2012) solved the real world portfolio optimization problem by considering multiple objectives, taking into account cardinality constraints, transaction cost and compliance norms. Xidonas, Mavrotas and Psarras (2010) proposed a multi-objective mixed integer programming model for the construction of portfolio in stock market.

2.6.Behavioral portfolio theory

Process of portfolio selection based upon Markowitz mean-variance approach further compared with investor portfolio selection by Kroll, Levy and Rapoport (1988) and Lipe (1998). It was evident that there was gap between theory and practice regarding portfolio selection. Benartzi (1999) and Joos and Kilka (1999) also argued that recommendation of portfolio theory reasonably different from that of portfolio holding in practice. Siebenmorgen and Weber (2003) argued that Markowitz portfolio selection was counterintuitive. Shefrin and Statman (2000) developed behavioral portfolio theory. They developed an efficient frontier for behavioral portfolio theory and also made comparison of this frontier with traditional efficient frontier based upon mean-variance. Result uncovered that these efficient frontiers did not matched with each other. It provided basis for selection of portfolio in the context of behavioral portfolio theory. It considered expected wealth, desire for security, aspiration level etc. for construction of portfolios.

Portfolio advisor did not recommend the portfolio selection based upon mean-variance optimization (Siebenmorgen & Weber 2003). In 2003, new behavioral portfolio theory was presented by Siebenmorgen and Weber and denied to follow the traditional mean-variance approach because they believed that investor did not based their portfolio construction as the literature described. Siebenmorgen and Weber (2003) firmly believed that behavioral aspects played central part in portfolio decision making. They further identified three parts in the decision of optimal portfolio construction. Investors considered expected return, pure risk and naive diversification in portfolio selection. Major emphasis in behavioral portfolio theory was the way in which risk is treated in traditional mean-variance approach. Siebenmorgen and

Weber (2003) finally concluded that recommendations made by advisors were better explained by behavioral portfolio theory than that of Markowitz mean-variance portfolio theory.

2.7.Utility based asset allocation

Asset allocation into various investment opportunities mainly determined from the goal of investors. Objective of investor may be the maximization of expected utility. Canakoglu and Ozekici (2010) analyzed the construction of portfolio based upon the maximization of expected utility. Utility functions were used to measure the risk preferences of investors. Sharpe (2007) studied a way for asset allocation on the basis of maximizing expected utility based upon some utility function. Sharp (2007) focused on the marginal expected utility and described that standard mean-variance was special case of general expected utility formulation and asset pricing assumptions were used as input for analysis. Zhang, Zhang and Xiao (2009) presented portfolio selection model with the maximum utility. Cohen and Natoli (2003) described criteria for risk and utility in portfolio theory.

2.8.Non-theory based diversification

Beside different quantitative strategies for optimal asset allocation into different investment opportunities, their existed evidence in the use of non-theory based diversification. Demiguel, Garlappi and Uppal (2009) compared different strategies for asset allocation with naive diversification. Perhaps the reason to call this strategy as 'non-theory based diversification' was that this diversification completely ignored the characteristics of data and there was no optimization or estimation phenomenon under naive diversification (1/N). Non-theory based diversification i.e naive diversification could be defined as invest equal in proportion to all available investment opportunities.

Existing literature on subject of asset allocation across different investment opportunities can be categorized on the basis of tools used for asset allocation. There may be mean-variance framework, safety first portfolio optimization, Black-Litterman model for asset allocation, expected utility asset allocation, behavioral portfolio theory for construction of portfolios, multi-stage and dynamics strategies for portfolio diversification, multi-objective optimization, other sophisticated strategies and rules for diversification and non-theory based diversification. Prime focus of Markowitz (1952) was on portfolio rather on single asset and this idea opened new avenues for investors. The principle suggested by Roy (1952) for portfolio construction termed as safety-first principle. He, Grant and Fabre (2012) argued that there are practical problems in the mean-variance framework suggested by Markowtiz (1952) and researcher agreed that this choice for asset allocation was unrealistic. Michaud (1989) even called it "enigma". Black and Litterman (1992) proposed a model for asset allocation which was more intuitive by suggesting healthy estimates for expected return vector. Portfolio selection problem may be extended from single period problem to multiple period problems and further it may be extended from single objective problem to multiple objective problems. Shefrin and Statman (2000) developed behavioral portfolio theory. This study provides comprehensive framework and facilitate the investors for asset allocation in Pakistan and in global perspective. For this, it compare and contrast the performance of portfolio based upon Markowitz Mean-Variance optimization, Black-Litterman model and augmented Black-Litterman model with the naively diversified portfolio on emerging Asian countries, by using global proxies for asset classes and asset classes in Pakistan.

Chapter 3

3. DATA DESCRIPTION AND RESEARCH METHODOLOGY

This study provides a comprehensive framework for asset allocation to investors in Pakistan as well in global perspective. For this comprehensive framework, study considers various asset classes as investment opportunities in emerging Asian countries, in global environment and in Pakistan. Data set consists of time series data associated with each asset class. Study uses the Bloomberg database for data collection related to emerging Asian countries, global asset classes and equity market of Pakistan. Further the data related to listed companies at Karachi stock exchange (KSE) have been collected form official website of KSE in Pakistan[1].

3.1 Asset classes

Investor needs to identify the universe of potential investment avenues for asset allocation. This universe of potential investment may call investment asset classes. Investors have various investment opportunities for asset allocation in emerging Asian countries, across the globe and in Pakistan. In emerging Asian countries study identifies ten (10) asset classes, in global environment study consider four (4) asset classes and in Pakistan it focus on the twenty two (22) asset classes while keeping in view the following points. First, assets within the asset class are homogenous. Second is the mutually exclusive property of asset classes. And finally being a separate asset class, it has enough ability to take a reasonable part of wealth of all investors. Following is the detail of data sets considered in emerging Asian countries, global and Pakistani perspective.

[1] http://www.kse.com.pk/

3.1.1. Emerging Asian countries

This study selects five emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand. The sample starts from January 9, 2009 to October 30, 2015. This sample is further divides into two subsamples i.e. 01/09/2009 to 06/01/2012 and 06/08/2012 to 10/30/2015. For the estimation of covariance matrix study use the first sub-sample and accuracy of this covariance matrix is evaluated as ex-post in the second sub-sample. For the estimation of inputs to Black-Litterman model this study uses the second sub-sample. Investor needs to identify the universe of potential investment avenues for asset allocation. For this purpose study uses the global industry classification standard (GICS) developed by MSCI and Standard & Poor's in 1999. It consists of 10 sectors. This study develops the weekly equally weighted indices in emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand. The detail of sectors with relative capitalization weight and total number of constituents stocks are presented in Table 3.1.

Table 3.1

Market capitalization weights of asset classes on the basis of GICS

Sectors	India	Indonesia	Pakistan	Philippines	Thailand
Consumer Discretionary	0.1203	0.1303	0.0809	0.0633	0.0789
Consumer Staples	0.0979	0.2651	0.1706	0.0952	0.1066
Energy	0.1088	0.0331	0.1610	0.0216	0.1173
Financials	0.1945	0.3109	0.2303	0.3859	0.2774
Health Care	0.0847	0.0302	0.0335	0.0006	0.0472
Industrials	0.1004	0.0625	0.0341	0.2335	0.1002
Information Technology	0.1283	0.0060	0.0016	0.0057	0.0246
Materials	0.0962	0.0553	0.1965	0.0360	0.1036
Telecommunication Services	0.0299	0.0921	0.0123	0.0645	0.1059
Utilities	0.0391	0.0146	0.0792	0.0937	0.0382
Total	3000	506	438	237	647

This study uses the 6-month T-bill rate as a proxy of risk-free rate in each emerging country. To measure the country risk, study uses the default spread between the yields on an international bond issued by each country with the bond issued by United States of America as a comparable risk free asset to capture the country risk dynamics in all the emerging Asian countries. The details of these selected international bonds are as follows. For Indonesia, study uses the US dollar denominated international bonds issued by the republic of Indonesia on 2/3/2006 due 9/3/2017 at coupon interest rate of 6.875% with ISIN no. USY20721AF61. It is listed on the Singapore exchange (SGX). For India this study uses the US dollar denominated international bonds issued by the NTPC (National thermal power corporation which is the largest state-owned power generating company in India and supply power in throughout the India) on 2/3/2006 due 2/3/2016 at coupon interest rate of 5.875% with ISIN no. XS0245398226. It is listed on the Singapore exchange.

For Pakistan, study uses the US dollar denominated international bonds issued by the Islamic republic of Pakistan on 05/24/2007 due 1/6/2017 at coupon interest rate of 6.875% with ISIN no. USY8793YAM40. It is listed on the Luxembourg S.E. For Philippines study uses the US dollar denominated international bonds issued by the republic of Philippines on 7/9/2005 due 01/15/2016 at coupon interest rate of 8% with ISIN no. US718286BA41. It is listed on the Luxembourg S.E. For Thailand this study uses the US dollar denominated international bonds issued by the IRPC (IRPC is public limited company in Thailand and together with its subsidiaries also provide products in the Asia Pacific) on 05/22/2007 due 05/25/2017 at coupon interest rate of 6.375% with ISIN no. XS0302863914. It is listed on the Singapore exchange.

Table 3.2 provides the detail of representative equity indices of all the selected emerging Asian countries use in this study.

Table 3.2

Details of selected equity indices and international bonds

Country	Equity market indices
Indonesia	The JCI is capitalization weighted index of the Indonesia stock exchange with base value 100 in 1982.
India	The S&P BSE Sensex is a market-weighted index of the Bombay stock exchange with base value 100 in 1978-79.
Pakistan	The KSE-100 is capitalization-weighted index of Karachi stock exchange with base value 1,000 in 1991.
Philippines	The PSEi is a capitalization-weighted index of Philippine stock exchange with base value of 1022 in 1990.
Thailand	The SET is a capitalization-weighted index of stock exchange of Thailand with base value of 100 in 1975.

3.1.2. Broad asset classes in global perspective

Global investment opportunities for investors can be described in the form of broad asset classes. Study considers the equity, bond, commodity and real estate as the broad asset classes in global perspective. The data period starts from March 2009 to June 2014 on monthly basis. Broad asset classes available to investor for asset allocation are given below:

1. Equity
2. Bond
3. Commodity
4. Real Estate

The detail of these asset classes are describe as follows.

3.1.2.1.Equity

Equity is considered as an asset class locally as well as globally. This study uses the Morgan Stanley capital international all countries world (MSCI ACWI) index as a proxy for equity in global perspective. It is a free float-adjusted market capitalization weighted index that is designed to measure the equity market performance of developed and emerging markets. It consists of total 45 country indices comprising 24 developed and 21 emerging market country indices.

3.1.2.2.Commodity

Investment in commodities is also treated as investment opportunity. The study uses S&P GSCI (Goldman Sachs Commodity Index) as a proxy for investment in the commodities as an asset class. S&P GSCI consists of 24 commodities from 5 major commodity sectors: energy (69%), agriculture (15.6%), industrial metals (6.9%), livestock (5%) and precious metals (3.6%).

3.1.2.3.Bond

Study uses the Barclays Global Aggregate Index as a proxy for global bond market. It includes fixed-rate treasury, government-related, corporate and securitized bonds from both developed and emerging markets issuers. It has three major components i.e. the US aggregate, the Pan-European aggregate and Asian-Pacific aggregate index.

3.1.2.4.Real Estate

For the asset allocation into real estate this study uses the FTSE EPRA/NAREIT Global Index as a proxy for world's real estate market. This index tracks the performance of real estate companies and REITS in both developed and emerging markets. It comprises of 36 countries both form developed and developing economies.

3.1.3. Asset classes in Pakistan

In Pakistani perspective, study considers the equally weighted indices of twenty two (22) sectors in equity market. These indices are treated as asset classes. Data period started from January 2000 to August 2014 on monthly basis. Study applied the asset allocation strategies in comparison with naïvely diversified portfolio. This study uses relatively longer data horizon i.e.

January 2000 to August 2014. This data is further divided into two subsamples. First sub-sample starts from January 2000 to August 2007 and second starts from September 2007 to August 2014. Dynamics of asset classes are also changes due to factors such as globalization, openness in economy, political and other economic factors. Therefore it is more appropriate to divide the whole sample into two subsamples. Further it also allows making rigorous comparisons over time. Hence study also compares the patterns of asset allocation in two subsamples in Pakistan and also with the output of global environment.

Pakistan has three stock markets namely Karachi stock exchange, Lahore stock exchange and Islamabad stock exchange. This study considers the Karachi stock exchange as equity market in Pakistan. Karachi stock exchange (Guarantee) limited established on September 18, 1947 with a paid up capital of 37 million rupees. KSE introduced its first index called KSE-50 and later on November 1, 1991 a KSE-100 index was introduced. KSE has been declared as the best emerging and best performing market due to its record breaking performance in 2013. The following Table 3.3 shows the detail of selected sectors, number of listed stocks and number of selected stocks in each sector in equity market in Pakistan.

Table 3.3
Detail of selected sectors in equity market in Pakistan

Sectors	Listed	Selected	Selected Percentage
Automobile and Parts	15	15	100%
Beverages	3	3	100%
Chemicals	34	31	91%
Construction and Materials (Cement)	36	34	94%
Electricity	17	15	88%
Electronic and Electrical Goods	3	3	100%
Engineering	10	10	100%
Fixed Line Telecommunication	4	4	100%
Food Producers	53	48	91%
Forestry (Paper and Board)	4	4	100%
General Industrials	13	13	100%
Health Care Equipment and Services	2	2	100%
Household Goods	13	11	85%
Industrial metals and Mining	10	7	70%
Industrial Transportation	4	2	50%
Multi-utilities (Gas and water)	2	2	100%
Oil and Gas	13	12	92%
Personal Goods (Textile)	180	171	95%
Pharma and Bio Tech	9	8	89%
Real Estate Investment and Services	2	2	100%
Tobacco	3	3	100%
Travel and Leisure	5	5	100%

The number of listed stock was taken as on 8/23/2014. All the data related to KSE is taken from official website of KSE i.e. http://www.kse.com.pk.

3.2.Research methodology

This study provides a comprehensive framework to investors for asset allocation into different asset classes. Investment opportunities in the form of asset classes are described in data description section. This session discusses the methodology uses for the estimation of inputs to portfolio optimization, strategies uses for asset allocation and the evaluation dimensions for asset allocation into various asset classes.

Study uses the assumption of continuously compounding instead of discretely compounding. Continuously compounded formula based upon the formula of $\text{Future value} = \text{Present value} * \mathrm{e}^{\mathrm{in}}$. Continuously compounded returns are calculated for each asset class by using the following formula.

$$R_t = \ln({}^{P_t}/_{P_{t-1}}) \dots \dots \dots (1)$$

Where:

R_t = Continuously compounded return

P_t = Price at period "t"

P_{t-1} = Price at period "t-1"

ln = Natural logarithm

3.2.1. Expected excess returns vector

For asset allocation into various asset classes, there is need to estimate the expected return vector associated with each asset class. This estimated return then uses as input for portfolio optimization. Excess return of each asset class is calculated by taking the difference of estimated return with 6-month government Treasury bill rates in respective country. Study uses the following alternative ways for estimation of expected return vector.

1. Historical average estimation
2. Auto-Regressive based estimation
3. ARIMA (p,d,q) model for estimation
4. ARIMA-Reg based estimation
5. CAPM based estimation

6. Implied equilibrium excess return vector
7. Black-Litterman (BL) model
8. Augmented Black-Litterman (BL-CR) model

3.2.1.1.Historical average estimation

For future return estimates $E\ (R_i)$, simple arithmetic mean over the studied period (M) of each asset class is calculated by using following formula. Here R_i is the historical return on asset class 'i'.

$$E\ (R_i) = \frac{1}{M}\sum_{t=1}^{M} R_{i,t} \dots\dots\dots.(2)$$

Where:

$E\ (R_i) =$ Estimated return for the asset class 'i'

$R_i =$ Historical return on asset class 'i'

$M =$ Number of periods

3.2.1.2.Auto-regressive based estimation

For the estimation of expected return, the order (p) in $AR_i(p)$ model is selected on the basis of rolling and non-rolling regressions. Under non-rolling regression, study estimates the $AR_i(p)$ model over the period 6 through j, then 6 through $j+1$ and up to 6 through $j+(n-1)$, n is the last observation under each asset class. Study compare the one-step ahead mean square prediction errors (MSPE) of $AR_i(p)$, p=1, 2,3,4,5 and select the one with lowest MSPE. To further asses the stability of model, $AR_i(p)$ is also estimated on the basis of rolling regressions through the sample and each time the model estimated on fix usable observations. Under majority (70%) of cases, both regressions arrived at the same order of $AR_i(p)$. In case of contradiction, select the one suggesting lower order of $AR_i(p)$. The following equation (3) is used for auto-regression model.

$$\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{R_{t-p}} + \varepsilon_\mathrm{t} \qquad p = 1,2,3,4,5 \ldots \ldots (3)$$

3.2.1.3.ARIMA (p,d,q) model

Box and Jenkins (1976) introduced the $ARIMA\ (p,d,q)$model. $ARIMA\ (p,d,q)$ model is used to forecast the future return of each asset class 'i′. Here AR represents autoregressive, 'I' represents integrated and MA represents moving average. The general form of $ARIMA\ (p,d,q)$ which is the combination of AR(p) and MA (q) process is as follow:

$$Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \theta_3 Y_{t-3} + \cdots + \theta_p Y_{t-p} + \mu_t + \beta_1 \mu_{t-1} + \beta_2 \mu_{t-2} + \beta_3 \mu_{t-3} + \cdots + \beta_q \mu_{t-q}$$

The above equation can be written in summation form as follow:

$$Y_t = \sum_{i=1}^{p} \theta_i Y_{t-i} + \mu_t + \sum_{j=1}^{q} \beta_j \mu_{t-j} \ldots \ldots \ldots. (4)$$

The equation (4) is estimated only if Y_t is stationary. Unit root test like augmented Ducky-fuller is carried out to check the order of integration of Y_t. For the selection of appropriate order of AR and MA, we run all possible combination of 'p' and 'q' up to p,q=4 and select the one which overall minimizes the AIC and BIC with adjusted R^2 model (the model which minimizes AIC, BIC and has highest adjusted R^2. Further these sixteen combinations of AR and MA is estimated on the basis of rolling and non-rolling windows for each asset class and select the order which consistently minimizes the criteria in rolling and non-rolling model specifications. In case of conflict study prefers the order which is more consistent in rolling window. Further Gauss-

Newton algorithm is used to estimate coefficients of $ARIMA\ (p, d, q)$model and selected models also checked to ensure that the estimation process converged.

Study also added explanatory variable in $ARIMA\ (p, d, q)$to estimate future return of each asset class in Pakistan and in global perspective. In global perspective, study uses the European economic policy uncertainty index and Europe Brent spot price (Dollars per Barrel) into the $ARIMA\ (p, d, q)$ model. European economic policy uncertainty index is composed of European news index, European CPI and European budget balance. In Pakistani perspective, study added KSE-100 index, 6-month government Treasury bill rates and exchange rate of direct quotation of US dollar (USD against Pakistani rupee (PKR, Rs) in Pakistan.

3.2.1.4.CAPM based estimation

Ordinary least square (OLS) is a way to analyze the dependency of one variable on one or more other variables, with a view to estimate the mean value of dependent variable in term of known values of regressor (s). Capital asset pricing model attributed simultaneously to William Sharp (1964), Jack Treynor (1961), John Linter (1965) and Jan Mossin (1966). It assumes that there is linear relationship between the return of asset class and market premium.

Study estimates the expected return for asset class 'i' by considering the return of market portfolio with the help of following equation 5:

$$\mathrm{R_{it}} = \gamma_0 + \gamma_1 R_{\mathrm{mt}} + \varepsilon_{\mathrm{t}} \dots\dots\dots (5)$$

$\mathrm{R_{it}}$ = Return on asset class 'i′

$\mathrm{Y_{it}}$ = Return on asset class 'i′ of market portfolio

ε_{t} = Error term

3.2.2. Black-Litterman model

The process of portfolio construction on the basis of mean-variance analysis is milestone in modern finance. He, Grant and Fabre (2012) argues that there are practical problem in the men-variance framework suggests by Markowtiz (1952). Black and Litterman working at Goldman Sachs try to work on an improve model than mean-variance model for asset allocation. Black and Litterman (1992) propose a model for asset allocation which is more intuitive by suggesting healthy estimates for expected return vector. These expected return vector then uses for the calculation of weights to assets in the portfolio. Critical review of literature and criticism on the traditional model for asset allocation has been presented in the literature review session.

Black and Litterman (1992) identifies two sources of information regarding the future returns. One is the quantitative view on expected returns and other is about the investor's opinion regarding the expected returns. Black and Litterman (1992) assume that it is difficult to beat the benchmark portfolio and originates expected returns of asset classes from benchmark portfolio in the first step. This quantitative view suggests a reference point and commonly called benchmark portfolio, equilibrium view or neutral view. The other view based upon the opinion of investment manager about asset class. Since manager have access to information, Black and Litterman (1992) give opportunity to present views in relative (Asset class 'B' will underperform/outperform the asset class 'A') and in absolute sense (Asset class B will have expected return equal to say 10%). Scowcroft and Sefton (2003) confirms that views in relative and absolute ways are more practical and closer to the thinking process of investors. Framework combines both sources of information into one formula. This formula then applies for estimation of expected return.

Black-Litterman model uses equilibrium returns as a neutral starting point. These equilibrium returns are computed with the following formula (6). For this assume that 'N' is number of asset in the portfolio, "Π" is "N×1" vector, "S" is "N×N" covariance matrix and "w_{mkt}" is a matrix of order "N×1".

$$\Pi = \lambda\, S\, w_{mkt} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Π = Implied excess equilibrium return vector

λ = Risk aversion coefficient

S = Covariance matrix of excess returns

w_{mkt} = Market capitalization weight of assets

Equation (6) can be derived by a maximization problem of investor with quadratic utility function. Suppose investor has the utility function $U = W^T \Pi - \frac{1}{2}\lambda\, W_m^T S\, W_m$ s.t. $W^T = 1$. By taking its first order derivative w.r.t. "w" and put it equal to zero, it gives $\Pi - \lambda\, Sw = 0$, which implies that $w = (\lambda S)^{-1}\,\Pi$. Black-Litterman assumes that w" is optimal and by using reverse optimization, solve it for vector of implied excess equilibrium return, it results $\Pi = \lambda\, S\, w$. Intuitively equation (6) link with capital asset pricing model. Hence CAPM formula $E(r_i) = r_f + \beta[E(r_m) - r_f]$ can be transform in the form of $\Pi = \lambda\, S\, w_{mkt}$ by following way. For this first transform the CAPM into vector notation. That is, $E(r) - r_f = \beta[E(r_m) - r_f] = \frac{Cov(r\,, r^T w_m)}{\sigma_m^2}[E(r_m) - r_f] \;\Rightarrow\; \Pi = \lambda\, S\, w_{mkt}$ where $\Pi = E(r) - r_f$, $\lambda = \frac{[E(r_m) - r_f]}{\sigma_m^2}$, $S = Cov(r\,, r^T)$.

In the computation of implied excess equilibrium return vector from equation (6), Grinold and Kahn (1999) and Idzorek (2002) uses the following formula for the estimation of risk aversion coefficient. It can be calculated by dividing the expected excess return by variance of market excess returns.

$$\lambda = \frac{\mathrm{E(r)} - \mathrm{r_f}}{\sigma^2} \ldots\ldots\ldots (7)$$

Where:

$\mathrm{E(r)}$ = Expected market or benchmark return

$\mathrm{r_f}$ = Risk free rate

σ^2 = Variance of market excess return

Black-Litterman formula for combined return vector along with description is as follow.

$$E(R) = [(\tau S)^{-1} + P^T \Omega^{-1} \mathrm{P}]^{-1} [(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q] \ldots\ldots\ldots (8)$$

Equation (8) has two parts i.e. $[(\tau S)^{-1} + P^T \Omega^{-1} \mathrm{P}]^{-1}$ and $[(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q]$.

If "N" is the number of assets in the portfolio and "K" denotes the number of views then:

$E(R)$ = "N×1" vector of combined expected return

τ = Scalar

S = "N×N" matrix of covariance of excess returns

P =Link matrix of order "K×N" and it shows the asset about which investor has views

P^T = Transpose of link matrix

Ω = Diagonal covariance matrix of order "K×K" representing the uncertainty associated with each view. Assumption of independent views forces it to diagonal matrix.

Ω^{-1} = Matrix showing the level of confidence about views

Π = "N×1" vector of implied excess equilibrium return

Q = "K×1" vector of views

$(\tau S)^{-1}$ = Weight on Π

$P^T\Omega^{-1}$ = Weight on Q

$[\,(\tau S)^{-1}\,\Pi + \,P^T\Omega^{-1}Q]$= Second part of the formula showing the weighted Π & Q

$[(\tau S)^{-1} + \,P^T\Omega^{-1}\mathrm{P}]^{-1}$ = First part of the formula ensuring sum of all weight must equal to one

In mean-variance optimization the optimal weight of BL asset allocation model (Lee 2000) is

$$\acute{w} = \frac{(M_{mix})^{-1}.\mathbf{I}}{\mathbf{I}^T.(M_{mix})^{-1}.\mathbf{I}} + (\lambda_0 M_{mix})^{-1}.\left\{E(R) - \frac{\mathbf{I}^T.(M_{mix})^{-1}.E(R)}{\mathbf{I}^T.(M_{mix})^{-1}.\mathbf{I}}.\mathbf{I}\right\}$$

3.2.2.1.Investor's view

Remarkable feature of Black and Litterman model is that it gives opportunity to incorporate the investor/analysts views about any or all asset classes in the portfolio for the calculation of expected return vector. Since investor/analysts have access to information, their views may be in relative or in absolute sense about asset classes. Most complex stage in the model is the incorporation of nake views of investor about asset classes into the model for the determination of expected return vector. Remember investor's views are just views and these are not facts. Hence there may be an error with each view. Value of error term is directly link with level of confidence of investors about each view. As Q = "K×1" vector of views then a view can be written as "$Q + \varepsilon$". In matrix notation it can be written as follows.

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

Here "ε" represents the error term vector associated with views. Further it is assume that "ε" is independently-normally-distributed with zero mean and uncertainty equal to matrix "Ω" which is diagonal covariance matrix (Zero in all off-diagonal positions). It can be written as follows.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_k \end{bmatrix} \sim N \left[\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1k} \\ \omega_{21} & \omega_{22} & \omega_{23} & \cdots & \omega_{2k} \\ \omega_{31} & \omega_{32} & \omega_{33} & \cdots & \omega_{3k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega_{k1} & \omega_{k2} & \omega_{k3} & \cdots & \omega_{kk} \end{bmatrix} \right]$$

Here "Ω" represents diagonal covariance matrix of order "K×K" representing the uncertainty associated with each view. Assumption of independent views forces it to diagonal matrix. The error term may be positive or negative other than zero except the extreme case where investor is fully (100%) confident about views. The error term vector associated with views doesn't directly place into the formula but variance of error does enter into equation. Variance of error terms is represented by "ω" and it forms a "K×K" matrix denoted by "Ω". Uncertainties in views expressed by investor have direct relation with variance of error term.

$$\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1k} \\ \omega_{21} & \omega_{22} & \omega_{23} & \cdots & \omega_{2k} \\ \omega_{31} & \omega_{32} & \omega_{33} & \cdots & \omega_{3k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega_{k1} & \omega_{k2} & \omega_{k3} & \cdots & \omega_{kk} \end{bmatrix} \Rightarrow \Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}$$

The determination of element of matrix "Ω" needs to form a new matrix called "link matrix" denoted by "P". Matrix "P" links the views from matrix "Q" to assets in the portfolio. Each view represented by a row vector and hence forms the row of matrix "P". Therefore "K" views form a "K×N" matrix. In matrix "K×N", row one represents view 1 and so on. View may be relative or in absolute sense. Most common view is relative view. In relative view, investor has positive opinion about one asset class with respect to some other asset classes. The value for view 1 in matrix "P" has positive weight for outperforming asset class and negative for underperforming but sum of each row must be equal to one. Zero is used for asset classes for which investor have no view. After defining the link matrix next step is to compute the variance of each individual portfolio. This variance represents uncertainty regarding each view and its inverse can be used to find the level of confidence about view. Uncertainty associated with view can be calculated by the formula $\Omega = \tau \text{ P S } P^T$.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{k1} & p_{k2} & p_{k3} & \cdots & p_{kn} \end{bmatrix}$$

Black-Litterman model is actually the weighted average of vector of implied excess equilibrium return and vector of views. Relative weightings are function of scalar and uncertainty of views. Black and Litterman (1992) suggest the value of scalar close to 1. But Idzorek (2002) describe that to specify this value is highly abstract and complex. Satchell and Scowcroft (2000) suggested that its value could be one. This study sets the value of scalar (τ) equal to one and uses absolute views about asset classes. As uncertainty in view about any asset increases then it is directly reflected from the vector of combined expected return and it should closer to Π.

3.2.2.1.1. Estimation of investor’s views

This study also develops market model for the estimation of investor’s views as input to Black-Litterman model in Pakistan and in global perspective. This study uses market model as a proxy for investor’s views. Study estimates separate model for each asset class locally as well as internationally. The views related to global asset class “i” is estimated by considering the global macroeconomic variables (Europe Brent spot price and Economic Uncertainty index) as well as other studied asset classes. Study uses European uncertainty index as a proxy of world uncertainty index which includes European news index, European CPI and European budget balance. For each specific asset class 'i', other asset classes are treated as explanatory variables for the estimation of views about that asset class 'i'. In emerging Asian countries views are aided by a market model in each country. For this study estimate the asset classes by its lag value, equity market index of respective country, yield on treasury bills of respective country and index of the world oil and gas prices.

Ordinary least square (OLS) is a way to analyze the dependency of one variable on one or more other variables, with a view to estimate the mean value of dependent variable in term of known values of regressor (s). General equation for estimation of return for each asset can be written as follow.

$$R_{it} = \gamma_0 + \sum_{j=1}^{n} \gamma_j Y_{jt} + \varepsilon_t$$

R_{it} = Return on asset class 'i'

Y_{jt} = Explanatory variable

ε_t = Error term

In Pakistani standpoint the following market model is used to forecast the quantitative views for each asset class 'i'.

$$R_{it} = \gamma_0 + \gamma_1 R_{it-1} + \gamma_2 R_{mt-1} + \gamma_3 BB_{t-1} + \gamma_4 D_{t-1} + \gamma_5 S_{t-1} + \gamma_6 AO_{t-1} + \gamma_7 GR_{t-6} + \gamma_8 ND_{t-1} + \gamma_9 BA_{t-6} + \gamma_{10} O_{t-1} + \varepsilon_t \ldots\ldots\ldots (9)$$

Where:

R_{mt} = Return on equity market

BB= Number of casualties in Bomb Blast in Pakistan

D= Number of casualties in Drone Attack in Pakistan

S=Number of casualties in Suicide Attacks

AO=Dummy for Army Operation in Pakistan

GR= Dummy for Government Regime in Pakistan

ND=Dummy for Natural Disasters

BA=Dummy for Budget Announcement

O= Change in Oil Prices

Equation (8) can be used to find the vector of combined expected return which also incorporates investor specific views about asset classes in the portfolio. These return then uses as input to determine the optimal weight for asset allocation into various investment opportunities.

3.2.3. The augmented Black-Litterman formula

The novel contribution of this study is the addition of country risk factor in the Black-Litterman formula. This study explores country risk as a risk factor and incorporates it into the Black-Litterman formula. The resultant model is named as augmented Black-Litterman formula. As described above in equation (6), the Black-Litterman model uses equilibrium returns as a neutral starting point. For this, assume that 'N' is number of assets in the portfolio, "Π" is "N×1" vector, "S" is "N×N" covariance matrix and "w_{mkt}" is a matrix of order "N×1".

$$\Pi = \lambda \, S \, w_{mkt}$$

Π = Implied excess equilibrium return vector

λ = Risk aversion coefficient

S = Covariance matrix of excess returns

w_{mkt} = Market capitalization weight of assets

Equation (6) can be derived by a maximization problem of investor with quadratic utility function. The Black-Litterman formula assumes that risk can be completely characterized by covariance. Hence it has the same assumption as the capital asset pricing model i.e. CAPM. But this single index model faces many criticisms. In this scheme Krishnan and Mains (2005) considered the recession as a risk factor and named the new model as two factor BL model. Ross (1976) suggested that expected return for any specific asset is a function of the sensitivity of that asset with one or more systematic factors. It can be observe that emerging economies are supposed to be more risky than the develop markets in term of economic, financial and political risk factors. So the investors of emerging countries are supposed to bear additional risk called country risk. Also Harvey (2004) concludes that country risk should be additionally rewarded and finds that there is high association between the expected return and country risk in emerging markets. Hence it is still possible to earn premium for taking on systematic exposures that are uncorrelated to the market but undesirable for certain investors.

This study explores country risk as a risk factor and incorporates it into the Black-Litterman formula. The resultant model is named as augmented Black-Litterman formula. This factor should generate positive returns and this premium cannot be accounted by the CAPM.

Black-Litterman uses the quadratic utility function but as described above, risk cannot describe completely by a single factor model. But investor may earn premium by assuming the country risk. A natural generalization in the utility function can be made in the following way:

$$\widehat{U} = W^T \Pi - \frac{1}{2} \lambda W_m^T S W_m - \lambda_1 W^T \beta_1$$

Here $\widehat{U}$ represents the utility under the country risk. In the above equation β_1 actually measure the responsiveness of each asset class to the country risk as a risk factor. This framework can be extended for multifactor in the following way.

$$\widehat{U} = W_m^T \Pi - \tfrac{1}{2}\lambda W_m^T S W_m - \sum_{j=1}^{n} \lambda_j W^T \beta_j \ldots\ldots\ldots (10)$$

Where:

$\widehat{U}$ = New utility under the country risk

W_m^T = "1×N" matrix of weight

Π = "N×1" vector of implied excess equilibrium return

W_m^T = Transpose matrix of market capitalization weight of assets

S = "N×N" matrix of covariance of excess returns

W_m = Market capitalization weight of assets

λ_j = Risk aversion coefficient of the 'j^{th}' independent risk factor

β_j = Response of each asset class to the factor

For the calculation of "N×1" vector of implied excess equilibrium return, take the first derivative of equation (10) with respect to 'w' and put it equal to zero, it gives:

$$\frac{\partial \widehat{U}}{\partial W^T} = W_m^T \Pi - \tfrac{1}{2}\lambda W_m^T S W_m - \sum_{j=1}^{n} \lambda_j W^T \beta_j = 0$$

Solving the above equation for 'Π', we have:

$$\Pi = \lambda\, S\, W_m + \sum_{j=1}^{n} \lambda_j\, \beta_j = \widehat{\Pi} \ldots\ldots\ldots (11)$$

After the incorporation of country risk factor, $\widehat{\Pi}$ represents the new "N×1" vector of implied excess equilibrium return. The above equation is intuitive and shows that when any asset expose to alternative risk then investor expects more return for taking additional country risk. Every investor should be rewarded for taking additional risk as defined by the utility function.

The few following changes are made for the application of BL formula directly. The above equation can be written as follows:

$$\Pi = \widehat{\Pi} - \sum_{j=1}^{n} \lambda_j \beta_j$$

After multiplication of above equation with "P" it becomes:

$P\Pi = \widehat{P\Pi} - \sum_{j=1}^{n} \lambda_j P \beta_j$ and $Q = \widehat{Q} - \sum_{j=1}^{n} \lambda_j P \beta_j$

By substituting these values in the original BL equation, it gives

$$\widehat{E(R)} = [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}P]^{-1}\left[(\tau\Sigma)^{-1}\widehat{\Pi} + \acute{P}\Omega^{-1}\widehat{Q}\right]$$

$$\widehat{E(R)} = [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}P]^{-1}\left[(\tau\Sigma)^{-1}(\Pi + \sum_{j=1}^{n} \lambda_j \beta_j) + \acute{P}\Omega^{-1}(Q + \sum_{j=1}^{n} \lambda_j P \beta_j)\right]$$

$$= [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}P]^{-1}\left[\left\{(\tau\Sigma)^{-1}\Pi + \acute{P}\Omega^{-1}Q\right\} + (\tau\Sigma)^{-1}\sum_{j=1}^{n} \lambda_j \beta_j) + \acute{P}\Omega^{-1}\sum_{j=1}^{n} \lambda_j P \beta_j)\right]$$

$$= [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}P]^{-1}\left\{(\tau\Sigma)^{-1}\Pi + \acute{P}\Omega^{-1}Q\right\} + [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}P]^{-1}\left[(\tau\Sigma)^{-1}\sum_{j=1}^{n} \lambda_j \beta_j) + \acute{P}\Omega^{-1}\sum_{j=1}^{n} \lambda_j P \beta_j)\right]$$

$$= E(R) + [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}\mathrm{P}]^{-1}\big[(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}\mathrm{P}\big]\left[\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}}\beta_{\mathrm{j}})\right]$$

$$\widehat{E(R)} = E(R) + \left[\sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}}\beta_{\mathrm{j}})\right] \ldots\ldots\ldots(12)$$

The equation (12) is used for the expected excess return of augmented BL formula under two variable frameworks. Moreover, the posterior variance is unchanged i.e. $M_{mix} = [(\tau\Sigma)^{-1} + \acute{P}\Omega^{-1}\mathrm{P}]^{-1}$.

Appendix B describe the mathematics of Black-Litterman model under country risk, it can be easily observe that the expected return of Black-Litterman model under country risk are higher than the expected return under original BL model.

For the calculation of risk aversion coefficient of the 'j$^{\text{th}}$' independent risk factor (λ_{j}), suppose r_m denotes the return of market portfolio, $\mathrm{f}_j$denotes the time series of return for factor and $\mathrm{r}_j$denotes the return for replicating portfolio against the risk factor 'j'. Since market has zero exposure to the f_j then we can find a weight factor v_j s.t $\acute{v}_j\beta_{\mathrm{j}} = 0$. For v_{j}, it requires a least square fit of $\|\mathrm{f}_{\mathrm{j}} - \acute{v}_j\Pi\|$ subject to the constraints $\acute{v}_j\beta_{\mathrm{j}} = 0$. Also v_0 is the market portfolio and further, for all factor f_j, $\mathrm{v}_0\beta_{\mathrm{j}} = 0$. There are numerous values of λ by multiplying the equation (11) by $'\mathrm{v}'$ and solving it for λ_0, it gives:

$$\acute{v}_0\Pi = \lambda_0\acute{v}_0\Sigma\mathrm{v}_0 + \sum_{\mathrm{j}=1}^{\mathrm{n}} \lambda_{\mathrm{j}}\acute{v}_0\beta_{\mathrm{j}}$$

Since $\mathrm{v}_0\beta_{\mathrm{j}} = 0$ and $\acute{v}_0\Pi = \mathrm{r}_m$, it gives

$$\lambda_0 = \frac{r_m}{\acute{v}_0 \Sigma v_0}$$

For any $j \geq 1$, after the multiplication of equation (11) with v_j and putting the value of λ_0, it gives

$$\acute{v}_j \Pi = \lambda_0 \acute{v}_j \Sigma v_j + \sum_{i=1}^{n} \lambda_i \acute{v}_j \beta_i$$

Since $v_i \beta_j = 0$ for all $i \neq j$, hence each λ_j can be written as:

$$\lambda_j = \frac{r_j - \lambda_0 \acute{v}_j \Sigma v_j}{\acute{v}_j \beta_j} \ldots \ldots \ldots (13)$$

Following Krishnan and Mains (2005), it can be argued that the above formula (13) for the calculation of λ_j is only approximation as $\left\| f_{j-} \acute{v}_j \Pi \right\|$ may be larger than zero. It is because the fact that $v_i \beta_j = 0$ for all $i \neq j$ may also be not fulfilled $\forall\ i\ \&\ j$. Walters (2008) confirmed that it can be ignored when study is dealing with single factor.

3.2.3.1.Country risk

Country risk may be the chances of loss associate with instability in borrower's country resulting inability to meet obligation. There are three approaches which are frequently used in literature to measure the country risk. These include country's sovereign credit rating, country risk scores and market based approach. Different rating agencies provide ratings to measure the country's default risk. But this approach is more skew towards default risk and ignores some other aspect for asset allocation. Since Damodaran (2011) claim that Moodey's did not update the India's rating from the period 2004-2007, therefore the rating from the rating agencies may lag behind the market fluctuations. Also, the rating agencies do not disclose all the procedures and steps adopted to assign the ratings to the countries. One other point in favor of not using this measure is existence of reasonable proportion of the lag period's credit rating information in the next periods rating. The other measure in this stream is country risk scores. There are many specialize companies that develops comprehensive framework to assign numerical scores (0-100) to countries based upon overall investment environment, economical, financial and political risk factors. But Damodaran (2011) also put his critic on this choice to measure the country risk. He argues that it's not easy to compare the country risk within the countries due to linearity in measures and finally there is also lack of transparency in this methodology.

In contrast to other measures for country risk, market based approach is consider to be more appropriate as it reflect most instant information. Researchers like Damodaran (2011) and Porras (2011) uses the bond default spread to measure the country risk. It can be define as the spread between yield to maturity of sovereign bond denominated in e.g. US dollar and the yield of a comparable US dollar bond respectively. Both securities must be issue in same currency and also have same maturity. This spread is more appropriate measure for risk premium against the country's overall position. If the sovereign international bond is not available for any country then study uses corporate international bond as a proxy for sovereign bond. This study uses the US 10-year bond as comparable risk free asset to capture the country risk dynamics in all the emerging Asian countries. Appendix C includes the details of sovereign credit ratings by the leading credit rating agencies i.e. Moody's, S & P and Fitch credit rating agency to the selected emerging Asian countries. Table 3.4 provides the detail of the alternative return estimation methods that are used in this study.

Table 3.4

Summary of the estimation of return vector

Abbreviation	Estimation of return vector
Hist	Historical Averages
AR	Auto-Regressive (p) Model Based Estimation
ARIMA	ARIMA (p,d,q) Based Estimation
ARIMA-Reg	ARIMA-Reg (p,d,q) Based Estimation
CAPM	Capital Asset Pricing Model Based Estimation
IEER	Implied Equilibrium Excess Return Based Estimation
BL	Black-Litterman Based Estimation
BL-CR	Black-Litterman under Country Risk Based Estimation

3.2.4. Variance-covariance matrix

For the calculation of invested proportion in each asset class by portfolio optimization, investor needs to come up with two most fundamental ingredients i.e. expected return of asset class and covariance matrix. The estimation of covariance matrix has its prominence towards portfolio optimization (Elton & Gruber (1973)). Among others Ledoit and Wolf (2004) called it the stickiest point in the whole process of asset allocation. Demiguel, Garlappi and Uppal (2009) empirically concluded the outperformance of naïve diversification over the mean-variance optimization. This is, perhaps, due to the estimation error and numerically instability in mean-variance asset allocation. This study compares the estimation of covariance matrices under four categories i.e. conventional methods, factor models, portfolio of estimators and shrinkage approach. The detail of these covariance matrices are given below.

3.2.4.1.Sample variance-covariance matrix

Assume ***"R"*** be a $\boldsymbol{p}\ \boldsymbol{x}\ \boldsymbol{k}$ matrix of p return on k observation, then sample variance-covariance matrix $\Sigma_{\boldsymbol{s}}$ can be computed by the following formula.

$$\hat{\Sigma}_{\boldsymbol{s}} = \frac{\mathbf{1}}{\boldsymbol{k}-\mathbf{1}}\ \boldsymbol{R}(\boldsymbol{I}-\frac{\mathbf{1}}{\boldsymbol{k}}\mathbf{1}\acute{\mathbf{1}})\acute{\boldsymbol{R}} \dots \dots \dots (14)$$

Here in equation (14) $\boldsymbol{I}$ is the identity matrix having order $\boldsymbol{T}$ and $\boldsymbol{1}$ is the $\boldsymbol{k}$ $\boldsymbol{x}$ $\boldsymbol{1}$ matrix of ones. The matrix Σ has rank T-1 when $p \geq T$ which is same as that of the rank of matrix $\boldsymbol{I} - \frac{\boldsymbol{1}}{\boldsymbol{k}}\boldsymbol{1}\acute{\boldsymbol{1}}$**,** thus it is not invertible. Further**,** Σ is rank deficit when $p > T$. The attractive feature of sample variance-covariance matrix is being the maximum likelihood under normality assumption. As the size of sample decreases it upsurges the chances of over fitting the data. Therefore its performance is superior for in-sample as compare to out-of-sample. Assume there are 'N' asset classes and further 'K' be the number of observations, then the covariance between the rates of return for asset class 'i'and asset class 'j'can be calculated by applying the following formula:

$$\boldsymbol{S_{ij}} = (K-1)^{-1} \sum_{t=1}^{K} [R_{i,t} - \overline{R_i}] [R_{j,t} - \overline{R_j}], \quad i,j = 1,2,3, \dots, N$$

Where $\overline{R_i} = \frac{1}{K} \sum_{t=1}^{K} R_{i,t} \quad , i = 1,2,3, \dots, N$

3.2.4.2.The market model for variance covariance matrix

This model is based upon the assumption that return on each asset class is generated by explicit exogenous variables. These exogenous variables can be originated from financial theory. In financial market, generally return of market is considered more relevant for any specific asset class. Sharp (1963) assumed that return on any asset class can be linearly regressed on market return. This is named sharp's single index model and can be formulated as follows.

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

With the assumption of $(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}, R_{mt}) = 0$ and $E\left(\varepsilon_{it}\varepsilon_{jt}\right) = 0$ then:

$$\sigma_i^2 = V(\beta_i R_m + \varepsilon_i) = \beta_i^2 \sigma_m^2 + 2Cov(\beta_i R_m, \varepsilon_i) + V(\varepsilon_i) = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon i}^2$$

$$\sigma_{ij} = Cov\left(\beta_i R_m + \varepsilon_i, \beta_j R_m + \varepsilon_j\right) = \beta_i \beta_j \sigma_m^2$$

In matrix notation, for all N asset, we have,

$$\hat{\Sigma}_{\boldsymbol{m}} = \boldsymbol{\beta}\sigma_m^2\acute{\boldsymbol{\beta}} + \boldsymbol{A}_{\boldsymbol{\varepsilon}}$$

As compare to the sample variance-covariance matrix, here the study only estimates the 2N+1 parameters. Therefor it is expected that estimation error may be decreases in single index model. Anyhow, this was done by introducing a specification error that asset returns are only depends on the market.

3.2.4.3.Constant correlation approach for variance covariance matrix

Elton and Gruber (1973) calculated the variance-covariance matrix by assuming that the variances of asset class are the sample returns. But 'same correlation coefficient' is used for all covariance. We know that $\sigma_{ij} = \mathrm{Cov}\left(R_i, R_j\right) = \rho_{i,j}\sigma_i\sigma_j$, therefore:

$$\sigma_{ij} = \begin{cases} \sigma_{ii} = \sigma_i^2 & when\ i = j \\ \sigma_{ii} = \rho_{i,j}\sigma_i\sigma_j & when\ \ i \neq j \end{cases} \dots\dots.\dots.(15)$$

The above formula (15) is also applied for the computation of variance-covariance matrix. Chan, Korceski and Lakonishok (1999) argued that constant correlation based method for variance-covariance worked better than its competing methods. For this, study used the average correlation coefficient among the asset classes for covariance.

3.2.4.4.Estimation of covariance matrix by principal component model

Principal component analysis (PCA) applies to forecast the underlying drivers of asset classes. Anyhow PCA performs this without any theoretical assumptions. With singular value decomposition (SVD) of sample covariance, PCA transform the vector space of *m* assets into another *m* factors. Hence each factor represents the linear combination of the original asset classes. The returns on asset class and covariance matrix can be written as

$$R_i^e = \beta_{i1}F_1 + \beta_{i2}F_2 + \beta_{i3}F_3 + \cdots + \beta_{im}F_m$$

$$\hat{\Sigma} = \beta A_F \acute{\beta}$$

Here β is M columns of eigenvectors and A_F represents the diagonal matrix of eigenvalues ($m*m$) . Since PCA is a dimension reduction technique and if first K factors govern reasonable variability of asset returns then the rest factors (*M-K)* can be drop and we have

$$\hat{\Sigma}_{\boldsymbol{pca}} = \tilde{\beta}\widetilde{A_F}\tilde{\acute{\beta}} \ \ + A_\varepsilon \dots\dots\dots (16)$$

Here in equation 16, $\tilde{\beta}$ is *M *K* matrix of factor loadings and $\widetilde{A_F}$ is the diagonal matrix (*K*K)* of eigenvalues and A_ε is *M*M* diagonal matrix of unexplained variance of idiosyncratic components by *K* factors.

3.2.4.5. Estimation of covariance matrix by Portfolio of estimators

Under this category, we estimate the covariance matrix with equal weights to different covariance matrices. The detail of these is described as follow.

1. A equally weighted portfolio of sample matrix & diagonal matrix
2. A equally weighted portfolio of sample matrix & single index matrix
3. A equally weighted portfolio of Sample matrix & constant correlation matrix
4. A equally weighted portfolio of Sample matrix, single index matrix & constant correlation matrix
5. A equally weighted portfolio of sample matrix, single index matrix, constant correlation matrix & diagonal

3.2.4.6.Shrinkage method

The minimization of quadratic loss function provides foundation to shrinkage approach as primarily familiarize by Stein (1956). It provides optimal mix between the precision matrix and the target matrix. Study use the sample covariance matrix as a starting point because it is easy to compute. But it also contains lot of estimation errors. Ledoit and Wolf (2003) argue that sample covariance matrix is occasionally applied because it imposes too little structure. The shrinkage estimators are base upon the combination of optimal weight between the sample covariance and the prior. For this, let S_s be sample covariance matrix, T denotes the target covariance matrix and λ be the weight assign to D then we have

$$\widehat{\Sigma}_{Shrinkage} = (1-\lambda)\widehat{\Sigma}_{\boldsymbol{s}} + \lambda T \dots\dots\dots(17)$$

If $\lambda = 0$ then, there is no shrinkage to $\widehat{\Sigma}_s$ and we returned $\widehat{\Sigma}_{Shrinkage} = \widehat{\Sigma}_s$. In case of complete shrinkage i.e. $\lambda = 1$ we have $\widehat{\Sigma}_{Shrinkage} = T$. And the possibility where $\lambda < 0$ or $\lambda > 0$ are meaningless from shrinkage viewpoint. Therefore the intended value of shrinkage intensity ranges between $0 < \lambda < 1$.

This study shrunk the sample covariance matrix to three different targets: the diagonal matrix, the single index model and constant correlation model.

a. Shrinkage towards diagonal target

For optimal shrinkage intensity, let the sample covariance is $S_{ij}, i \neq j$ and σ_{ij} denotes the corresponding true covariance. Also assume that weighted average lies between zero and S_{ij} with weights beings λ and $1-\lambda$ respectively then the squared deviation $\left[(1-\lambda)S_{ij} - \sigma_{ij}\right]^2$represents the loss. With S_{ij} being random variable, we are looking such a value of λ so that we have the lowest value of $E\left\{\left[(1-\lambda)S_{ij}-\sigma_{ij}\right]^2\right\}$. Same intuition is drawn-out for each individual covariance. Since covariance matrix is symmetric, so we only need to take into account the $n(n-1)/2$ covariances in upper triangle, where$j>i$. Therefore the loss function $E\left\{\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[(1-\lambda)S_{ij}-\sigma_{ij}\right]^2\right\}$, for $i=1,2,\dots,n-1$ & $j=i+1,i+2,\dots,n$, can be represent as $E\left\{\sum_{j>1}\left[(1-\lambda)S_{ij}-\sigma_{ij}\right]^2\right\}$. Base on the minimization of loss function optimal shrinkage intensity $(\lambda)$is

$$\lambda=\frac{\sum_{j>i}Var(S_{ij})}{\sum_{j>i}\left[Var\left(S_{ij}\right)+\sigma_{ij}^2\right]}\dots\dots\dots(18)$$

Here denominator is greater than numerator and also both are positive, therefore $0<\lambda<1$ satisfy. For further details on this optimal shrinkage intensity see the kwan (2011). For covariance estimation we put this value of λ along with diagonal covariance matrix as a target matrix in equation 17.

b. Shrinkage towards single index covariance target

Consistent with Ledoit and Wolf (2003), this study also shrinks the sample covariance matrix towards single index covariance matrix. Practically there are two boundaries: single factor model base on one-factor and a traditional estimator can be interpreted as N-factor model. The basic idea is existence of optimal level between the estimation and specification error. Study follow the Ledoit and Wolf (2003) procedure for the computation of optimal shrinkage intensity λ and is calculate as follow.

$$\lambda=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}Var\left(s_{ij}\right)-cov(\widehat{\varphi_{ij}},s_{ij})}{\sum_{i=1}^{n}\sum_{j=1}^{n}[var\left(\widehat{\varphi_{ij}}-s_{ij}\right)+\left(\varphi_{ij}-\sigma_{ij}\right)^2]}\dots\dots\dots(19)$$

Here s_{ij} is the (i,j)the element in the sample covariance matrix $\hat{\Sigma}_s$ and other can be define similarly. For healthy details on the computation of optimal shrinkage intensity we refer the Ledoit and Wolf (2003). For covariance estimation, put this estimated λ (Eq. 19) along with single index covariance matrix ($\hat{\Sigma}_{\boldsymbol{m}}$) as a target T in equation 17.

c. Shrinkages towards constant correlation based covariance target

For the calculation of optimal shrinkage intensity or shrinkage constant between the sample covariance matrix and constant correlation based covariance matrix we follow the procedure suggest by Ledoit and Wolf (2004). They use the quadratic measure of distance between true and estimated covariance which is based on Frobenius norm. For further details on optimal shrinkage intensity study refer the Ledoit and Wolf (2004). Table 3.5 summarizes the above describe covariance matrices. These are considered as input for portfolio optimization. On the basis of this estimation of inputs, study applies the Mean-Variance optimal framework, global minimum variance portfolio, forced diversification and naïve diversification for asset allocation.

The structure of alternative covariance matrices can include conventional methods, factor models, a portfolio of estimators and the shrinkage approach (Table 3.5). The sample matrix is based on historical covariances, but has a lower structure than other covariance estimators. Elton and Gruber (1973) recommend using the historical degree of association to estimate covariance estimators. Similarly, Sharpe (1963) uses systemic risk factors to determine the covariance matrix, although this is criticized on the grounds that it relies on a single systematic risk factor. Arguably, the single-index covariance matrix is more appropriate than the sample covariance on the basis of estimation errors, but it can lead to specification errors.

Ledoit and Wolf (2003, 2004) use an optimal combination of two covariance matrices to yield one covariance estimator by shrinking the sample covariances to the target matrix. Jagannathan and Ma (2003) challenge this approach and propose a simpler, equally weighted average of two or more covariance estimators. Ledoit and Wolf's (2003, 2004) method is theoretically more rigorous, but its empirical results are questionable (Disatnik & Benninga, 2007). To check the robustness of covariance matrices, these alternative estimators are used in this study. Table 3.5 summarizes the alternative covariance matrices. It also includes the diagonal method of estimating covariances, which is the basis for other covariance estimators under the categories of "portfolio of estimators" and "shrinkage approaches".

Table 3.5

Summary of the variance-covariance methods

Category	Variance-covariance matrices
Conventional methods	Diagonal method
	Sample matrix
	Constant correlation model
Factor Model	Single Index matrix
	Principal component analysis based model
Portfolio of estimators	Portfolio of sample matrix & diagonal matrix
	Portfolio of sample matrix & single index matrix
	Portfolio of Sample matrix & constant correlation matrix
	Portfolio of Sample matrix, single index matrix & constant correlation matrix
	Portfolio of sample matrix, single index matrix, constant correlation matrix & diagonal
Shrinkage approaches	Shrinkage to the diagonal matrix
	Shrinkage to the single index model
	Shrinkage to the constant correlation model

3.2.5. Alternative asset allocation strategies

3.2.5.1.Traditional mean –variance framework

Markowitz (1952, 1959) presents a statistical procedure for portfolio selection and this contribution is term as modern portfolio theory. Markowitz assumes that investors are risk averse i.e. for specific return level, investor always prefer minimum risk. The detail of the mean-variance framework is as follow.

3.2.5.2.Risk and return of portfolio

Assume there are 'N' asset classes and 'w_i' be the proportion of funds invest in asset class 'i', then expected return of portfolio can be computed as follow:

$$E(R_p) = w_1 E(R)_1 + w_2 E(R)_2 + \cdots + w_N E(R)_N = \sum_{i=1}^{N} w_i E(R)_i$$

Where:

$E(R_p)$ = Expected Return of Portfolio

w_i = The proportion of funds invest in asset class 'i'

$E(R)_i$ = Expected Return on asset class 'i'

In matrix notation the expected return of portfolio can be written as:

$$E(R_p) = \sum_{i=1}^{N} w_i E(R)_i = W^T E(R) \ldots\ldots\ldots (20)$$

Where:

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_N \end{bmatrix} \;\; \& \;\; E(R) = \begin{bmatrix} E(R_1) \\ E(R_2) \\ E(R_3) \\ \vdots \\ E(R_N) \end{bmatrix}$$

Return of the portfolio is actually the weighted average return of individual asset classes in the portfolio. But according to the logarithm property, logarithm of a sum is not equal to sum of logarithms. Therefore continuously compounded portfolio return is not exactly (but approximately) equal to the weighted averages of continuously compounded returns of asset classes. It is assume that past sample period data represent the distribution of returns for next month.

The variance (measure of risk) of portfolio is not simply the weighted average of variances of individual asset class in the portfolio. This is mainly due to the phenomena of diversification. The formula of variance of portfolio also considers the covariance between each pair of asset class in the portfolio. Assume there are 'N' asset classes and 'w_i' be the proportion of funds invest in asset class 'i', then variance of portfolio can be computed as follow:

$$Var(R_p) = \sum_{i=1}^{N}(w_i)^2\, Var\,(R_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j\, Cov\,(R_i, R_j)$$

Where:

$Var(R_p)$ = Standard deviation of portfolio

w_i, w_j = The proportion of funds invest in asset class 'i'& 'j'

$Cov(r_i, r_j) =$ Covariance between the rates of return for assets class 'i'& 'j'

The above equation can be written in the following way:

$$Var(R_p) = \sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j\ \sigma_{ij} \dots\dots\dots (21)$$

Where:

$\sigma_{ij} = Cov\,(R_i, R_j)$ while $\sigma_{ii} = Var(R_i)$ and also $\rho_{i,j}\,\sigma_i\sigma_j = \sigma_{ij}$

In matrix notation the formula for the computation of variance of portfolio can be written as follow:

$$Var(R_p) = [w_1 \quad w_2 \quad w_3 \quad \cdots \quad w_N]\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3N} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_N \end{bmatrix}$$

$$Var(R_p) = W^T SW$$

In matrix notation W^T shows the transpose of matrix 'W' and matrix 'S' is the variance-covariance matrix.

Variance, standard deviation and correlation coefficient for asset classes 'i' & 'j' can be computed by using the following formulas:

$$Var = \sigma_i^2 = \frac{1}{N}\sum_{t=1}^{N}[(R_{i,t} - E(R_i)]^2$$

$$Std = \sigma_i = \sqrt{Var}$$

Here 'Var' stands for variance while 'Std' stands for standard deviation of asset class 'i'. The correlation coefficient between the rates of return on asset class 'i' & 'j'can be calculated by using the following formula. In this formula 'Corr' stands for correlation coefficient between the rates of returns on asset class 'i' & 'j'.

$$Corr = \rho_{i,j} = \frac{\sum_{t=1}^{N}[R_{i,t} - E(R_i)]\,[R_{j,t} - E(R_j)]}{\sqrt{\sum_{t=1}^{N}[R_{i,t} - E(R_i)]^2 \sum_{t=1}^{N}[R_{j,t} - E(R_j)]^2}}$$

The Covariance between the rates of return for assets class 'i' & 'j'can be calculated by applying the following formula:

$$Cov\,(r_i, r_j) = \frac{1}{N}\sum_{t=1}^{N}[R_{i,t} - E(R_i)]\,[R_{j,t} - E(R_j)]$$

The relationship between covariance and correlation coefficient can be described with the following formula.

$$Corr = \rho_{i,j} = \frac{Cov(r_i, r_j)}{\sigma_i \sigma_j}$$

Assume investor's expected utility function depends only on the expected return and risk. Investor also wants to maximize it i.e. Max : $\mathrm{E}[U(x)] = \mathrm{E}(\mathrm{r_p} - \mathrm{r_f}) - \lambda\sigma_p^2 \;\; \mathrm{s.t.}\, \acute{w} = 1$. More formally, risk averse investor ($\lambda > 0$) needs to change the proportion invests in each asset class to maximize the expected utility which is given by

$$\mathrm{Max}_w\!: \mathrm{E(U)} = \acute{w}E(R) - \lambda\acute{w}_m\Sigma\mathrm{w_m} \;\; \mathrm{s.t.}\, w\acute{}\mathbf{1} = 1$$

By differentiating the expected utility function with respect to 'w' and put it equal to zero, it gives

$$\widehat{w}_i = \frac{\hat{\Sigma}_i^{-1}E(R)}{Sum[\hat{\Sigma}_i^{-1}E(R)]} \dots\dots\dots(22)$$

Here $E(R)$ is the set of expected excess return of all asset classes in the portfolio. Under force diversification, study first compute the optimal weights by imposing the 'no short' constraints and secondly, it compute the weight for each asset class by imposing the constraint $0.1 < w_i < 0.35$ for global and $0 < w_i < 0.25$ for Pakistani perspective.

3.2.5.3.Global minimum variance (GMV) portfolio

As per the Markowitz framework on asset allocation, the portfolio selection depends upon the expected return, risk and covariances among the involve asset classes. Portfolio selection is all about maximization of return with respect to specific level of risk of investment or conversely it is minimizing the risk with respect to a specific level of return. Global minimum variance portfolio (GMVP) is the left most point on the mean variance efficient frontier. Below this point the inefficient tail of frontier starts. It is the portfolio which offers highest return with least risk. It considers the return, variances and correlation coefficient of all the involve investment opportunities. Mathematically the GMVP for two asset class i.e. A and B can be computed as follow.

Let w_1 be the proportion of funds invests in asset class A and $(1-\mathrm{w}_1)$ be the proportion of funds invests in asset class B, and then variance of portfolio can be written as:

$$\sigma_p^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2\,w_1(1-w_1)\sigma_1\sigma_2\rho_{1,2}$$

For GMVP, the weights are computed in such a way that based upon these chosen weights, the variance of portfolio should be minimum. Therefore it can be written as:

$$\underset{w}{\mathrm{Min}}\ \ \sigma_p^2 = w_1^2\sigma_1^2 + (1-w_1)^2\sigma_2^2 + 2\,w_1(1-w_1)\sigma_1\sigma_2\rho_{1,2}\ \ldots\ldots\ldots(23)$$

The equation (23) deal with minimization of the variance of portfolio subject to the constraint $\sum_{\mathrm{j}=1}^{2}\mathrm{w}_\mathrm{j} = 1$ i. e sum of proportion of fund invests in all the asset classes must equal to one. The second constraint is $\sum w_\mathrm{j}\ \geq 0$ where j = 1,2.

$$\Longrightarrow\ \sigma_p^2 = w_1^2\sigma_1^2 + \sigma_2^2 + w_1^2\sigma_2^2 - 2\,w_1\sigma_2^2 + 2\,w_1\sigma_1\sigma_2\rho_{1,2} - 2\,w_1^2\sigma_1\sigma_2\rho_{1,2}$$

For minimum variance portfolio, take its first order derivative with respect to w_1 and put it equal to zero i.e. $\frac{\mathrm{d}\sigma_\mathrm{p}^2}{\mathrm{dw}_1} = 0$

$$\Longrightarrow \frac{d\sigma_p^2}{dw_1} = \ 2w_1\sigma_1^2 + 2w_1\sigma_2^2 - 2\sigma_2^2 + 2\sigma_1\sigma_2\rho_{1,2} - 4w_1\sigma_1\sigma_2\rho_{1,2} = 0$$

$$\Longrightarrow\ w_1 = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{1,2}}$$

Since $\sigma_{ij} = \text{Cov}\left(R_i, R_j\right) = \rho_{i,j}\sigma_i\sigma_j$ then above can be written as:

$$w_1 = \frac{\sigma_2^2 - Cov(r_1, r_2)}{\sigma_1^2 + \sigma_2^2 - 2Cov(r_1, r_2)}$$

As the last equation give the weight to one asset class and the equation $w_2 = (1 - w_1)$ gives the value of w_2. The weight in GMVP for 'N' asset classes can also be computed by minimizing the Lagrange function C for portfolio variance.

$$\text{Min } \text{Var}\left(R_p\right) = \sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j \, \sigma_{ij}$$

$$\text{Subject to } \textstyle\sum_{j=1}^{N} w_j = 1$$

$$C = \sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j \, \sigma_{ij} + \lambda_1 \left(1 - \sum_{i=1}^{N} w_i\right)$$

Here w_i, w_j , σ_{ij} and $\lambda_1$are the weights, covariance and Lagrange multiplier respectively.

Merton (1980) argued that it's hard to estimate the expected return on asset classes. Jorion (1991) claimed that investor can get suboptimal portfolio due to estimation error. Therefore, consistent with Ledoit and Wolf (2003) and many others, this study compute the weight for global minimum variance portfolio (GMVP). Since GMVP is the only portfolio on the efficient frontier that depends upon covariance matrix. The weight for GMVP of 'n' risky asset universe can be computed as the followings

$$\text{Min } \text{Var}\left(R_p\right) = W^T \Sigma W \text{ s.t } \acute{w}\mathbf{1} = 1$$

Where '***1***' is column vector on ones and Wrepresents the vector of weight in portfolio. The weight for GMVP are computed as

$$\hat{w}_{mv} = \frac{\hat{\Sigma}_i^{-1}\mathbf{1}}{\acute{\mathbf{1}}\hat{\Sigma}_i^{-1}\mathbf{1}} \ldots\ldots\ldots (24)$$

Non-theory based diversification i.e naive diversification could be defined as invest equal in proportion in all available investment opportunities. If 'N' is total number of asset classes for investment opportunities then weights are calculated by using the 1/N fallacy i.e. invest equally in all available investment opportunities.

Table 3.6

Summary of Asset allocation strategies

Abbreviation	Asset allocation strategies
MVP	Efficient portfolios based on mean-variance criteria
GMVP	Global minimum variance portfolio
FD	Constrained portfolio (no short sale)
FD	Constrained portfolio (no short sale)
EWP	Equally weighted portfolio

3.2.6. Evaluation dimensions

For evaluation of covariance estimators the use of root mean square error (RMSE) and risk analysis of minimum variance portfolio (GMVP) are frequent in literature. To make the result more compatible with the existing literature this study also use these two criteria for comparison among 12 covariance estimators. In consistent with Liu and Lin (2010) study uses the following formula for the calculation of RMSE to compare the pair wise estimation accuracy of co-variances

$$RMSE = \sqrt{\frac{N(N-1)}{2}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}(\hat{\sigma}_{ij} - \sigma_{ij})^2}$$

Here $\sigma_{ij}$and $\hat{\sigma}_{ij}$represents the pair wise actual and estimated covariance while $\frac{N(N-1)}{2}$ represents the number of pair wise covariance of covariance matrix of order N*N. For the estimation of covariance matrix study use the first subsample and accuracy of this covariance matrix is evaluated as ex-post in the second subsample. A relatively low value of RMSE shows the relatively high pair wise accuracy of covariance estimator.

In consistent with Chan, Karceski, Lakonishok (1999) this study also analyzes the covariance matrix on the basis of minimum variance portfolios. As GMVP are independent of the choice of expected return vector so its performance can give relatively better insight about the estimation of covariance matrix. Anyhow GMVP gives limited information about the covariance estimator as it is based upon only one portfolio. Jagannathan and Ma (2003) describes that imposing the constraints on the GMVP improves the performance of sample covariance but the focus of our paper is on the errors in covariance estimator in forecasting the variance of portfolios. Therefore we are not putting any restriction on the GMVP. This study calculates the weights on the basis of GMVP by using each covariance estimators. With the help of this weight of MVP, the next step is to compute and note the out-of-sample return of the GMVP. This series of return of portfolios leads towards the computation of average mean and standard deviation of GMVP. The RMSE analyze the accuracy of pair wise estimator while minimum variance portfolio tests the effectiveness of covariance estimator towards selecting the minimum risk portfolios. For comparison among the future return estimation techniques this study uses the paired sample t-test, correlation analysis, descriptive statistics and mean square prediction error.

In consistent with Fernandes, Ornelas and Cusicanqui (2012) this study compares the allocation strategies on the basis of two evaluation dimensions: financial efficiency and diversification.

Conventional Sharp ratio is computed for financial performance while variance of weights and Herfindahl index is used for diversification. Excess Sharp ratio is calculated by dividing the expected excess portfolio return to the portfolio excess standard deviation. Herfindahl index is simply the sum of square of weights.

Herfindahl index is computed as: $HI_i = w_{1i}^2 + w_{2i}^2 + w_{3i}^2 + \cdots + w_{ni}^2$

Further, sharp ratio, variance and Herfindahl index is computed and subsequently compared by changing the estimation methods of forecasting the expected return, variance-covariance matrix and asset allocation strategies. Study also calculates the number of positive weights, number of negative weights, minimum value of weight, maximum value of weight and range of weights. Table 3.7 gives the details of the evaluation parameters used in this study.

Table 3.7

Summary of the evaluation strategies

Abbreviations	Evaluation Dimensions
Evaluation dimensions for asset allocation strategies	
ESR	Excess sharp ratio
HI	Herfindahl index
Var	Variance of weights
DS	Descriptive statistics of weights
Evaluation dimensions for variance-covariance matrix	
RMSE	Root mean square error
SD-GMVP	Risk profile of minimum variance portfolios
M-GMVP	Return profile of minimum variance portfolios
Evaluation dimensions for future return estimates	
	Paired sample t-test
	Correlation analysis
	Descriptive statistics
	Mean square prediction error (MSPE)

Chapter 4

4. EMPIRICAL RESULT

This section presents the empirical findings of the study. It contains the finding about asset allocation framework in emerging Asian countries, asset allocation by using the global proxies and asset allocation framework in Pakistan. The details of these are as follow.

4.1. Empirical evidence from emerging Asian countries

Table 4.1 presents the root mean square error (RMSE) of the pair wise covariance estimation of all consider covariance estimators against corresponding out-of-sample values. Study estimates the covariance matrices in the first subsample and second subsample is used for the computation of out-of-sample values.

Table 4.1

Root mean square error (RMSE) results

Covariance matrices	India	Indonesia	Pakistan	Philippines	Thailand
Sample matrix	0.0344	0.0278	0.0230	0.0168	0.0145
Constant correlation model	0.0311	0.0247	0.0168	0.0158	0.0130
Single Index matrix	0.0288	0.0245	0.0165	0.0122	0.0127
PCA method	0.0006	0.0003	0.0003	0.0001	0.0001
Portfolio of sample & diagonal	0.0172	0.0139	0.0115	0.0084	0.0072
Portfolio of Sample & constant correlation	0.0327	0.0259	0.0194	0.0149	0.0137
Portfolio of sample & single index	0.0316	0.0258	0.0193	0.0142	0.0135
Portfolio of Sample, single index & correlation	0.0314	0.0252	0.0182	0.0137	0.0133
Portfolio of sample, single index, correlation & D	0.0235	0.0189	0.0136	0.0103	0.0100
Shrinkage to diagonal	0.0342	0.0277	0.0228	0.0167	0.0144
Shrinkage to single index	0.0370	0.0271	0.0217	0.0166	0.0141
Shrinkage to constant correlation	0.0336	0.0271	0.0202	0.0161	0.0136

From the Table 4.1, it is evident that factor models as a group outperform the competing covariance estimators (conventional methods, Portfolio of estimators, and shrinkage approaches) in all the emerging Asian countries. In consistent with Liu and Lin (2010) sample covariance estimator proves worse among all the 12 consider covariance estimators in all the emerging countries. But when study takes the equally weighted average of sample covariance with diagonal matrix then it comes in the second place just after the PCA in all the emerging countries. Further single index covariance matrix also has relatively low value of RMSE than the constant correlation based covariance matrix in all the emerging Asian countries. It's also worth mentioning that the performance of sample covariance improves by just taking the simple average with single index covariance matrix.

When study compares the so complex shrinkage approach with the simple average of different covariance estimators then it is clearly reveal that simpler is better. Therefore the estimators suggested by Ledoit and Wolf (2003) and Ledoit and Wolf (2004) are not able to outperform the simple weighted averages suggest by Jagannathan and Ma (2003) and Disatnik and Benninga (2007). So in consistent with the findings of Jagannathan and Ma (2003), Liu and Lin (2010) and many other studies this study also confirms that most complex shrinkage estimator does not work well against simple averages of estimators in all the emerging countries. Potential reason for this may be the estimation error in the estimators. As discussed by Demiguel, Garlappi and Uppal (2009) that this is mainly due to estimation errors and numerical instability in estimators which may outweighs the potential benefits.

Table 4.2 reports the average standard deviation of the minimum variance portfolios (GMVP) under 12 different covariance estimators. Study finds some consistent evidence of covariance estimators under RMSE and risk of GMVP. Here again the equally weighted portfolios of covariances as suggest by Jagannathan and Ma (2003) and Disatnik and Benninga (2007), on an average, outperform the most complex shrinkage estimators suggested by Ledoit and Wolf

(2003) and Ledoit and Wolf (2004) in all the emerging countries. Moreover there is mix evidence of performance between the single index covariance estimator and constant correlation based covariance matrix in all the emerging Asian countries.

Table 4.2

Average standard deviation of the GMVP results

Covariance matrices	India	Indonesia	Pakistan	Philippines	Thailand
Sample matrix	0.0200	0.0184	0.0279	0.0138	0.0178
Constant correlation model	0.0189	0.0185	0.0250	0.0144	0.0184
Single Index matrix	0.0202	0.0166	0.0261	0.0134	0.0192
PCA method	0.0454	0.3770	0.1247	0.1871	0.0363
Portfolio of sample & diagonal	0.0213	0.0159	0.0240	0.0131	0.0196
Portfolio of Sample & constant correlation	0.0174	0.0172	0.0259	0.0131	0.0181
Portfolio of sample & single index	0.0184	0.0171	0.0262	0.0135	0.0186
Portfolio of Sample, single index & correlation	0.0181	0.0169	0.0256	0.0130	0.0184
Portfolio of sample, single index, correlation & D	0.0201	0.0161	0.0244	0.0130	0.0191
Shrinkage to diagonal	0.0190	0.0183	0.0279	0.0138	0.0178
Shrinkage to single index	0.0193	0.0179	0.0272	0.0138	0.0181
Shrinkage to constant correlation	0.0176	0.0174	0.0263	0.0134	0.0180

From Table 4.2, overall it can be said that there are some differences in performance of covariance estimators under both the criteria i.e. RMSE and risk of GMVP. In consistent with Liu and Lin (2010) this study also reveals that the performance of complex estimators improves under the GMVP than RMSE. It means that the complex estimators have more constraints than the simpler one. Like Jagannathan and Ma (2003) argue that the constraints can lower the risk of estimated portfolios, no matter whether the constraints are impose in right or wrong way. For the purpose of comparison of the average standard deviation with the results of average mean of the GMVP, study also report the results of average mean of the GMVP in all the emerging countries in appendix D.

Table 4.3 reports the quantitative features of the optimal weights under mean-variance framework (MV-model), original Black-Litterman model (BL-model) and Black-Litterman model under country risk (BL-CR). It reports the value of Herfindahl index, standard deviation of optimal portfolios, the difference between maximum and minimum weights (Range) and variance of weights under different inputs to portfolio optimization in all the emerging countries.

Table 4.3

Comparison among optimal portfolio weights under alternative models

Country	Characteristics of weights	MV-Model	BL-Model	BL-CR Model
India	Herfindahl index	5.6275	0.4231	0.3764
	Standard deviation of portfolio	0.0556	0.0180	0.0169
	Range	2.7260	0.6282	0.5374
	Variance of weights	0.6142	0.0359	0.0307
Indonesia	Herfindahl index	10.7010	16.9331	9.7263
	Standard deviation of portfolio	0.0534	0.0690	0.0563
	Range	3.1401	3.7566	2.7499
	Variance of weights	1.1779	1.8703	1.0696
Pakistan	Herfindahl index	0.9648	0.2780	0.2713
	Standard deviation of portfolio	0.0307	0.0205	0.0204
	Range	0.9844	0.4254	0.4256
	Variance of weights	0.0961	0.0198	0.0190
Philippines	Herfindahl index	12.9369	0.3379	0.3374
	Standard deviation of portfolio	0.0793	0.0145	0.0145
	Range	3.4073	0.4835	0.4700
	Variance of weights	1.4263	0.0264	0.0264
Thailand	Herfindahl index	4.7126	0.9204	0.7887
	Standard deviation of portfolio	0.0386	0.0225	0.0184
	Range	2.2449	0.9709	0.7090
	Variance of weights	0.5125	0.0912	0.0765

It is evident from Table 4.3 that quantitative measures strictly depend upon the choice of inputs. It is also observe that portfolios under mean variance framework are highly concentrated, mostly counterintuitive, and highly sensitive to the choice of inputs.

From the results, it is evident that the BL-model has relatively low value of Herfindahl index than MV-model and it further decreases under BL-CR models. Similarly the BL-model has relatively low value of standard deviation of portfolio than MV-model and it further decreases under BL-CR models. Further BL-CR model also have a lower value of range and variance of portfolio weights than the simple BL model under all the emerging countries. Therefore study safely conclude that BL-CR model outperform the original BL and MV-model. Also the portfolios under BL-CR model are less concentrated than its competing BL-model in all the emerging countries.

4.1.1. Discussion on empirical findings

From the results of both the criteria regarding the comparison of 12 covariance matrices, the equally weighted portfolio of estimators outperforms the complicated shrinkage covariance estimators. Anyhow the performance of shrinkage estimator improves under GMVP than RMSE for all the emerging Asian countries and even in Thailand the Ledoit and Wolf (2004) outperform the equally weighted portfolio of estimators which are consistent with the findings of Jagannathan and Ma (2003) and Liu and Lin (2010). It is also clear that PCA based covariance matrix has minimum RMSE in all the emerging Asian countries but the drawback of using PCA is that we may lose the economic definition of selected factors. Further it gives exactly opposite result under the criteria of GMVP. As a whole the sample covariance matrix proves poor estimator under both criteria.

The BL-CR model considers country risk as one of the additional risk factor so it gives more reasonable advice to the potential investors for tactical asset allocation than original BL model in all the emerging Asian countries. For the risk aversion investor, if country risk sensitivity coefficient of any asset classes is less than zero then BL-CR model estimates lower return than BL model and if country risk sensitivity coefficient is positive then model estimates higher return than original BL model. These results are opposite for risk preference investors. Further for risk aversion investors, BL-CR propose more weights to the asset classes having positive association with country risk and it proposes less weights to the asset classes having negative association with country risk. These results are opposite for risk preference investors. Asset classes which can resist against the country risk results more weight in the BL-CR model than BL model. Also the optimal weights under BL-CR model are more dependent on the responsiveness of country risk and risk aversion coefficient of the country risk factor. On practical ground investor should consider the country risk for tactical asset allocation and ultimately investor demands more return for bearing this additional risk.

4.2.Empirical evidence from global perspective

Theoretically there should be a positive and linear ex ante relationship between the risk and return of any asset class. Figure 4.1 shows the trade-off between risk and return among the studied global asset classes. It is evident that bond market has lowest return while REIT offering the highest returns. The commodity market observes to be most risky asset class. Apparently, positive and linear relation is observe between risk and return in global studied asset classes.

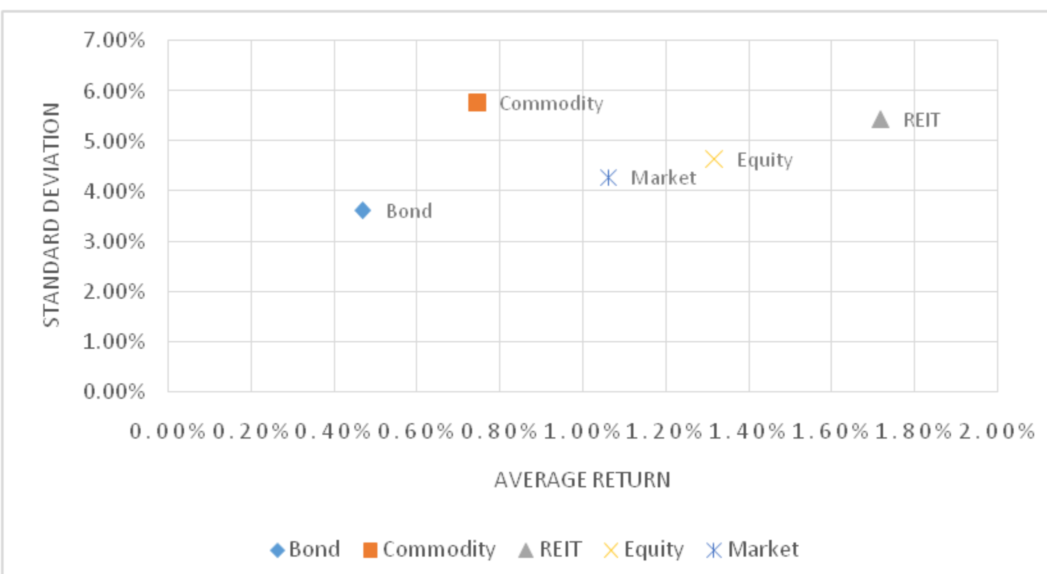

Figure 4.1: Risk-return trade-off in global perspective

Result of descriptive statistics of global considered variables are presented at Table 4.4. Commodity is more risky asset class with a standard deviation of 0.0362 while bond market has lowest standard deviation. The maximum average return is offer by the commodity market and lowest average return is by the bonds. Return series of all the asset classes are negatively skewed expect REIT. From the values of Jarque-Bera, one cannot reject the null hypothesis of normality expect for the bond market. Further the quantitative feature of oil prices and economic policy uncertainty index also presented at Table 4.4.

Table 4.4

Descriptive statistics

	Bond	Commodity	REIT	Equity	Oil	EPU
Mean	0.0047	0.0075	0.0172	0.0132	0.0142	-0.0045
Std. Dev.	0.0362	0.0577	0.0544	0.0464	0.0590	0.1066
Maximum	0.0789	0.1452	0.1915	0.1142	0.1801	0.2363
Minimum	-0.1620	-0.1567	-0.1388	-0.1050	-0.1480	-0.2661
Skewness	-1.6783	-0.4058	0.1239	-0.3400	0.0005	-0.1451
Kurtosis	8.7281	3.8929	4.2953	3.4125	3.4613	3.2974
JB-[P]	117.54[0]	3.882[0.14]	4.638[0.1]	1.687[0.43]	0.5676[0.75]	0.4603[0.79]

The first step for traditional asset allocation is the estimation of future return vector and variance covariance matrix. Study compared the out-of-sample performance of different future return estimation methods with the actual returns on one year window of monthly returns of each asset class. Table 4.6 shows the selected order of ARIMA (p,d,q) for the estimation of future return vector. The same order of ARIMA (p,d,q) has been used for the ARIMA-Reg estimation. Further Table 4.5 shows the mean square prediction error under the autoregressive model up to 5 lags for all the global asset classes with rolling and non-rolling basis. Study selects the order of AR having lowest MSPE and used this selected AR (q) model for the estimation of future return vector.

Table 4.5

Selected order of ARIMA (p,d,q) model

Asset Class	ARIMA (p,d,q) order
Bond	(3,0,4)
Commodity	(4,0,3)
REIT	(3,0,2)
Equity	(4,0,3)

Table 4.6

Forecasted performance of auto-regressive models

	Bond	Commodity	REIT	Equity
RMSP with 'No Rolling' Regression				
AR(1)	0.00049	0.00174	0.00121	0.00098
AR(2)	0.00048	0.00181	0.00119	0.00099
AR(3)	0.00051	0.00191	0.00122	0.00110
AR(4)	0.00048	0.00196	0.00124	0.00113
AR(5)	0.00051	0.00198	0.00123	0.00120
RMSP with 'Rolling' Regression				
AR(1)	0.00049	0.00179	0.00121	0.00104
AR(2)	0.00048	0.00189	0.00117	0.00104
AR(3)	0.00050	0.00205	0.00121	0.00114
AR(4)	0.00048	0.00208	0.00116	0.00115
AR(5)	0.00057	0.00216	0.00126	0.00125

Table 4.7 shows the inputs require for the estimation of future return vector using the Black-Litterman model.

Table 4.7

Inputs to the BL estimation

Asset Classes	Market Capitalization (M)	Matrix Q	Link Matrix P			
			Bond	Commodity	REIT	Equity
Bond	938716.796	0.0057	1	0	0	0
Commodity	3,782,728.79	-0.0015	0	1	0	0
REIT	1,242,490	0.007	0	0	1	0
Equity	41,612,744.07	0.0089	0	0	0	1

Table 4.7 further includes the market capitalization of global asset classes in United State dollar (USD), the matrix Q and link matrix P. Matrix P and matrix Q are used for the calculation of matrix omega which is further used as an input for the estimation of return by using the Black-Litterman model.

Table 4.8 depicts the sample variance-covariance matrix among the global asset classes. From this table the diagonal element shows the variance of asset classes while off-diagonal elements show the covariance of among the global asset classes. Covariance actually measures how two asset classes move together while variance only shows the scatter-ness or dispersion from the mean of an asset class. Maximum covariance has been observe between REIT and commodity i.e. 0.00252 while the minimum covariance observe among the REIT and bond market i.e. 0.00068. Further BL model also requires the average excess return of the market, variance of market and value of lambda which is price of risk. The computed values of average excess return are 0.0106, average risk free rate on monthly basis is 0.00421, variance of market is 0.0018, and lambda i.e. risk aversion coefficient is 5.8061.

Table 4.8

Sample variance-covariance matrix

	Bond	Commodity	REIT	Equity
Bond	0.0013	0.0010	0.0007	0.0007
Commodity	0.0010	0.0033	0.0025	0.0025
REIT	0.0007	0.0025	0.0030	0.0023
Equity	0.0007	0.0025	0.0023	0.0022

Since this study estimates the future return vector by using 7 alternative ways for global asset classes. This research also compares the out-of-sample performance of different future return estimation methods with the actual return on one year window of monthly returns of each asset class. Table 4.9 shows the results of future return vector for global asset classes by using alternative ways of estimations.

Table 4.9

Forecasted return under alternative estimation

Asset Class	Hist	AR(p)	ARIMA (p,d,q)	ARIMA-Reg (p,d,q)	CAPM	IEER	BL
Bond	0.0075	0.0075	-0.0066	-0.0081	0.0054	0.0085	0.0071
Commodity	0.0065	0.0065	-0.0224	-0.0204	0.0136	0.0293	0.0139
REIT	0.0136	0.0136	0.0143	0.0126	0.0123	0.0267	0.0168
Equity	0.0121	0.0121	-0.0031	-0.0057	0.0111	0.0251	0.0170

Study uses 4 different techniques i.e. paired sample t-test, correlation matrix, descriptive statistics and mean square prediction error (MSPE) to evaluate the performance consistencies of alternative future return estimation dimensions to make out of sample comparison among future return estimation techniques with actual return. This comparison is made on a sample of one year on monthly basis.

Table 4.10

Correlation analysis

	Average Correlation	Ave. Significance
Hist	0.0989	0.5085
AR	-0.0465	0.5834
ARIMA	0.0442	0.2248
ARIMA_Reg	0.7956	0.0494
CAPM	0.9045	0.0002

Table 4.10 shows the results of correlation coefficient among estimated return and actual return of the global asset classes. CAPM based estimation has average correlation 0.90 (0.00) with actual return under all considered asset classes in global environment. While the average value of correlation coefficient of future return estimates under different estimation techniques i.e. Hist,

AR, ARIMA and ARIMA-Reg with actual returns are 0.09 (0.50), -0.05 (0.58), 0.04 (0.22) and 0.79 (.04) respectively. CAPM based future return estimation has highest average correlation with actual return and it also attains lowest P-value i.e. 0.0002. Similarly ARIMA-Reg based estimation has second highest average correlation while Hist based estimation has third highest average correlation coefficient. Estimation on the basis of auto-regressive models reveals the weak negative average correlation with the actual return vector.

Table 4.11 reports the descriptive statistics of estimated return and actual returns. CAPM based estimated return vector has average value 0.0109 while average value of actual return has 0.0109. Similarly the average values of future return estimation techniques under Hist, AR, ARIMA and ARIMA-Reg are 0.0075, 0.0107, -0.0731 and 0.0143 respectively. It also reports the average standard deviation under all the future return estimation technique and actual returns. Therefore it can be inferred that CAPM based estimation has almost same pattern of forecasting as that of the actual returns and it also even has the low average standard deviation as compare to the actual average standard deviation.

Table 4.11

Descriptive statistics

	Mean of Mean	Mean Standard Deviation
Actual	0.0109	0.0254
Hist	0.0075	0.0029
AR	0.0107	0.0026
ARIMA	-0.0731	0.0698
ARIMA-Reg	0.0143	0.0261
CAPM	0.0109	0.0230

Results of mean square prediction error (MSPE) under global asset classes have been presented at Table 4.12. On the basis of minimum MSPE, CAPM based estimation has been selected 3 times out of 4 while ARIMA-Reg result minimum MSPE for forecasting the REITS. Along these evaluation dimensions, Paired sample t-test also applies to compare the estimated return vector with actual returns. It suggests that there is no statistical difference between these alternative estimation techniques on the basis of future return estimation.

Therefore on the basis of correlation analysis, descriptive statistics, mean square predication error and paired sample t-test it is reveal that CAPM based future return estimation outperform the other studied ways for future return estimation in global perspective.

Table 4.12

Mean square prediction error

	Hist	AR	ARIMA	ARIMA-Reg	CAPM
Bond	0.00020	0.00028	0.00014	0.00022	0.00009
Commodity	0.00097	0.00090	0.00240	0.00027	0.00020
REIT	0.00087	0.00095	0.00067	0.00016	0.00045
Equity	0.00074	0.00074	0.15546	0.00010	0.00006

Table 4.13 reports the result of excess Sharp ratio, Herfindahl index and variance of weights under minimum variance portfolio, equally weighted portfolio, efficient portfolios and forced diversification. Forced diversification includes the ‘no short’ constraints and constrained diversifications having limits from 10% weights to 35% in one asset class. It also reports the above measures under seven alternative ways for future returns estimates and sample variance-covariance matrices in global perspective.

Under GMVP, AR based future return estimation results higher excess sharp ratio (0.4186) as compare to other competing return estimation technique while BL based future return estimation also produces very close excess sharp ratio as of AR estimation. Since global minimum variance portfolio weights are independent from the choice of future return estimates therefore it only depends upon the variance covariance matrix. Therefore study has the same value of Herfindahl index and variance of weights under each return estimation technique. ARIMA-Reg based estimation produces lowest and even negative excess sharp ratio and ARIMA estimation results second lowest return per unit of risk ratio. Overall there are inconsistencies in term of sharp ratio among the return estimation technique under GMVP. Again there is large variation in excess sharp ratios of mean variance portfolios under alternative ways for future return estimation. From the variance and HI measures of the mean-variance framework it is clear that resultant portfolios are concentrated, counterintuitive and highly sensitive to the choice of input in the shape of future return estimates. Similarly the financial efficiency of the portfolios in the shape of ESR also highly sensitive to the future return estimates. From the output of forced diversification, it is evident that as investor imposes the constraints on the weights then resultant portfolios become less concentrated and ESR also decreases.

Table 4.13

Financial efficiency and diversification measure under s-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.3488	0.4186	0.1453	-0.0290	0.1735	0.3472	0.4112
	HI	2.4172	2.4172	2.4172	2.4172	2.4172	2.4172	2.4172
	Var	0.7224	0.7224	0.7224	0.7224	0.7224	0.7224	0.7224
Equally Weight	ESR	0.2479	0.2327	-0.1044	-0.1266	0.2478	0.5243	0.3210
	HI	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.4800	0.4967	1.0553	-0.9412	0.2478	0.5400	0.4875
	HI	6.6638	6.9619	88.71	1608.84	0.2500	0.7724	6.6055
	Var	2.1379	2.2373	29.4870	536.1980	0.0000	0.1741	2.1185
FD-NS	ESR	0.3156	0.2859	0.2628	0.2308	0.2478	0.5400	0.3687
	HI	0.8261	0.3553	1.0000	1.0000	0.2500	0.7724	0.7159
	Var	0.1920	0.0351	0.2500	0.2500	0.0000	0.1741	0.1553
FD 10%-35%	ESR	0.2845	0.2639	0.0081	-0.0292	0.2478	0.5339	0.3406
	HI	0.2950	0.2839	0.2950	0.2950	0.2500	0.2850	0.2838
	Var	0.0150	0.0113	0.0150	0.0150	0.0000	0.0117	0.0113

Table 4.14 shows the financial efficiency and diversification measure under single index variance covariance matrix in global perspective. Here minimum variance portfolio produces low value of sharp ratio, Herfindahl index and variance of weights as compare to sample covariance and highest value of ESR comes under the AR based return estimation. BL model also produces highest value of ESR under equally weighted portfolios. From the variance and HI measures of the mean-variance framework it is clear that resultant portfolios are concentrated, counterintuitive and highly sensitive to the choice of input in the shape of future return estimates. Similarly the financial efficiency of the portfolios in the shape of ESR also highly sensitive to the future return estimates. From the output of forced diversification, it is evident that forced diversification produces low value of ESR as compare to 'no short' constraints.

Table 4.14

Financial efficiency and diversification measure under si-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.2645	0.3217	0.0516	-0.0567	0.1700	0.3189	0.3174
	HI	1.1669	1.1669	1.1669	1.1669	1.1669	1.1669	1.1669
	Var	0.3056	0.3056	0.3056	0.3056	0.3056	0.3056	0.3056
Equally Weight	ESR	0.2405	0.2257	-0.1012	-0.1228	0.2404	0.5086	0.3113
	HI	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.5006	0.4458	1.2848	-1.1404	0.2434	0.5510	0.4344
	HI	10.72	6.24	1810.0	1085.6	0.356	1.293	4.824
	Var	3.4912	1.9961	603.24	361.78	0.0352	0.3477	1.5245
FD-NS	ESR	0.3153	0.2726	0.2628	0.2308	0.2434	0.5414	0.3652
	HI	0.8070	0.3609	1.0000	1.0000	0.3556	0.6631	1.0000
	Var	0.1857	0.0370	0.2500	0.2500	0.0352	0.1377	0.2500
FD 10%-35%	ESR	0.2777	0.2534	0.0079	-0.0285	0.2430	0.5273	0.3314
	HI	0.2950	0.2844	0.2950	0.2950	0.2943	0.2894	0.2950
	Var	0.0150	0.0115	0.0150	0.0150	0.0148	0.0131	0.0150

Table 4.15 shows the financial efficiency and diversification measure under constant correlation variance covariance matrix in global perspective. From the minimum variance portfolio, it is

quite evident that value of Herfindahl index decreases from 1.1669 to 0.5188 as compare to single index model. But still the sensitivity of mean variance framework with inputs to portfolio optimization has been observed. The value of ESR (0.2987) of BL based estimation under GMVP is higher than the competing ways i.e. CAPM and Hist based estimation. Similarly BL also outperform the competing strategies under mean variance framework in term of financial efficiency and diversification. Again the forced diversification produces low value of ESR and Herfindahl index as compare to ‘no short’ constraints.

Table 4.15

Financial efficiency and diversification measure under cc-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.2200	0.2628	-0.1385	-0.1887	0.2152	0.4106	0.2987
	HI	0.5188	0.5188	0.5188	0.5188	0.5188	0.5188	0.5188
	Var	0.0896	0.0896	0.0896	0.0896	0.0896	0.0896	0.0896
Equally Weight	ESR	0.2764	0.2595	-0.1164	-0.1412	0.2763	0.5847	0.3579
	HI	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.3630	0.3111	-0.6687	-0.6232	0.2894	0.6595	0.3975
	HI	1.3661	0.5778	17.7155	8.5938	0.4872	1.0733	0.6462
	Var	0.3720	0.1093	5.8218	2.7813	0.0791	0.2744	0.1321
FD-NS	ESR	0.3481	0.3005	0.2628	0.2308	0.2879	0.6343	0.3958
	HI	0.5030	0.3613	1.0000	1.0000	0.3419	0.3528	0.4949
	Var	0.0843	0.0371	0.2500	0.2500	0.0306	0.0343	0.0816
FD 10%-35%	ESR	0.3166	0.2880	0.0091	-0.0325	0.2850	0.6185	0.3791
	HI	0.2950	0.2939	0.2950	0.2950	0.2843	0.2841	0.2906
	Var	0.0150	0.0146	0.0150	0.0150	0.0114	0.0114	0.0135

Table 4.16 shows the financial efficiency and diversification measure under shrinkage variance covariance matrix in global perspective. The shrinkage covariance matrix builds upon the optimal shrinkage intensity. As the computation of optimal shrinkage intensity in global perspective applies very low level of shrinkage on the sample covariances so the results are almost same to that of sample covariance matrix.

Table 4.16

Financial efficiency and diversification measure under sh-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.3212	0.3832	0.1079	-0.0368	0.1810	0.3536	0.3797
	HI	1.6845	1.6845	1.6845	1.6845	1.6845	1.6845	1.6845
	Var	0.4782	0.4782	0.4782	0.4782	0.4782	0.4782	0.4782
Equally Weight	ESR	0.2492	0.2340	-0.1049	-0.1273	0.2491	0.5271	0.3227
	HI	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.4526	0.4576	1.0178	-0.9117	0.2491	0.5414	0.4536
	HI	4.9971	5.0134	140.0083	914.8257	0.2501	0.5903	4.5247
	Var	1.5824	1.5878	46.5861	304.8586	0.0000	0.1134	1.4249
FD-NS	ESR	0.3157	0.2871	0.2628	0.2308	0.2491	0.5414	0.3690
	HI	0.8141	0.3543	1.0000	1.0000	0.2501	0.5903	0.7072
	Var	0.1880	0.0348	0.2500	0.2500	0.0000	0.1134	0.1524
FD 10%-35%	ESR	0.2859	0.2653	0.0082	-0.0294	0.2491	0.5367	0.3423
	HI	0.2950	0.2838	0.2950	0.2950	0.2501	0.2848	0.2838
	Var	0.0150	0.0113	0.0150	0.0150	0.0000	0.0116	0.0113

Table 4.17 shows some quantitative feature of weights under varying input to portfolio optimization. It computes the weights by applying the mean variance criteria with 4 different ways of covariance matrix and 7 ways for future return estimation techniques. Under each covariance matrix, result includes number of positive weights, number of negative weights, maximum proportion in asset class and minimum weight in asset class. Constant correlation based covariance outperform the competing ways under GMVP as it has highest number of

positive weights (4), lowest number of negative weights (0), lowest value of standard deviation (0.300), and low value of range of weights (max-min). Further single index based estimation outperform the competing estimation alternatives.

Table 4.17

Descriptive statistics of weights under varying inputs to portfolio optimization

Portfolio characteristics	GMVP	Portfolios under Mean-variance Framework						
		Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
Sample variance covariance matrix								
No of Positive Weights	2	3	2	3	2	4	4	2
No of Negative Weights	2	1	2	1	2	0	0	2
Maximum	1.2	2	2.1	5.3	22.8	0.3	0.9	2.2
Minimum	-0.7	-1.6	-1.5	-7.4	-31.5	0.2	0	-1.2
Standard Deviation	0.850	1.462	1.496	5.430	23.156	0.002	0.417	1.456
Single Index variance covariance matrix								
No of Positive Weights	2	2	3	2	2	4	3	3
No of Negative Weights	2	2	1	2	2	0	1	1
Maximum	0.8	2	1.7	26.3	23.1	0.5	1.1	1.9
Minimum	-0.4	-2.2	-1.7	-31.9	-23.1	0	-0.4	-1.1
Standard Deviation	0.553	1.868	1.413	24.561	19.021	0.188	0.590	1.235
Constant correlation variance covariance matrix								
No of Positive Weights	4	2	3	2	3	3	3	3
No of Negative Weights	0	2	1	2	1	1	1	1
Maximum	0.7	0.8	0.5	2.6	1.6	0.5	0.7	0.7
Minimum	0	-0.3	-0.2	-3	-2.1	-0.2	-0.5	-0.1
Standard Deviation	0.300	0.610	0.331	2.413	1.668	0.281	0.524	0.363
Shrinkage variance covariance matrix								
No of Positive Weights	2	3	2	3	3	4	4	2
No of Negative Weights	2	1	2	1	1	0	0	2
Maximum	0.9	1.7	1.7	7.2	17.8	0.3	0.7	1.9
Minimum	-0.5	-1.4	-1.3	-9.1	-23.6	0.2	0	-1
Standard Deviation	0.691	1.258	1.260	6.825	17.460	0.006	0.337	1.194

4.2.1. Discussion on empirical findings

The above section report the results of mean variance portfolios, minimum variance portfolios, equally weighted portfolios and forced diversification under alternative ways of estimation of inputs to portfolio optimization; future return vector and covariance matrix. It also reports different evaluation dimensions that are computed in this study. On the basis of Herfindahl index, variance of weights, no of positive and negative positions, it is evident that constant correlation based covariance matrix outperform the competing covariance matrices in global framework. Shrinkage base covariance matrix with optimal shrinkage intensity produces higher value of excess sharp ratio under GMVP as compare to other variance covariance matrices. Table E1 and E2 at appendix E shows the detail of excess sharp ratio with varying degree of shrinkage intensity i.e. from 0 to 1. Study uses four dimensions to make out of sample comparison among future return estimation techniques with actual return on a sample of one year window. CAPM based future return estimation outperform the other considered ways for future return estimation in global perspective.

On the basis of financial efficiency and diversification dimensions, the portfolios under mean-variance framework are concentrated, counterintuitive and highly sensitive to the choice of input to portfolio optimization. Relatively BL based future return estimates produces less concentrated portfolios in term of Herfindahl Index. Overall study observes much competitiveness among equally weighted portfolio and mean variance portfolio in term of financial efficiency and diversification in global perspective.

4.3.Empirical evidence from Pakistan

The following figure 4.2 shows the risk-return characteristics of 22 sectors in equity market for the whole sample period in Pakistan. It is obvious from the plot that there is a positive relation between risk and return in equity market in Pakistan. Risk is measure in term of historical standard deviation while return is characterize by historical arithmetic averages of the continuously compounded return for the entire data period. However health care equipment and services, real estate and transportation prove relatively higher risky sectors respectively. Most of the equity sectors offer monthly return about 0.5% with a standard deviation about 10%. Further each point in the Cartesian plane shows the risk-return combination for each consider equity sector in Pakistan.

Figure 4.2

Risk-return trade-off in equity market in Pakistan

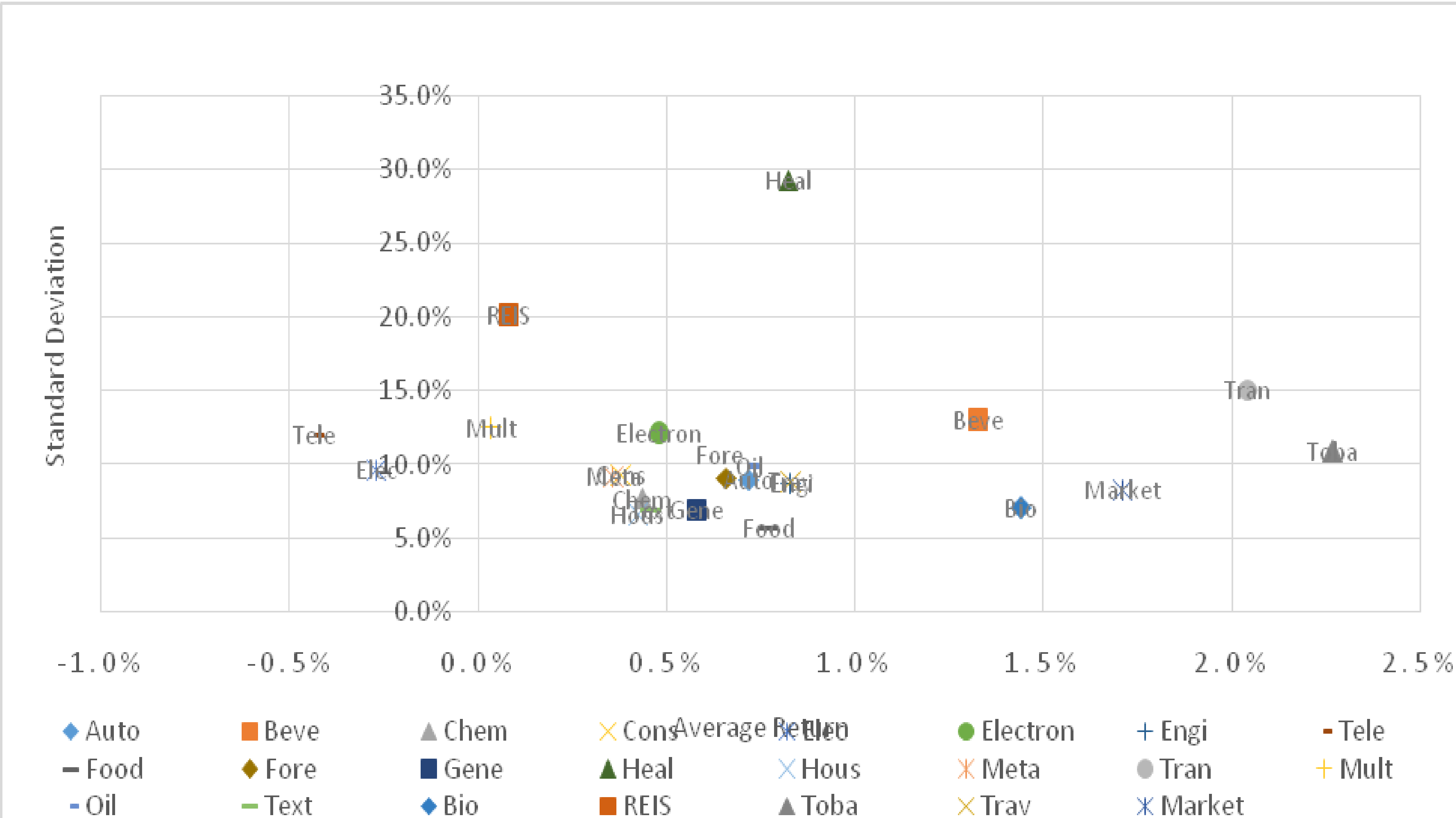

Table 4.18 reports the result of quantitative feature of considered sectors as asset classes and other explanatory variables in Pakistan for the whole sample period which starts from January 2000 to August 2014. Generally there is a positive relationship between return and standard deviation of asset classes. Health care equipment and services sector offer the maximum return in one month i.e.161.38% while maximum loss is again suffer by health care equipment and services i.e. 170.25%.

The value of skewness is generally about zero and returns of most of the asset classes are positively skewed. It depicts frequent small negative returns and extremely bad situation are not as likely. While a long left tail is also observe for few asset classes and depicting frequent small profits and few extreme losses are observe in only few sectors. A leptokurtic distribution is observed for most of the sectors in equity markets, so generally there is fatter tails and lesser risk of extreme outcomes. Few distributions of returns are simultaneously less peaked with thinner tails are observe. JB statistics mostly rejects the null hypothesis of normal distribution. On average about 219 person kill in bomb blast per month in Pakistan and maximum number of casualties was 1198 on October 2009. Similarly Pakistan experienced about 16 casualties in each month from January 2000 to august 2014 in Drone attacks and it reached maximum of 162 casualties on September 2010.

Table 4.18 reports the results of descriptive statistics for each asset class and other explanatory variables in Pakistan for the whole sample period. In this table, SD represents the standard deviation, Max & Min shows the maximum value and minimum value, Skew shows the skewness while Kurt stands for Kurtosis. JB-[P] denotes the Jarque-Bera test statistics while P value is written in parenthesis. Exchange rate is the direct quotation of US dollar (USD against Pakistani rupee (PKR, Rs) in Pakistan and while BSP represents the Brent spot oil prices.

Table 4.18

Results of descriptive statistics in Pakistan

	Mean	SD	Max	Min	Skew	Kurt	JB-[P]
Automobile and Parts	0.0072	0.0895	0.2358	-0.2352	-0.1651	3.0154	0.8 [0.67]
Beverages	0.0133	0.1303	0.4160	-0.4179	0.2002	4.3381	14 [0.00]
Chemicals	0.0144	0.0706	0.1821	-0.1498	0.1701	2.7666	1.2 [0.54]
Construction and Materials	0.0044	0.0768	0.2605	-0.2709	-0.3512	4.8653	29 [0.00]
Electricity	0.0038	0.0930	0.3115	-0.2646	0.2036	4.0177	8.8 [0.01]
Electronic and Electrical Goods	-0.0027	0.0962	0.4172	-0.3584	0.3583	6.1394	76 [0.00]
Engineering	0.0048	0.1214	0.3747	-0.3468	0.4392	4.0288	13 [0.00]
Fixed Line Telecommunication	0.0083	0.0873	0.2575	-0.2555	0.0451	3.4760	1.7 [0.42]
Food Producers	0.0077	0.0566	0.1988	-0.1223	0.4741	3.7109	10 [0.01]
Forestry (Paper and Board)	0.0066	0.0907	0.3458	-0.3744	0.4938	6.4364	93 [0.00]
General Industrials	0.0058	0.0692	0.2094	-0.2133	0.0676	3.6801	3.5 [0.17]
Health Care Equipment and Services	0.0082	0.2920	1.6138	-1.7025	0.0079	17.1825	14 [0.00]
Household Goods	0.0042	0.0662	0.3052	-0.2137	0.3910	5.8410	63 [0.00]
Industrial metals and Mining	0.0036	0.0915	0.3446	-0.3650	0.1796	5.7533	56 [0.00]
Industrial Transportation	0.0003	0.1248	0.3826	-0.5164	0.0592	5.2994	38 [0.00]
Multi utilities (Gas and water)	0.0072	0.0989	0.3945	-0.4453	-0.6128	7.3776	151 [0.0]
Oil and Gas	0.0008	0.2016	1.0960	-1.1296	0.5410	15.4086	1137 [0]
Personal Goods (Textile)	-0.0044	0.1200	0.3538	-0.5560	-0.1477	5.1486	34 [0.00]
Pharma and Bio Tech	0.0046	0.0690	0.2536	-0.1889	0.7050	5.0714	46 [0.00]
Real Estate Investment and Services	0.0227	0.1089	0.4396	-0.2837	0.3410	4.4736	19 [0.00]
Tobacco	0.0204	0.1501	0.7558	-0.4300	0.7818	6.8485	126 [0.0]
Travel and Leisure	0.0083	0.0885	0.2887	-0.4075	-0.0385	6.4703	88 [0.00]
KSE-100 Index	0.0171	0.0828	0.2411	-0.4488	-1.0604	8.5018	255 [0.0]
Exchange Rate	0.0037	0.0123	0.0610	-0.045	1.1739	9.0536	309 [0.0]
T-bills	0.0073	0.0029	0.0115	0.001	-0.7011	2.6289	15 [0.00]
Killed in Bomb Blasts	219.07	228.42	1198.0	0.000	1.331	4.692	73 [0.00]
Killed in Drone	16.44	29.49	162.00	0.000	2.259	8.418	365 [0.0]
BSP	0.0079	0.0877	0.1979	-0.3110	-1.0315	4.7288	53 [0.00]

The first step for traditional asset allocation is the estimation of future return vector and covariance matrix. Study compare the out-of-sample performance of different future return estimation methods with the actual returns on one year window of monthly returns of each asset class in Pakistan. Table 4.19 and Table 4.20 show the mean square prediction error under the autoregressive model up to 5 lags for all the asset classes in Pakistan with rolling and non-rolling basis. Study selects the order of AR having lowest MSPE and use this selected AR (q) model for the estimation of future return vector. If there is any conflict on the order of AR (q) model in rolling and non-rolling auto-regressive models then study prefer the one which minimizes the MSPE with lower order of AR(q).

Table 4.19

Forecasted performance of auto-regressive models (No-rolling)

RMSP	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Automobile and Parts	0.0068	0.0069	0.0069	0.0071	0.0071
Beverages	0.0191	0.0193	0.0195	0.0203	0.0206
Chemicals	0.0051	0.0051	0.0052	0.0053	0.0053
Construction and Materials (Cement)	0.0073	0.0073	0.0073	0.0077	0.0077
Electricity	0.0090	0.0090	0.0091	0.0094	0.0096
Electronic and Electrical Goods	0.0161	0.0164	0.0166	0.0168	0.0170
Engineering	0.0063	0.0062	0.0064	0.0064	0.0064
Fixed Line Telecommunication	0.0166	0.0167	0.0168	0.0173	0.0170
Food Producers	0.0028	0.0028	0.0029	0.0030	0.0030
Forestry (Paper and Board)	0.0077	0.0078	0.0078	0.0083	0.0085
General Industrials	0.0058	0.0057	0.0058	0.0058	0.0058
Health Care Equipment and Services	0.1083	0.1081	0.1066	0.1072	0.1094
Household Goods	0.0041	0.0041	0.0042	0.0042	0.0043
Industrial metals and Mining	0.0064	0.0063	0.0061	0.0062	0.0062
Industrial Transportation	0.0133	0.0133	0.0139	0.0139	0.0140
Multi utilities (Gas and water)	0.0125	0.0126	0.0130	0.0131	0.0133
Oil and Gas	0.0090	0.0093	0.0095	0.0094	0.0096
Personal Goods (Textile)	0.0048	0.0049	0.0051	0.0051	0.0051
Pharma and Bio Tech	0.0050	0.0051	0.0052	0.0053	0.0051
Real Estate Investment and Services	0.0374	0.0419	0.0426	0.0436	0.0444
Tobacco	0.0125	0.0125	0.0127	0.0129	0.0128
Travel and Leisure	0.0062	0.0063	0.0063	0.0065	0.0065

Table 4.19 presents the result of forecast performance in term of mean square prediction error of auto-regressive models on non-rolling samples in Pakistan. The mean square prediction error is minimum at first lag for all the sector expect electricity, engineering, general industrials, health care equipment and services and industrial metals and mining under non-rolling auto-regressive models. Table 20 presents the mean square prediction error of AR (q) model on rolling samples in Pakistan. Again the MSPE is minimum at first lag for all the sectors expect the general industrials, industrial metals and mining and industrial transportation. From Table 4.19 and Table 4.20 it is reveal that for 19 asset classes the auto-regressive models base on rolling and non-rolling sample arrive at the same order of AR (q) while for the electricity, engineering and health care equipment and services both produces different order of AR. The study selects the one which give minimum MSPE at lower order of AR (q).

Table 4.20

Forecast performance of Auto-Regressive models (with-rolling)

RMSP	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Automobile and Parts	0.0070	0.0071	0.0074	0.0074	0.0076
Beverages	0.0201	0.0205	0.0216	0.0222	0.0230
Chemicals	0.0052	0.0053	0.0055	0.0055	0.0057
Construction and Materials (Cement)	0.0075	0.0076	0.0077	0.0080	0.0080
Electricity	0.0092	0.0093	0.0092	0.0096	0.0099
Electronic and Electrical Goods	0.0168	0.0173	0.0176	0.0179	0.0181
Engineering	0.0066	0.0066	0.0067	0.0068	0.0071
Fixed Line Telecommunication	0.0173	0.0177	0.0180	0.0190	0.0191
Food Producers	0.0029	0.0030	0.0032	0.0032	0.0034
Forestry (Paper and Board)	0.0076	0.0077	0.0078	0.0083	0.0085
General Industrials	0.0058	0.0056	0.0058	0.0058	0.0059
Health Care Equipment and Services	0.1139	0.1151	0.1154	0.1173	0.1210
Household Goods	0.0043	0.0043	0.0044	0.0045	0.0047
Industrial metals and Mining	0.0067	0.0066	0.0065	0.0068	0.0067
Industrial Transportation	0.0136	0.0135	0.0146	0.0146	0.0149
Multi utilities (Gas and water)	0.0135	0.0137	0.0140	0.0145	0.0147
Oil and Gas	0.0093	0.0096	0.0100	0.0100	0.0104
Personal Goods (Textile)	0.0051	0.0052	0.0058	0.0058	0.0058
Pharma and Bio Tech	0.0053	0.0055	0.0057	0.0057	0.0056
Real Estate Investment and Services	0.0378	0.0425	0.0434	0.0448	0.0457
Tobacco	0.0127	0.0129	0.0128	0.0130	0.0134
Travel and Leisure	0.0065	0.0066	0.0068	0.0069	0.0070

For the estimation of future return vector study also estimates the ARIMA (p,d,q) and ARIMA-Reg (p,d,q) models for each asset class in Pakistan on the basis of rolling and non-rolling samples. Table 4.21 shows the selected order of ARIMA (p,d,q) for the estimation of future return vector on the basis of minimizing the the AIC and BIC with adjusted R^2 model (the model which minimizes AIC, BIC and has highest adjusted R^2). Further Gauss-Newton algorithm is use to estimate coefficients of ARIMA (p, d, q)model and selectd models also check to ensure that the estimation process converge. As in Pakistani perspective, study also adds KSE-100 index, 6-month government Treasury bill rates and exchange rate of direct quotation of US dollar (USD against Pakistani rupee (PKR, Rs) in Pakistan for the ARIMA-Reg estimation procedure.

Table 4.21

Selected order of ARIMA (p,d,q) model

Asset Classes	ARIMA (p,d,q)
Automobile and Parts	(4,0,4)
Beverages	(3,0,2)
Chemicals	(3,0,3)
Construction and Materials (Cement)	(3,0,2)
Electricity	(3,0,3)
Electronic and Electrical Goods	(3,0,3)
Engineering	(2,0,2)
Fixed Line Telecommunication	(2,0,4)
Food Producers	(3,0,3)
Forestry (Paper and Board)	(3,0,3)
General Industrials	(4,0,3)
Health Care Equipment and Services	(3,0,4)
Household Goods	(4,0,2)
Industrial metals and Mining	(3,0,4)
Industrial Transportation	(3,0,2)
Multi-utilities (Gas and water)	(3,0,2)
Oil and Gas	(4,0,4)
Personal Goods (Textile)	(2,0,3)
Pharma and Bio Tech	(2,0,3)
Real Estate Investment and Services	(3,0,3)
Tobacco	(4,0,4)
Travel and Leisure	(4,0,4)

Table 4.22 contains the detail of market capitalization of each asset class in equity market in Pakistan. Oil and gas sector has highest market capitalization i.e. 37.992% of the total market capitalization of Pakistani equity market. The food producers and chemical sector has second and third highest share on the basis of market capitalization that is 13.036% and 10.665% respectively while real estate investment and services has lowest market capitalization that is 0.022%. The row vector (Q) shows the views of investor about each asset class which are estimated by quantitative model in Pakistan. The absolute views with market capitalization then uses as input for estimation of future return vector by Black-Litterman model. The computed market risk premium is 0.9829%, average monthly risk free is 0.727%, the variance of market is 0.6863% and price of risk (Lambda) is 1.4320.

Table 4.22
Market capitalization and estimated views of asset class

Asset Classes	Market Capitalization	Matrix Q
Automobile and Parts	2.587%	-0.0620
Beverages	0.620%	0.0342
Chemicals	10.665%	-0.0054
Construction and Materials (Cement)	7.042%	0.0207
Electricity	4.036%	0.0234
Electronic and Electrical Goods	0.051%	0.0331
Engineering	0.878%	0.0020
Fixed Line Telecommunication	1.867%	0.0042
Food Producers	13.036%	-0.0259
Forestry (Paper and Board)	0.205%	-0.0878
General Industrials	1.656%	-0.0246
Health Care Equipment and Services	0.127%	-0.0866
Household Goods	0.768%	-0.0085
Industrial metals and Mining	0.444%	-0.0255
Industrial Transportation	1.021%	-0.0021
Multi-utilities (Gas and water)	0.716%	-0.0516
Oil and Gas	37.992%	0.0153
Personal Goods (Textile)	6.251%	-0.0049
Pharma and Bio Tech	2.694%	0.0152
Real Estate Investment and Services	0.022%	0.0244
Tobacco	6.550%	0.0344
Travel and Leisure	0.771%	-0.0429

As the estimation of future return vector is central to portfolio optimization and study estimates the future return vector by 7 alternative ways in Pakistan. Table 4.23 shows the one period estimation by alternative methods for all the considered asset classes in Pakistan. This research also compares the out-of-sample performance of different future return estimation methods with the actual returns on one year window of monthly returns of each asset class.

Table 4.23

Forecasted return under alternative estimation methods

	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
Automobile and Parts	-0.0001	-0.0052	-0.0267	-0.0456	0.0064	0.0140	-0.0240
Beverages	0.0060	-0.0112	0.0673	0.0280	0.0035	0.0087	0.0214
Chemicals	-0.0029	-0.0067	-0.0048	-0.0161	0.0065	0.0137	0.0041
Construction and Materials	-0.0035	-0.0173	0.0122	-0.0133	0.0070	0.0145	0.0176
Electricity	-0.0100	-0.0137	-0.0060	-0.0666	0.0071	0.0142	0.0188
Electronic and Electrical Goods	-0.0025	-0.0066	-0.0100	-0.0589	0.0032	0.0090	0.0210
Engineering	0.0010	-0.0045	-0.0215	-0.0274	0.0051	0.0118	0.0069
Fixed Line Telecommunication	-0.0116	-0.0128	-0.0545	-0.0546	0.0098	0.0177	0.0109
Food Producers	0.0004	-0.0064	0.0016	-0.0027	0.0029	0.0077	-0.0091
Forestry (Paper and Board)	-0.0007	-0.0021	-0.0061	-0.0925	0.0032	0.0089	-0.0394
General Industrials	-0.0015	-0.0171	-0.0303	-0.0396	0.0038	0.0091	-0.0077
Health Care Equipment and Services	0.0010	0.0255	0.0652	-0.0611	0.0070	0.0150	-0.0358
Household Goods	-0.0030	-0.0055	-0.0083	-0.0203	0.0029	0.0078	-0.0003
Industrial metals and Mining	-0.0037	-0.0144	-0.0162	-0.0300	0.0051	0.0115	-0.0070
Industrial Transportation	0.0131	-0.0020	0.0150	-0.0096	0.0086	0.0169	0.0074
Multi-utilities (Gas and water)	-0.0069	0.0345	-0.0004	-0.0211	0.0099	0.0162	-0.0177
Oil and Gas	-0.0001	-0.0042	0.0118	-0.0477	0.0086	0.0182	0.0168
Personal Goods (Textile)	-0.0027	-0.0117	-0.0089	0.0069	0.0034	0.0094	0.0023
Pharma and Bio Tech	0.0071	-0.0062	-0.0327	-0.0278	0.0042	0.0097	0.0124
Real Estate Investment and Services	-0.0065	-0.0329	-0.0500	-0.0470	0.0064	0.0135	0.0189
Tobacco	0.0154	0.0166	-0.0010	-0.0566	0.0024	0.0082	0.0213
Travel and Leisure	0.0010	-0.0080	0.0244	-0.0230	0.0039	0.0075	-0.0177

For the comparison among future return estimation techniques, the correlation analysis, Paired sample t-test, descriptive statistics and mean square prediction error is employed. Table 4.24 presents the average correlation between the actual returns and estimated return by alternative ways for future return estimation techniques. Further it also shows the average significance value in each case. From the Table 4.24, it is evident that CAPM based estimation has highest average correlation with actual returns that is 0.5451 along with lowest average significance value i.e. 0.1399. Estimation with historical averages results on an average negative correlation with actual returns. Auto-regressive estimation has average correlation 0.1621 with actual returns while estimation with ARIMA (p,d,q) almost results no relation with actual returns. Further ARIMA-Reg (p,d,q) results more association with actual returns than simply ARIMA (p,d,q). On the basis of correlation analysis CAPM based estimation shows more association with actual returns in Pakistan.

Table 4.24

Correlation analysis

	Average Correlation	Average Significance
Hist	-0.3062	0.3944
AR	0.1621	0.5162
ARIMA	0.0406	0.6203
ARIMA_Reg	0.1370	0.3682
CAPM	0.5451	0.1399

Table 4.25 reports some quantitative features of estimation of returns under alternative ways and actual returns. It reports the mean of average values and mean of standard deviation of actual returns and other estimated returns on the basis of one year monthly window. It is evident that mean of average of actual return is 0.0156 and the mean of average of forecasted returns under CAPM based estimation that is 0.0076. Similarly the mean of mean under Hist, AR (p), ARIMA

(p,d,q), ARIMA-Reg (p,d,q) are 0.0059, 0.0065, 0.0031 and 0.0001 respectively. Therefore again CAPM based estimation results closer to the actual return in Pakistan.

Table 4.25

Descriptive statistics

	Mean of Mean	Mean Standard Deviation
Actual	0.0156	0.0893
Hist	0.0059	0.0015
AR	0.0065	0.0114
ARIMA	0.0031	0.0140
ARIMA_Reg	0.0001	0.0217
CAPM	0.0076	0.0241

Mean square prediction error (MSPE) which is the average of the square of difference of actual returns with estimated returns under each asset class on one year sample window is computed and results are presented at Table 4.26. The estimation technique with consistent lower mean square prediction error outperformed the other competing return estimation techniques. For 17 out of 22 asset classes, the MSPE under CAPM base estimation is lowest. Any how the ARIMA-Reg (p,d,q) base estimation results minimum MSPE for electricity, health care equipment and services and household goods that is 0.0025, 0.0085, 0.0043 respectively. MSPE is lowest under AR (p) base estimation for engineering sector (0.0036) and it is lowest under Hist base estimation for Tobacco sector (0.0310). Therefore CAPM base estimation outperforms the competing tools for future return estimation on the basis of mean square predication error in Pakistan. Along these evaluation dimensions, Paired sample t-test also applies to compare the estimated return vector with actual returns. It suggests that there is no statistical difference between these alternative estimation techniques on the basis of future return estimation.

Therefore on the basis of correlation analysis, descriptive statistics, mean square predication error and paired sample t-test the CAPM base future return estimation outperforms the other consider ways for future return estimation in Pakistan.

Table 4.26

Mean square prediction error

	Hist	AR	ARIMA	ARIMA-Reg	CAPM
Automobile and Parts	0.0048	0.0042	0.0085	0.0040	0.0030
Beverages	0.0377	0.0382	0.0382	0.0393	0.0361
Chemicals	0.0043	0.0040	0.0045	0.0039	0.0034
Construction and Materials	0.0054	0.0048	0.0052	0.0050	0.0034
Electricity	0.0031	0.0030	0.0028	0.0025	0.0032
Electronic and Electrical Goods	0.0229	0.0227	0.0244	0.0212	0.0190
Engineering	0.0041	0.0036	0.0040	0.0046	0.0040
Fixed Line Telecommunication	0.0095	0.0094	0.0095	0.0100	0.0082
Food Producers	0.0033	0.0032	0.0033	0.0027	0.0025
Forestry (Paper and Board)	0.0083	0.0082	0.0084	0.0098	0.0066
General Industrials	0.0108	0.0107	0.0104	0.0134	0.0090
Health Care Equipment and Services	0.0090	0.0107	0.0139	0.0085	0.0088
Household Goods	0.0053	0.0052	0.0053	0.0043	0.0046
Industrial metals and Mining	0.0026	0.0026	0.0028	0.0049	0.0016
Industrial Transportation	0.0066	0.0053	0.0066	0.0061	0.0036
Multiutilities (Gas and water)	0.0059	0.0065	0.0058	0.0074	0.0034
Oil and Gas	0.0045	0.0043	0.0068	0.0046	0.0023
Personal Goods (Textile)	0.0103	0.0102	0.0101	0.0089	0.0081
Pharma and Bio Tech	0.0082	0.0083	0.0089	0.0078	0.0067
Real Estate Investment and Services	0.0028	0.0026	0.0024	0.0042	0.0022
Tobacco	0.0310	0.0315	0.0316	0.0336	0.0327
Travel and Leisure	0.0016	0.0024	0.0017	0.0023	0.0009

Table 4.27 reports the result of excess Sharp ratio, Herfindahl index and variance of weights under minimum variance portfolio, equally weighted portfolio, efficient portfolios and forced diversification. Forced diversification includes the ‘no short’ constraint and constraint diversifications with limits from 0% weights to 25% in one asset class. It also reports the above measures under seven alternative ways for future returns estimates and sample variance-covariance matrix in Pakistani perspective.

Table 4.27

Financial efficiency and diversification measure under s-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.0196	-0.1132	-0.1416	-0.3973	0.0543	0.1360	-0.0583
	HI	0.4178	0.4178	0.4178	0.4178	0.4178	0.4178	0.4178
	Var	0.0177	0.0177	0.0177	0.0177	0.0177	0.0177	0.0177
Equally Weight	ESR	-0.0072	-0.0768	-0.0547	-0.4980	0.0890	0.1804	0.0145
	HI	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.3558	-0.6659	-1.7353	-1.9979	0.1062	0.1992	-1.2273
	HI	272.543	17.780	117.669	21.187	0.318	0.190	413.862
	Var	12.9761	0.8445	5.6011	1.0067	0.0130	0.0069	19.7056
FD-NS	ESR	0.1587	0.2948	0.5683	0.2151	0.1050	0.1991	0.2828
	HI	0.4097	0.5654	0.4722	1.0000	0.1819	0.1869	0.2052
	Var	0.0173	0.0248	0.0203	0.0455	0.0065	0.0067	0.0076
FD 0%-25%	ESR	0.1348	0.1861	0.4524	0.0755	0.1049	0.1981	0.2813
	HI	0.2195	0.2115	0.1981	0.2500	0.1594	0.1315	0.1819
	Var	0.0083	0.0079	0.0073	0.0097	0.0054	0.0041	0.0065

From table 4.27, minimum variance portfolio suggests a positive value of sharp ratio under Hist, CAPM and IEER based future return estimation techniques while it reveals a negative value of sharp ratio under AR, ARIMA (p,d,q), ARIMA-Reg (p,d,q) and BL based estimation techniques. Since global minimum variance portfolio weights are independent from the choice of future return estimates therefore Table 4.27 reports the same value of Herfindahl index and variance of weights under each return estimation technique. The equally weighted portfolio results relatively high value of sharp ratio under CAPM based estimation (0.0890) and it gives negative ESR under other return estimation techniques. Mean variance efficient portfolios results relatively high value of ESR under historical averages based future estimation.

There is large variation in excess sharp ratio of mean variance portfolios under alternative ways of future return estimation. From the diversification dimension of mean variance framework it is pretty visible that resultant portfolios are concentrated, counterintuitive and highly sensitive to the choice of inputs to portfolio optimization. Similarly the financial efficiency of the portfolios in the shape of ESR also highly sensitive to the future return estimates. Further as investor imposes the constraints on the weights then resultant portfolios become less concentrated and sharp ratio also decreases.

Table 4.28 reports the result of excess Sharp ratio, herfindahl index and variance of weights under minimum variance portfolio, equally weighted portfolio, efficient portfolios and forced diversification. It also reports the above measures under seven alternative ways for future returns estimates and single index variance-covariance matrices in Pakistani perspective.

Table 4.28

Financial efficiency and diversification measure under si-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.0604	-0.2869	-0.2054	-0.5834	0.0649	0.1995	-0.1436
	HI	0.1939	0.1939	0.1939	0.1939	0.1939	0.1939	0.1939
	Var	0.0071	0.0071	0.0071	0.0071	0.0071	0.0071	0.0071
Equally Weight	ESR	-0.0088	-0.0931	-0.0663	-0.6033	0.1078	0.2185	0.0176
	HI	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.3114	-0.6859	-1.3068	-1.5813	0.1135	0.2444	-0.9389
	HI	4.722	0.983	7.828	1.232	0.066	0.096	8.601
	Var	0.2227	0.0446	0.3706	0.0565	0.0010	0.0024	0.4074
FD-NS	ESR	0.1750	0.3013	0.5922	0.2284	0.1135	0.2413	0.3366
	HI	0.3452	0.5043	0.4405	0.5150	0.0665	0.0813	0.1692
	Var	0.0143	0.0219	0.0188	0.0224	0.0010	0.0017	0.0059
FD 0%-25%	ESR	0.1551	0.2066	0.5032	0.0901	0.1135	0.2413	0.3366
	HI	0.2002	0.1986	0.2074	0.2193	0.0665	0.0813	0.1690
	Var	0.0074	0.0073	0.0077	0.0083	0.0010	0.0017	0.0059

Table 4.28 depicts that CAPM based estimation outperform the other future return estimations on the basis of financial efficiency under single index variance covariance matrix. From equally weighted portfolios, the values of sharp ratio of the CAPM and IEER base estimation are (0.1078), (0.2185) respectively. Mean variance efficient portfolios results higher ESR under

historical averages base future estimation. Mean variance portfolio prove sensitive and concentrated to the choice of input and strictly depends on alternative ways for future return estimation and covariance matrix. Further as investor imposes the constraints on the weights then resultant portfolios become less concentrated and sharp ratio also decreases.

Table 4.29 presents the result of financial efficiency and diversification measures under minimum variance portfolio, equally weighted portfolio, efficient portfolios and forced diversification. Forced diversification includes the ‘no short’ constraint and constraint diversifications with limits from 0% weights to 25% in one asset class. It also reports the above measures under seven alternative ways for future returns estimates and constant correlation variance-covariance matrices in Pakistani perspective.

Table 4.29

Financial efficiency and diversification measure under cc-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	-0.0193	-0.2840	-0.4143	-0.3699	0.0835	0.2095	-0.1241
	HI	0.2524	0.2524	0.2524	0.2524	0.2524	0.2524	0.2524
	Var	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099	0.0099
Equally Weight	ESR	-0.0068	-0.0724	-0.0516	-0.4693	0.0838	0.1700	0.0137
	HI	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	-0.3440	-0.6838	-1.4248	-1.6583	0.1594	0.2873	-1.0184
	HI	80.818	1.300	3.086	5.478	0.566	0.278	19.663
	Var	3.8463	0.0597	0.1448	0.2587	0.0248	0.0111	0.9342
FD-NS	ESR	0.1518	0.2805	0.5234	0.2159	0.1250	0.2365	0.2657
	HI	0.4400	0.6864	0.6587	0.7442	0.1680	0.1610	0.1650
	Var	0.0188	0.0305	0.0292	0.0333	0.0058	0.0055	0.0057
FD 0%-25%	ESR	0.1257	0.1618	0.3968	0.0757	0.1250	0.2365	0.2657
	HI	0.2239	0.1936	0.1981	0.2191	0.1679	0.1566	0.1650
	Var	0.0085	0.0071	0.0073	0.0083	0.0058	0.0053	0.0057

From Table 4.29, with constant correlation variance covariance, again the CAPM base return estimation outperforms the other future return estimations in term of financial efficiency of optimal portfolios. From equally weighted portfolios, the excess sharp ratio of the CAPM and IEER base estimation is (0.0838), (0.1700) respectively. Mean variance portfolio again prove sensitive and concentrated to the choice of input and strictly depends on alternative ways for future return estimation and variance covariance matrix.

Table 4.30 presents the result of excess Sharp ratio, Herfindahl index and variance of weights under minimum variance portfolio, equally weighted portfolio, efficient portfolios and force diversification. Force diversification includes the 'no short' constraints and constraint diversifications having limits from 0% weights to 25% in one asset class.

Table 4.30

Financial efficiency and diversification measure under sh-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
GMVP	ESR	0.0195	-0.1136	-0.1424	-0.3987	0.0507	0.1367	-0.0595
	HI	0.4105	0.4105	0.4105	0.4105	0.4105	0.4105	0.4105
	Var	0.0174	0.0174	0.0174	0.0174	0.0174	0.0174	0.0174
Equally Weight	ESR	-0.0073	-0.0770	-0.0549	-0.4992	0.0829	0.1808	0.0146
	HI	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455	0.0455
	Var	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MVP	ESR	0.3541	-0.6635	-1.7242	-1.9882	0.0989	0.1996	-1.2189
	HI	267.780	17.238	112.868	20.417	0.312	0.186	384.260
	Var	12.7493	0.8187	5.3725	0.9701	0.0127	0.0067	18.2959
FD-NS	ESR	0.1588	0.2949	0.5686	0.2151	0.0978	0.1996	0.2833
	HI	0.4092	0.5651	0.4716	1.0000	0.1798	0.1852	0.2043
	Var	0.0173	0.0247	0.0203	0.0455	0.0064	0.0067	0.0076
FD 0%-25%	ESR	0.1350	0.1863	0.4530	0.0756	0.0977	0.1986	0.2818
	HI	0.2194	0.2113	0.1981	0.2500	0.1569	0.1298	0.1849
	Var	0.0083	0.0079	0.0073	0.0097	0.0053	0.0040	0.0066

Table 4.30 also reports the above measures under seven alternative ways for future returns estimates and shrinkage variance-covariance matrices with optimal shrinkage intensity in Pakistani perspective. The computed value of lambda i.e. of price of risk which we use for calculation of future return estimates under BL model in Pakistan is 1.4320. For the computation of shrinkage variance covariance matrix, optimal shrinkage intensity in Pakistan is 0.006. As the optimal shrinkage intensity is relatively low and it put very low level of shrinkages on the sample covariance matrix. Therefore the results under this variance covariance matrix are almost the same as of the sample variance covariance matrix.

Table 4.31 compares the variance-covariance matrices under the minimum variance portfolios in full sample period in Pakistan. It uses the diversification and financial efficiency as comparison criteria. It only reports the number of asset classes with positive weights, number of asset classes with negative positions, standard deviation of weights, maximum and minimum value of weights under each covariance matrix. There are relatively greater numbers of positive weights and the concentration of weights on each asset class also decreases under single index covariance matrix as compare to sample covariance matrix. Also the number of short position increases under constant correlation base covariance matrix than the single index covariance matrix.

Table 4.31

Descriptive statistics of weights under varying inputs to optimization

Portfolio characteristics	GMVP	Portfolios under Mean-variance Framework						
		Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL
Sample variance covariance matrix								
No of Positive Weights	12	11	11	11	12	15	22	7
No of Negative Weights	10	11	11	11	10	7	0	15
Maximum	0.46	10.34	1.98	3.97	2.2	0.33	0.38	11.5
Minimum	-0.19	-5.35	-2.34	-4.4	-1.85	-0.21	0	-7.53
Standard Deviation	0.13	3.6	0.92	2.37	1	0.11	0.08	4.44
Single Index variance covariance matrix								
No of Positive Weights	15	11	15	11	15	22	20	9
No of Negative Weights	7	11	7	11	7	0	2	13
Maximum	0.26	1.31	0.45	1.26	0.61	0.12	0.14	1.32
Minimum	-0.08	-0.86	-0.67	-1	-0.34	0	-0.06	-0.86
Standard Deviation	0.08	0.47	0.21	0.61	0.24	0.03	0.05	0.64
Constant variance covariance matrix								
No of Positive Weights	14	12	13	13	12	13	15	9
No of Negative Weights	8	10	9	9	10	9	7	13
Maximum	0.33	3.32	0.59	0.86	1.34	0.35	0.26	2.46
Minimum	-0.06	-4.89	-0.6	-0.67	-0.98	-0.21	-0.1	-1.21
Standard Deviation	0.1	1.96	0.24	0.38	0.51	0.16	0.11	0.97
Shrinkage variance covariance matrix								
No of Positive Weights	12	11	11	11	12	15	22	7
No of Negative Weights	10	11	11	11	10	7	0	15
Maximum	0.46	10.27	1.96	3.91	2.17	0.32	0.37	11.02
Minimum	-0.18	-5.3	-2.31	-4.28	-1.82	-0.2	0	-7.25
Standard Deviation	0.13	3.57	0.9	2.32	0.98	0.11	0.08	4.28

A detail of ESR under GMVP with varying degree of shrinkage intensity i.e. from 0 to 1 are presented at Table F1 at appendix F. The above describe patterns in term of number of positive, negative weights and maximum, minimum values of invested proportion in each asset class are same under different future return estimates. Further this generalization is independent on the choice of calculation of weights i.e. minimum variance portfolio weights or efficient portfolio weights. Therefore minimum short positions and relatively small range of minimum variance portfolio weight is observed under single index covariance matrix.

Any portfolio optimizer requires two main inputs i.e. future return vector and covariance matrix. The above section describes the output under mean variance portfolios, minimum variance portfolios, equally weighted portfolios and force diversification by changing different ways for estimation of future return vector and covariance matrix. It also reports the sharp ratio, Herfindahl index of weights, variance of weights of asset classes, number of positive and negative positions in asset allocation framework in Pakistan. The values of Herfindahl index under minimum variance portfolios under s-cm, si-cm, cc-cm and sh-cm are 0.4178, 0.1939, 0.2524 and 0.4105 respectively. Therefore single index covariance outperforms the other consider covariance matrices on the basis of diversification in full sample period in Pakistan. For financial efficiency there is no consistent pattern observe in ESR under minimum variance portfolio of all the return estimation techniques. Anyhow, si-cm and cc-cm mostly produces the competing ESR under different future return estimates in Pakistan.

From the number of positive and negative weights to asset classes, maximum and minimum value of weights, other diversification measures i.e. variance and HI measures of the mean-variance framework, it is reveal that resultant portfolios are concentrated, mostly counterintuitive, results more short positions and highly sensitive to the choice of input in Pakistan. Similarly the financial efficiency of these portfolios in the shape of ESR also highly sensitive to the input estimates. Overall CAPM base future return estimates and si-cm outperforms the competing alternatives, therefore on the basis of these two inputs, the ESR under mean variance framework, GMVP and EWP are 0.1135, 0.0649 and 0.1078 respectively.

4.4. Empirical evidence from Pakistan (Sub-sample)

In Pakistani perspective, study applies the asset allocation strategies in comparison with naïvely diversified portfolio in two subsamples: first sub-sample starts from January 2000 to August 2007 and second starts from September 2007 to August 2014. The following are the results under the sub-sample starts from September 2007 to August 2014.

Table G1 to G4 in appendix G presents the mean square prediction error under auto-regressive models up to lag 5 on rolling and non-rolling regression, selected order of ARIMA (p,d,q) for future estimations and mean square prediction error of the return estimation method on one year out of sample window. Mostly the sig (2-tailed) value of paired samples t-test is greater than 0.05 show that one can not reject the null hypothesis that there is no statistical difference between the actual return and estimated future return vector.

The average correlation coefficient (average p-values in parentheses) of actual returns with estimated returns under Hist, AR, ARIMA, ARIMA-Reg and CAPM base estimations are -0.3062 (0.39), 0.1621 (0.52), 0.0406 (0.62), 0.1370 (0.37) and 0.5451 (0.14) respectively. It is quite visible that CAPM base estimation has relatively strong positive relations with actual returns in subsample period for Pakistan. The out-of-sample average value of actual return for all the asset classes is 0.0156. The comparable average values for forecasted returns under Hist, AR, ARIMA, ARIMA-Reg and CAPM base estimation are 0.0156, 0.0059, 0.0065, 0.0031, 0.0001 and 0.0076 respectively. Again CAPM base estimation relatively outperforms the other future return estimation techniques in subsample. The mean square prediction error of CAPM base estimation is lowest for nineteen asset classes out of 22 asset classes. Again in align with the full sample period, CAPM based future return estimation outperforms the other competing models on the basis of Paired sample t-test, descriptive statistics, correlation matrix and MSPE. Table 4.32 shows the forecasted return vector for all the asset classes in Pakistan.

As the estimation of future return vector is central to portfolio optimization Table 4.32 shows the estimated return under alternative method of forecasting for all the consider asset classes in Pakistan. This research also compares the out-of-sample performance of different future return estimation methods with the actual returns on one year window of monthly returns of each asset class.

Table 4.32

Forecasted return under alternative estimation methods

	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
Automobile and Parts	-0.0087	-0.0069	-0.0099	-0.0224	0.0020	0.0010	-0.0021	0.0264
Beverages	0.0057	0.0049	0.0519	0.0395	0.0014	0.0009	0.0282	0.0734
Chemicals	-0.0070	-0.0080	-0.0126	0.0044	0.0021	0.0010	0.0016	0.0154
Construction and Materials	-0.0139	-0.0141	-0.0179	0.0299	0.0022	0.0011	0.0133	0.0375
Electricity	-0.0098	-0.0111	-0.0087	0.0134	0.0021	0.0010	-0.0206	-0.0060
Electronic &Electrical Goods	-0.0129	-0.0134	-0.0158	-0.0079	0.0010	0.0006	0.0178	0.0455
Engineering	-0.0097	-0.0109	-0.0625	-0.0017	0.0013	0.0007	0.0164	0.0411
Fixed Line Telecommunication	-0.0212	-0.0223	0.0126	-0.0435	0.0034	0.0015	-0.0022	0.0200
Food Producers	-0.0042	-0.0045	-0.0062	-0.0116	0.0009	0.0005	0.0009	0.0124
Forestry (Paper and Board)	-0.0136	-0.0134	-0.0092	-0.1123	0.0010	0.0006	0.0003	0.0190
General Industrials	-0.0099	-0.0112	0.0289	0.0188	0.0014	0.0007	0.0006	0.0237
Health Care Equipment	0.0008	-0.0083	-0.0022	-0.0004	0.0018	0.0008	0.0166	0.0340
Household Goods	-0.0095	-0.0093	-0.0002	-0.0198	0.0009	0.0005	0.0127	0.0324
Industrial metals and Mining	-0.0189	-0.0193	-0.0307	-0.0228	0.0019	0.0009	-0.0046	0.0141
Industrial Transportation	-0.0021	-0.0011	0.0016	-0.0057	0.0023	0.0011	0.0005	0.0215
Multiutilities (Gas and water)	-0.0150	-0.0179	0.0189	0.0504	0.0024	0.0010	0.0017	0.0174
Oil and Gas	-0.0098	-0.0108	0.0077	-0.0483	0.0031	0.0014	0.0105	0.0279
Personal Goods (Textile)	-0.0046	-0.0049	0.0109	0.0230	0.0010	0.0006	0.0052	0.0280
Pharma and Bio Tech	0.0002	-0.0010	0.0507	-0.0015	0.0013	0.0007	0.0154	0.0400
Real Estate Investment	-0.0250	-0.0273	-0.0412	-0.0524	0.0032	0.0014	0.0278	0.0566
Tobacco	0.0131	0.0087	0.0254	0.0970	0.0005	0.0005	0.0015	0.0374
Travel and Leisure	-0.0081	-0.0120	-0.0213	-0.0371	0.0009	0.0004	-0.0012	0.0149

Table 4.33 demonstrates the result of correlation analysis among the out of sample future return vector by return estimated techniques in Pakistan. The correlation coefficient between Hist and AR (p) base estimation is 0.96 which suggests that there is strong positive association between these return estimation techniques. Similarly there exist positive moderate level of association between Hist and ARIMA (p,d,q) base estimation, Hist and ARIMA-Reg (p,d,q) base estimation, AR (p) and ARIMA (p,d,q) base estimation, AR (p) and ARIMA-Reg (p,d,q) base estimation, ARIMA (p,d,q) and ARIMA-Reg (p,d,q) base estimation, CAPM and IEER base estimation. While there also exists some evidence of negative correlation among return estimation alternatives. The correlation coefficient between BL and BL-CR model is 0.924 which shows that there is strong positive association between the original BL model and BL-CR model.

Table 4.33

Correlation matrix among estimated return vectors

	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
Hist	1.000	0.966	0.543	0.581	-0.571	-0.481	0.081	0.217
AR	0.966	1.000	0.551	0.537	-0.581	-0.467	0.040	0.204
ARIMA	0.543	0.551	1.000	0.399	-0.128	-0.060	0.037	0.176
ARIMA-Reg	0.581	0.537	0.399	1.000	-0.254	-0.191	0.014	0.174
CAPM	-0.571	-0.581	-0.128	-0.254	1.000	0.977	-0.003	-0.067
IEER	-0.481	-0.467	-0.060	-0.191	0.977	1.000	0.051	0.043
BL	0.081	0.040	0.037	0.014	-0.003	0.051	1.000	0.924
BL-CR	0.217	0.204	0.176	0.174	-0.067	0.043	0.924	1.000

Table 4.34 presents the results of financial efficiency and diversification measures under minimum variance portfolio, equally weighted portfolios and mean variance portfolios with alternative ways of future return estimation and sample covariance matrix. The Black-Litterman model under country risk produces higher excess sharp ratio under minimum variance portfolios and equally weighted portfolios that is 0.1571 and 0.4515 respectively. The BL-CR model

outperform the BL model under mean variance portfolios as it has higher excess sharp ratio, low value of Herfindahl index and less variance of weights. The calculated value of risk aversion coefficient to the market (Price of risk) is 0.1128 the risk aversion coefficient of the country risk is 1.84. Further these risk aversion coefficient are the inputs for the estimation of asset allocation strategies under alternative ways of variance covariance matrices.

Table 4.34

Financial efficiency and diversification measure under s-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
GMVP	ESR	-0.145	-0.192	-0.260	0.024	0.017	0.009	-0.049	0.157
	HI	0.590	0.590	0.590	0.590	0.590	0.590	0.590	0.590
	Var	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026
Equally weighted	ESR	-0.131	-0.153	-0.021	-0.079	0.027	0.014	0.100	0.452
	HI	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
	Var	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
MVP	ESR	-0.601	-0.592	-2.471	3.443	0.034	0.015	-0.788	1.149
	HI	19.64	10.48	90.970	22379	0.360	0.220	205	48.940
	Var	0.933	0.497	4.329	1065	0.015	0.008	9.805	2.328

Table 4.35 reports the results of financial efficiency and diversification measures under minimum variance portfolios, equally weighted portfolios and mean variance portfolios with alternative ways of future return estimation and single index variance covariance matrix. Again the Black-Litterman model under country risk produces higher excess sharp ratio under minimum variance portfolios and equally weighted portfolios that is 0.8043 and 0.5274 respectively. When we compare the BL model with BL-CR model under mean variance portfolios then BL-CR model outperform the BL model as it has higher excess sharp ratio (0.6098), low value of Herfindahl index (0.3406) and less variance of weights (0.0141). The portfolios under BL-CR model also outperform the CAPM base estimation in subsample in Pakistan. The excess sharp ratio also increases in single index variance covariance matrix than

sample covariance matrix under minimum variance portfolios of BL-CR model, while the value of Herfindahl index decreases from 0.5896 to 0.2174.

Table 4.35

Financial efficiency and diversification measure under si-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
GMVP	ESR	-0.148	-0.177	-0.108	0.083	0.017	0.013	0.138	0.804
	HI	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
	Var	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008
Equally weight	ESR	-0.154	-0.179	-0.025	-0.093	0.032	0.016	0.117	0.527
	HI	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
	Var	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
MVP	ESR	-0.410	-0.418	-1.625	2.349	0.035	0.017	0.610	1.169
	HI	1.399	0.993	51.537	166.205	0.097	0.077	3.735	0.341
	Var	0.065	0.045	2.452	7.912	0.002	0.002	0.176	0.014

The results of financial efficiency and diversification measures under asset allocation strategies with 8 alternative ways of future return estimation and constant correlation base covariance matrix are presented at Table 4.36.

Table 4.36

Financial efficiency and diversification measure under cc-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
GMVP	ESR	-0.185	-0.187	-0.126	-0.354	0.027	0.015	0.047	0.473
	HI	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225
	Var	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009
Equally Weight	ESR	-0.121	-0.141	-0.020	-0.073	0.025	0.012	0.092	0.415
	HI	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
	Var	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
MVP	ESR	-0.479	-0.466	-1.740	-2.533	0.051	0.023	0.605	0.998
	HI	1.430	1.300	63.904	13.774	0.605	0.346	44.554	0.985
	Var	0.066	0.060	3.041	0.654	0.027	0.014	2.120	0.045

In Table 4.36, the Black-Litterman model under country risk produces highest excess sharp ratio (0.4733) under minimum variance portfolios and equally weighted portfolios (0.4149). Also BL under country risk outperform the original BL model under mean variance portfolios as it has higher excess sharp ratio (0.9976), low value of Herfindahl index (0.9849) and less variance of weights (0.0447). Further the mean variance portfolios are highly sensitive and counterintuitive under alternative future return estimation techniques. Sharp ratio of minimum variance portfolios of BL-CR model also decreases under constant correlation base covariance matrix than the single index base covariance matrix and further the value of Herfindahl index increases under constant correlation base covariance matrix.

Table 4.37 depicts the results of financial efficiency and diversification measures under minimum variance portfolios, equally weighted portfolios and mean variance portfolios with 8 alternative ways of future return estimation and shrinkage variance covariance matrix. The value of optimal shrinkage intensity is 0.0554 therefore it suggest a relatively low level of shrinkage to the sample variance covariance matrix therefore the output under sample variance covariance and shrinkage variance covariance matrix are almost same.

Table 4.37

Financial efficiency and diversification measure under sh-vcm

	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
GMVP	ESR	-0.145	-0.188	-0.201	-0.038	0.018	0.010	-0.022	0.233
	HI	0.427	0.427	0.427	0.427	0.427	0.427	0.427	0.427
	Var	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018
Equally Weight	ESR	-0.135	-0.157	-0.022	-0.081	0.028	0.014	0.102	0.462
	HI	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
	Var	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
MVP	ESR	-0.550	-0.543	-2.274	-3.153	0.034	0.016	-0.733	1.077
	HI	11.630	6.530	105.490	5808.140	0.278	0.161	706.450	13.670
	Var	0.552	0.309	5.021	276.576	0.011	0.006	33.639	0.649

Table 4.37 shows that the Black-Litterman model under country risk produces higher excess sharp ratio (0.2330) under minimum variance portfolios and equally weighted portfolios (0.4621). Anyhow here the excess sharp ratio increases as compare to excess sharp ratio under sample variance covariance i.e. 0.1571. Also the BL-CR model under mean variance portfolios

outperform the BL model as it has higher excess sharp ratio (1.0769), low value of Herfindahl index (13.67) and less variance of weights. The mean variance portfolios are highly sensitive and counterintuitive under alternative future return estimation techniques. Table G6 to Table G9 in appendix G includes further details about asset allocation under shrinkage base covariance matrix in Pakistan.

Table 4.38 demonstrates the excess sharp ratio, Herfindahl index and variance of weights under force diversification. It further shows these measures under four variance covariance matrices: sample variance covariance, single index variance covariance, constant correlation variance covariance and shrinkage method of variance covariance matrix. Each variance covariance matrix contains two type of constraints that is ‘no short’ and constraint diversification (maximum weight in one asset class is 25% and minimum weight is 0%). From the Table 4.38 it is reveal that BL-CR model outperform the original BL model on the basis of financial efficiency and diversification under alternative variance covariance matrices.

Table 4.38

Sharp and Herfindahl measures under constrained portfolios

Covariances	Constraints	Measure	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
s-vcm	No Short	ESR	0.0876	0.0835	0.7069	1.0029	0.0334	0.0152	0.2649	0.6785
		HI	0.4097	0.7070	0.9866	0.4937	0.3392	0.2110	0.2272	0.1669
		Var	0.0173	0.0315	0.0448	0.0213	0.0140	0.0079	0.0087	0.0058
	FD 0%-25%	ESR	0.0484	0.0373	0.7069	0.8024	0.0327	0.0151	0.2644	0.6785
		HI	0.2195	0.2500	0.9866	0.2188	0.1365	0.1280	0.2149	0.1642
		Var	0.0083	0.0097	0.0448	0.0083	0.0043	0.0039	0.0081	0.0057
si-vcm	No Short	ESR	0.0948	0.0863	0.7930	1.0065	0.0345	0.0173	0.3475	0.9373
		HI	0.3452	0.6491	0.4202	0.3544	0.0966	0.0731	0.2043	0.1547
		Var	0.0143	0.0287	0.0178	0.0147	0.0024	0.0013	0.0076	0.0052
	FD 0%-25%	ESR	0.0533	0.0443	0.6871	0.8786	0.0345	0.0172	0.3475	0.9373
		HI	0.2002	0.2500	0.2096	0.1906	0.0966	0.0713	0.2042	0.1548
		Var	0.0074	0.0097	0.0078	0.0069	0.0024	0.0012	0.0076	0.0052
cc-vcm	No Short	ESR	0.0913	0.0806	0.7245	0.9093	0.0400	0.0184	0.2808	0.7249
		HI	0.4400	0.9014	0.6548	0.7943	0.1983	0.1772	0.2573	0.2305
		Var	0.0188	0.0408	0.0290	0.0357	0.0073	0.0063	0.0101	0.0088
	FD 0%-25%	ESR	0.0372	0.0362	0.5452	0.7219	0.0399	0.0184	0.2790	0.7224
		HI	0.2239	0.2500	0.2153	0.2139	0.1817	0.1727	0.2147	0.2100
		Var	0.0085	0.0097	0.0081	0.0080	0.0065	0.0061	0.0081	0.0078
sh-vcm	No Short	ESR	0.1239	0.0836	0.7072	1.0047	0.0339	0.0155	0.2696	0.6916
		HI	0.7486	0.7018	0.9357	0.4788	0.2762	0.1629	0.2277	0.1637
		Var	0.0335	0.0313	0.0424	0.0206	0.0110	0.0056	0.0087	0.0056
	FD 0%-25%	ESR	0.0602	0.0378	0.5591	0.8121	0.0334	0.0154	0.2690	0.6916
		HI	0.2241	0.2500	0.2166	0.2176	0.1314	0.1220	0.2128	0.1626
		Var	0.0085	0.0097	0.0081	0.0082	0.0041	0.0036	0.0080	0.0056

Table 4.39 reports the result under different variance covariance matrices in sub sample period in Pakistan. It reports the number of positive and negative positions in asset classes, standard deviation of weights, maximum and minimum value of weights under each variance covariance matrix and future return estimation tools. The value of optimal shrinkage intensity in the second subsample in Pakistan is 0.0554. The calculated value of risk aversion coefficient to the market (price of risk) is 0.1128. The computed value of risk aversion coefficient of the country risk is 1.84. From the descriptive statistics of weights under alternative variance covariance matrices and future return estimation techniques it is evident that BL-CR model base estimation outperforms the original BL base estimation under financial efficiency and diversification dimensions. As it results more number of positive weights, less concentration, low range and relatively less standard deviation of optimal weights in Pakistan.

Tabe 4.39

Descriptive statistics of weights under varying inputs to optimization

Portfolio characteristics	GMVP	Efficient portfolios under mean-variance framework (MVP)							
		Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
Sample variance covariance matrix									
No of Positive Weights	13.00	10.00	9.00	11.00	13.00	13.00	22.00	11.00	14.00
No of Negative Weights	9.00	12.00	13.00	11.00	9.00	9.00	0.00	11.00	8.00
Maximum	0.56	3.12	2.30	5.65	58.45	0.53	0.42	5.63	2.57
Minimum	-0.23	-2.02	-1.32	-2.87	-72.26	-0.09	0.00	-6.11	-4.01
Standard Deviation	0.16	0.97	0.70	2.08	32.64	0.12	0.09	3.13	1.53
Single index variance covariance matrix									
No of Positive Weights	17.00	14.00	14.00	12.00	12.00	22.00	20.00	11.00	15.00
No of Negative Weights	5.00	8.00	8.00	10.00	10.00	0.00	2.00	11.00	7.00
Maximum	0.26	0.69	0.57	3.58	4.51	0.23	0.14	0.74	0.26
Minimum	-0.22	-0.48	-0.41	-3.87	-6.77	0.00	-0.02	-0.94	-0.19
Standard Deviation	0.09	0.25	0.21	1.57	2.81	0.05	0.04	0.42	0.12
Constant correlation variance covariance matrix									
No of Positive Weights	14.00	15.00	14.00	11.00	10.00	11.00	14.00	9.00	12.00
No of Negative Weights	8.00	7.00	8.00	11.00	12.00	11.00	8.00	13.00	10.00
Maximum	0.35	0.65	0.62	4.51	2.56	0.37	0.29	2.78	0.49
Minimum	-0.05	-0.60	-0.50	-4.65	-1.47	-0.23	-0.15	-3.28	-0.41
Standard Deviation	0.09	0.26	0.24	1.74	0.81	0.16	0.12	1.46	0.21
Shrinkage variance covariance matrix									
No of Positive Weights	13.00	11.00	11.00	11.00	9.00	14.00	18.00	13.00	13.00
No of Negative Weights	9.00	11.00	11.00	11.00	13.00	8.00	4.00	9.00	9.00
Maximum	0.49	2.40	1.80	6.17	39.32	0.44	0.34	10.97	1.42
Minimum	-0.17	-1.28	-0.85	-3.72	-29.95	-0.10	-0.01	-10.02	-1.81
Standard Deviation	0.13	0.74	0.56	2.24	16.63	0.11	0.07	5.80	0.81

The above section reports the asset allocation framework in Pakistani equity markets. It gives detailed comparison among asset allocation strategies, input to portfolio optimizers and different portfolio evaluation dimensions in sub-samples in Pakistan. Financial efficiency and diversification dimensions are used to evaluate the asset allocation strategies. It is clear that single index base covariance matrix outperform the competing ways for estimation of covariance matrices under minimum variance portfolios in sub-sample period in Pakistan. Overall mean variance portfolios are very sensitive and counterintuitive in sub sample period in Pakistan. Further equally weighted portfolios are very competitive with mean variance portfolio in Pakistan. The results of second subsample period are similar to that of the first sub samples in equity market in Pakistan. Table G1 to Table G9 in appendix G includes further details about asset allocation in Pakistan.

Since this study also introduce the country risk into the BL model and uses this augmented model i.e. BL-CR for future return estimates. As a first step study estimates the individual sensitivities of each asset class with the country risk (See Table G5 in appendix G). The regression coefficient shows that all asset classes have positive gradient with country risk. Beverages (0.025), Tobacco (0.019) and real estate investment and services (0.016) have respectively higher regression coefficient while regression coefficient for food producer is lowest that is 0.0062.

The estimated value of risk aversion coefficient of the country risk is 1.84 and further all the asset classes have positive gradient so country risk impacts the expected return. For the risk aversion investor i.e. λ_j greater than zero, if country risk sensitivity coefficient is greater than zero then expected return will be higher than the original expected return. The BL-CR model outperform the original BL model as it has less short position, more number of positive weight, less variance, low value of Herfindahl index and high value of ESR. Therefore BL-CR model outperform the BL model on the basis of both financial efficiency and diversification under all alternatives of estimation of variance covariance matrices.

Chapter 5

5. SUMMARY, CONCLUSION AND RECOMMENDATIONS

There are two main streams to deal with traditional asset allocation strategies i.e. theoretical approach and implementation approach. These approaches are the prime focus of this study. Portfolio optimization is based upon two fundamental ingredients i.e. estimation of return vector and covariance matrix. Investors need to estimate these fundamental ingredients because these are unknown to investors. This study compare the 12 covariance matrix under four categories i.e conventional methods, factor models, portfolio of estimators and shrinkage approach. It includes the diagonal method, sample matrix, constant correlation model, single index matrix, principal component analysis based model, portfolio of sample matrix & diagonal matrix, portfolio of sample matrix & single index matrix, portfolio of sample matrix & constant correlation matrix, portfolio of sample matrix, single index matrix & constant correlation matrix, portfolio of sample matrix, single index matrix, constant correlation matrix & diagonal, shrinkage to the diagonal matrix, shrinkage to the single index model and shrinkage to the constant correlation model.

This study also compares the performance of 7 alternative ways for estimation of return vectors which includes historical average estimation, auto-regressive estimation, auto-regressive integrated moving average based estimation, auto-regressive integrated moving average-regression based estimation, capital asset pricing model based estimation, implied equilibrium excess return and Black-Litterman model. Study also develops portfolios based on mean-variance optimization, minimum variance portfolios and constrains portfolios. These strategies for asset allocation then compare with naïve diversification. The comparison of asset allocation strategies are base upon the financial efficiency and diversification dimensions i.e. sharp ratio, Herfindahl index of weights, variance of weights of asset classes, number of asset classes with positive weights, number of asset classes with negative positions, standard deviation of weights, maximum and minimum value of weights. For comparison purpose within the covariance matrices study uses the root mean square error and risk of minimum variance portfolios. Further study uses four different criteria to evaluate the performance consistencies of alternative future return vector estimation techniques i.e. paired sample t-test, correlation matrix, descriptive statistics and mean square prediction error.

This study first time investigates the ‘country risk’ as unprice risk factor in the Black-Litterman model and proposes augmented Black-Litterman formula (BL-CR) for the estimation of expected return vector. Study also develops a market model for the estimation of investor’s views as input into the Black-Litterman model.

The broader contest of this study is on the asset allocation framework especially on Markowitz portfolio selection, constrained optimization, naïve diversification, minimum variance portfolios, Black-Litterman framework, augmented Black-Litterman model (propose), alternative ways for the estimation of inputs (variance covariance matrix and expected return vector) to portfolio optimization and different ways for comparison of asset allocation strategies and inputs to portfolio optimizations. Summarizing, this study provides a comprehensive framework to investors for asset allocation in Pakistan as well in global perspective.

For asset allocation framework, study considers various asset classes as investment opportunities in emerging Asian countries, in global environment and in Pakistan. Data set consists of time series data associated with each asset class. In emerging Asian countries study identifies ten (10) asset classes, in global environment study consider four (4) asset classes and in Pakistan it focus on the twenty two (22) asset classes. This study selects 5 emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand and uses the global industry classification standard

(GICS) develop by MSCI and Standard & Poor's in 1999 which consists of 10 sectors. In global perspective study considers the equity, bond, commodity and real estate as the broad asset. In Pakistani perspective, study considers the equally weighted indices of twenty two (22) sectors in equity market of Pakistan and these indices are treated as asset classes.

Study reveal that factor models as a group outperform the competing covariance estimators (conventional methods, Portfolio of estimators, and shrinkage approaches) in all the emerging countries. Also it is clear that equally weighted portfolio of covariance estimators outperforms the complicated shrinkage covariance estimators in all the selected emerging Asian countries and hence 'simpler is better'. As a whole, sample covariance matrix proves poor estimator under root mean square error and risk profile of minimum variance portfolios. It further reveals that Black-Litterman model under country risk performs better than the original Black-Litterman and mean-variance model in all the emerging Asian countries. The BL-CR model considers country risk as one of the additional risk factor so it gives more reasonable advice to the potential investors for tactical asset allocation than original BL model in emerging Asian countries.

On the basis of Herfindahl index, variance of weights, number of positive and negative positions, it is evident that constant correlation base covariance matrix outperform the competing covariance matrices in global framework. CAPM base future return estimation outperform the other consider ways for future return estimation in global perspective. On the basis of financial efficiency and diversification dimensions, the portfolios under mean-variance framework are concentrated, counterintuitive and highly sensitive to the choice of input to portfolio optimization. Relatively BL base future return estimates produces less concentrated portfolios in global perspective. Overall study observes much competitiveness among equally weighted portfolio and mean variance portfolio in term of financial efficiency and diversification in global perspective.

In Pakistani perspective, single index covariance matrix outperforms the other consider covariance matrices on the basis of diversification. For financial efficiency there is no consistent pattern observe under minimum variance portfolio of all the return estimation techniques. From the number of positive and negative weights to asset classes, maximum and minimum value of weights, other diversification measures of the mean-variance framework, it is reveal that resultant portfolios are concentrated, mostly counterintuitive, results more short positions and highly sensitive to the choice of input in Pakistan. Similarly the financial efficiency of these portfolios in the shape of Sharp ratio also highly sensitive to the input estimates. Overall CAPM base future return estimates and single index covariance matrix outperforms the competing alternatives in Pakistan. Further these results are also consistent within the sub-samples in Pakistan.

When study compare the financial efficiency and diversification of mean variance portfolios with minimum variance portfolios and equally weighted portfolios then, on an average, equally weighted portfolios results a competitive strategy with competing portfolios in Pakistan. This result is also consistent with global environment. Therefore study also recommends that investment managers and academia should at least consider the naïve diversification as a first obvious benchmark in comparisons with other asset allocation strategies in Pakistan and global perspective.

Study also draws the following major empirical conclusions about the Black-Litterman model under country risk. For the risk aversion investor, when country risk sensitivity coefficient is greater than zero then expected return is higher than the original expected return. These results are opposite for risk preference investors. Further for risk aversion investors, BL-CR propose more weights to the asset classes having positive association with country risk and it proposes less weights to the asset classes having negative association with country risk. These results are opposite for risk preference investors. Further BL-CR outperform the original BL model as it has less short positions, more number of positive weights, less variance, low value of Herfindahl index and high value of excess sharp ratio. Asset classes which can resist against the country risk results more weight in the BL-CR model than BL model. Also the optimal weights under BL-CR model are more dependent on the responsiveness of country risk and risk aversion coefficient of the country risk factor. The Black-Litterman model under country risk is also more appropriate than original BL model to disperse country risk in Pakistan. Therefore BL-CR model also outperform the BL model on the basis of both financial efficiency and diversification under all alternatives of estimation of variance covariance matrices. On practical ground investor should consider the country risk for tactical asset allocation and ultimately investor demands more return for bearing this additional risk.

The study on the subject of framework for global and domestic asset allocation has following recommendations:

1. Study suggests that investor should use the factor models for estimation of variance covariance matrix as an input to portfolio optimization in emerging Asian countries i.e. India, Indonesia, Pakistan, Philippines & Thailand. It also recommends that equally weighted portfolio of covariance estimators is a better way than the complicated shrinkage covariance estimators in all the selected emerging Asian countries and hence 'simpler is better'.
2. Capital asset pricing model base future return estimation technique outperforms the other competing models for estimation of return vector in Pakistan as well in global perspective. Therefore study provide guidelines that investor should use capital asset pricing model base return estimation as input to portfolio optimization.
3. In consistent with emerging Asian countries, single index covariance matrix consistently outperform the other consider covariance matrices in Pakistan. Therefore study also recommends the use of single index variance covariance for asset allocation in Pakistan.
4. Resultant portfolios base on mean-variance framework are concentrated, mostly counterintuitive, result more short positions and highly sensitive to the choice of input. Similarly the financial efficiency of these portfolios also highly sensitive to the input estimates. So investor should be vigilant about this enigma in traditional asset allocation technique.
5. There is no consistent pattern observe in excess sharp ratio under minimum variance portfolio among the inputs to optimization in Pakistan while shrinkage based variance

covariance with optimal shrinkage intensity produce higher value of excess sharp ratio in global perspective.

6. Study also recommends that investment managers and academia should at least consider the naïve diversification as first obvious benchmark in comparison with other asset allocation strategies. This is due to the fact that the financial efficiency and diversification of mean variance portfolios in comparison with equally weighted portfolios, on an average, results competitive strategy with competing portfolios.
7. The augmented Black-Litterman model outperform the original model as it has relatively less short positions, more number of positive weight, less variance, low value of Herfindahl index and high value of excess sharp ratio in all emerging Asian countries as well in Pakistan. Therefore BL-CR model is more appropriate on mathematical and empirical grounds in asset allocation than original model to disperse country risk. Study also recommends that investment managers and academia should consider the Black-Litterman model under country risk for tactical asset allocation decisions in emerging Asian countries and in Pakistan.

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APPENDIX

Appendix A: Basics about asset allocation

1. Return and Risk

Generally individuals invest some amount in one or more asset classes for the purpose of some reward relative to the initial invested amount. The ‘return’ or rate of return can be describe mathematically as follows

$$Return = \frac{Profit}{Invested\ Amount} = \frac{End\ Value - Starting\ Value + Cash\ Flows}{Starting\ Value}$$

The above definition of return purely related to the ‘past’ but under asset allocation investors are primarily concerned about the future behavior of assets that is future return. Markowitz (1959) expressed the expected value of returns as future return. As investor desires to predict the future return of asset class (s) so the term expected return describe the forecasted return.

There are two main characteristics of assets under modern portfolio theory i.e. expected return and risk. Intuitively, risk is the uncertainty about future unfavorable outcome or probability of loss. But it is quite challenging job to define the above definition of risk in mathematically rigorous fashion. Markowitz (1952) describe the variance of return as measure of risk. Under investment philosophy, variance of return basically measures the deviation in return around the expected return. Therefore in this definition of variance as a measure of risk, it includes both the positive and negative deviation around the expected return. But theoretically investors should only considers unfavorable outcome i.e. negative deviation as risk. Therefore this traditional measure of risk looks counter-intuitive. If we take the square-root of variance then it results standard deviation called volatility in financial literature.

1.1. **Expected Return**

Modern portfolio theory heavy based upon on expected value, variance and covariance. Briefly these are described as follows.

1.1.1. Expected Value (Return)

The sample expected value is the average value of the sample. If $x_i, i = 1,2,3, \dots, n$ $then$

$$\boldsymbol{E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

1.2.Variability in Return (Risk)

Markowitz (1952) used the variance as a measure of risk. Variance is actually the variation around the expected value. It can be defined as

$$Var(X) = \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}[(x_i - E(X)]^2$$

The square root of Var (X) is called standard deviation and considered more intuitive than variance as a measure of variability i.e.

$$Std = \sigma = \sqrt{Var}$$

1.3.Correlation Coefficient

Correlation coefficient measure the degree of association among two random variable. More specifically it measure the strength and direction of relationship among variables. Its values varies from +1 to -1. That is there may be perfect positive relation to perfect negative relationship. If correlation coefficient is zero implies there is no relationship among the studied variables. It can be calculated as follows

$$Corr = \rho_{i,j} = \frac{\sum_{t=1}^{N}[X_{i,t} - E(X_i)]\,[X_{j,t} - E(X_j)]}{\sqrt{\sum_{t=1}^{N}[X_{i,t} - E(X_i)]^2 \sum_{t=1}^{N}[X_{j,t} - E(X_j)]^2}}$$

The Covariance between the rates of return for assets class ‘i’ & ‘j’ can be calculated by applying the following formula:

$$Cov\,(x_i, x_j) = \frac{1}{N}\sum_{t=1}^{N}[X_{i,t} - E(X_i)]\,[X_{j,t} - E(X_j)]$$

The relationship between covariance and correlation coefficient can be described with the following formula.

$$Corr = \rho_{i,j} = \frac{Cov(x_i, x_j)}{\sigma_i \sigma_j}$$

If two variables are independent of each other, then covariance among these is zero which implies that there is zero correlation.

1.4.Some Basic Properties

The followings are few properties related to the expected value, variance and covariance. Here *c* and *d* are real valued scalar and *Y*, *Z* denoted the random variables then

$$cov(Y, Z) = cov(Z, Y) \dots\dots 1$$

$$cov(Z, Z) = var(Z) \dots\dots 2$$

$$cov(cY, dZ) = cd\ cov(Z, Y) \dots\dots 3$$

$$E(cY + d) = cE(Y) \dots\dots 4$$

$$E\left(\sum_{j=1}^{n} Y_j\right) = \sum_{j=1}^{n} E(Y_j) \dots\dots 5$$

$$var(cY + dZ) = c^2 var(Y) + d^2 var(Z) + 2\text{cd cov(Y, Z)} \dots\dots 6$$

$$var\left(\sum_{j=1}^{n} Y_j\right) = cov\left(\sum_{i=1}^{n}\sum_{j=1}^{n} Y_i Y_j\right) \dots\dots 7$$

$$var(AZ) = A\ var(Z)\acute{A}\ where\ A\ is\ real\ valued\ matrix \dots\dots 8$$

1.5.Risk and Return of Portfolio

Portfolio is basically the combination of two or more asset classes. For *n* asset class portfolio, we can write a vector$\boldsymbol{W} \in \mathbb{R}^n\ s.t. \sum_{j=1}^{n} w_j = 1$.

Proposition 1.

The expected return and variance of portfolio are $\acute{\boldsymbol{w}}E(r)$*and* $\acute{\boldsymbol{w}}\Sigma\boldsymbol{w}$ *respectively.*

Proof. Let r_i be the return on asset class i, $E(\mathrm{r}_i)$ be the expected return, σ_i^2 denotes the variance, σ_{ij}be the covariance between i and j and $\Sigma \in \mathbb{R}^{nxn}$ be the variance covariance matrix. Since the covariance is symmetrical so variance covariance matrix is symmetrical. For the return of portfolio

$$E(\mathrm{r}_p) = E\left(\sum_{j=1}^{n} w_j r_j\right) = \sum_{j=1}^{n} E(w_j r_j)$$

$$E(\mathrm{r}_p) = \sum_{j=1}^{n} w_j \, E(r_j) = \acute{\boldsymbol{w}} \, E(r)$$

For variance of portfolio we can write as follows

$$var(\mathrm{r}_p) = var\left(\sum_{j=1}^{n} w_j r_j\right) = \sum_{k=1}^{n}\sum_{j=1}^{n} cov(\, w_k r_k, w_j r_j)$$

$$= \sum_{k=1}^{n}\sum_{j=1}^{n} w_k w_j \, cov(\, r_k, r_j) = \sum_{k=1}^{n}\sum_{j=1}^{n} w_k w_j \sigma_{kj}$$

$$var(\mathrm{r}_p) = \acute{\boldsymbol{w}}\Sigma\boldsymbol{w}$$

The above derived working of expected return (ẃE(r)) and variance of portfolio (ẃΣw) based upon some basic properties as describe in section 1.4.

1.6.Rationale for Diversification

Diversification considered most desirable way for asset allocation. It is generally believe that it reduces risk. To understand rationale for diversification consider two situations. Under first instance, there are *n* assets having same expected returns and variances i.e. E(r) & σ^2 with$\sigma_{kj} = 0 \; \forall k \neq j$. Further also give equal weights to all the assets in the portfolio i.e. $w_i = \frac{1}{n} \; for \; each \; i$.

$$E(\mathrm{r}_p) = \sum_{j=1}^{n} w_j\, E(r_j) = \frac{1}{n}\sum_{j=1}^{n} E(r_j) = E(r)$$

This implies that estimated return of portfolio is equal to estimated return of individual assets in the portfolio. Therefore in this case the estimated return is independent of total number of asset classes that constitutes the portfolios. Postulates

For variance of portfolios

$$var(\mathrm{r}_p) = \sum_{k=1}^{n}\sum_{j=1}^{n} w_k w_j \sigma_{kj} = \frac{1}{n^2}\sum_{k=1}^{n}\sum_{j=1}^{n} \sigma_{kj}$$

$$var(\mathrm{r}_p) = \frac{1}{n^2}\sum_{k=1}^{n} \sigma^2 = \frac{1}{n}\sigma^2$$

For the last equation, as oppose to return of portfolio the variance of portfolio directly depends upon the number of assets in the portfolio. Therefore the variance of portfolio decreases as number of assets in the portfolio increases and it approaches zero when number of assets in the portfolio approaches infinity.

$$\lim_{n\to\infty} var(\mathrm{r}_p) = \lim_{n\to\infty}\left(\frac{1}{n^2}\sum_{k=1}^{n}\sigma^2\right) = \lim_{n\to\infty}\left(\frac{1}{n}\sigma^2\right) = 0$$

It can be safely said that investor can reap the benefit of diversification and even the variance of portfolio reached to zero as number of uncorrelated assets in the portfolio approaches to infinity.

Under second instance, assume there are *n* assets having same expected returns and variances i.e. E(r) & σ^2 with$\sigma_{kj} = a\sigma^2\ \forall k \neq j, a \in \mathbb{R}$. Further also give equal weights to all the assets in the portfolio i.e. $w_i = \frac{1}{n}\ for\ each\ i,$ then expected return of portfolio again equal to

$$E(\mathrm{r}_p) = \sum_{j=1}^{n} w_j\, E(r_j) = \frac{1}{n}\sum_{j=1}^{n} E(r_j) = E(r)$$

But the variance of portfolio is as follow:

$$var(\mathrm{r}_p) = \sum_{k=1}^{n}\sum_{j=1}^{n} w_k w_j \sigma_{kj} = \frac{1}{n^2}\sum_{k=1}^{n}\sum_{j=1}^{n} \sigma_{kj}$$

$$var(\mathrm{r}_p) = \frac{1}{n^2}\left(\sum_{k=j}\sigma_{kj} + \sum_{k\neq j}\sigma_{kj}\right) = \frac{1}{n^2}\left(\sum_{k=j}\sigma^2 + \sum_{k\neq j} a\sigma^2\right)$$

$$var(\mathrm{r}_p) = \frac{1}{n^2}(n\sigma^2 + (n^2 - n)a\sigma^2) = \frac{\sigma^2}{n} + \left(1 - \frac{1}{n}\right)a\sigma^2$$

$$var(\mathrm{r}_p) = \frac{(1-a)\sigma^2}{n} + a\sigma^2$$

$$\lim_{n\to\infty} var(\mathrm{r}_p) = \lim_{n\to\infty}\left(\frac{(1-a)\sigma^2}{n} + a\sigma^2\right) = a\sigma^2$$

Therefore when assets in the portfolio are mutually correlated i.e. $\sigma_{kj} = a\sigma^2 \ \forall k \neq j, a \in \mathbb{R}$ then variance of portfolio cannot be zero by the increase of asset classes in the portfolio. Generally diversification reduced the risk somehow at the cost of lowering expected return. But this trade-off between reduction in risk by diversification and lowering the return of portfolio required considerable attentions.

Appendix B: Mathematical working of the BL formula under country risk

As the views of investors are not sure, therefore the goal of the BL model under country risk could be written as:

$$\underbrace{Min}_{E(R)}\,[\widehat{E(R)}-\widehat{\Pi}]'.(\tau\Sigma)^{-1}.\left[\widehat{E(R)}-\widehat{\Pi}\right]\; s.t. P\widehat{E(R)}=\widehat{Q}+\varepsilon_1$$

Given $\widehat{\Pi}=\widehat{E(R)}+\varepsilon_2\,,\widehat{Q}=P\widehat{E(R)}+\varepsilon_3$

Now set $Y=\binom{\widehat{\Pi}}{V}, X=\binom{I}{\acute{P}}, W=\begin{pmatrix}\tau\Sigma & 0\\ 0 & \Omega\end{pmatrix}\quad \mu\sim N\,(0,W)\quad then$

$$Y=X.\widehat{E(R)}+\varepsilon_4$$

With the help of generalized least squares method, we have

$\widehat{E(R)}=(\acute{X}W^{-1}X)^{-1}\acute{X}W^{-1}Y$ and by substituting the above we have

$$\widehat{E(R)}=\left[\binom{I}{\acute{P}}'\begin{pmatrix}\tau\Sigma & 0\\ 0 & \Omega\end{pmatrix}\binom{I}{\acute{P}}\right]^{-1}.\left[\binom{I}{\acute{P}}'\begin{pmatrix}\tau\Sigma & 0\\ 0 & \Omega\end{pmatrix}^{-1}\binom{\widehat{\Pi}}{V}\right]$$

$$=\left[\left((\tau\Sigma)^{-1}\quad \acute{P}\Omega^{-1}\right)\binom{I}{\acute{P}}\right]^{-1}.\left[\left((\tau\Sigma)^{-1}\quad \acute{P}\Omega^{-1}\right)\binom{\widehat{\Pi}}{V}\right]$$

$$=[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}P)]^{-1}.\left[(\tau\Sigma)^{-1}\widehat{\Pi}+\acute{P}\Omega^{-1}\widehat{Q}\right]$$

$$=[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}\mathrm{P}]^{-1}\left[(\tau\Sigma)^{-1}(\Pi+\sum_{j=1}^{n}\lambda_j\beta_j)+\acute{P}\Omega^{-1}(\mathrm{Q}+\sum_{j=1}^{n}\lambda_j P\beta_j)\right]$$

$$=[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}\mathrm{P}]^{-1}\left[(\tau\Sigma)^{-1}\Pi+(\tau\Sigma)^{-1}\sum_{j=1}^{n}\lambda_j\beta_j)+\acute{P}\Omega^{-1}\acute{\mathrm{Q}}+P\Omega^{-1}\sum_{j=1}^{n}\lambda_j P\beta_j)\right]$$

$$=[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}\mathrm{P}]^{-1}.\left\{(\tau\Sigma)^{-1}\Pi+\acute{P}\Omega^{-1}\mathrm{Q}\right\}+[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}\mathrm{P}]^{-1}.[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}P]\left[\sum_{j=1}^{n}\lambda_j\beta_j\right]$$

$$\widehat{E(R)}=[(\tau\Sigma)^{-1}+\acute{P}\Omega^{-1}\mathrm{P}]^{-1}\left[(\tau\Sigma)^{-1}\Pi+\acute{P}\Omega^{-1}\mathrm{Q}\right]+\sum_{j=1}^{n}\lambda_j\beta_j$$

Appendix C: Credit rating to emerging Asian countries

Credit rating is the credit worthiness of the sovereign entity. Credit rating of any county normally use by the investors who are willing to invest in a country and generally it depicts overall investing environment of a country. Normally it depends upon different factors. Table B1 shows the detail of the rating assign to emerging Asian countries (India, Indonesia, Pakistan, Philippines & Thailand) by three big credit rating agencies. It includes Moody's Investor Service, Standard & Poor's Financial Services LLC (S&P) and Fitch Ratings. These ratings are collected from the Bloomberg L.P. in October 2015.

Table C1

Credit Rating to emerging Asian countries

	Moody's	S&P	Fitch
India	Baa3	BBB-	BBB-
Indonesia	Baa3	BB+	BBB-
Pakistan	B3	B-	B
Philippines	Baa2	BBB	BBB-
Thailand	Baa1	BBB+	BBB+

Appendix D: Details of the results of minimum variance portfolios

Study also analyzes the covariance matrix on the basis of minimum variance portfolios. With the help of weight of GMVP, study compute and note the out-of-sample return of the GMVP. This series of return of portfolios leads towards the computation of average mean of GGMVP which are presented at the below Table D1.

Table D1

Mean of mean of the minimum variance portfolios (GMVP)

Covariance matrix	India	Indonesia	Pakistan	Philippines	Thailand
Sample matrix	0.0046	0.0034	0.0099	0.0011	0.0040
Constant correlation model	0.0050	0.0000	0.0089	0.0007	0.0034
Single Index matrix	0.0031	0.0021	0.0089	0.0006	0.0038
PCA method	0.0088	0.0617	0.0084	0.0031	0.0085
Portfolio of sample & diagonal	0.0026	0.0009	0.0082	0.0008	0.0035
Portfolio of Sample & constant correlation	0.0047	0.0013	0.0094	0.0011	0.0038
Portfolio of sample & single index	0.0035	0.0025	0.0093	0.0008	0.0039
Portfolio of Sample, single index & correlation	0.0039	0.0017	0.0092	0.0009	0.0038
Portfolio of sample, single index, correlation & D	0.0030	0.0010	0.0085	0.0008	0.0035
Shrinkage to diagonal	0.0046	0.0033	0.0099	0.0011	0.0040
Shrinkage to single index	0.0046	0.0031	0.0097	0.0010	0.0040
Shrinkage to constant correlation	0.0047	0.0025	0.0095	0.0011	0.0039

Appendix E: Varying degree of shrinkage intensity in global perspective

Table E1 describes the comparison on the basis of varying the shrinkage intensity lambda. From equation 17, if $\lambda = 0$ then, there is no shrinkage to $\hat{\Sigma}_s$ and if $\lambda = 1$, it results full shrinkage to variance covariance matrix. Since GMVP is the only portfolio on the efficient frontier that depends upon covariance matrix and is independent from the choice of future return estimates. Therefore the Herfindahl index and variance of weights are independent from the future return estimation techniques.

Table E1

Comparisons on the basis of varying degree of shrinkage intensity

	ESR under GMVP								
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	HI	Var
0	0.3488	0.4186	0.1453	-0.0290	0.1735	0.3472	0.4112	2.4172	0.7224
0.1	0.2709	0.3196	0.0067	-0.0835	0.2014	0.3828	0.3312	0.7866	0.1789
0.2	0.2581	0.3013	-0.0505	-0.1214	0.2173	0.4147	0.3258	0.5965	0.1155
0.3	0.2588	0.2978	-0.0844	-0.1464	0.2322	0.4472	0.3331	0.5138	0.0879
0.4	0.2658	0.3009	-0.1076	-0.1646	0.2477	0.4821	0.3462	0.4593	0.0698
0.5	0.2774	0.3085	-0.1251	-0.1791	0.2647	0.5205	0.3637	0.4170	0.0557
0.6	0.2930	0.3198	-0.1395	-0.1917	0.2838	0.5637	0.3854	0.3820	0.0440
0.7	0.3127	0.3350	-0.1524	-0.2037	0.3059	0.6133	0.4118	0.3522	0.0341
0.8	0.3373	0.3548	-0.1649	-0.2163	0.3321	0.6717	0.4441	0.3268	0.0256
0.9	0.3684	0.3805	-0.1783	-0.2306	0.3641	0.7425	0.4844	0.3054	0.0185
1	0.4087	0.4146	-0.1939	-0.2481	0.4046	0.8319	0.5362	0.2877	0.0126

Table E2

Comparisons on the basis of varying degree of shrinkage intensity

	ESR under Equal Weight								
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	HI	Var
0	0.2479	0.2327	-0.1044	-0.1266	0.2478	0.5243	0.3210	0.250	0.000
0.1	0.2566	0.2409	-0.1080	-0.1310	0.2565	0.5427	0.3322	0.250	0.000
0.2	0.2662	0.2499	-0.1121	-0.1360	0.2662	0.5632	0.3447	0.250	0.000
0.3	0.2771	0.2601	-0.1167	-0.1415	0.2770	0.5861	0.3588	0.250	0.000
0.4	0.2894	0.2717	-0.1218	-0.1478	0.2893	0.6121	0.3747	0.250	0.000
0.5	0.3035	0.2849	-0.1278	-0.1550	0.3034	0.6419	0.3929	0.250	0.000
0.6	0.3198	0.3003	-0.1347	-0.1633	0.3197	0.6765	0.4141	0.250	0.000
0.7	0.3392	0.3184	-0.1428	-0.1732	0.3391	0.7174	0.4391	0.250	0.000
0.8	0.3625	0.3403	-0.1526	-0.1851	0.3624	0.7667	0.4693	0.250	0.000
0.9	0.3914	0.3675	-0.1648	-0.1999	0.3913	0.8279	0.5068	0.250	0.000
1	0.4286	0.4023	-0.1804	-0.2189	0.4284	0.9065	0.5549	0.250	0.000

See the note of Table E1

Appendix F: Varying degree of shrinkage intensity in Pakistani perspective

Table F1 describes the comparison on the basis of varying the shrinkage intensity lambda in Pakistan. From equation 17, if $\lambda = 0$ then, there is no shrinkage to $\hat{\Sigma}_s$ and if $\lambda = 1$, it results full shrinkage to variance covariance matrix. Since GMVP is the only portfolio on the efficient frontier that depends upon covariance matrix and is independent from the choice of future return estimates. Therefore the Herfindahl index and variance of weights are independent from the future return estimation techniques.

Table F1

Comparisons on the basis of varying degree of shrinkage in Pakistan

	ESR under GMVP								
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	HI	Var
0	0.0196	-0.1132	-0.1416	-0.3973	0.0505	0.1360	-0.0583	0.4178	0.0177
0.1	0.0180	-0.1202	-0.1510	-0.4235	0.0553	0.1477	-0.0707	0.3212	0.0131
0.2	0.0168	-0.1276	-0.1570	-0.4550	0.0609	0.1604	-0.0747	0.2566	0.0101
0.3	0.0155	-0.1359	-0.1628	-0.4916	0.0674	0.1747	-0.0748	0.2089	0.0078
0.4	0.0138	-0.1456	-0.1700	-0.5342	0.0751	0.1914	-0.0725	0.1719	0.0060
0.5	0.0117	-0.1575	-0.1795	-0.5852	0.0845	0.2116	-0.0686	0.1424	0.0046
0.6	0.0087	-0.1724	-0.1929	-0.6488	0.0965	0.2372	-0.0630	0.1185	0.0035
0.7	0.0046	-0.1920	-0.2123	-0.7326	0.1124	0.2712	-0.0557	0.0989	0.0025
0.8	-0.0015	-0.2200	-0.2421	-0.8521	0.1353	0.3200	-0.0460	0.0830	0.0018
0.9	-0.0113	-0.2654	-0.2933	-1.0462	0.1722	0.3992	-0.0326	0.0703	0.0012
1	-0.0305	-0.3617	-0.4059	-1.4587	0.2494	0.5663	-0.0110	0.0610	0.0007

Table F2

Comparisons on the basis of varying degree of shrinkage in Pakistan

	ESR under Equal Weight								
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	HI	Var
0	-0.0072	-0.0768	-0.0547	-0.4980	0.0827	0.1804	0.0145	0.045	0.000
0.1	-0.0076	-0.0803	-0.0572	-0.5206	0.0864	0.1886	0.0152	0.045	0.000
0.2	-0.0079	-0.0843	-0.0601	-0.5466	0.0907	0.1980	0.0160	0.045	0.000
0.3	-0.0084	-0.0890	-0.0634	-0.5770	0.0958	0.2090	0.0169	0.045	0.000
0.4	-0.0089	-0.0946	-0.0674	-0.6131	0.1018	0.2221	0.0179	0.045	0.000
0.5	-0.0095	-0.1014	-0.0722	-0.6569	0.1091	0.2380	0.0192	0.045	0.000
0.6	-0.0103	-0.1098	-0.0782	-0.7117	0.1181	0.2578	0.0208	0.045	0.000
0.7	-0.0114	-0.1208	-0.0861	-0.7830	0.1300	0.2836	0.0229	0.045	0.000
0.8	-0.0128	-0.1360	-0.0969	-0.8811	0.1463	0.3192	0.0257	0.045	0.000
0.9	-0.0149	-0.1587	-0.1131	-1.0288	0.1708	0.3727	0.0301	0.045	0.000
1	-0.0187	-0.1989	-0.1417	-1.2892	0.2140	0.4670	0.0377	0.045	0.000

See the note of Table F1

Appendix G: Inputs to portfolio optimization in Pakistan (Subsample)

Table G1 presents the mean square prediction error of AR (5) model with no-rolling samples in Pakistan. Study selects the order of AR having lowest MSPE and use this selected AR (q) model for the estimation of future return vector. If there is any conflict on the order of AR (q) model in rolling and non-rolling auto-regressive models then study prefer the one which minimizes the MSPE with lower order of AR (q).

Table G1

Forecast performance of auto-regressive models under MSPR (No Rolling)

	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Automobile and Parts	0.0059	0.0061	0.0063	0.0063	0.0066
Beverages	0.0267	0.0278	0.0291	0.0299	0.0308
Chemicals	0.0045	0.0046	0.0047	0.0048	0.0050
Construction and Materials	0.0069	0.0070	0.0072	0.0068	0.0067
Electricity	0.0097	0.0099	0.0106	0.0112	0.0117
Electronic and Electrical Goods	0.0191	0.0194	0.0201	0.0209	0.0219
Engineering	0.0058	0.0058	0.0057	0.0058	0.0058
Fixed Line Telecommunication	0.0157	0.0157	0.0166	0.0168	0.0174
Food Producers	0.0031	0.0032	0.0033	0.0033	0.0034
Forestry (Paper and Board)	0.0057	0.0059	0.0060	0.0060	0.0059
General Industrials	0.0061	0.0058	0.0059	0.0058	0.0060
Health Care Equipment and Services	0.0962	0.0948	0.0853	0.0902	0.0926
Household Goods	0.0048	0.0049	0.0049	0.0050	0.0053
Industrial metals and Mining	0.0043	0.0043	0.0043	0.0047	0.0046
Industrial Transportation	0.0086	0.0087	0.0087	0.0087	0.0094
Multiutilities (Gas and water)	0.0096	0.0098	0.0098	0.0104	0.0106
Oil and Gas	0.0050	0.0050	0.0051	0.0051	0.0053
Personal Goods (Textile)	0.0064	0.0069	0.0072	0.0072	0.0072
Pharma and Bio Tech	0.0059	0.0059	0.0059	0.0058	0.0059
Real Estate Investment and Services	0.0183	0.0184	0.0184	0.0184	0.0198
Tobacco	0.0149	0.0152	0.0149	0.0153	0.0159
Travel and Leisure	0.0050	0.0049	0.0050	0.0051	0.0052

Table G2 presents the mean square prediction error of AR (5) model with rolling samples in Pakistan. Study selects the order of AR having lowest MSPE and use this selected AR (q) model for the estimation of future return vector. If there is any conflict on the order of AR (q) model in rolling and non-rolling auto-regressive models then study prefer the one which minimizes the MSPE with lower order of AR (q).

Table G2

Forecast performance of auto-regressive models under RMSE (Rolling)

Asset Classes	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Automobile and Parts	0.0057	0.0059	0.0062	0.0064	0.0068
Beverages	0.0266	0.0282	0.0280	0.0302	0.0309
Chemicals	0.0045	0.0048	0.0047	0.0050	0.0049
Construction and Materials (Cement)	0.0066	0.0070	0.0073	0.0076	0.0077
Electricity	0.0099	0.0104	0.0113	0.0122	0.0129
Electronic and Electrical Goods	0.0187	0.0193	0.0200	0.0205	0.0215
Engineering	0.0053	0.0056	0.0058	0.0061	0.0064
Fixed Line Telecommunication	0.0165	0.0172	0.0177	0.0187	0.0207
Food Producers	0.0031	0.0033	0.0035	0.0035	0.0036
Forestry (Paper and Board)	0.0056	0.0059	0.0059	0.0061	0.0065
General Industrials	0.0058	0.0062	0.0064	0.0067	0.0068
Health Care Equipment and Services	0.0961	0.0941	0.0841	0.0879	0.0903
Household Goods	0.0042	0.0044	0.0047	0.0050	0.0053
Industrial metals and Mining	0.0038	0.0040	0.0039	0.0041	0.0041
Industrial Transportation	0.0091	0.0093	0.0099	0.0103	0.0110
Multiutilities (Gas and water)	0.0097	0.0107	0.0105	0.0112	0.0117
Oil and Gas	0.0048	0.0050	0.0050	0.0052	0.0056
Personal Goods (Textile)	0.0064	0.0070	0.0067	0.0068	0.0069
Pharma and Bio Tech	0.0053	0.0058	0.0058	0.0059	0.0062
Real Estate Investment and Services	0.0195	0.0207	0.0212	0.0198	0.0207
Tobacco	0.0146	0.0155	0.0157	0.0163	0.0178
Travel and Leisure	0.0049	0.0049	0.0051	0.0051	0.0053

Table G3 shows the selected order of ARIMA (p,d,q) for the estimation of future return vector on the basis of minimizing the the AIC and BIC with adjusted R^2 model (the model which minimizes AIC, BIC and has highest adjusted R^2). Further Gauss-Newton algorithm is use to estimate coefficients of $ARIMA\ (p, d, q)$model and selectd models also check to ensure that the estimation process converge.

Table G3

Selected order of ARIMA (p,d,q) model

Asset Classes	ARIMA (p,d,q)
Automobile and Parts	(2,0,2)
Beverages	(2,0,2)
Chemicals	(2,0,2)
Construction and Materials (Cement)	(2,0,3)
Electricity	(2,0,2)
Electronic and Electrical Goods	(2,0,2)
Engineering	(3,0,3)
Fixed Line Telecommunication	(2,0,2)
Food Producers	(2,0,2)
Forestry (Paper and Board)	(2,0,2)
General Industrials	(2,0,2)
Health Care Equipment and Services	(2,0,3)
Household Goods	(2,0,2)
Industrial metals and Mining	(2,0,4)
Industrial Transportation	(2,0,2)
Multiutilities (Gas and water)	(2,0,2)
Oil and Gas	(2,0,2)
Personal Goods (Textile)	(4,0,2)
Pharma and Bio Tech	(2,0,3)
Real Estate Investment and Services	(2,0,2)
Tobacco	(2,0,2)
Travel and Leisure	(3,0,2)

Mean square prediction error (MSPE) which is the average of the square of difference of actual returns with estimated returns under each asset class on one year sample window is computed and results are presented at Table G 4. The estimation technique with consistent lower mean square prediction error outperforms the other competing return estimation techniques.

Table G4

Mean square prediction error

Asset Classes	Hist	AR	ARIMA	ARIMA-Reg	CAPM
Automobile and Parts	0.0048	0.0042	0.0085	0.0040	0.0030
Beverages	0.0377	0.0382	0.0382	0.0393	0.0361
Chemicals	0.0043	0.0040	0.0045	0.0039	0.0034
Construction and Materials (Cement)	0.0054	0.0048	0.0052	0.0050	0.0034
Electricity	0.0031	0.0030	0.0028	0.0025	0.0032
Electronic and Electrical Goods	0.0229	0.0227	0.0244	0.0212	0.0190
Engineering	0.0041	0.0036	0.0040	0.0046	0.0040
Fixed Line Telecommunication	0.0095	0.0094	0.0095	0.0100	0.0082
Food Producers	0.0033	0.0032	0.0033	0.0027	0.0025
Forestry (Paper and Board)	0.0083	0.0082	0.0084	0.0098	0.0066
General Industrials	0.0108	0.0107	0.0104	0.0134	0.0090
Health Care Equipment and Services	0.0090	0.0107	0.0139	0.0085	0.0088
Household Goods	0.0053	0.0052	0.0053	0.0043	0.0046
Industrial metals and Mining	0.0026	0.0026	0.0028	0.0049	0.0016
Industrial Transportation	0.0066	0.0053	0.0066	0.0061	0.0036
Multiutilities (Gas and water)	0.0059	0.0065	0.0058	0.0074	0.0034
Oil and Gas	0.0045	0.0043	0.0068	0.0046	0.0023
Personal Goods (Textile)	0.0103	0.0102	0.0101	0.0089	0.0081
Pharma and Bio Tech	0.0082	0.0083	0.0089	0.0078	0.0067
Real Estate Investment and Services	0.0028	0.0026	0.0024	0.0042	0.0022
Tobacco	0.0310	0.0315	0.0316	0.0336	0.0327
Travel and Leisure	0.0016	0.0024	0.0017	0.0023	0.0009

Since this study also introduce the country risk into the BL model and uses this augmented model i.e. BL-CR for future return estimates. As a first step study estimates the individual sensitivities of each asset class with the country risk and presented at Table G5. The regression coefficient shows that all asset classes have positive gradient with country risk.

Table G5

Sensitivities of each asset class towards country risk in Pakistan

Asset class	Sensitivities/Slope
Automobile and Parts	0.0155
Beverages	0.0246
Chemicals	0.0075
Construction and Materials (Cement)	0.0132
Electricity	0.0079
Electronic and Electrical Goods	0.0151
Engineering	0.0135
Fixed Line Telecommunication	0.0121
Food Producers	0.0063
Forestry (Paper and Board)	0.0102
General Industrials	0.0125
Health Care Equipment and Services	0.0095
Household Goods	0.0107
Industrial metals and Mining	0.0102
Industrial Transportation	0.0115
Multiutilities (Gas and water)	0.0085
Oil and Gas	0.0095
Personal Goods (Textile)	0.0124
Pharma and Bio Tech	0.0134
Real Estate Investment and Services	0.0157
Tobacco	0.0196
Travel and Leisure	0.0088

Table G6 describes the comparison on the basis of varying the shrinkage intensity lambda in Pakistan. Since GMVP is the only portfolio on the efficient frontier that depends upon covariance matrix and is independent from the choice of future return estimates. Therefore the Herfindahl index and variance of weights are independent from the future return estimation techniques. From equation 17, if $\lambda = 0$ then, there is no shrinkage to $\hat{\Sigma}_s$ and if $\lambda = 1$, it results full shrinkage to variance covariance matrix.

Table G6

Comparisons on the basis of varying degree of shrinkage

	ESR under GMVP									
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR	HI	Var
0	-0.145	-0.192	-0.260	0.024	0.017	0.009	-0.04	0.157	0.590	0.026
0.1	-0.145	-0.186	-0.173	-0.067	0.019	0.010	-0.008	0.275	0.357	0.015
0.2	-0.147	-0.185	-0.136	-0.110	0.021	0.011	0.015	0.346	0.265	0.010
0.3	-0.152	-0.189	-0.117	-0.138	0.023	0.013	0.034	0.405	0.209	0.008
0.4	-0.161	-0.197	-0.106	-0.160	0.026	0.014	0.050	0.463	0.170	0.006
0.5	-0.173	-0.208	-0.098	-0.180	0.029	0.015	0.067	0.524	0.140	0.005
0.6	-0.189	-0.224	-0.094	-0.200	0.033	0.017	0.085	0.594	0.117	0.003
0.7	-0.213	-0.248	-0.091	-0.222	0.038	0.020	0.106	0.680	0.097	0.003
0.8	-0.248	-0.284	-0.091	-0.250	0.045	0.023	0.133	0.798	0.082	0.002
0.9	-0.307	-0.346	-0.097	-0.294	0.057	0.029	0.171	0.979	0.069	0.001
1	-0.432	-0.478	-0.120	-0.384	0.081	0.041	0.246	1.350	0.061	0.001

Table G7

Comparisons on the basis of varying degree of shrinkage

	ESR under equal weight									
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR	HI	Var
0	-0.131	-0.153	-0.021	-0.079	0.027	0.013	0.100	0.157	0.045	0.000
0.1	-0.137	-0.160	-0.022	-0.083	0.028	0.014	0.104	0.275	0.045	0.000
0.2	-0.144	-0.167	-0.023	-0.087	0.030	0.015	0.109	0.346	0.045	0.000
0.3	-0.151	-0.176	-0.024	-0.091	0.031	0.016	0.115	0.405	0.045	0.000
0.4	-0.160	-0.186	-0.026	-0.097	0.033	0.016	0.122	0.463	0.045	0.000
0.5	-0.171	-0.199	-0.028	-0.103	0.035	0.018	0.130	0.524	0.045	0.000
0.6	-0.184	-0.214	-0.030	-0.111	0.038	0.019	0.140	0.594	0.045	0.000
0.7	-0.201	-0.234	-0.033	-0.121	0.042	0.021	0.153	0.680	0.045	0.000
0.8	-0.224	-0.260	-0.036	-0.135	0.046	0.023	0.170	0.798	0.045	0.000
0.9	-0.256	-0.298	-0.041	-0.154	0.053	0.026	0.195	0.979	0.045	0.000
1	-0.309	-0.359	-0.050	-0.186	0.064	0.032	0.235	1.350	0.045	0.000

See the note of Table G6

Table G8 describes the comparison of optimal weights on the basis of varying the shrinkage intensity lambda in Pakistan. It further describe the financial efficiency in term of Sharp ratio under alternative inputs to portfolio optimization in Pakistan. From equation 17, if $\lambda = 0$ then, there is no shrinkage to $\hat{\Sigma}_s$ and if $\lambda = 1$, it results full shrinkage to variance covariance matrix.

Table G8

Comparisons on the basis of varying degree of shrinkage

	ESR under optimal weights							
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
0	-0.6012	-0.5917	-2.4705	3.4432	0.0336	0.0152	-0.7880	1.149
0.1	-0.5218	-0.5159	-2.1577	-2.9868	0.0346	0.0157	-0.7002	1.039
0.2	-0.4798	-0.4764	-1.9668	-2.7234	0.0360	0.0164	0.6452	0.988
0.3	-0.4534	-0.4522	-1.8311	-2.5434	0.0377	0.0173	0.6064	0.962
0.4	-0.4363	-0.4373	-1.7271	-2.4104	0.0397	0.0183	0.5776	0.953
0.5	-0.4259	-0.4292	-1.6438	-2.3074	0.0422	0.0196	0.5558	0.959
0.6	-0.4217	-0.4275	-1.5751	-2.2255	0.0455	0.0213	0.5395	0.980
0.7	-0.4245	-0.4333	-1.5173	-2.1594	0.0499	0.0236	0.5279	1.020
0.8	-0.4374	-0.4501	-1.4680	-2.1063	0.0564	0.0269	0.5215	1.091
0.9	-0.4697	-0.4883	-1.4258	-2.0653	0.0669	0.0323	0.5228	1.223
1	-0.5580	-0.5884	-1.3907	-2.0417	0.0894	0.0437	0.5441	1.531

Table G9

Comparisons on the basis of varying degree of shrinkage

	Herfindahl Index under optimal weights							
Lambda	Hist	AR	ARIMA	ARIMA-Reg	CAPM	IEER	BL	BL-CR
0	19.64	10.48	91	22379	0.36	0.22	206.0	48.9
0.1	8.57	4.93	114	1377	0.24	0.14	4437	7.48
0.2	4.96	2.96	120	315	0.19	0.11	804.3	3.01
0.3	3.09	1.92	114	135	0.16	0.09	118.7	1.55
0.4	1.97	1.27	101.9	71.12	0.13	0.08	39.00	0.90
0.5	1.26	0.84	87.36	41.33	0.11	0.07	16.47	0.55
0.6	0.78	0.55	71.85	24.87	0.10	0.07	7.70	0.34
0.7	0.47	0.35	55.60	14.82	0.08	0.06	3.70	0.22
0.8	0.27	0.21	38.63	8.28	0.07	0.06	1.72	0.14
0.9	0.15	0.12	21.69	3.96	0.06	0.06	0.71	0.09
1	0.08	0.07	7.29	1.26	0.06	0.06	0.22	0.06

See the note of Table G8