CAPITAL UNIVERSITY OF SCIENCE AND
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State and Parameter Estimation of Induction Motor through Adaptive High Gain Observer

by

Mohsin Ullah

A thesis submitted in partial fulfillment for the
degree of Master of Science

in the

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Department of Electrical Engineering

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Dedicated to my Teachers who enlighten my soul and directed me on Sirat-e-Mustaqeem.

And

Dedicated to my Parents who pray for me and always pave the way to success for me

CERTIFICATE OF APPROVAL

State and Parameter Estimation of Induction Motor through Adaptive High Gain Observer

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Abstract

Induction motor is the most widely used electric motor in industry, EVs and HEVs due to high reliability, ruggedness, low cost of maintenance, high torque during starting and wide range of operating speed. However, highly non-linear dynamics of induction motor make it difficult to control as compare to other motors. With the development of field oriented control, the induction motor can be controlled like separately excited dc motor. Field oriented control requires the exact information of rotor flux. Since the rotor flux is not easy to estimate due to the large variations in the parameters of induction motor due to the surrounding and operating temperature. Rotor resistance and rotor inductance are crucial parameters for estimation of flux. The objective of simultaneous estimation of fluxes along with critical parameter is achieved through adaptive high gain observer. The observer estimate the actual values of rotor resistance and rotor inductance under persistence excitation condition which guarantees the exact estimation of rotor flux.

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Abbreviations

EV	Electric Vehicle
HEV	Hybrid Electric Vehicle
IM	Induction Machine
SRRF	Synchronously Rotating Reference Frame
SRF	Stationary Reference Frame
d-axis	Direct axis
q-axis	Quadrature axis
SMC	Sliding Mode Control
ANN	Artificial Neural Network
LTV	Linear Time Varying
MIMO	Multi Input Multi Output
FOSM	First Order Sliding Mode
HOSM	Heigher Order Sliding Mode
RMF	Rotating Magnetic Field

Symbols

n_r	Rotor Angular Speed
n_e	Synchronous Speed
f	Frequency of Voltage
i_{ds}	d-Axis Stator Current
i_{qs}	q-Axis Stator Current
u_{ds}	d-Axis Stator Voltage
u_{qs}	q-Axis Stator Voltage
ψ_{dr}	d-Axis Rotor Flux
ψ_{qr}	q-Axis Rotor Flux
τ_e	Electromagnetic Torque
τ_l	Load Torque
r_s	Stator Resistance
r_r	Rotor Resistance
χ_s	Stator Inductance
χ_r	Rotor Inductance
M	Mutual Inductance
p	Number of Pole Pairs
j	Inertia
b	Damping
χ_{lr}	Rotor Leakage Inductance

Chapter 1

Introduction

1.1 Background

In the past decades, induction motors have been controlled from the grid under a fixed frequency/speed, but later there is much improvement in the technology of induction machine. Due to the development in the field of digital signal processing boards and power electronics devices, the control of induction motor become possible without the need of maintenance. Induction machines are used in hybrid and electric vehicles and industries due to its low cost, robustness, reliability, high torque during starting, less maintenance and wide range of operating speed. Induction machine has much higher power to weight ratio as compared to the DC machines. They are small in size and its construction is quite simple due to absence of commutator and carbon brushes. The absence of carbon brushes enable induction motor to operate even in the hazardous environment. Induction motor is highly recommended for the automobile industry, as it can be inherently de-excited with respective to inverter fault and need less maintenance due to the absence of carbon rushes. Moreover, the speed of the induction motor remains same over wide range despite, the changes occur in load. Wide range of speed can be obtained by flux-weakening with the help of invariable power.

1.2 Construction of Induction Motor

An induction machine comprised of largely two parts known as stator and rotor. The static part is known as *stator*, while the rotating part is known as *rotor*. The construction of stator is same as synchronous motors while the construction of rotor is different. Its construction is explained below.

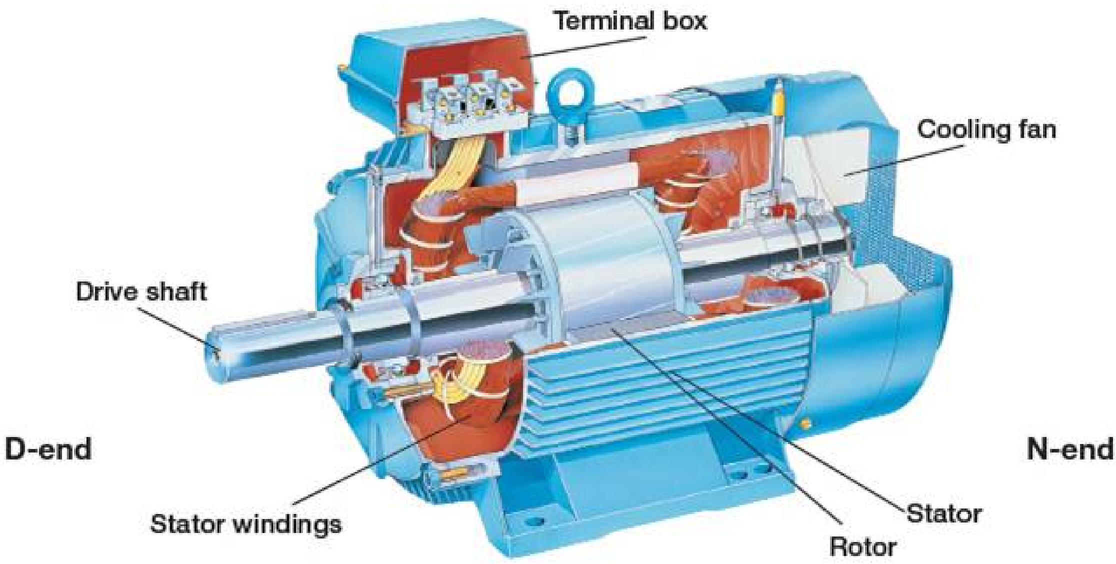

FIGURE 1.1: Construction of an Induction Motor

1.2.1 Stator Construction

The stator is comprised of alloy of high-grade steel laminations in order to reduce losses due to eddy current. It has mainly three parts, i.e. outer frame, stator core and stator winding.

1.2.1.1 Outer Frame

The outermost part of the stator is called stator frame. It provides support and also protect the core of the stator and winding wound on it. It also gives mechanical strength and support to the induction motor inner parts. It is manufactured using fabricated steel. The stator frame of induction machine needs to be strong and rigid.

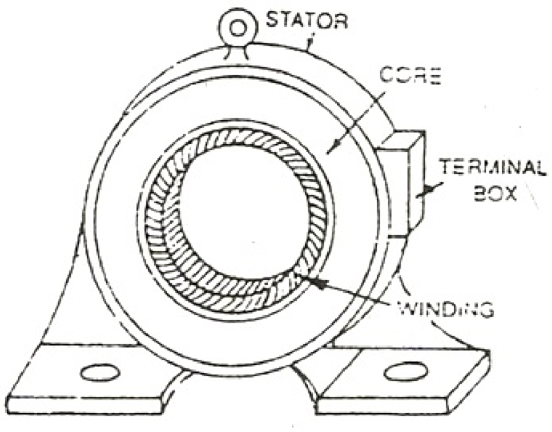

FIGURE 1.2: Stator Frame

1.2.1.2 Stator Core

The stator core is fabricated from silicon steel stampings of high-grade. It carries alternating magnetic flux. A thin varnish layer is used to insulate stamping from one another to minimize losses due to eddy current. The stamping is about 0.3 mm to 0.5mm thick. Slots are then fixed on the inner side of the stampings. The stator core is contained in the stator frame.

1.2.1.3 Stator Winding

Three-phase stator windings are put in the slots which are usually connected with three phase power supply. Speed of induction motor depends upon the wounding of stator for a fixed number of pair of poles. The speed of induction motor increases by reducing the number of pair of poles. Both star and delta configurations can be used for connecting three-phase windings which depends upon the type of starting method.

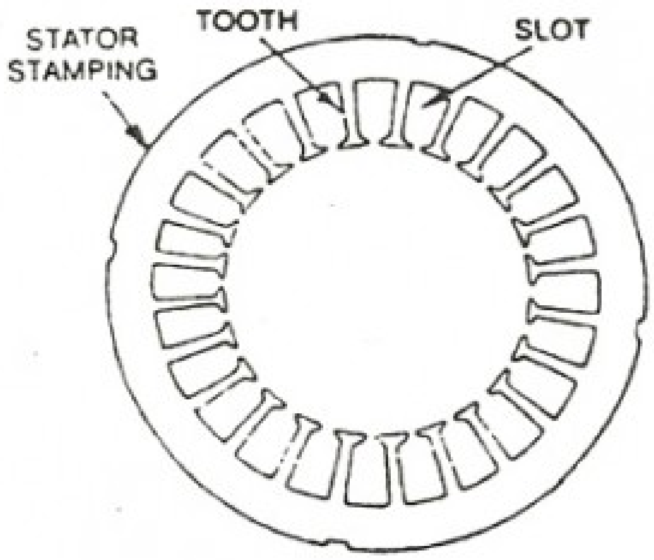

FIGURE 1.3: Stator Core

1.2.2 Rotor Construction

The Rotor is rotating part of induction machine. A mechanical load is attached to the rotor with the help of shaft. Generally, rotor is categorized as follow:

1.2.2.1 Squirrel Cage Rotor

The construction of rotor is just like squirrel cage wheel, i.e. the exercise wheel that hamster or squirrels run on, and motors using this type rotors are called as squirrel cage motors. This type of rotor is comprised of laminated cylindrical core with semi enclosed slots on its periphery. These slots have un-insulated rotor conductors which consist of bars of aluminum or copper or alloys instead of wires. The rotor bars are permanently short circuited at both ends by rings or plates to which the rotor bars are brazed or welded.
The resistance of the rotor is constant and any addition of external resistance in series is not possible. In other words, we cannot use rotor rheostatic method for starting such type of induction motor. The rotor slots are not placed quite parallel to the shaft and are made skewed for the following reason:

1. It makes the motor to operate very quietly by reducing magnetic hum.

2. It makes the torque uniform.

3. It reduces the tendency of locking.

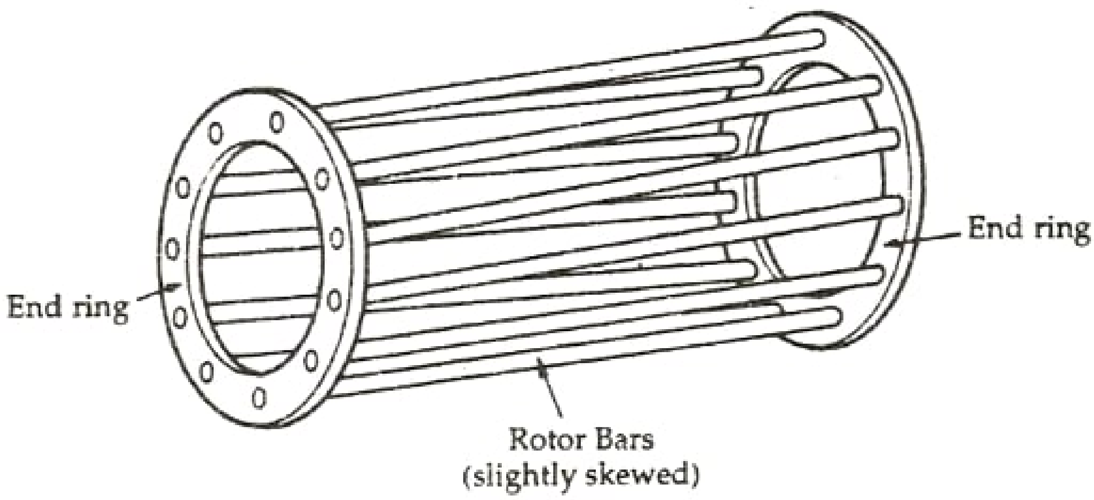

FIGURE 1.4: Squirrel Cage Rotor

1.2.2.2 Wound Rotor

This type of rotor contain three-phase windings and motor having this type of rotor is called wound rotor induction motors. The three terminals of the three phase windings can be brought out and connected to the insulated slip rings mounted on the shaft of the rotor with brush resting on each of them. Therefore, we can access rotor current of the wound rotor induction motors at the carbon brushes. So one can examine and insert an extra resistance to the rotor circuit. The benefit of this feature is that the torque-speed characteristics can be modified. The starting torque of motor can also be increased using this feature. Squirrel cage motor is less expansive than wound-cage induction motor and require less maintenance, because there is no wear and tear associated with the carbon brushes and also slip rings. As a result, squirrel cage motors are normally used.

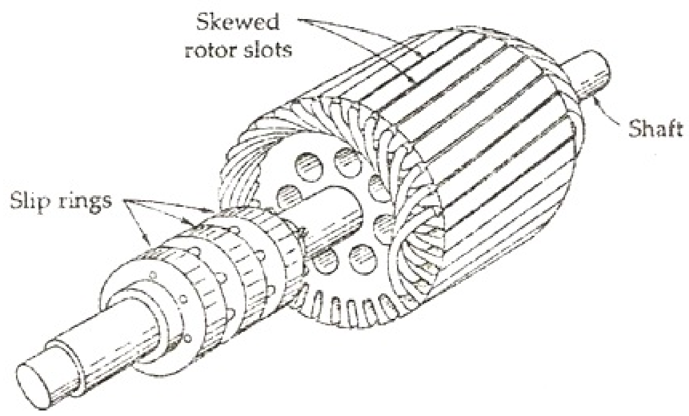

FIGURE 1.5: Wound Rotor

Difference Between Cage Rotor Motor and Wound rotor Motor

1. In squirrel cage motor, rotor circuit has constant resistance. Moreover, it is short-circuited. Whereas in wound-rotor motor the resistance of the rotor circuit can be varied for starting and speed control purposes and it is short circuited under normal conditions.

2. Squirrel cage motor has cheaper cost as compared to the the induction motor having wound rotor.

3. The starting torque is low for squirrel cage motor while wound-rotor induction motor has high starting torque due to addition of extra resistance.

4. Squirrel cage motor does not require carbon brushes as well as slip rings. Whereas, wound-rotor motor has carbon brushes and also slip rings.

1.2.3 Bearings

It is the mechanical part of induction motor which is responsible for relative motion between different parts such as shaft.

1.2.4 Cooling Fan

Heat is generated by electrical machines as a result of mechanical and electrical losses inside the machine. Normally high losses occurs during starting period, overload or dynamic braking. Therefore, it is necessary to cool down the temperature inside the machine and transfer the heat energy to the external medium. In case of AC induction machine, cooling air is circulated internally with the help of fan mounted on the rotor shaft. In addition, external frame of motors are designed with cooling ribs to increase the surface area for heat radiations.

1.3 Principle of Operation

The working principal of induction motor is electromagnetic mutual induction. Three phase windings of stator are uniformly distributed in space and displaced at 120° from each other as show in figure 1.6. When AC supply is connected to stator windings, it produces an alternating magnetic flux around the stator. This alternating magnetic flux will revolve with synchronous speed. This flux is termed as Rotating Magnetic Field. This flux produce around the stator will also cut the rotor conductors and will induced an emf according to electromagnetic induction. Current will be produced in the rotor due to induced emf. The current that is induced in the rotor circuit will then generate alternating magnetic flux around the rotor which lags behind the magnetic flux of stator. According to Lenz's law, direction of current induced in the rotor circuit will tend to oppose its cause. The relative velocity between rotor and rotating flux in the stator is the cause and it will produce torque in the rotor circuit. The rotor starts rotation and tries to catch up the RMF in the stator. The rotor is free to move and rotates in the similar direction as that of RMF to minimize the relative velocity. However, rotor speed will not be equal to the synchronous speed. Therefore the rotor speed will be comparatively less than synchronous speed. The torque produced in the rotor will depends upon the rotating flux, rotor current and rotor power factor.

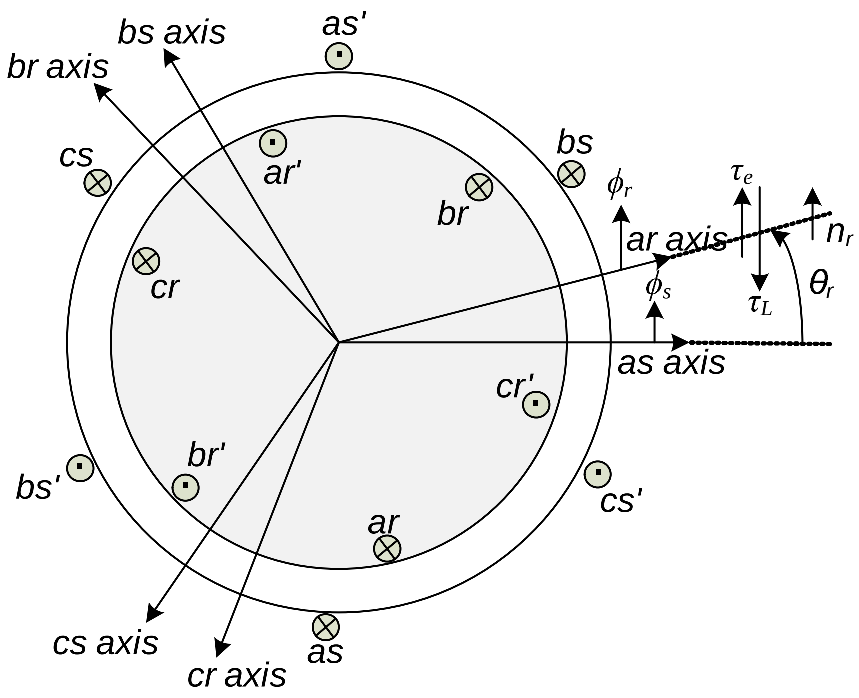

FIGURE 1.6: Idealized Two Pole Three-Phase Wound-Rotor Induction Machine [1]

1.4 Terminologies

1.4.1 Synchronous Speed

The speed of the alternating magnetic flux in the stator of induction motor is known as synchronous speed. Mathematically,

$$n_e = \frac{120f}{p}$$

1.4.2 Slip Speed

Slip speed is the difference between actual speed of induction motor and synchronous speed. Per unit slip is shown below:

$$S = \frac{n_e - n_r}{n_e} \times 100$$

1.5 Applications of Induction Motor

Low cost, ruggedness, robustness, reliability, high torque during starting, less maintenance and wide range of operating speed are the main reason due to which induction motor is highly adopted in EVs, HEVs and Industry. Induction machines can be inherently de-excited with respect to inverter fault therefore it is highly suitable for automobile industries. Its small size and high power to weight ratio make it more effective as compared to DC motors. Squirrel cage motor do not require carbon brushes as well as slip rings. Thats why the the researchers of automotive have keen interest in squirrel-cage motor for EVs and HEVs. Induction motors are widely used as a propulsion machine in hybrid and electric vehicles applications. Tesla Model (2012), Honda Fit EV (2012), Toyota RAV4 EV (2012), Renault/Kangoo (1998), BMW/X5 (Germany), Chevrolet (USA), Durango (USA) [2]. Chris Mi proposed squirrel-cage motor as a propulsion machine in hybrid and electric vehicles due to its small size as compared to DC machine, high efficiency and ease of fabrication [3]. Wide range of speed can be obtained by flux-weakening with the help of invariable power. As a result of breakdown of torque, the wide invariable power operation of induction motor becomes limited. This problem can be solved by using multi-phase pole adjusting induction motors for the application of propulsion. Tesla Motors are using induction motors for almost all types of EVs and HEVs.

1.6 Drawbacks

With the advantages, the induction motor has also some drawbacks. The machine model is highly non linear. There are some machines parameters (i.e. rotor resistance, stator resistance and rotor inductance etc.) which can vary significantly during the operating conditions. Components of rotor flux which are the two states of the machines model are not easily available. Problem of speed control is also difficult. Efficiency becomes poor at low loads.

1.7 Thesis Organization

The thesis comprises five chapters along with introduction.

Chapter 2: This chapter includes literature survey of estimation techniques proposed by different researchers. The literature survey describes the past effort of the researchers to estimate flux, torque and speed of induction machine.

Chapter 3: This chapter includes mathematical model of a induction motor. It provides necessary details for understanding the performance of induction machine. The same model is also used for observer design. A two-phase representation of a three-phase induction motor is shown. The dynamics of induction motor is then simulated in MATLAB®/Simulink.

Chapter 4: This chapter deals with the estimation of missing states and unknown parameters of the induction motor. The estimation of flux along with critical parameters is crucial for efficient control of induction machine. The objective of missing states and time varying parameters is achieved through adaptive high gain observer.

Chapter 5: It includes the final conclusion and direction that can be obtained in future work.

Chapter 2

Literature Review

2.1 Background

Parameter estimation problem is one of the most active research field during the last two decades. Some techniques are proposed for specific applications with the assumption that the estimated parameters should be kept constant during the identification process. But in practical plant most of the parameters vary with time such as rotor resistance of an induction machine which can vary up to 150% of its nominal value due to the operating temperature. So, we need an accurate algorithm of joint estimation of missing states and unknown parameters.

The challenging task to control induction motor is exact estimation of rotor flux, electromagnetic torque and angular speed. Since these quantities are not easily estimated due to large variations in parameters with surrounding and operating temperature, aging and magnetic saturation in induction motor. So, the knowledge of induction machine parameters are necessary to tune the controller motor drive system. Therefore, accurate estimation of parameters is necessary for efficient control. Generally this fact has urged the development of parameter estimation techniques. Parameter estimation in nonlinear systems play a key role in modeling and diagnosis.

2.2 Parameter Estimation Techniques

Different techniques have been used by different researchers which are as follows:

1. Conventional Observers
2. Sliding Mode Control Observer
3. Artificial Neural Networks
4. Adaptive Observer
5. Adaptive High Gain Observer

2.2.1 Conventional Observers

Most Commonly used observers for induction motor are Kalman and Luenberger observers [4–6]. In [7] three full order extended Luenberger observers are used for estimation of both internal states and unknown parameters of induction motor. Observer I is used to estimate rotor resistance and rotor flux, observer II estimate rotor speed and rotor flux while observer III estimate load torque, rotor speed and rotor flux. Its gain matrix L can be computed from the machine model. That's why, its working is not efficient due to excessive variations in the parameters. The extended Kalman filter approach in [8] is used for simultaneous identification of various parameters of induction motor using measurements of stator voltages, currents and rotor speed. The estimated parameters include mutual inductance, rotor time constant, stator resistance and leakage inductance. In case of induction motor, the estimated parameters may violate the physical ranges. Therefore in [9] a modified form of extended Kalman filter is proposed, such that if physical constraint is not satisfied by parameters, then the estimation will be adjusted by quadratic programming. The basic drawback of Kalman filter is not esay to guarantee error convergence for time-varying systems.

2.2.2 Sliding Mode Control

The advantages of sliding mode observers are robustness, parameter invariance and order reduction [10]. A Luenberger-sliding based observer is proposed in [11] for parameter estimation. It is basically a closed loop observer where estimated parameters are fed back for correction. The observer has two parts. Its Luenberger part is determined from machine dynamics while sliding based part causes insensitivity to disturbances and mismatches in parameters. In [12] Utkin proposed two observers i.e. first and second order sliding mode observer (FOSM and HOSM) which provide joint estimation of rotor speed, rotor flux and rotor resistance using stator currents measurements. The dynamics of induction machine in stationary reference frame is used. The rotor time constant is estimated through sliding mode observer in [13]. Rotor time constant is essential to guarantee decouple control of flux and torque in stator flux oriented approach. The dynamics of induction motor based on stator flux was used for synthesizing sliding based observer in [13]. Godpromesse in [14] proposed an on-line technique for estimation of both states and time varying parameters of an induction motor based on variable structure control. Rotor flux, angular speed and rotor resistance are estimated using measurements of only stator currents. The condition of persistently exciting inputs is not required by the error convergence proof. Sliding mode is rarely used for the problem of control and estimation problem of induction machine, besides its insensitivity to disturbance and parameter variations due to chattering phenomena [15]. Also gain computation is tedious task.

2.2.3 Artificial Neural Network

Artificial neural network theory is also a wide and active research for the parameter estimation problems in recent years. Neural networks can be used in control and identification problems due to the adaptive ability of learning process. In the literature, normally two approaches are used to tackle adaptive control problems.

The first approach is off-line method in which parameters are learned by measuring input-output signal and observing motor behavior. The second approach is on-line and adaptive learning is implemented. The control input is determined as the output of neural network. In [16] induction machine parameters are estimated on-line during transients and remembered, based on artificial neural network. The estimated parameters are rotor resistance and mutual inductance. The method was applied in closed loop control of sensor-less induction motors. Direct torque control method was proposed alternative to the vector oriented control in the late 1980s. In [17] stator resistance is estimated on-line based on artificial neural network to enhance the performance of direct torque control method. Vector Control algorithm requires an accurate estimation of parameters of induction motor for efficient control and rotor resistance is one of the most critical parameter. Therefore in [18] an adaptive algorithm is proposed, to estimate rotor resistance online based on artificial neural network. The artificial neural network lack joint estimation of both states and parameters simultaneously.

2.2.4 Adaptive Observers

Parameter estimation is the main purpose of foundation of adaptive control. The most common methods to estimate dynamics of parameters include recursive least square and gradient algorithms. These are basically update laws, to adjust estimates in real time. Its convergence criteria is satisfied through Lyapunov function. In [19] an error equation of state estimation is integrated with parameter adaptation algorithm and proposed an adaptive observer. After several years, Kreisselmeier [20] proposed three adaptive observers based on minimization of particular criterion with exponential convergence. In the recently developments, an original system is dynamically transformed into some other canonical form where the unknown parameter is simplified to some extent [21–24]. Using the same approach Q.Zhang [25] in 2002 proposed an adaptive observer for simultaneous estimation of missing states and unknown parameters. Global exponential convergence is achieved for noise free systems.

2.2.5 Adaptive High Gain Observer

As far as high gain observer is concerned, there are two independent schools of research on this subject. The first school is led by Hassan K.Khalil in terms of output feedback stabilization problem [26]. The second school, lead mostly by French researchers (Gauthier, Hammouri and Farza) focused on calculating global results by assuming condition of global Lipschitz [27–30]. High gain observer proposed by French researchers in [30], which can be used to estimate rotor flux of an induction motor. Its gain is calculated from Lyapunov algebraic equation. On-line estimation of rotor and stator resistance based on high gain observer is proposed in [31]. High Gain Observer for rotor flux estimation in case of sensor-less induction motor is proposed in [32].
As discussed earlier that Q.Zhang presented an adaptive observer for joint estimation of states and unknown parameters. Besancon in [33] puts forward that an adaptation law for estimating dynamics of parameter in [25] can also be derived by using high gain observer approach. T.Matoug in [34] presented adaptive high gain observer for estimation of missing states and parameter for non-linear system with coupled structures using the same idea of Besancon. Later on, Mondher Farza add considerable developments to adaptive high gain observer. He proposed Adaptive high gain observer for nonlinearly parametrized class of uniformly observable non-linear systems in [35, 36]. A survey of different techniques shows that the Induction machine parameters can vary due to many factors such as operating temperature, machine aging magnetic saturation and so forth. Therefore, a simultaneous estimation of missing states and time varying parameters of induction motor is inevitable to control it efficiently. That's why the parameter estimation problem of the induction motor captured so much attention. Therefor the task of simultaneous estimation of internal states and parameters is achieved through adaptive high gain observer due to following reasons.

1. The gain of adaptive high gain observer can be adjusted through single parameter.

2. The observer structure is simple and can give rise to different observers like adaptive sliding high gain observer.
3. The convergence proof of proposed observer is guaranteed under well define condition of persistent excitation .

2.3 Motivation

Induction motor is widely used in EVs, HEVs and industries due to its low cost, less maintenance, robustness and high torque during starting. The efficient control of induction machine requires an accurate knowledge of rotor flux. Since it varies due to variations in parameters due to the surrounding and operating temperature. Hence it also requires to estimate parameters along with the states of induction machine.

2.4 Problem Statement

The main of this work is to estimate state and parameter of induction motor through adaptive high gain observer.

2.5 Research Methodology

In this work, below stated research methodology is followed.

- Literature review of different estimation techniques.
- Study and analyze the dynamical model of induction machine.
- Implementation of dynamics of an induction motor in MATLAB®/Simulink.
- Estimation of states and parameters through an adaptive high gain observer.

2.6 Chapter Summary

In this chapter, different techniques proposed for the estimation of states and parameters are discussed. Sliding mode observers are extensively used for states and parameter estimation but are not preferred due to chattering problems in case of electric motors. Adaptive high gain observer gains much attention for joint estimation of internal states and parameters due to its simple structure and easy calibration of gain. So, in this work, adaptive high gain observer is chosen for joint estimation of non-measured state and some unknown parameters.

Chapter 3

Mathematical Model of 3-Phase Induction Motor

3.1 Background

This chapter is devoted to the development of dynamics of the Induction Machine. Considerable attention has been paid to mathematical modeling of induction motors under transient and steady state operating condition. The mathematical model provides necessary details to understand performance and working of induction motor. The equivalent two phase d-q model of a three-phase induction machine in both synchronously rotating and stationary reference frame are discussed. The stator currents, rotor fluxes, rotor speed, electromagnetic torque and stator voltage equations, which are helpful in obtaining the induction motor model in synchronously rotating and stationary reference frame are presented. At the end, the dynamics induction machine is also simulated in MATLAB®/Simulink to observer its motoring behaviour. Relation between load torque, electromagnetic torque and angular rotor speed are also shown graphically. The d-q model in stationary reference frame will be then used to estimate parameters and internal states of induction motor.

3.2 Induction Motor Modeling

It is quiet difficult to understand the performance of induction motor, because the three-phase stator winding move with respective rotor winding. As stator and rotor circuits in the induction machine is electromagnetically coupled. Therefore, the continuously change in the rotor position θ_r is responsible for the change in coupling coefficient. To overcome this problem, a mathematical model of a three-phase induction motor should be converted to the equivalent two phase induction motor [37, 38]. Another reason to go towards the dynamic model of induction motor is that the per phase equivalent circuit model is only valid for steady state performance of induction motor. All the electrical transients are ignored during the variations in the load and stator frequency. In the variable speed drives, such variation cannot be ignored. Hence such a model is required to capture both the steady state and transient behavior of the induction motor.

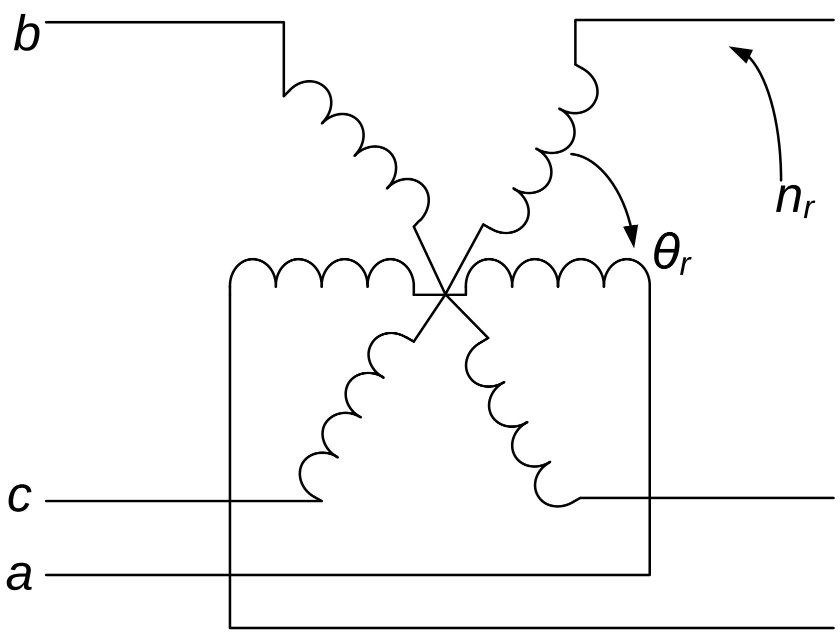

FIGURE 3.1: Coupling Effect in Three-Phase Stator and Rotor Windings of Induction Machine [38].

3.3 d-q Model of Induction Machine

The $d-q$ model captures both the transient and steady state behavior of induction motor. In $d-q$ model, the three phase induction motor can be represented in equivalent two phase machine shown in figure 3.1 and 3.2 respectively.

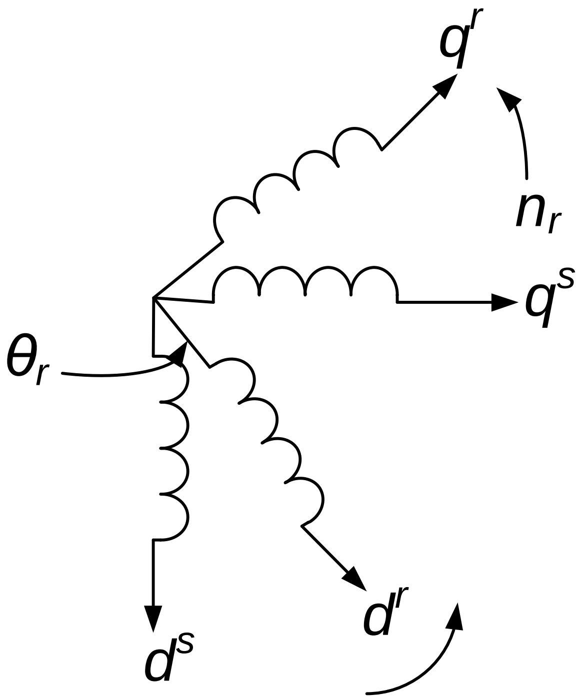

FIGURE 3.2: Equivalent Two-Phase Machine

The variables in the equivalent two phase machine are called as direct (d) axis and quadrature (q) axis as shown in figure 3.2. Stator direct and stator quadrature axes are denoted by d^s and q^s respectively, while rotor direct axis and and rotor quadrature axis is denoted by d^r and q^r respectively. The stator abc reference frame is denoted by the axis a,b,c with subscript s while rotor abc reference frame is denoted by the axis a,b,c with subscript r in the figure 1.6. The speed of the $d-q$ axis can be arbitrary, although the two preference speeds or reference frames are as follow.

1. Stationary reference frame (d-q axes is stationary).

2. Synchronuously Rotating Reference Frame (d-q axes rotate at synchronous speed).

3.3.1 Axis Transformation

Our goal is to transform the variables of three phase reference frame $(as - bs - cs)$ into equivalent two phase stationary reference frame $(d^s - q^s)$ and then transform into synchronously rotating reference frame $(d^e - q^e)$ and vice versa. The stator a, b, c axis variables can be transformed to the $(d^s - q^s)$ axes given in the figure 3.4 by the following sets of equations.

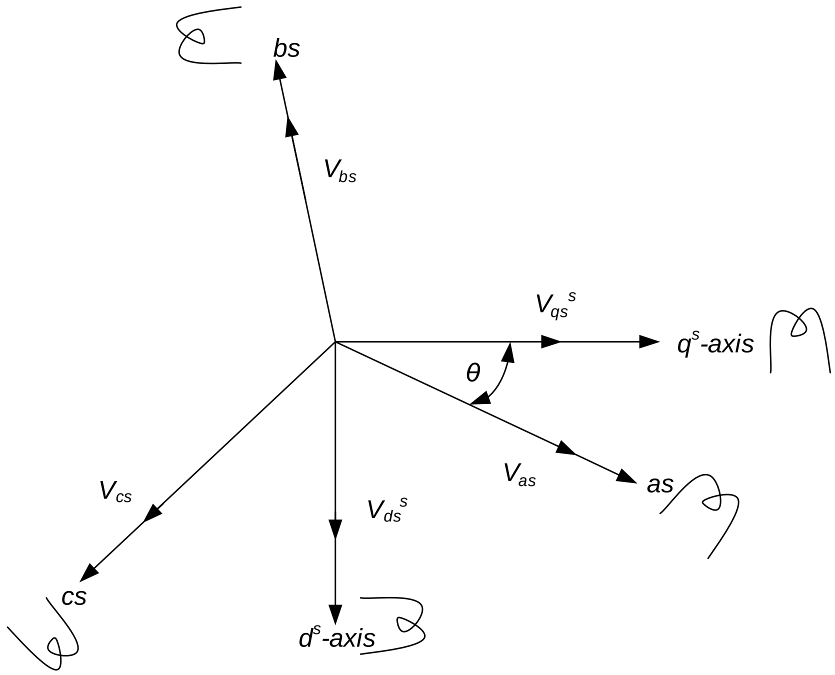

FIGURE 3.3: Transformation of Axes from Three Phase $a - b - c$ to $d^s - q^s$.

$$f_{qs} = \frac{2}{3}[f_{as}cos\theta + f_{bs}cos(\theta - 120^0) + f_{cs}cos(\theta + 120^0)] \tag{3.1}$$

$$f_{ds} = \frac{2}{3}[f_{as}sin\theta + f_{bs}sin(\theta - 120^0) + f_{cs}sin(\theta + 120^0)] \tag{3.2}$$

Where f denotes the voltage or current of the three-phase and two-phase stator circuit. f_{qs} is the stator q-axis voltage or current, f_{ds} is the stator d-axis voltage or current, f_{as} is the stator a phase voltage or current, f_{bs} is the stator b phase voltage or current and f_{cs} is the stator c phase voltage or current. The stationary $d^s - q^s$ axes quantities can be converted to synchronously rotating $d^e - q^e$ axes quantities as represented in figure 3.4 by the following set of equations.

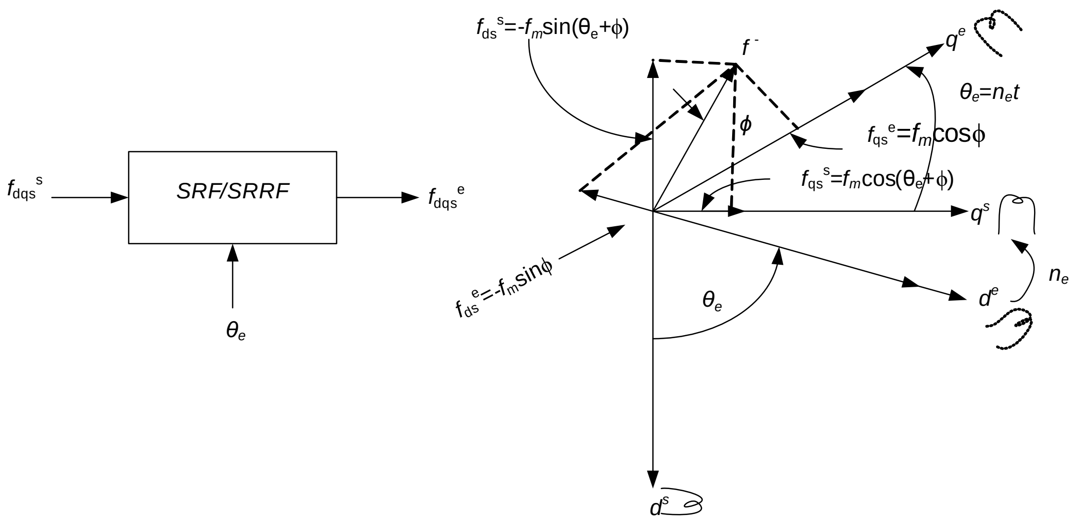

FIGURE 3.4: Rotating $d^e - q^e$ Axes Relative to Stationary $d^s - q^s$ Axes.

$$f_{ds}^{e} = f_{qs}^{s} sin\theta_e + f_{ds}^{s} cos\theta_e \tag{3.3}$$

$$f_{qs}^{e} = f_{qs}^{s} cos\theta_e - f_{ds}^{s} sin\theta_e \tag{3.4}$$

The inverse transformation for synchronously rotating reference frame $d^e - q^e$ to stationary reference frame $d^s - q^s$ is as follow;

$$f_{ds}^{s} = -f_{qs}^{e} sin\theta_e + f_{ds}^{e} cos\theta_e \tag{3.5}$$

$$f_{qs}^{s} = f_{qs}^{e} cos\theta_e + f_{ds}^{e} sin\theta_e \tag{3.6}$$

After the substitution of three-phase stator sinusoidal and balanced voltages in the (3.1) and (3.2), following equations yields;

$$f_{qs}^{s} = f_m cos(\omega_e t + \phi) \tag{3.7}$$

and

$$f_{ds}^{s} = -f_m sin(\omega_e t + \phi) \tag{3.8}$$

Again, substituting (3.1) and (3.2) in (3.4) and (3.5), yields;

$$f_{qs}^{s} = f_m cos\phi \tag{3.9}$$

and

$$f_{ds}^{s} = -f_m sin\phi \tag{3.10}$$

(3.7) and (3.8) shows that f_{qs}^{s} and f_{ds}^{s} are two phase balanced voltage or current whose peak values are equal to each other and the voltage is at phase lead of angle $\pi/2$ with respect to the current. (3.9) and (3.10) shows that in synchronously rotating frame, sinusoidal variables of stationary frame appears as dc quantities. The reason is that the synchronously rotating frame is moving with synchronous speed.

3.3.2 d-q Model of a Three-phase Induction Machine in Synchronously Rotating Reference Frame (SRRF)

The synchronously rotating frame rotates with synchronous speed. The equivalent circuits of two phase induction motor in $d-q$ axes axes are shown in figure 3.5. Electrical differential equations can be obtained from the equivalent circuit of $d-q$ axes of two-phase induction machine. Stator voltage equations of $d-q$ axes are shown below.

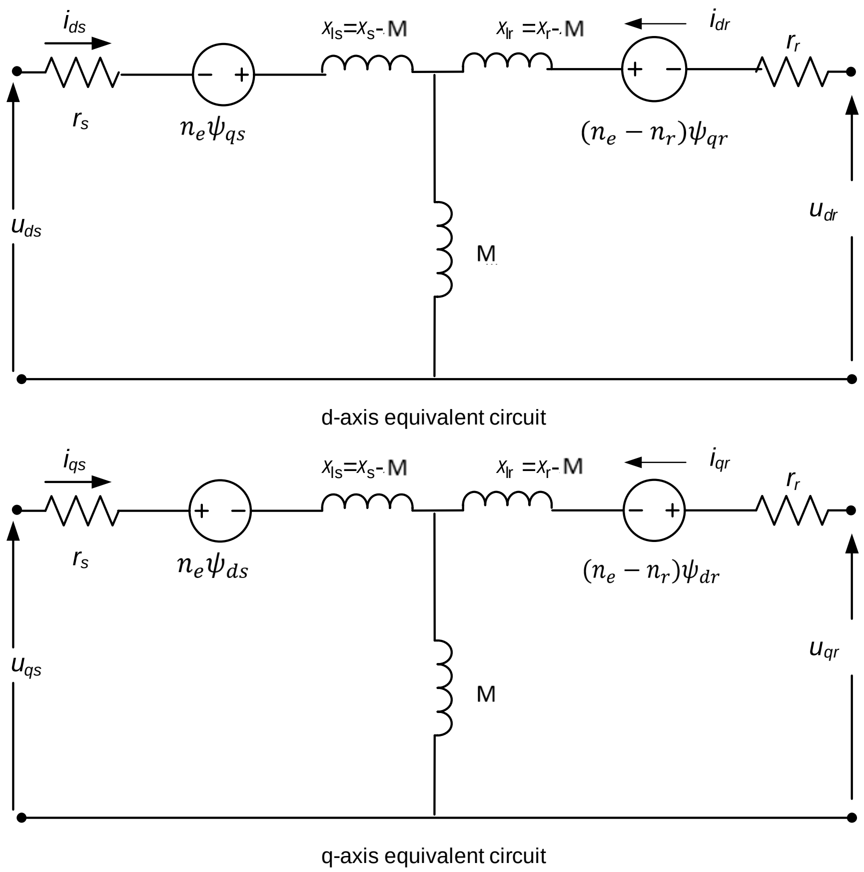

FIGURE 3.5: Dynamic $d - q$ $(2 - \phi)$ Equivalent Circuits of a $3 - \phi$ Induction motor [38].

$$u_{ds} = r_s i_{ds} + \frac{d\psi_{ds}}{dt} - n_e \psi_{qs} \tag{3.11}$$

$$u_{qs} = r_s i_{qs} + \frac{d\psi_{qs}}{dt} + n_e \psi_{ds} \tag{3.12}$$

where,

$$\psi_{qs} = \chi_{ls} i_{qs} + M(i_{qs} + i_{qr}) \tag{3.13}$$

$$\psi_{ds} = \chi_{ls} i_{ds} + M(i_{ds} + i_{dr}) \tag{3.14}$$

All the variables are rotating at synchronous speed denoted by n_e thats why they appears as dc quantities in SRRF. The stator fluxes of $d - q$ axes are denoted by ψ_{ds} and ψ_{qs}. The synchronous speed, stator resistance, magnetizing inductance and stator leakage inductance are denoted by n_e, r_s, M and χ_{ls} respectively. u_{ds}

and u_{qs} denote stator voltage of d and q axis respectively as well as inputs of the system. i_{ds} and i_{qs} denote stator current of d and q axis respectively as well as outputs of the system. i_{dr} and i_{qr} denote rotor currents of d and q axis respectively.

Since rotor of induction motor is rotating at the speed of n_r, therefore rotor $d-q$ axes move at the speed of $(n_e - n_r)$ relative to the motion of synchronously rotating reference frame. So, rotor equations are given by,

$$u_{dr} = r_r i_{dr} + \frac{d\psi_{dr}}{dt} - (n_e - n_r)\psi_{qr} \tag{3.15}$$

$$u_{qr} = r_r i_{qr} + \frac{d\psi_{qr}}{dt} + (n_e - n_r)\psi_{dr} \tag{3.16}$$

where,

$$\psi_{qr} = \chi_{lr} i_{qr} + M(i_{qs} + i_{qr}) \tag{3.17}$$

$$\psi_{dr} = \chi_{lr} i_{dr} + M(i_{ds} + i_{dr}) \tag{3.18}$$

ψ_{dr} and ψ_{qr} denote rotor flux of d and q axis respectively. r_r, n_e, n_r, M and χ_{lr} are the rotor resistance, synchronous speed, rotor speed, magnetizing inductance and rotor leakage inductance respectively. u_{dr} and u_{qr} denote rotor voltage of d and q axis respectively.

The d-q representation presented above is used to formulate model of induction machine in synchronously rotating frame. The definitions of control variables and states are presented in Table 3.1.

With the help of (3.13)- (3.18) and manipulation of these equations for the state (control) variable yields the following fourth-order electrical dynamics of the machine.

Rotor Flux Equations:

$$\frac{d\psi_{dr}}{dt} = -\frac{r_r}{\chi_r}\psi_{dr} + p(n_e - n_r)\psi_{qr} + \frac{Mr_r}{\chi_r} i_{ds} \tag{3.19}$$

$$\frac{d\psi_{qr}}{dt} = -\frac{r_r}{\chi_r}\psi_{qr} - p(n_e - n_r)\psi_{dr} + \frac{Mr_r}{\chi_r} i_{qs} \tag{3.20}$$

TABLE 3.1: State Variables Used in the Model of the Induction Machine

Symbol	**Description**	**Units**
n_r	Angular speed of Rotor	r/s
ψ_{dr}	Rotor flux of d-axis	Wb
ψ_{qr}	Rotor flux of q-axis	Wb
i_{ds}	Stator current of d-axis (state and output)	A
i_{qs}	Stator current of q-axis (state and output)	A
u_{ds}	Stator voltage of d-axis (input)	V
u_{qs}	Stator voltage of q-axis (input)	V

Stator Current Equations:

$$\frac{di_{ds}}{dt} = -\frac{(M^2 r_r + \chi_r^2 r_s)}{\sigma \chi_s \chi_r^2} i_{ds} + n_e i_{qs} + \frac{M r_r}{\sigma \chi_s \chi_r^2} \psi_{dr} + \frac{pMn_r}{\sigma \chi_s \chi_r} \psi_{qr} + \frac{1}{\sigma \chi_s} u_{ds} \quad (3.21)$$

$$\frac{di_{qs}}{dt} = -\frac{(M^2 r_r + \chi_r^2 r_s)}{\sigma \chi_s \chi_R^2} i_{qs} - n_e i_{ds} + \frac{M r_r}{\sigma \chi_s \chi_r^2} \psi_{qr} + \frac{pMn_r}{\sigma \chi_s \chi_r} \psi_{dr} + \frac{1}{\sigma \chi_s} u_{qs} \quad (3.22)$$

Equations (3.19)- (3.20) are the rotor flux and (3.21)- (3.22) are stator currents of $d-q$ axis in the SRRF.

3.3.3 d-q Model of a Three-phase Induction Machine in Stationary Reference Frame (SRF)

The synchronous speed in SRF is zero. Hence by putting $n_e = 0$ in equations obtained for the three-phase machine moving in a synchronously rotating frame, we get a model of a equivalent $2-\phi$ induction machine in SRF. The model in SRF is used to implement the observer for the efficient operation of induction machine. Therefore, corresponding stationary reference frame equations at stator can be represented as;

$$u_{ds} = r_s i_{ds} + \frac{d\psi_{ds}}{dt} \quad (3.23)$$

$$u_{qs} = r_s i_{qs} + \frac{d\psi_{qs}}{dt} \quad (3.24)$$

where,

$$\psi_{qr} = \chi_{lr} i_{qr} + M i_{qs} \tag{3.25}$$

$$\psi_{dr} = \chi_{lr} i_{dr} + M i_{ds} \tag{3.26}$$

At rotor:

$$0 = r_r i_{dr} + \frac{d\psi_{dr}}{dt} + n_r \psi_{qr} \tag{3.27}$$

$$0 = r_r i_{qr} + \frac{d\psi_{qr}}{dt} - n_r \psi_{dr} \tag{3.28}$$

where the voltage of the rotor are, $u_{dr} = u_{qr} = 0$, for squirrel cage motor.

Now, d-q representation presented above is used to formulate the induction motor model in stationary reference frame. The definition of control variables and states are presented in table 3.1.

Rotor flux equations:

Eliminating i_{dr} from (3.23) with the aid of (3.22) gives;

$$\frac{d\psi_{dr}}{dt} = -\frac{r_r}{M}\psi_{dr} - pn_r\psi_{qr} + \frac{Mr_r}{\chi_r} i_{ds} \tag{3.29}$$

Similarly, eliminating i_{qr} from (3.21) with the aid of (3.18) gives;

$$\frac{d\psi_{qr}}{dt} = -\frac{r_r}{\chi_r}\psi_{qr} + pn_r\psi_{dr} + \frac{Mr_r}{\chi_r} i_{qs} \tag{3.30}$$

To obtain the stator current equations, substituting the (3.21)- (3.22) and (3.25)- (3.26) in (3.19)- (3.20) respectively gives,

Stator current equations:

$$\frac{di_{ds}}{dt} = -\frac{(M^2 r_r + \chi_r^2 r_s)}{\sigma\chi_s\chi_r^2} i_{ds} + \frac{Mr_r}{\sigma\chi_s\chi_r^2}\psi_{dr} + \frac{pMn_r}{\sigma\chi_s\chi_r}\psi_{qr} + \frac{1}{\sigma\chi_s} u_{ds} \tag{3.31}$$

$$\frac{di_{qs}}{dt} = -\frac{(M^2 r_r + \chi_r^2 r_s)}{\sigma\chi_s\chi_r^2} i_{qs} + \frac{Mr_r}{\sigma\chi_s\chi_r^2}\psi_{qr} - \frac{pMn_r}{\sigma\chi_s\chi_r}\psi_{dr} + \frac{1}{\sigma\chi_s} u_{qs} \tag{3.32}$$

Mechanical Equations:

The speed n_r in (3.28)- (3.31) can not be generally taken as a invariable. It has relation with the torque of the machine as [38, 39].

$$\tau_e = \tau_L + J\frac{dn_M}{dt} + bn_r \tag{3.33}$$

and

$$n_M = \frac{2}{p}J\frac{dn_r}{dt} \tag{3.34}$$

Therefore, by substituting (3.34) in (3.33) yields;

$$\frac{dn_r}{dt} = \frac{p}{2J}(\tau_e - \tau_L - bn_r) \tag{3.35}$$

In (3.35), the electromagnetic generated torque (τ_e) is given as;

$$\tau_e = \frac{3}{2}(\frac{p}{2})\frac{M}{\chi_r}(\psi_{dr}i_{qs} - \psi_{qr}i_{dr}) \tag{3.36}$$

where b, J, n_M, τ_L and τ_e is the rotor damping, rotor inertia, and rotor mechanical speed, load torque and electromagnetic generated torque respectively. The (3.29)-(3.32) represent the fourth order nonlinear electrical dynamics of the three-phase induction motor in stationary $d^s - q^s$ reference frame. The (3.35) represent the mechanical dynamics of the machine which include differential equation of rotor speed. Therefore, nonlinear dynamical model of the three-phase induction motor is of fifth-order.

3.3.4 Complete Dynamics of Induction Motor

In case of induction machine, stator currents are usually taken as measured output, stator voltages are considered as input. The rotor flux is not available through measurements and can be estimated. The complete dynamics of induction motor can be given as;

Electrical Equations:

$$\frac{di_{ds}}{dt} = -\gamma i_{ds} + \frac{\beta}{\alpha}\psi_{dr} + \beta p n_r \psi_{qr} + \frac{1}{\sigma \chi_s} u_{ds} \tag{3.37}$$

$$\frac{di_{qs}}{dt} = -\gamma i_{qs} + \frac{\beta}{\alpha}\psi_{qr} - p n_r \beta \psi_{dr} + \frac{1}{\sigma \chi_s} u_{qs} \tag{3.38}$$

$$\frac{d\psi_{dr}}{dt} = -\frac{1}{\alpha}\psi_{dr} - p n_r \psi_{qr} + \frac{M}{\alpha} i_{ds} \tag{3.39}$$

$$\frac{d\psi_{qr}}{dt} = -\frac{1}{\alpha}\psi_{qr} + p n_r \phi_{dr} + \frac{M}{\alpha} i_{qs} \tag{3.40}$$

Mechanical equation:

$$\frac{dn_r}{dt} = \frac{p}{2J}(\tau_e - \tau_L - b n_r) \tag{3.41}$$

$$\tau_e = \frac{3}{2}(\frac{p}{2})\frac{M}{\chi_r}(\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) \tag{3.42}$$

Where,

$$\gamma = \frac{(M^2 r_r + \chi r^2 r_s)}{\sigma \chi_s \chi_r^2}, \quad \alpha = \frac{\chi_r}{r_r}, \quad \beta = \frac{M}{\sigma \chi_s \chi_r}, \quad \sigma = 1 - \frac{M^2}{\chi_s \chi_r}$$

Differential equation of stator currents of $d-q$ axes are given by (3.37) and (3.38) respectively. Differential equation of rotor fluxes of $d-q$ axes are given by (3.39) and (3.40) respectively. Electromagnetic torque expression is denoted by (3.41). In compact form the above model can be written as;

$$\begin{aligned} \dot{i} &= \beta \mathcal{H}(n_r)\psi - \gamma i + u/(\sigma \chi_s) \\ \dot{\psi} &= -\mathcal{H}(n_r)\psi + (\beta/\alpha) i \\ \dot{n_r} &= \frac{p}{2J}(\tau_e - \tau_L - b n_r) \end{aligned} \tag{3.43}$$

Where

$$i = (i_{ds}, i_{qs})^T, \quad \psi = (\psi_{dr}, \psi_{qr})^T \quad \text{and} \quad u = (u_{ds}, u_{qs})^T$$

$$\mathcal{H}(n_r) = \frac{1}{\alpha} I_2 - p n_r J_2 \quad \text{with} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \quad J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The differential equation of stator currents and rotor fluxes are shown by vector form in (3.43).

3.4 Model Simulation

The equivalent two-phase d-q model of three phase Induction motor is simulated using MATLAB®/Simulink. 30 KW induction motor with 380 line to line voltage and 50 Hz electrical frequency is considered. Values of parameters are shown in table 3.2. The simulation model has been developed for the machine to analyze different characteristics of induction motor such as fluxes, phase currents, angular speed and torque under different conditions.

TABLE 3.2: Parameters Values of Induction Motor

Symbol	**Description**	**Value/Units**
p	Number of pole pair	2
r_r	Rotor resistance	0.4 Ω
χ_r	Rotor inductance	0.091 H
r_s	Stator resistance	0.63 Ω
χ_s	Stator inductance	0.097 H
M	Mutual inductance	0.091 H
J	Inertia	0.22 $kg.m^2$
b	Damping	0.01 $N.m.s.r^{-1}$

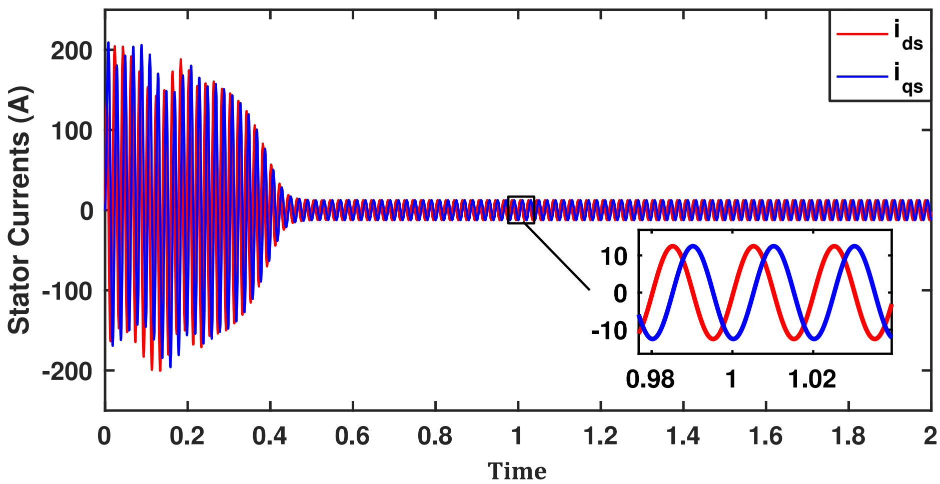

FIGURE 3.6: d-axis and q-axis Stator Currents of Induction Machine

The d-axis and q-axis stator currents are shown in figure 3.6. Stator currents are taken as output of the induction motor. d-axis and q-axis rotor flux are shown in figure 3.7. Normally rotor fluxes are not available through measurement and it needs accurately estimation for efficient control of induction motor.

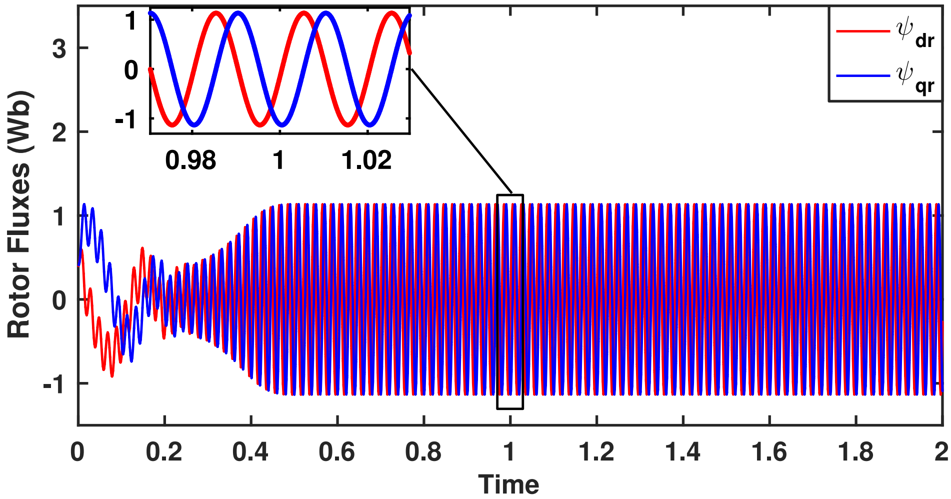

FIGURE 3.7: d-axis and q-axis Rotor Flux of Induction Machine

A positive load torque of $100Nm$ is applied at t = 2s and it becomes zero again at t = 3s in figure 3.8 to simulate presented model of induction motor and observing its motoring behavior. It can be noted that electromagnetic torque changes with respective to the load torque variations. It can be also analyzed that as load torque is applied, the rotor's speed slows down.

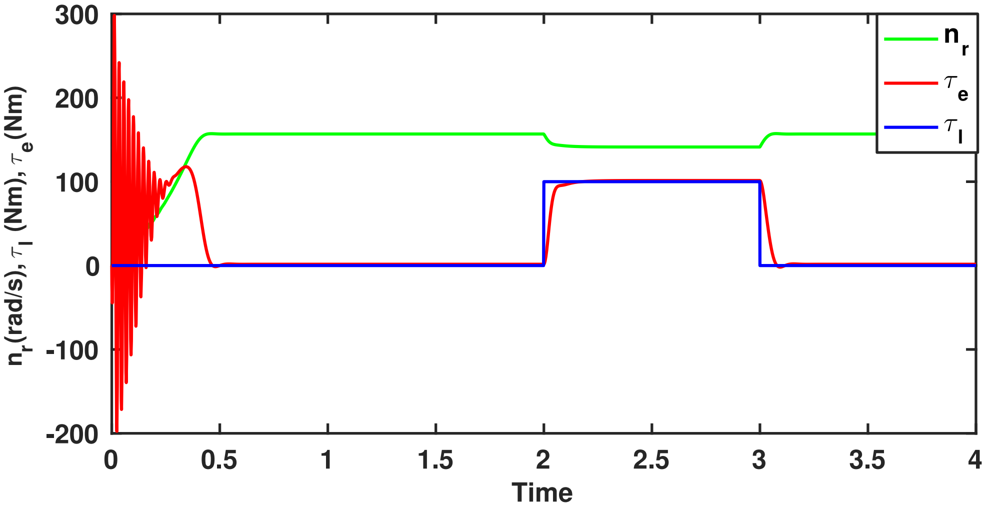

FIGURE 3.8: Induction Machine Load Torque, Electromagnetic Generated Torque and Rotor Angular Speed

The reference voltage signals are shown in figure 3.9

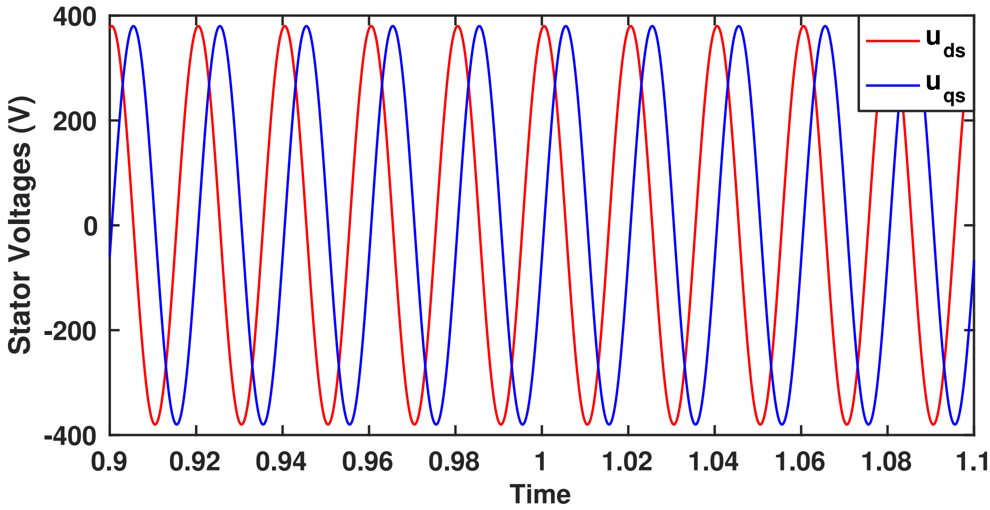

FIGURE 3.9: d-axis and q-axis Stator Voltages of Induction Motor

3.5 Chapter Summary

In this chapter, dynamics of three-phase induction motor is explained in detail. The equivalent two phase model is termed as d-q model. The d-q model both in SSRF and SRF frame is presented. The dynamic equations of rotor fluxes, stator currents and angular speed are derived. Simulation of the model is performed in MATLAB®/Simulink to show the effectiveness of the presented model.

Chapter 4

Adaptive High Gain Observer For Induction Motor

4.1 Introduction

This chapter deals with state and parameter estimation of induction motor. The estimation of rotor fluxes is very crucial for efficient control of induction machine. Since it is not an easy task to estimate rotor flux in the presence of vast variations in the parameters of an induction motor. The reason behind the variations in the parameters is operating temperature of induction motor. Rotor resistance and inductance are the most critical parameters which can vary up to 150%. Due to this reason we will also estimate the parameter of induction motor i.e rotor resistance and rotor inductance. The objective of joint estimation of state and parameter is achieved through adaptive high gain observer for uniformly observable non linear system. Three main features of the proposed observer are to be mentioned. Firstly, the gain of observer can be calibrated through single parameter. Secondly, the observer can give rise to different observers like adaptive sliding high gain observer. Thirdly, the error convergence of both state and parameter is achieved through Lyapunov function under persistence excitation of inputs.

4.2 Problem Formulation

Let consider non-linear MIMO system which is diffeomorphic to the form as shown below:

$$\begin{aligned} \dot{z} &= Az + h(z,u,\theta) + \varphi(z,u)\theta \\ y &= Cz^1 \end{aligned} \tag{4.1}$$

where

$$z = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^n \end{pmatrix}; \; z^k = \begin{pmatrix} z_1^k \\ z_2^k \\ \vdots \\ z_q^k \end{pmatrix}; \; \theta = \begin{pmatrix} \theta^1 \\ \theta^2 \\ \vdots \\ \theta^p \end{pmatrix}; \; h(z,u,\theta) = \begin{pmatrix} h^1(z^1,u,\theta) \\ h^2(z^1,z^2,u,\theta) \\ \vdots \\ h^{n-1}(z^1,\cdots,z^{n-1},u,\theta) \\ h^n(z,u,\theta) \end{pmatrix}$$

$$\varphi^T(z,u) = \begin{pmatrix} \varphi_1^T(z,u) \\ \varphi_2^T(z,u) \\ \vdots \\ \varphi_p^T(z,u) \end{pmatrix}; \; \varphi_j(z,u) = \begin{pmatrix} \varphi_j^1(z^1,u) \\ \varphi_j^2(z^1,z^2,u) \\ \vdots \\ \varphi_j^{n-1}(z^1,\cdots,z^{n-1},u) \\ \varphi_j^n(z,u) \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & I_{(n-1)q} \\ 0 & 0 \end{bmatrix}; \; C = [I_q, 0_q, \cdots, 0_q]$$

where $y \in \mathbb{R}^q$ is the output; $z \in \mathbb{R}^s$ with $z^k \in \mathbb{R}^n$ and $z_j^k \in \mathbb{R}$, $k = 1, \cdots, n$ and $j = 1, \cdots, q$ is state vector; $u \in \mathbb{R}^t$ is input; $\theta \in \mathbb{R}^p$ is unknown parameter vector, $h(z,u,\theta) \in \mathbb{R}^s$ with $h^k(z,u,\theta) \in \mathbb{R}^s$, $k = 1. \cdots, n$; $\varphi(z,u)$ is a $s \times p$ matrix and each $\varphi_j \in \mathbb{R}^s$, $j = 1, \cdots, p$ denotes its jth column with $\varphi_j^k(z,u) \in \mathbb{R}^n$, $k = 1, \cdots, n$. The notation I_h and 0_h denotes the identity and null matrix respectively where h is positive integer and denotes $h \times h$ matrix.

The system (4.1) must be uniformly observable when parameter vector θ is known. This is a necessary condition for the observer design. The condition of persistently exciting inputs is also require when parameter vector θ is unknown which is discussed later.

4.2.1 Assumptions

The observer requires following assumptions must be satisfied by the system.

A-1: $u(t)$, $z(t)$ and missing parameters θ are bounded for $t \geq 0$. let $z(t) \in Z \subset \mathbb{R}^s$, $u(t) \in U \subset \mathbb{R}^t$, and $\theta \in \Omega \subset \mathbb{R}^p$ for $t \geq 0$ in compact sets Z, U and Ω.

A-2: The function $\varphi(z, u)$ are of class C^1 with respective to z and function $h(z, u, \theta)$ are of class C^1 with respective to z and θ. The functions $\varphi(z, u)$ and $h(z, u, \theta)$ are piecewise continuous with respective to u. The input $u(t)$ is piecewise continuous.

4.2.2 Definition and Notations

Some definitions and notations related to the high gain observer are given below.

- Δ_ε is the diagonal gain matrix which is define as follow:

$$\Delta_\varepsilon = diag[I_q, \frac{1}{\varepsilon} I_q, \cdots, \frac{1}{\varepsilon^{n-1}} I_q] \tag{4.2}$$

where ε is a positive scalar.

- Following identities must be satisfied by the system:

$$\Delta_\varepsilon A \Delta_\varepsilon^{-1} = \varepsilon A, \qquad C\Delta_\varepsilon^{-1} = C \tag{4.3}$$

- Let S be a symmetric and positive definite matrix which will satisfy the following Lyapunov equation:

$$\varepsilon S + A^T S + SA - C^T C = 0 \tag{4.4}$$

and ε is a positive constant, which is kept high to overcome system bounds. The explicit solution of (4.4) is given as:

$$S(i,j) = \frac{(-1)^{i+j} C_{i+j-2}^{j-1}}{\varepsilon^{i+j-1}} \quad \text{for} \quad 1 \leq i,j \leq s, \\ C_n^r = \frac{n!}{(n-r)!r!} \tag{4.5}$$

S is SPD matrix for every $\varepsilon > 0$. see [30].

- The p characteristic indices ζ_j is associated to each unknown parameters θ_j. The characteristic index ζ_j is equal to smallest positive integer i such that $\frac{\partial \dot{z}_i}{\partial \theta_j} \neq 0$ and the following property holds:

$$\frac{\partial \dot{z}_i}{\partial \theta_j} = 0 \ \text{ for } \ i = 1, ..., \zeta_{j-1} \ \text{ and } \ \frac{\partial \dot{z}_{\zeta_j}}{\partial \theta_j} \neq 0$$

Let each entry of $\varphi(z,u)$ is located at $\varphi_i^j(z,u)$ where i denotes the row and j denoted the column. According to the system (4.1), one can show that $\frac{\partial \dot{z}_i(t)}{\partial \theta_j} = \varphi_i^j(z,u)$ and entries of the matrix $\varphi(z,u)$ satisfy following property:

$$\varphi_i^j(z,u) = 0 \ \text{ if } \ i \leq \zeta_j - 1 \\ \varphi_{\zeta_j}^j(z,u) \neq 0 \quad \text{otherwise}$$

Now define Ω_ε be $p \times p$ diagonal matrix:

$$\Omega_\varepsilon = diag[1/\varepsilon^{\zeta_1}, 1/\varepsilon^{\zeta_2}, ..., 1/\varepsilon^{\zeta_p}] \tag{4.6}$$

Set $J(z,u) \cong \Delta_\varepsilon \varphi(z,u)\Omega_\varepsilon^{-1}$ and from (4.2) and (4.6) we get:

$$J_i^j(z,u) = \varepsilon^{-(i-1)} \varphi_i^j(z,u) \varepsilon^{\zeta_j} = \varepsilon^{\zeta_j - i + 1} \varphi_i^j(z,u)$$

The following decomposition of $J(z,u)$ can be hold:

$$\Delta_\varepsilon \varphi(z,u)\Omega_\varepsilon^{-1} = \varepsilon \Phi(z,u) + W(z,u,1/\varepsilon)$$

where $\Phi(z,u)$ and $W(z,u,1/\varepsilon)$ are rectangular matrices both of order $s \times p$. Their respective entries $\Phi_i^j(z,u)$ and $W_i^j(z,u,1/\varepsilon)$ where $i \in [1,s]$ and $j \in [1,p]$ can be defined as follows:

$$\begin{cases} \Phi_i^j(z,u) = 0 \ \text{ if } \ i \neq \zeta_j \\ \Phi_{\zeta_j}^j(z,u) = \varphi_{\zeta_j}^j(z,u) \ \text{ otherwise.} \end{cases}$$
$$\begin{cases} W_i^j(z,u) = 0 \ \text{ if } \ i \leq \zeta_j \\ W_i^j(z,u,1/\varepsilon) = 1/\varepsilon^{i-1-\zeta_j}\, \varphi_i^j(z,u) \ \text{ otherwise.} \end{cases} \tag{4.7}$$

From (4.7) it can be shown that:

$$\begin{aligned} &\Delta_\varepsilon \varphi(z,u)\Omega_\varepsilon^{-1} = \varepsilon\Phi(z,u) \quad \text{and} \\ &W_1^j(z,u,1/\varepsilon) = 0 \quad \text{for} \quad j = 1 \cdots p \end{aligned} \tag{4.8}$$

where ε appears only with nagative powers in $W(z,u,1/\varepsilon)$.

4.3 Observer Design

The design of adaptive high gain observer is explained in this section.

4.3.1 Proposed Adaptive High Gain Observer

The dynamical system, which is shown below, is an appropriate candidate for high gain adaptive observer, which will be discussed later in the theorem.

$$\begin{aligned} \dot{\hat{z}}(t) &= A\hat{z}(t) + h(\hat{z}(t),u(t),\hat{\theta}(t)) + \varphi(\hat{z}(t),u(t))\hat{\theta}(t) \\ &\quad - \varepsilon\Delta_\varepsilon^{-1}(S^{-1}C^T + \Gamma(t)\Lambda(t)\Gamma^T(t)C^T)(C\hat{z}(t) - y(t)) \\ \dot{\hat{\theta}}(t) &= -\varepsilon\, \Omega_\varepsilon^{-1}\Lambda(t)\Gamma^T(t)C^T(C\hat{z}(t) - y(t)) \end{aligned} \tag{4.9}$$

$$\dot{\Gamma}(t) = \varepsilon(A - S^{-1}C^TC)\Gamma(t) + \varepsilon\Phi(\hat{z}(t),u(t)) \quad \text{with} \quad \Gamma(0) = 0 \tag{4.10}$$

$$\dot{\Lambda}(t) = -\varepsilon\Lambda(t)\Gamma^T(t)C^TC\Gamma(t)\Lambda(t) + \varepsilon\Lambda(t) \quad \text{with} \quad \Lambda(0) = \Lambda^T(0) \geq 0 \tag{4.11}$$

where $\hat{z} \in \mathbb{R}^s$ with $\hat{z}^k \in \mathbb{R}^n$ and $\hat{z}_j^k \in \mathbb{R}$, $k = 1, \cdots, n$ and $j = 1, \cdots, q$ denote the estimate of state; $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_p) \in \mathbb{R}^p$ denote the estimate of parameter; y and u are respectively output and input to a system.

Comments: $\Gamma(t) \in \mathbb{R}^{s\times p}$ and $\Lambda(t) \in \mathbb{R}^{p\times p}$ are used to balance the convergence speed of state and parameter estimation error. The parameter estimation dynamics has firstly given in [25], where the author suggested an adaptive observer for LTV MIMO systems. The parameter dynamics involve an adaptation law that was motivated from classic recursive least square algorithm. The same adaptation law is derived using high gain design in [33]. Using the same approach of [33], it can be shown that dynamics of observer (4.9) can be obtained through a high gain approach by using augmented model approach having original system combine with unknown dynamics of parameter i.e. $\dot{\rho}(t) = 0$

4.3.2 An Assumption on Input of a System: Persistently Exciting

The assumption explained below defines those admissible input signals which make the state and parameter estimation possible.

A-3: The input u should be such that for every trajectory $\hat{z}$ of a system (4.9) starting from $(\hat{z}(0), \hat{\theta}(0)) \in Z \times \Omega$, the matrix $C\Gamma(t)$ should be persistently exciting, i.e

$\exists \eta_1, \eta_2 > 0;\ \forall T > 0;\ \forall t \geq 0$:

$$\eta_1 I_m \leq \int_t^{t+T} \Gamma^T(\tau) C^T C \Gamma(\tau) d\tau \leq \eta_2 I_m \tag{4.12}$$

The above inequality gives persistent excitation condition stated in [40]. However this assumption does not give us information about generating such input u. The problem of generating such persistently exciting input is still an open problem for nonlinear systems while some particular cases exists for linear systems.

4.3.3 Theorm 4.1

Consider the system (4.1) is subjected to assumptions A-1 and A-2, then for any bounded input which will satisfy assumption A-3, a constant ε_0 will exist, such that for every $\varepsilon > \varepsilon_0$, system (4.9) is adaptive high gain observer for system (4.1) having exponential convergence of error to the origin for values of ε chosen sufficiently high i.e for any initial conditions $(\theta(t_0), \hat{\theta}(t_0)) \in \Omega^2$, $(z(t_0), \hat{z}(t_0)) \in Z^2$, the errors $\hat{\theta}(t) - \theta$ and $\hat{z}(t) - z(t)$ exponentially goes to zero as $t \to \infty$. [36]

Comments: The observer dynamics are given below.

$$\begin{aligned}\dot{\hat{z}}(t) &= A\hat{z}(t) + h(\hat{z}(t), u(t), \hat{\theta}(t)) + \varphi(\hat{z}(t), u(t))\hat{\theta}(t) \\ &- \varepsilon\Delta_\varepsilon^{-1} S^{-1} C^T (C\tilde{z}) + \Delta_\varepsilon^{-1}\Gamma(t)\Omega_\varepsilon\dot{\hat{\theta}}(t)\end{aligned}$$

The equation of observer dynamics contains a copy of system (4.1) and two corrective terms. The first one is $\varepsilon\Delta_\varepsilon^{-1} S^{-1} C^T (C\tilde{z})$ and met with classical high gain observer [30] while the second term, $\Delta_\varepsilon^{-1}\Gamma(t)\Omega_\varepsilon\dot{\hat{\theta}}(t)$ is inspired from adaptive observer in [25], where the similar expression is used for updating unknown parameters.

4.3.4 Convergence Proof

Set $e(t) = \hat{z}(t) - z(t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. From (4.1) and (4.9) we get:

$$\begin{aligned}\dot{e}(t) &= Ae(t) - \varepsilon\Delta_\varepsilon^{-1} S^{-1} C^T (Ce(t)) - \Delta_\varepsilon^{-1}\Gamma(t)\Omega_\varepsilon\dot{\hat{\theta}}(t) \\ &+ [h(\hat{z}, u, \hat{\theta}) - h(z, u, \theta)] + [\varphi(\hat{z}, u)\hat{\theta} - \varphi(z, u)\theta]\end{aligned} \tag{4.13}$$

$$\dot{\hat{\theta}}(t) = -\varepsilon\Omega_\varepsilon^{-1}\Lambda(t)\Gamma^T(t)C^T(Ce(t)) \tag{4.14}$$

Add and subtract $h(\hat{z}, u, \theta)$ and $\varphi(\hat{z}, u)\theta$ to (4.13).

$$\begin{aligned}\dot{e}(t) &= Ae(t) - \varepsilon\Delta_\varepsilon^{-1} S^{-1} C^T (Ce(t)) - \Delta_\varepsilon^{-1}\Gamma(t)\Omega_\varepsilon\dot{\hat{\theta}}(t) \\ &+ [h(\hat{z}, u, \hat{\theta}) - h(\hat{z}, u, \theta)] + [h(\hat{z}, u, \theta) - h(z, u, \theta)] \\ &+ [\varphi(\hat{z}, u) - \varphi(z, u)]\theta + \varphi(\hat{z}, u)\tilde{\theta}(t)\end{aligned} \tag{4.15}$$

Set $\bar{e} = \Delta_\varepsilon e$ and $\bar{\theta} = \Omega_\varepsilon \tilde{\theta}$ (t is excluded for convenience). Using identities (4.3) and decomposition (4.8) of $\Delta_\varepsilon \varphi \Omega_\varepsilon^{-1}$ under the form we get:

$$\begin{aligned}
\dot{\bar{e}} &= \varepsilon(A - S^{-1}C^T C)\bar{e} + \Gamma\dot{\bar{\theta}} + \Delta_\varepsilon \tilde{h}_1(z, \hat{z}, u, \theta) + \Delta_\varepsilon \tilde{h}_2(\hat{z}, u, \theta, \hat{\theta}) \\
&\quad + \Delta_\varepsilon \tilde{\varphi}(z, \hat{z}, u)\theta + \varepsilon\Phi(\hat{z}, u)\bar{\theta} + W(\hat{z}, u, 1/\varepsilon)\bar{\theta} \\
\dot{\bar{\theta}} &= -\varepsilon\Lambda\Gamma^T C^T (C\bar{e})
\end{aligned} \tag{4.16}$$

Where $\tilde{h}_1(z, \hat{z}, u, \theta) = h(\hat{z}, u, \theta)) - h(z, u, \theta)$; $\tilde{h}_2(\hat{z}, u, \theta, \hat{\theta}) = h(\hat{z}, u, \hat{\theta})) - h(\hat{z}, u, \theta)$ and $\tilde{\varphi}(z, \hat{z}, u) = \varphi(\hat{z}, u) - \varphi(z, u)$.

Now set $\xi(t) = \bar{e}(t) - \Gamma(t)\bar{\theta}(t)$. The $\Gamma(t)$ is shown by the ODE in (4.10), so we get:

$$\begin{aligned}
\dot{\xi} &= \varepsilon(A - S^{-1}C^T C)\xi + \Delta_\varepsilon \tilde{h}_1(z, \hat{z}, u, \theta) + \Delta_\varepsilon \tilde{h}_2(\hat{z}, u, \theta, \hat{\theta}) \\
&\quad + W(\hat{z}, u, 1/\varepsilon)\bar{\theta} + \Delta_\varepsilon \tilde{\varphi}(z, \hat{z}, u)\theta
\end{aligned} \tag{4.17}$$

Set $V_1(\xi(t)) = \xi^T(t)S\xi(t)$ and $V_2(\bar{\theta}(t)) = \bar{\theta}^T(t)\Lambda^{-1}(t)\bar{\theta}(t)$, where S and Λ^{-1} are shown in (4.4) and (4.11).The Lyapunov function will become:

$$V(\xi(t), \bar{\theta}(t)) = V_1(\xi(t)) + V_2(\bar{\theta}(t)) \tag{4.18}$$

Deriving time derivative of $V_1(\xi(t)) = \xi^T(t)S\xi(t)$ and using (4.4), one gets:

$$\begin{aligned}
\dot{V}_1(\xi) &= -\varepsilon V_1(\xi) - \varepsilon\xi^T C^T C\xi + 2\xi^T S\Delta_\varepsilon \tilde{h}_1(z, \hat{z}, u, \theta) + 2\xi^T S\Delta_\varepsilon \tilde{h}_2(\hat{z}, u, \theta, \hat{\theta}) \\
&\quad + 2\xi^T SW(\hat{z}, u, 1/\varepsilon)\bar{\theta} + 2\xi^T S\Delta_\varepsilon \tilde{\varphi}(u, z, \hat{z})\theta
\end{aligned} \tag{4.19}$$

According to the definition of ξ:

$$\|\bar{e}\| \leq \|\xi\| + \|\Gamma(t)\|\|\bar{\theta}\| \leq \|\xi\| + \gamma_M\|\bar{\theta}\| \tag{4.20}$$

with $\gamma_M = \sup_{t\geq 0}\|\Gamma(t)\|$

Using mean value theorm, one gets:

$$\Delta_\varepsilon \tilde{h}_1(z, \hat{z}, u, \theta) = \Delta_\varepsilon \frac{\partial \tilde{h}_1}{\partial z}(z, \hat{z}, u, \theta)(\hat{z} - z) \tag{4.21}$$

$$\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) = \Delta_\varepsilon \frac{\partial \tilde{h}_1}{\partial z}(z,\hat{z},u,\theta)\Delta_\varepsilon^{-1}\bar{e} \tag{4.22}$$

Also,

$$\|\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta)\| \leq \|\Delta_\varepsilon \frac{\partial \tilde{h}_1}{\partial z}(z,\hat{z},u,\theta)\Delta_\varepsilon^{-1}\|\|\bar{e}\| \tag{4.23}$$

$$\text{where} \quad b_1 = \|\Delta_\varepsilon \frac{\partial \tilde{h}_1}{\partial z}(z,\hat{z},u,\theta)\Delta_\varepsilon^{-1}\|$$

$$\|\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta)\| \leq b_1\|\bar{e}\| \tag{4.24}$$

Using (4.20) and (4.24) we get:

$$\|\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta)\| \leq b_1\|\xi\| + b_1\gamma_M\|\bar{\theta}\| \tag{4.25}$$

$$\text{Where} \quad b_2 = b_1\gamma_M$$

$$\|\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta)\| \leq b_1\|\xi\| + b_2\|\bar{\theta}\| \tag{4.26}$$

$$2\xi^T S\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) \leq 2\|S\|\|\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta)\|\|\xi\| \tag{4.27}$$

Putting (4.26) in (4.27), we get:

$$2\xi^T S\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) \leq 2b_1\|S\|\|\xi\|^2 + 2b_2\|S\|\|\xi\|\|\bar{\theta}\| \tag{4.28}$$

$$\text{Where} \quad b_3 = 2b_1\|S\|, \quad b_4 = 2b_2\|S\|$$

,

$$2\xi^T S\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) \leq b_3\|\xi\|^2 + b_4\|\xi\|\|\bar{\theta}\| \tag{4.29}$$

$$2\xi^T S\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) \leq \frac{b_3}{\lambda_{min}(S)}V_1 + \frac{b_4}{(\sqrt{\lambda_{min}(S)\lambda_{min}(\Lambda)})}\sqrt{V_1}\sqrt{V_2} \tag{4.30}$$

$$\text{Where} \quad k_1 = \frac{b_3}{\lambda_{min}(S)} \quad \text{and} \quad c_1 = \frac{b_4}{(\sqrt{\lambda_{min}(S)\lambda_{min}(\Lambda)})}$$

,

$$2\xi^T S\Delta_\varepsilon \tilde{h}_1(z,\hat{z},u,\theta) \leq k_1V_1 + c_1\sqrt{V_1}\sqrt{V_2} \tag{4.31}$$

Where $b_1 = \|\Delta_\varepsilon \frac{\partial \tilde{h}_1}{\partial z}(z,\hat{z},u,\theta)\Delta_\varepsilon^{-1}\|$, $b_2 = b_1\gamma_M$, $b_3 = 2b_1\|S\|$, $b_4 = 2b_2\|S\|$, $k_1 = \frac{b_3}{\lambda_{min}(S)}$ and $c_1 = \frac{b_4}{(\sqrt{\lambda_{min}(S)\lambda_{min}(\Lambda)})}$, are positive scalars which do not depend on ε for $\varepsilon \geq 1$. $\lambda_{min}(.)$ denotes minimum eigen value of $(.)$.

In the same way,

$$2\xi^T S\Delta_\varepsilon\tilde{\varphi}(z,\hat{z},u)\theta \leq k_2V_1 + c_2\sqrt{V_1}\sqrt{V_2}$$
$$2\xi^T SW(\hat{z},u,1/\varepsilon)\bar{\theta} \leq c_3\sqrt{V_1}\sqrt{V_2}$$
$$2\xi^T S\Delta_\varepsilon\tilde{h}_2(\hat{z},u,\theta,\hat{\theta}) \leq c_4\sqrt{V_1}\sqrt{V_2}$$

Where k_2, c_2, c_3 and c_4 are positive scalar which do not depend on ϵ_0 for $\epsilon > 1$. From the above developments, we get:

$$\begin{aligned}\dot{V}_1(\xi) = &-\varepsilon V_1 - \varepsilon\xi^T C^T C\xi + k_1V_1 + c_1\sqrt{V_1}\sqrt{V_2} + k_2V_1 \\ &+ c_2\sqrt{V_1}\sqrt{V_2} + c_3\sqrt{V_1}\sqrt{V_2} + c_4\sqrt{V_1}\sqrt{V_2}\end{aligned} \tag{4.32}$$

Where $k = k_1 + k_2$ and $c = c_1 + c_2 + c_3 + c_4$.

$$\dot{V}_1(\xi) = -\varepsilon V_1 - \varepsilon\xi^T C^T C\xi + kV_1 + c\sqrt{V_1}\sqrt{V_2} \tag{4.33}$$

$$\dot{V}_1(\xi) \leq -(\varepsilon - k)V_1 - \varepsilon\xi^T C^T C\xi + c\sqrt{V_1}\sqrt{V_2}$$

The time derivative of $V_2(\bar{\theta}(t)) = \bar{\theta}^T(t)\Lambda^{-1}(t)\bar{\theta}(t)$ becomes:

$$\dot{V}_2(\bar{\theta}) = -\varepsilon V_2 - \varepsilon\bar{\theta}^T\Gamma^T C^T C\Gamma\bar{\theta} - 2\varepsilon\bar{\theta}^T\Gamma^T C^T C\xi$$

Using the Lyapunov function shown in (4.18), we get:

$$\begin{aligned}\dot{V}(t) &\leq -(\varepsilon - k)(V_1 + V_2) + (c/2)(V_1 + V_2) \\ &= -(\varepsilon - k - c/2)V\end{aligned}$$

By selecting $\varepsilon_0 = k + c/2$ required by the theorem will ends the proof. The bound of error can be made smaller by selecting large ε.

4.4 Example: Induction Motor

It is not an easy task to estimate rotor flux in presence of excessive variations in the parameters of induction motor. Among various parameters rotor resistance and inductance are the critical parameters and needs to be estimated simultaneously with non-measured state variables i.e. rotor fluxes. These parameters gives essential knowledge about the rotor flux. Therefore adaptive high gain observer is used for simultaneous estimation of rotor flux along with rotor resistance and inductance from measurements of stator voltages, stator currents and angular rotor speed. Dynamics of induction motor in SRF is given in (3.43). The dynamics should be transformed to such canonical form where unknown parameters is simplified to some extent. Before going towards transformation we analyze the observability property of induction machine.

4.4.1 Observability Property of Induction Machine

In [41] the observability property of induction machine is explained in detail. It has been shown that using stator current and speed measurements, the dynamics induction motor is observable. In case of sensor less control of induction motor (i.e using only stator current measurements), the observability rank condition is not satisfied for the following cases.

Case I: $n_r = 0$

Case II: $n_e = 0$

Case III: $\psi_{dr} = \psi_{qr} = n_e = 0$

Case IV: $\psi_{dr} = \psi_{qr} = n_e = n_r = 0$

In this work the observer uses the measurements of stator voltages, stator currents and angular rotor speed. The observer is designed upto an injections of the rotor speed measurements so that only electrical dynamical equations are considered [30].

4.4.2 Transformation

Let transform the system (3.43) to z-coordinates such that $z = T(i, \psi)$ and T must be invertible for all $z \in T(D)$ where D is the domain of T.

$$z = T(i, \psi) = \begin{bmatrix} i \\ \beta \mathcal{H}(n_r)\psi \end{bmatrix}$$

The above transformation puts the system (3.43) into a form given below:

$$\begin{aligned} \dot{z}^{(1)} &= z^{(2)} - \gamma z^{(1)} + u/(\sigma \chi_s) \\ \dot{z}^{(2)} &= pJ_2(z^{(2)} - (\beta M/\alpha) z^{(1)}) n_r - 1/\alpha (z^{(2)} - (\beta M/\alpha) z^{(1)}) + \varepsilon^{(2)} \\ y &= z^{(1)} = (i_{ds}, i_{qs})^T \quad\quad u = (u_{ds}, u_{qs})^T \end{aligned}$$

Where the function $\varepsilon^{(2)} : t \in \mathbb{R} \rightarrow \varepsilon^{(2)}(t) = -p\beta J_2 \psi \dot{n}_r$. This term can be considered as unknown bounded function and is assumed negligible under the following condition [14]:

$$\frac{dn_r}{dt} << \frac{2|n_r|}{\alpha}$$

Rotor flux is recovered by following inverse transformation:

$$\psi = \frac{1}{\beta} \mathcal{H}^{-1}(n_r) z^{(2)} = \frac{1}{\beta} \frac{(\frac{1}{\alpha} I_2 + pn_r J_2) z^{(2)}}{(\frac{1}{\alpha})^2 + (pn_r)^2} \tag{4.34}$$

4.4.3 Estimation of States and Parameter through Adaptive High Gain Observer

Now the χ_s, r_s and M are known parameters while r_r and χ_r are considered to be unknown and need to be estimated simultaneously with non-measured state variable ψ. The values of parameters are shown in table 3.2. The input sequence is set as $u = (u_{ds}, u_{qs})^T$ with $u_{ds} = 180cos(100\pi t)$ and $u_{qs} = 80sin(100\pi t)$
Set $\theta_1 = \gamma$ and $\theta_2 = 1/\sigma \chi_s$. We can express χ_r and r_r as a function of θ_1 and θ_2 and known parameters;

$$\chi_r = \frac{M^2 \theta_2}{\chi_s \theta_2 - 1} \quad\quad \text{and} \quad\quad r_r = M^2 \theta_2 \frac{\theta_1 - r_s \theta_2}{(\chi_s \theta_2 - 1)^2};$$

$$s = 4, \;\; n = q = p = 2$$

$$A = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix}; \quad \Phi(z,u) = \begin{bmatrix} -z^{(1)} & u \\ 0_{2\times 1} & 0_{2\times 1} \end{bmatrix}; \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ 1/\sigma\chi_s \end{bmatrix}$$

$$h(z,u,\theta) = \begin{bmatrix} 0_{2\times 1} \\ pJ_2(z^{(2)} - (\beta M/\alpha)z^{(1)})n_r - 1/\alpha(z^{(2)} - (\beta M/\alpha)z^{(1)}) \end{bmatrix}$$

$$\Delta_\varepsilon = diag[I_2, \frac{1}{\epsilon}I_2] \qquad \Omega_\varepsilon = diag[\frac{1}{\epsilon}, \frac{1}{\epsilon}]$$

$$\Gamma(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \Lambda(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Basically the observer estimate the $\theta_1 = \gamma$ and $\theta_2 = 1/\sigma\chi_s$. But as shown above the rotor resistance and inductance can be represented as a function of these estimated and known parameters.

4.4.4 Results and Discussion

Simulation results shows an excellent performance of adaptive high gain observer. The observer has exactly estimated rotor flux along with rotor resistance and rotor inductance. Three cases are shown below:

4.4.4.1 Case 1: Constant Parameters

The rotor flux of both $d-$axis and $q-$axis are shown in figure 4.1. The estimation error in figure 4.2 goes to zero after $0.2s$, shows the effectiveness of observer. Figure 4.3 shows the parameters of induction motor estimated by observer. The value of rotor resistance and rotor inductance is set as 0.4Ω and $0.091H$ respectively and the observer has exactly estimated the required parameters.

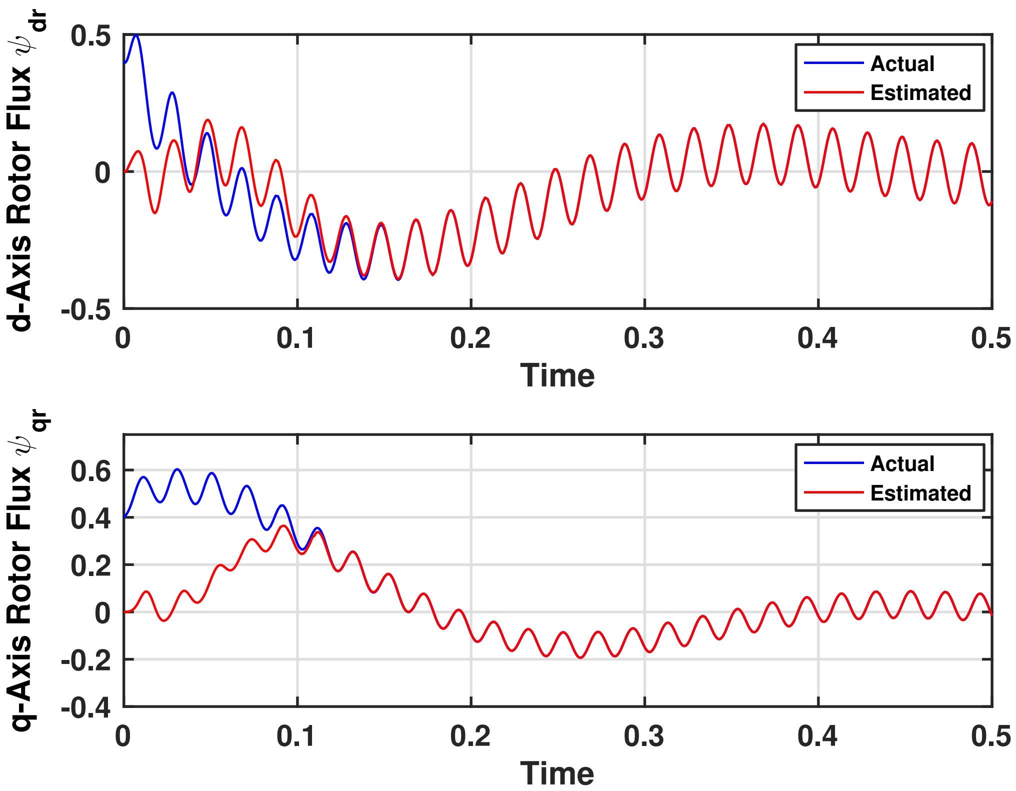

FIGURE 4.1: Estimation of d-axis (top) and q-axis (bottom) Rotor Flux

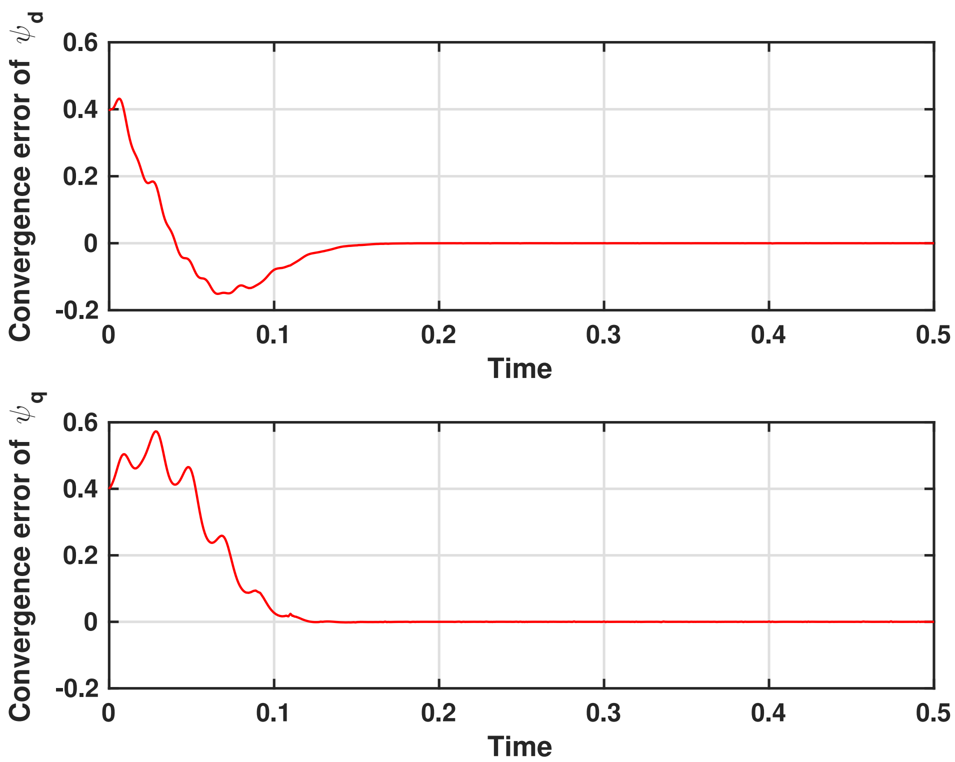

FIGURE 4.2: Convergence Error of d-axis (top) and q-axis (bottom) Rotor Flux

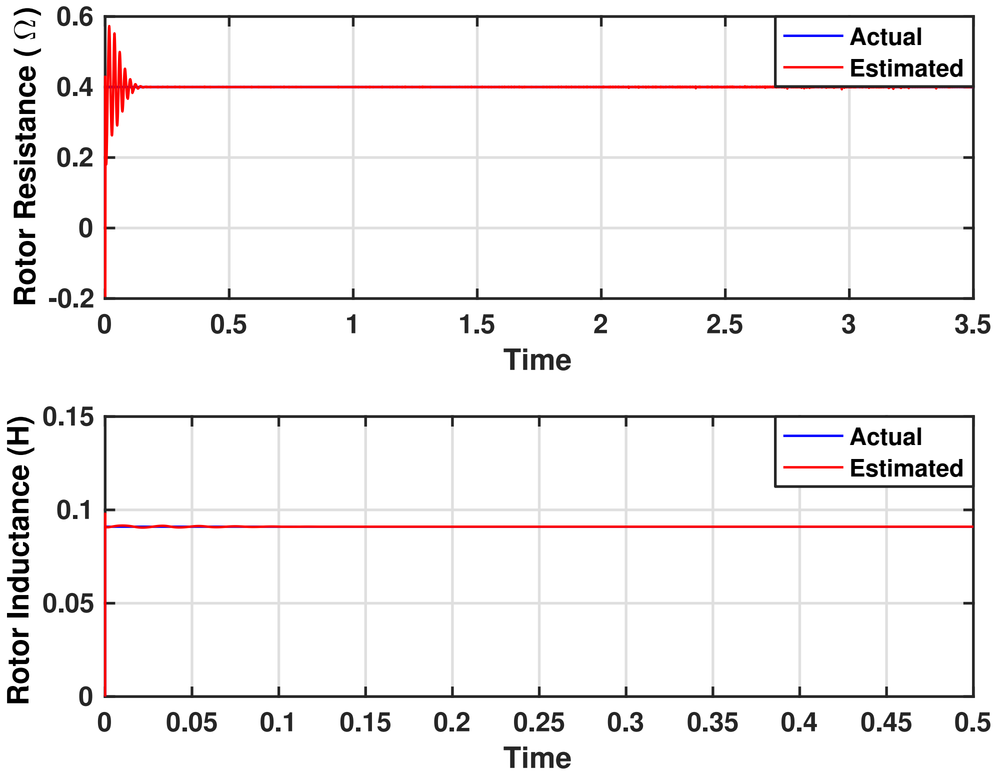

FIGURE 4.3: Estimated Parameters: Rotor Resistance (top) and Rotor Inductance (bottom)

4.4.4.2 Case 2: Time Varying Parameter

In this case, the value of rotor resistance is varied at certain points.

$$r_r = \begin{cases} 0.4 & 0 \leq t \leq 1 \\ 0.8 & 1 \leq t \leq 2 \\ 1.2 & 2 \leq t \leq 3 \\ 0.6 & otherwise \end{cases} \tag{4.35}$$

Figure 4.4 shows, that the observer has exactly estimated the time-varying rotor resistance. The spikes in the error convergence of rotor flux is due to abrupt variation in the rotor resistance. The rotor resistance estimated by the observer exactly followed the actual parameter despite changing at different time instants.

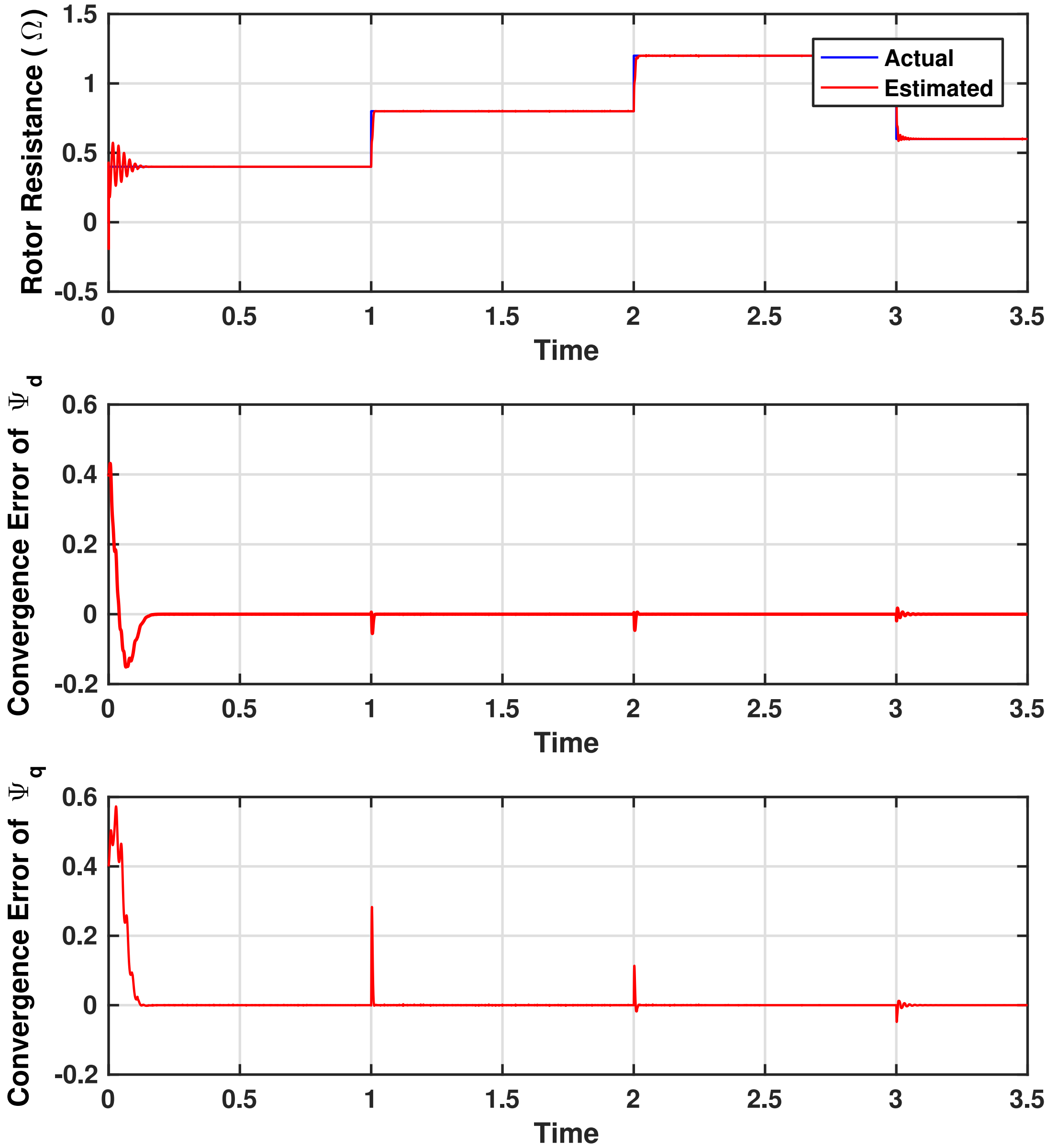

FIGURE 4.4: Rotor Resistance (top), Convergence Error of d-axis Rotor Flux (middle), Convergence Error of q-axis Rotor Flux (bottom)

4.4.4.3 Case 3: Fault Detection

A fault is generated at $t = 2.5s$ by making the rotor resistance equal to zero (short circuit).

$$r_r = \begin{cases} 0.4 & 0 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2.5 \\ 0 & otherwise \end{cases} \quad (4.36)$$

The short circuit of rotor circuit can damage the induction motor. The proposed observer successfully detects and estimates the parameter fault as shown in figure 4.5.

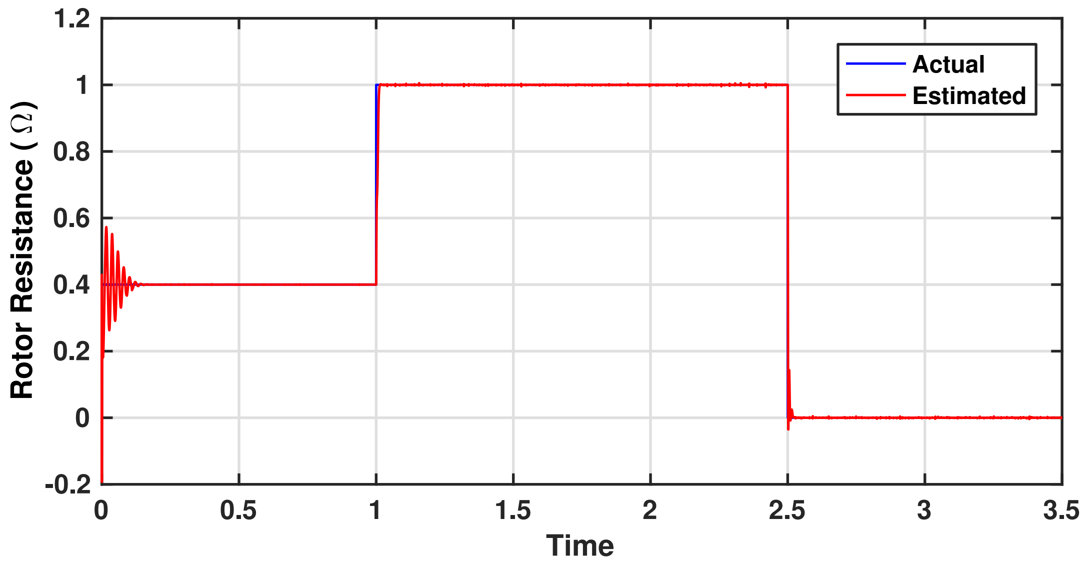

FIGURE 4.5: Estimated Rotor Resistance: Fault (short circuit) Generated at $t = 2.5s$

4.5 Comparative Study of Adaptive High Gain Observer and Adaptive Observer

Simulation results of state and parameters estimates of induction motor through adaptive high gain observer is compared with adaptive observer proposed by Qinghua Zhang in [25]. In figure 4.6 and 4.8 adaptive high gain observer is applied to estimate rotor flux and parameters of induction motor respectively. In fiugre 4.7 and 4.9 adaptive oberver is applied to estimate rotor flux and parameters of induction motor respectively. The results infer that the convergence of state and parameter estimation error using adaptive high gain observer is faster as compared to adaptive observer.

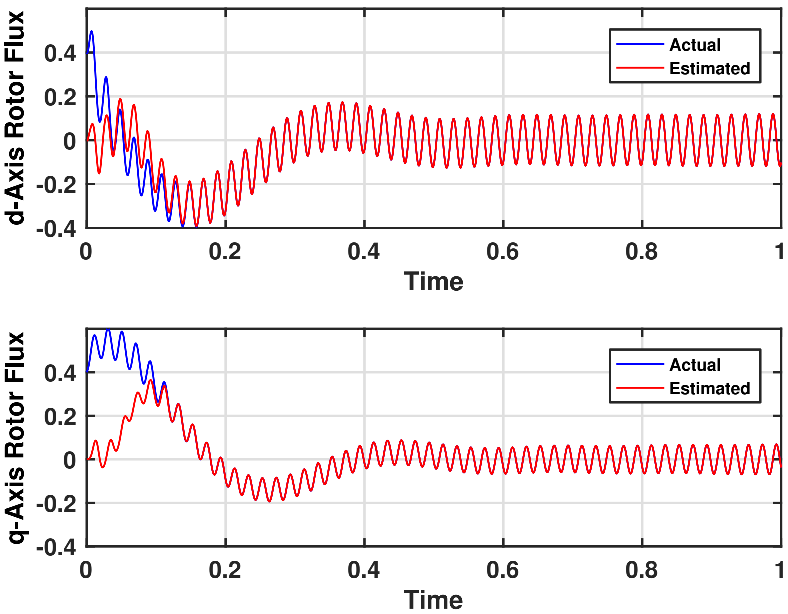

FIGURE 4.6: Rotor Flux Estimation through Adaptive High Gain Observer

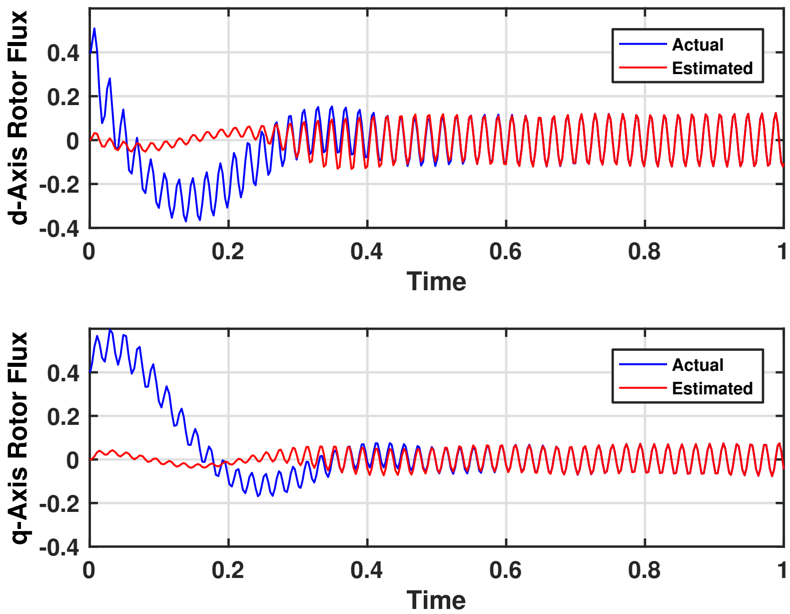

FIGURE 4.7: Rotor Flux Estimation through Adaptive Observer [25]

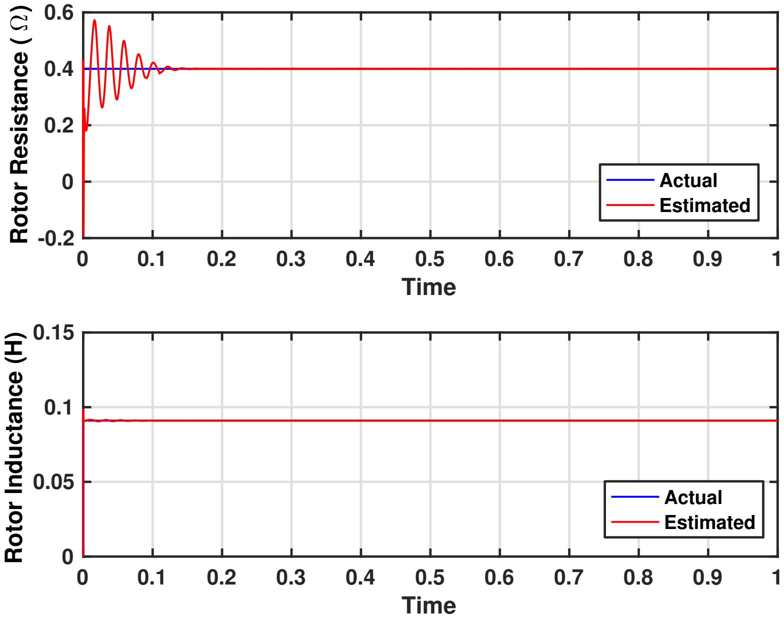

FIGURE 4.8: Parameters Estimation through Adaptive High Gain Observer

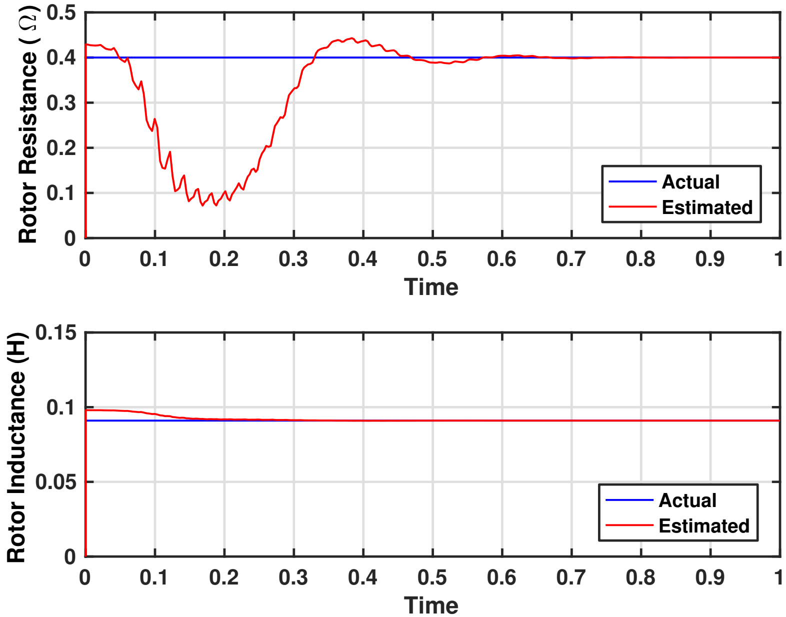

FIGURE 4.9: Parameters Estimation through Adaptive Observer [25]

4.6 Chapter Summary

In this chapter, an adaptive high gain observer is implemented on induction motor dynamics. The adaptive high gain observer jointly estimates the non-measured states (i.e. rotor fluxes) and unknown parameters (i.e. rotor resistance and inductance) of an induction motor. Error convergence proof is achieved through Lyapunov function candidate under well-defined persistently exciting inputs. The proposed observer can estimate the time-varying parameters and also capable of detecting parameter estimation fault.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

The parameters of induction machine may vary due to many reasons. So, it is difficult to exactly estimate rotor flux. Since, vector control algorithm requires the precise estimation of rotor flux to control the induction motor efficiently. Therefore it is needed to estimate the rotor flux along with parameters. Rotor resistanceas well as rotor inductance are crucial parameters for estimation of rotor flux. The objective of estimation of rotor flux along with unknown parameters is achieved through adaptive high gain observer for the following reasons. The observer gain is adjusted through the choice of single parameter. The convergence of the observer is proved through Lyapunov function candidate under persistence excitation of inputs.

5.2 Future Work

The work carried out in the thesis can be extended in several other ways. The observer can be made robust with respective to variations in the parameters that are not estimated. Some other parameters like stator resistance, stator inductance and mutual inductance are also necessary parameters that vary with temperature and need estimation for efficient control of induction motor.

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