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How Accurate Are the Maximum Drawdown at Risk Models in Pakistani Commercial Banks?

by

Tooba Ayaz

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To my parents, who have been there for me from day one. Thank you for all of the love, support, encouragement and dedication.

To my husband, who has been there for me throughout this period. Thank you for all of your love, support, help and patience.

This work is dedicated to the three of you.

CERTIFICATE OF APPROVAL

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MMS-153004

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Abstract

Risk measurement is an integral part of risk management process. The study employs a novel idea of maximum drawdown at risk. The sample of study consists of fifteen commercial banks listed at Pakistan stock exchange during 2000-2017.The maximum drawdown has been established by using historical simulations and GARCH based simulation techniques. The GARCH based models include GARCH, E-GARCH, GJR-GARCH with and without autoregressive moving average terms. The models are backtested to compare the expected MDD with actual drawdowns. The results indicate that the predictive power of GARCH and ARMA-GARCH is same. The violations reported by GJR-GARCH and ARMA-GJR-GARCH are on lower side in comparison to rest of the models. Therefore, these are better models for estimating maximum drawdown at risk. The duration of most of the shocks is less than 10 days. The longest duration of of 22 days is observed. Therefore, maximum drawdown method with GJR-GARCH based simulations can be applied by the investors to capture the draw down risk.

Key words: Risk management, Maximum drawdown, Parametric models, ARCH/-GARCH models, Historical simulations.

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Symbols

ABL	Allied Bank Limited
ACBL	Askari Bank Limited
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive Moving Average
BAF	Bank Al-Falah Limited
BIPL	Bank Islami Pakistan Limited
BOK	Bank of Khyber Limited
BOP	Bank of Punjab
CDaR	Conditional Drawdown at Risk
CDD	Conditional Drawdown
E-GARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
FBL	Faysal Bank Limited
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GJR-GARCH	Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedasticity
HS	Historical Simulation
JSB	JS Bank Limited
MBL	Meezan Bank Limited
MCB	Muslim Commercial Bank Limited
MDaR	Maximum Drawdown at Risk
MDD	Maximum Drawdown
PSX	Pakistan Stock Exchange
SBL	Summit Bank Limited

SCB	Standard Chartered Bank Limited
SLK	Silk Bank Limited
SMB	Samba Bank Limited
SNB	Soneri Bank Limited
VaR	Value at Risk

Chapter 1

Introduction

Modern portfolio theory had been considered contemporary for more than 50 years in the mathematical interpretation of asset allocation due to Markowitz (1959). Recently portfolio theory is generalized by Maier-Paape and Zhu (2018) in such a manner that now wide range of risk measures and utility functions can be measured through efficient portfolios. Such portfolios follow the Markowitz portfolio theory and imitate the relation between risk and utility. Besides the expected return of the portfolio, Markowitz used only the portfolios expected return but now general concave utility functions are also allowed, e.g., the log utility used for growth optimal portfolio theory (Hermes and Maier-Paape, 2017; Hermes 2016; Vince and Zhu ,2015; Zhu ,2007, 2012; Vince ,1992, 1995; Kelly ,1956). In a portfolio, the expected log returns play an important role in the creation of fastest compounded growth, which is done through growth optimal portfolios. In addition to the utility functions generalization another revolution is that the risk measures with more realistic approach are now allowed. Capital asset pricing model (CAPM) of Sharpe (1964) and Markowitz use the portfolio returns standard deviation as risk measure, the model of Maier-Paape and Zhu (2018) is appropriate for most of the convex risk measures. Mathematical finance has a long history of convex risk measures. Like the conditional value at risk (CVaR) provides a nontrivial convex risk measure in addition to standard deviation (Rockafellar and Uryasev ,2002; and Rockafellar et al. ,2006), but conversely the classical value at risk cannot be used in this context as it is not convex. Thus, the focus is the provision and analysis of numerous convex risk measures of portfolio returns pertain to the expected log drawdown. Practically, risk measures based on drawdown are considered to be superior in comparison to standard deviation risk measure for generation of risk averse strategies. Furthermore, the empirical simulations of Maier-Paape (2015) indicates that for growth optimal portfolios, risk averse strategies like drawdown are becoming highly important, as their recurrent use generates tremendous drawdowns. (Tharp,2008). Therefore, in portfolio optimization, the application of the risk measures related to construct drawdown is very important. Drawdown risk measures are discussed by number of authors. For instance, Chekhlov et al. (2003, 2005) applied the conditional value at risk on absolute drawdown processes, which is conditional drawdown at risk (CDaR). The properties such as positive homogeneity and convexity were shown for the CDaR. Later, Zabarankin et al. (2014) uses the conditional value at risk to create CDaRs new variant and now on rolling time frame drawdown. Goldberg and Mahmoud (2017) also used conditional value at risk and introduced the so-called conditional expected drawdown (CED), which is an alternate to a general deviation measure, and this time on path wise maximum absolute drawdowns. The results in Maier-Paape (2013, 2018), can be generalized by constructing the risk measure pertain to drawdown having one risky asset for a portfolio. In these papers, growth optimal portfolio theory (Vince ,2009) is used to construct randomly drawn equity curves, which allow the measurement of drawdowns. According to the theory by Maier-Paape and Zhu (2018), just like CAPM model, efficient portfolios can be structured by using utility function through risk measures which are positively homogeneous. Moreover, in such scenario a risky efficient portfolio like a market portfolio connected to drawdown risks can be created. Drawdowns are very helpful in the determination of the financial risk of an investment and are an expected part of trading.

Maximum drawdown that can be defined as the largest aggregate loss from peak to trough, is on one side, used as most common risk indicator in the industry, but on other side, it is one of the least developed in the context of probabilistic risk metrics.

Generally, a known recovery window length and diversification of portfolio are used, to mitigate the drawdown risks. In the economic growth of a country, banking is considered to be an important pillar, as governments and private sector gets major financial support from banks. It means that stability of financial sector of a country leads the path towards quick economic growth. The growth ratio of countries with stable financial sector is much higher than the countries having unstable or chaotic financial sector.

Banking sector in Pakistan is comparatively stable and commercial banks are the key players in this domain. The playing environment for the banking sector is highly competitive. Still in last two decades, the Pakistani banks are showing swift growth and outstanding performances.

A levered investor is likely to fall prey to liquidity trap and is not able to secure funding in the result of an unexpected decline in market, due to such harsh market conditions he would have to sell his valued assets. During the 2007-2009 financial crises, such an experience is convention and it played a great role in highlighting the drawdown, as investors both levered and unlevered, considered it as an important creator in liquidity trap and diverted their attention towards it. Volatility, Value-at-Risk, Expected Shortfall, and other common risk diagnostics are considered to be irrelevant at the end of the intended investment horizon in case of large drawdown. Maximum drawdown is the most widely used risk diagnostic when we talk about hedge funds and commodity trading advisors (CTAs). However, any specific methodology, which is generally acceptable in order to predict about expected drawdowns in future, does not exist in mathematics. The degree of attention paid to other conventional risk diagnostics could not be provided to Drawdown in this context (Artzner et al. ,1999).

1.1 Theoretical Background

In recent years, the emphasis on risk assessment for allocation of money or any investment is gradually increasing, through which new measures to assess risk are in discussion and have been incorporated not only in regulating authorities but also become the part of regular work of investment firms. Therefore, it has become really important and crucial for financial firms to know how they are supposed to perform and how to measure it. At the same time, they should be able to perform significant risk assessments and estimations, which are in their interest not only in terms of internal use but also to follow the regulations. Given this fact, this study aims at presenting maximum drawdown as a suitable and understandable risk measure analyzing both the advantages and shortcomings to demonstrate its performance as a safe measure to predict about future risk.

Optimal portfolio allocation is an age-old issue, not only in practical portfolio management but also in academic research on portfolio theory. Different methods that have been proposed and studied (Grinold and Kahn,1999) in this context assume measure of portfolio risk to some extent as starting point. Financial institutions use risk-adjusted performance measures (RAPMs) for both pre allocation and post allocation decisions. Such decisions not only involve the capital and asset allocation but also the performance of these allocations.

Risk can be defined as any ambiguity belongs to investments that have the capability to create negative impact on financial welfare. Financial institutions face two major problems while allocating funds: First is that the investment of the funds should be initiated with keeping in mind the main objective of business which is to maximize the expected utility of investors. And second is the optimal allocation of the risk capital of a financial institution, as it involves different risk level assumed by individual business activities. Same mathematical concepts are used in both these areas but still existing literature in finance treats the two research areas separately. This study bridges the gap between the two areas.

Although long term historical averages can guide decision-making about risk, still it is difficult to predict that they will play in ones favor in specific circumstances and particular goals and needs.

A drawdown is defined as the collected loss arises due to sequential decrease in the investments price. Durations and time intervals both are random variables in case of drawdowns. In the setup of fixed investment horizon, the maximum drawdown is a flexible measure and is able to create an entirely different perception about price flow and risk attached to an investment. Besides value at risk and standard deviation, drawdowns can be considered as a complementary approach. Maximum drawdown can be described as the largest cumulative loss from peak to trough, and is widely used in the industry of fund management as a risk assessing tool. In spite of this fact, maximum drawdown can be considered as the least explored risk measure.

An important characteristic of a drawdown is that it is defined over highly correlated data. In other words, drawdown is the highest value of the wealth of investors that has been lost and maximum drawdown is the historical highest value of lost wealth.

VaR is a mode of description for the magnitude of losses expected in a portfolio. By definition, an entitys VaR is the loss that has the expectation of exceeding in the holding period of t-days and the probability of such occurrence is X percent. Usually probability X is considered to be 5 percent, 2.5 percent, and 1 percent, and usual values for t day holding periods are 1 day, 2 days, 10 days, and one month. This fact employs that VaR cannot be compared among different entities without the coordinated adjustment of X and t. Regardless of the approach selected, the calculated VaR often assumes that the future will behave like the past, an assumption that can be quite risky in and of itself.

VaR is a preferred risk measure due to number of reasons. Firstly, because it gives a simple and clear figure in cash that how big a loss will be, and that is also with a convincingly high level of probability. This is a simple thing to understand by everyone unlike volatility, as it is difficult for a layman to understand what are the meanings of 10% volatility. Moreover, in simple straightforward cases, VaR can be calculated easily and hence makes the comparison of results simple. It really helps the banks to develop capital related rules and regulations on the basis of VaR figure for specific time period. Furthermore, in normal conditions, VaR also helps in specifying maximum possible loss, which is really difficult for other risk diagnostics to accomplish.

Value at risk measure also possess a lot of drawbacks. First of all, it is unable to predict the magnitude of loss. It means, even if one calculates VaR for different levels of confidence, still it is not possible to predict loss beyond VaR figure, so probability distribution cannot be fully considered. The fatness of the tails is ignored while calculating VaR, which leads that two portfolios having different distributions can still have same VaR figure. Which raises a question mark at the quality of VaR as a risk measure because two banks having same VaR figure dont need to keep same amount of money to cover the risk. Similarly, VaR is not capable enough to predict about the right hand distribution and simultaneously some other measures are also needed to make a better assessment. Finally, VaR is not reliable as it lacks the synergy attitude. This means that portfolio consists of number of assets may assume more risk than the added up risk of individual assets, which sounds as threat to investors as it contradicts the diversification principle. Still, VaR is the most sought after risk measuring technique, the attention that VaR caught in the research field could not be captured by Drawdown and this field is really needed to be explored, which we have tried to some extent in this study.

Classically, risk is defined as the probability of experiencing loss, incurred in the result of choosing a given alternative. In the scope of financial investments, this given alternative can be defined as an investment in portfolio of assets or in a single asset. Perception and preferences play important role in setting the amount of risk in addition to valuation. So, the perception of different investors about risk differs from each other, partially in accordance with the risk diagnostics they use and partially in accordance to their preferences. The loss is not as simple as that can be necessarily defined as a negative outcome. It can be defined as a result worse than a certain assumed target outcome or least accepted outcome, beyond that the outcomes are considered to be a gain. The assumed target outcome carries great importance and is dependent on the investors choice. The perception of risk starts to decline with the increasing potential gains. The reason behind this phenomenon is still not determined. This topic is covered by a modern area of finance i.e., behavioral finance that has emerged in later years. However, the level of risk of an investment is judged by different people in different ways and that are entirely different in case of judging the attractiveness of that investment. (Brachinger, 2002). Generally, two dimensions of risk are discussed i.e., possible loss that can be happened and probability of occurrence of that loss. Volatility is a common risk measure depicts that normally risk is discussed in terms of probabilities and possibilities of occurrence of loss but it is also helpful in measuring the deviations in the expected average returns.

Due to unidentified causes of different financial crises, risk management has become the focal point for experts of financial management. One problem is the risk measurement in such a way it can be used to predict quantifiable and optimal risk exposure. To answer this question financial managers have identified many risk measures to calculate risk exposures of different classes like operational risk, credit risk, market risk, liquidity risk etc. Drawdown has a vital characteristic that it is defined over highly correlated data. The existence of a process, which is dependent on time and also responsible for creating local dependence, is evident from the sequential decline in the stock price. This topic was first introduced by Mandelbrot (1967) in the context of modeling some aspects of a phenomenon showing an intermittent turbulence. According to Mandelbrot (1972, 1997), financial time series would possess fractal dimensions which would induce cycles of many different durations. This inherent characteristic would explain the turbulent cascades in stock markets, the fat tails returns distributions, and the presence of long memory in stock returns and squared stock returns. Here we focus in the statistical modeling of these sequences of losses (gains) in stock markets.

Probability distribution of drawdown was discussed in detail in the previous literature in several studies including Johansen and Sornette (2001), that modelled the drawdown severity from currencies, commodities and indexes by using the Stretched Exponential. It was found that with the exception of minor number of extreme observations, overall this distribution performs well for most of the data. Mendes and Brandi (2003) empirically showed that the drawdown is well captured through Modified Generalized Pareto distribution and its sub-models, including most of those observations previously found to be atypical.

The effectiveness of this measure is also discussed in a lot of other studies with focus on its worth financial applications. For instance, conditional Drawdown-at-Risk (CDaR) was introduced by Chekhlov et al. (2000), as an optimality constraint to obtain optimal portfolio allocation Palmquist et al (1999). Up to that time, different methods to create optimal portfolio with controlling drawdown were already established. Grossman and Zhou used a risky and a risk free asset in Black Scholes economy, to propose a portfolio optimization under drawdown constraints. Dynamic programming helped to solve the optimization problem. Cvitanic and Karatzas used martingale method to solve the optimization problem and generalized the results also for multi-risky assets. Conditional drawdown (CDD) was introduced by Chekhlov et al (2003) as a risk measure, which can be helpful in developing a portfolio optimization method. The method was used by Hakamada et al (2007) and Krokhmal et al (2002) to construct portfolio for hedge funds.

Previous literature also focused the properties of maximum drawdowns. The probability distribution of maximum drawdowns was discussed by Magdon-Ismail et al (2004) and Magdon-Ismail and Atiya for Brownian motion and geometric Brownian motion, respectively. For geometric Brownian motion, probability distribution of drawdown was also discussed by Belentepe, with discussion on the use of portfolio diversification to reduce the maximum drawdown. The relationship between maximum drawdown/draw up and directional trade was studied by Vecer (2006), and Vecer (2007) also discussed hedging contingent and pricing claims on maximum drawdown. Both of these studies used Monte Carlo simulation to conduct the analysis with the assumption that the geometric Brownian motion was followed by underlying asset. Pospisil and Vecer (2008) analyzed it by a partial differential equation (PDE) method under the same assumption.

MDD is also helpful in measuring the investment sustainability (Magdon-Ismail, 2004), and is considered as an important tool in industries like hedge funds. Besides the calculation of value, the length of an uninterrupted decrease or uninterrupted drawdown also grabs attention. Furthermore, MDD also guides about the recovery time, i.e. the time required to get the value of asset back to the initial level from where the drawdown started. MDD is advantageous for the investor not only due to its implicit nature but also its ability to define the ceiling for losses. It warns the investor about the riskiness of the asset before reoccurrence of the past losses by explaining the duration and past recovery of the drawdown.

In comparison to volatility, MDD is considered preferred due to its ability to measure downside risk, its less abstract and more intuitive nature and for the fact that regardless of return distribution MDD can be calculated for any time series. Whereas volatility takes upside risk into account as well.

However, MDD as a risk measure also has some drawbacks attached to it and needs some careful application (Lhabitant, 2004). First, the reporting intervals are crucial if comparison of MDDs of different assets time series is desirable, i.e. in such case all the time series should have same frequency of the measurement interval or should be adjusted accordingly. The reason is that MDD is inversely proportional to the frequency of measurement interval, if its smaller than MDD would be greater.

Second, for all assets that have to be compared, MDD should be calculated for equal periods because for longer time series, MDD also gets greater. Selection of a suitable investment horizon as a base to calculate the historical maximum drawdown is quite crucial and important. To address this issue, an attempt to generalize the industry standard is made, by declaring a three-year period as a ground in order to calculate present MDDs. Finally, this measure considers only the worst drawdown and explains nothing beyond that in terms of second or third largest drawdown. Furthermore, it is unable to describe the expected magnitude of loss or expected portfolio risk before its occurrence.

1.1.1 Banking Sector in Pakistan

In todays modern trade and commerce activities, the banking sector is considered to be the life blood, as it is mainly responsible to provide financing. The concept of efficiency has become highly important due to the increasing trend of globalization not only for non-financial institutions but equally for financial institutions including banks of course. Success and growth of banks is basically dependent on their competitive marketing strategy. In comparison to the previous years, new millennium has evolved the protocols of doing the banking business (Hussain and Bhatti, 2010). Commercial banks are the backbone of the financial system of Pakistan.

The nationalization of domestic banks and growth of public sector development financial institutions in early 1970s thoroughly changed the financial institutions scenario in Pakistan. By the end of 1980s, it became evident that nationalization was not the right way to attain the national socio-economic objectives. Financial inadequacy, declining asset quality and growing threats of downfall of financial institutions are a few benevolences given by public sector. By the end of 1990, 90 percent share in total assets owned by banking industry is occupied by public sector and the rest belonged to foreign banks due to non-existence of domestic private banks in that era. Besides this high shares existed for investments, deposits and advances. In the year 1997, the banking supervision process in Pakistan is got in-line with the international best practices, which brought substantial changes in the structure of banking sector in Pakistan. Structure, concentration and ownership of banking sector has gone through some notable changes due to continuous process of merger and acquisition in addition to the privatization of public sector banks (State Bank of Pakistan, 2009).

1.2 Problem Statement

The basic aim of the study is to deeply analyze the risk measures and their capability to assess risk for an asset or portfolio of assets. Also, back testing method is used in the study to evaluate the certain‘ risk measures through time series data of various types and rolling estimation windows. The main purpose of the study is to test how accurate the future risk can be predicted through the chosen risk measures. Given the background, the following issues are contributing towards the formulation of the problem. It presents a detailed significant comparison of risk measures and analyze their pros and cons in the most understandable manner. A study is performing on some chosen risk measures (GARCH, ARMA-GARCH, GJR-GARCH etc.) with regards to how well they predict risk. The econometric term of generalized autoregressive conditional heteroscedasticity (GARCH) process was developed in 1982 by Engle, an economist, to describe an approach in financial markets for volatility estimation. Now, number of different GARCH models exist. The prices and rates of financial instruments can best be predicted through GARCH models due to their more factual nature so experts normally prefer GARCH on other forms. The results of the study are analyzed, and a conclusion is extracted that how accurately future risk can be predicted through the risk measures chosen in the study.

1.3 Research Question

- Do GARCH type models really help in forecasting maximum drawdown?
- Which GARCH model is appropriate to explain MDD?
- How fit maximum drawdown is estimated through historical simulations?
- Which estimation model, suggests the best MDD for a selected group of commercial banks?

1.4 Objective of the Study

- To estimate the maximum drawdown in stocks of banking sector.
- To test the ability of chosen risk measures in prediction of future risk.
- To evaluate the quality of forecasting of various MDD models.
- To propose the best model to measure MDD in the banking sector.

1.5 Significance of Study

Risk management is one of the most important area in financial management. As in the expression of Benjamin Graham, the crux of investment management lies in risk management rather than management of returns. To understand the magnitude of risk, first we need to quantify it. The gravity of risk guides the investors that which line of action has to be chosen. So in order to make better financial decisions, risk measures play an important role. Various tools are used to measure risk including volatility, VaR & MDD.

The MDD captures very specific risk features of an asset and is one of the most widely used risk indicators among investors due to its easily understandable nature. MDD has the ability of measuring the loss over a period of time so it also warns the investors about the worst time for investment, i.e. If it is done during the drawdown before it gets full recovery. MDD is not an abstract measure but presents the physical reality of the risk attached to an asset. Moreover, irrespective of the distribution, it can be calculated for any time series. Value at risk, Volatility, Expected Shortfall and common risk measures are not of much significance in the end of the prospect investing field in case of large drawdowns.

Drawdown is unable to attract the attention of applied researchers, as a risk measure or measure of deviation, in comparison to the other conventional methods. This study develops and tests a simulation based methodology for estimating the MDD. This study pertains to implementation of maximum drawdown at risk as a tool to control risk and preserve investments capital. It fits an econometric model to the data and estimate the risk measure through Historical simulations.

It also offers contextual contribution regarding an emerging market that is Pakistan that has recently considered as part of emerging market index and have attracted international investors. MDD can be an important tool to safeguard the interest of foreign investors as well as the local investors by contributing valuably in the risk assessment and management. This study explores the avenue of drawdown at risk in the context of Pakistani market about risk dynamics.

Financial sector and especially the banking sector plays a vital role in overall growth in modern economic world. It not only helps in capital generation but also acts like a stimulant to move the economy cart forward. The strength and stability of this sector indicates the better performance of economy in future. So It is also the requirement of the moment to have better insight in this sector.

1.6 Plan of Study

This study is overall divided into five chapters. Chapter 2 describes the reviews on literature pertain to different risk diagnostics and importance of maximum drawdown as an effective risk measuring tool. Chapter 3 discusses the methodology followed to complete this study including how different models are used to calculate MDD. Chapter 4 consists of the results of GARCH type models and historical simulations applied to calculate MDD and their analysis. Chapter 5 includes the conclusion and recommendations for future studies.

Chapter 2

Literature Review

Jackson (1995) and Kupiec and OBrien (1995) debate risk measurement with reference to bank regulations. Dimson and Marsh (1995) argue in detail the effects of the building block approach. Different approaches to VaR appears in systematic explanation in Jackson (1995). The June 1996 special issue of Risk presents different practitioner point of views on VaR. The terms used to differentiate two systems of VaR analysis differs among authors in a slightly confusing manner. For instance, Laycock and Paxson (1995) mention parametric and simulation centered VaRs as simulation and backtesting approaches, correspondingly. The prior is also discussed as the variance-co variance approach. VaR models in some banks, for example, CSFBs Prime risk, relate various weighting schemes through asset classes. Lawrence and Robinson (1995) claim weighting schemes to be asset specific. Jackson et al (1997) follow Risk Metrics in using an identical weighting scheme.

U.S. commercial banks, early in 1998, may regulate their regulatory capital needs for risk exposure in financial market by means of value-at-risk (VaR) models i.e., portfolio returns based on the time varying distribution models. Lopez (1997) proposed three hypothesis-testing methods: the interval forecast method, the binomial method and the distribution forecast method for evaluating the accuracy of VaR models available for the regulators. These approaches use hypothesis tests to study whether the characteristic of accurate VaR forecasts exhibit VaR forecasts in question. Nevertheless, the low power often demonstrated by these assessments, these approaches may often misrepresent forecasts from erroneous models as precise. A fresh valuation technique is suggested that employ loss functions based on probability forecasts. Simulation results specify that this technique accomplished distinguishing facts among forecasts from exact and erroneous, alternative VaR models.

Value at risk models have become the priority of big banks to address the problem of risk involved in trading operations. The Basle Committee has recognized the use of these models by the banks to assess the capital needed to cover the volume of securities trading, which is a major revolution in regulatory world. Jackson et al (1998) used a large bank that holds equity security holdings, foreign exchange and actual fixed income, to assess and evaluate that how different VaR models performed. This study also evaluated the past performance of the bank in the context of these models and with implementation of the anticipated rules.

Most of the literature backs the point of view that in the environment of tough competition, banks normally prefer to opt the portfolios that have higher risk associated to them, which concludes that banks logically choose risky portfolios. Regulators and management of central banks support this school of thought significantly. Empirical literature is reviewed by Boyd and Nicolo (2005) and they made the conclusion that the best description for the evidence is to declare it as mixed. The existing theoretical analyses of this topic are fragile, since there exist fundamental risk-incentive mechanisms that operate in exactly the opposite direction, causing the markets to become concentrated and eventually banks are becoming riskier. Models that evaluate the competition in banks should hold such mechanisms.

In contrast Danielsson (2008) argued that despite all the stress testing, the presence of sophisticated models and all the numbers calculated financial crises could not be controlled and it surprised everyone. The financial institutions that have the best management are performing well in the game of survival in comparison to the institutions having best models to assess risk but not a better management. Risk assessment models can only perform better in managing risk if their drawbacks are also considered before application. These models are helpful in capturing risk only in small trading units and are unable to assess risk in large business units. The managers are responsible for the whole financial system, they have to take care of the accumulated risk, so the issue is bigger for them as these models are not reliable. In financial institutions those models are useful for internal risk management which helps in decision making process. The sample size of data is normally sufficient to estimate the events of high probability in smaller units correctly, so statistical models have the best applicability there. Levels of probability, testing and size of sample are needed to be synchronized in order to get a high quality modeling process. A risk model normally performs better for level of probability on which it is based in comparison to other levels of risk. For example, 95% VaR can be calculated accurately through exponentially weighted moving average (EWMA) but same cannot be considered for 99% VaR. Impracticable expectations from risk models effect the basic theory behind the scientific process, and damages the process of verification, which is generally done by back-testing. Despite all the mathematical procedures done, number of tail VaR and 99.9% models are not considered to be scientific because they cannot be back-tested. Some exceptions do exist but those methods are still mostly experimental, and not suitable for everyday use.

Topic of drawdown was first introduced by Mandelbrot (1967) in the context of modeling some aspects of a phenomenon showing an intermittent turbulence. According to Mandelbrot (1972, 1997), financial time series would possess fractal dimensions which would induce cycles of many different durations. This inherent characteristic would explain the turbulent cascades in stock markets, the fat tails returns distributions, and the presence of long memory in stock returns and squared stock returns. This study focuses in the statistical modeling of these sequences of losses (gains) in stock markets. Previous literature addressing the problem of obtaining the probability distribution of the drawdown includes Johansen and Sornette (2001), that used the Stretched Exponential to model the drawdown severity from indexes, commodities and currencies. They found that typically this distribution fits well the bulk of the data, but under-estimates one to ten extreme observations. A comparison of the model predictions using the price and the logarithm of the price, respectively, furthermore indicates according to this model that such large downward movements in the markets are nothing but depletions of the preceding bubble thus bringing the market back towards equilibrium. Mendes and Brandi (2003) empirically showed that the Modified Generalized Pareto distribution and its sub-models fit very well the drawdown data, including most of those observations previously found to be atypical. According to Harmantzist and Miaao (2005), the community of investment managers have started to pay attention to maximum drawdown as a useful risk measure, which is able to imitate the effect of VaR in terms of methodologies and results. This study discussed the modelling of MDD and suggested stable paretian distribution as the best tool. Time-series of global indices for developed and emerging markets are used to test the assumptions. MDD and daily returns for fat tailed distributions are used to compare risk methods. Different risk profiles are evaluated through various methods using different markets.

A drawdown is defined as the precentral accumulated loss which arises due to the sequential exceedance drop in an investments price. The duration of drawdown is a random variable and It is measured over varying time intervals. The maximum drawdown is a flexible measure so it presents an entirely different insight into risk and stream of price of the investment, when investment is done in fixed amount. The empirical study indicates that there may exist a relation between the pattern of the GARCH volatility of an investment and the fluctuations of the severity of the maximum drawdown and that, typically, extreme (but not outlying) maximum drawdowns occur during stress periods of high volatility. Applications for the maximum drawdown is suggested, including the computation of the Maximum Drawdown-at-Risk with probability, and the classification of investments according to their performance when controlling losses via the maximum drawdown.

The maximum drawdown of portfolio, in simplest words, is the longest drop a portfolio gets from the peak during its whole span of life. Optimal risky investment was analyzed by Grossman and Zhou (1993) in context of an investor, who does not want to experience a drawdown more than the maximum value of his whole wealth earned up to that time, exceeding a pre-determined percentage at any stage of investment.

The effectiveness of maximum drawdown as a risk measure in financial applications is discussed in number of studies. For example, Chekhlov et al. (2000) in his studies discussed that how an optimal constraint can be helpful in achieving the optimal portfolio allocation. For this purpose, he presented the conditional Drawdown-at-Risk (CDaR) as an optimality constraint. Gray and Vogel (2013) suggested that linear factor models are generally unable to capture the tail risk, which can be concealed through maximum drawdown due to its instinctive and easily understandable nature. Market strategies discussed in the literature, experience drawdowns at different points and by identifying those drawdowns, tail risk can be assessed. If risk identification gets delayed, they can lead the investors to liquidation in the worst case.

Acar and James (1997) scrutinized the performances of portfolios and funds through maximum drawdown. The study is helpful in opening new gateways for further research in the field of maximum drawdown. This study envisions the use of their estimated densities to discriminate the portfolios performances, like how the diversification in international scenario be helpful in creating the potential benefits. The focus of the study is not the detailed extensive research on the maximum drawdown and model applications but just to instigate new suggestions for future research.

Yang and Zhong (2012) gave the concept of Rolling economic drawdown(REDD) in their study conducted on data of three broad asset class indexes: Dow-Jones UBS Commodity Total Return Index (DJUBS), with 3-month US Treasury Bill as the risk-free asset, Barclays Capital 20+ Year US Treasury Bond Index (TLT) and S&P 500 Total Return Index (SPTR), to identify maximum loss limit and presented a discrete trading strategy which can be helpful not only in maximization of the growth rate of a portfolio but also focusses to control the maximum drawdown percentage in portfolio within some predetermined boundaries.

According to Johansen and Sornette (2001), drawdowns play a significant role for investors, due to their ability to measure investments collective loss that can occur. They are also able to show the clear picture to the investors, like if they invest at a high level and then sell at next minimum level, which can be the worst case. It is thus worthwhile to check the drawdowns in context of their distributions structure. It can be noted that drawdowns are created from runs of variations of same sign so they are indirectly dependent. Drawdowns are based on a non-fixed time scale. The time period keeps on varying, at times it would be for one day or six days or seven days and so on. Drawdown distribution measures the effect of successive drops that how they create an effect on whole distribution and can influence each other and form a continuous progression, which cannot be done by two-point correlation function or returns distribution.

Soo-Hyun Kim (2018) conducted a study on Korean stock market and noted that MDD is actually the maximum collective loss. Moreover, MDD depends on series of values and VaR estimates the maximum worst return only in single value so the risk captured by MDD can be quite different from that of VaR. If VaR has a large value, it can be due to one large negative value of return in the stream of positive returns. So large value of VaR does not indicate large MDD. Similarly, VaR can be a small value in the stream of small negative returns, still MDD can be large in this case. Which clearly depicts the diverse nature of MDD and VaR as risk measures. Moreover, VaR and MDD are not able to capture additional risk factors after volatility, they can be used to get some further information in addition to volatility.

Earlier studies regarding the estimation of VaR extensively used GARCH volatility model but it could not be specified that which volatility model is the best in terms of forecasting VaR. So the topic continues to be the centre of interest for researchers and a lot of work has been done later on. Vlaar (2000) used assumptions pertain to different distributions and then applied the GARCH model on Dutch bond portfolios, this study found that if GARCH is used under normal distribution, it has the ability to outperform the estimations calculated through historical simulations method.

Brooks & Persand (2003) conducted a study on stock indices of Southeast Asian countries. The study was about the effects of the models which consider asymmetry and models which do not consider the effect of asymmetry on VaR estimation. The study found that models considering the asymmetrical effect in returns have larger VaR estimates in comparison to the models overlooking the asymmetry and they have extremely small VaR estimates. Generally, it is assumed that equity returns show more volatility in response to the negative shocks than the positive ones. Yet, normal distribution (which is a symmetric distribution) assumption is the base used for maximum number of analysis regarding VaR. Angelidis et. al (2004) performed the study on some major stock indices and found that no single model is able to outperform others but overall leptokurtic distribution performs better than the normal distribution, which is an addition to the previous study. He also added that the length of estimation window is crucial for VaR as it influences the results of estimation. In a relatively current study, Orhan & Kksal (2012) tested a large number of models to measure volatility and concluded that for the estimation of Value at risk, the best results can be generated through combination of leptokurtic error distributions and ARCH model. On the basis of different studies, we can comment that this research on finding the best volatility models is an ongoing process and financial risk managers are still looking towards the researchers to nominate the optimal forecasting model. Engle (1982) modelled Auto Regressive Conditional Heteroscedasticity (ARCH) model to compute the most important factor in VaR estimates, i.e. volatility, Bollerslev (1986) and Taylor (1986) also discussed ARCH in their studies separately. The ARCH models are the most popular way of capturing the volatility clustering and parameterizing the dependence (Tersvirta, 2006).

Univariate GARCH models are used for measuring the risk and precise forecasting of volatility, whereas multivariate models are used for portfolio risk management. (Andersen et. al,1998 2007). Poon & Granger (2003) conducted a detailed survey and suggests that generally GARCH model has an edge over ARCH models, so now a large number of models are added up in GARCH. Moreover, the models that are able to capture asymmetric effect, like exponential GARCH by Nelson (1991) and GJR-GARCH by Glosten et. al (1993), show better performance and results in comparison to the original GARCH model. Another detailed study by Hansen & Lunde (2005) was done to compare a wide range of volatility forecasting models and suggested that best result is given by GARCH for exchange rates but in case of stocks, models that carry leverage effects are supposed to perform better. Kksal (2009) and by Hung-Chun & Jui-Cheng (2010) also suggested the same results in their studies. The models tested in this study are the ARCH, GARCH, EGARCH and GJR-GARCH. The error term in the financial time series modelled by GARCH, nonetheless needs to be assumed and Bollerslev (1987) proposed the Students t distribution rather than the Normal distribution originally assumed by Engle (1982) and Bollerslev (1986). Another distribution was suggested by Nelson (1991), that is capable to capture the effect of fat tails, namely the Generalized Error Distribution (GED) by Harvey (1981). But, Hung-Chun & Jui-Cheng (2010) tested different distributions in their study and found that the error distribution does not play any significant role in volatility estimation through GARCH model, e.g. the skewed generalized t distribution (SGT) by Theodossiou (1998). However, Wilhelmsson (2006), finds in his study that application of leptokurtic error distributions leaves a positive impact and show better results in comparison to Normal distribution. In the literature, very important studies have been done in the domain of maximum drawdown but still academically this field remain ignored and is unable to grab major attention in the literature in comparison to other common linear factor models such as the CAPM, the 3-factor, and the 4-factor models. Even then, maximum drawdown is persistently in use for measuring volatility. Like any other measure, MDD is also not impeccable. It is an in-sample realization of the worst-case scenario, and there does not exist any cordial connection between traditional statistical analysis and this measure. However, maximum drawdown has the ability set a yardstick in favor of investors through which they can assess their strategies in terms of expected loss. MDD is studied as a risk measure by Cvitani and Karatzas (1999) Chekhlov et al. (2000) went a step further and discussed the Conditional Expected Drawdown (CDaR) in terms of the mean of all drawdowns exceeding a particular drawdown level. Mendes and Brandi (2004) used Modified Generalized Pareto Distribution to make parametric estimations of the CDaR by applying the models to the extreme tail of drawdowns. The theory behind the conditional drawdown-at-risk is fairly similar to the idea behind the conditional value-at risk. Conditional drawdown-at-risk also requires a predefined quantile (the most common being 0.9, 0.95 or 0.99), where the average and maximum drawdowns are the two special cases of the conditional drawdown-at-risk. Hoesli and Hamelink (2004) concluded that mean variance approach is able to suggest a higher MDD in comparison to return-MDD approach while optimizing the portfolio. MDD was statistically discussed by Rebonato and Gaspari (2006) and Pospisil and Vecer (2008). Pospisil and Vecer (2010) also discussed a new area related to the derivatives of a financial contract value with respect to the MDD. Mean-variance analysis of Markowitz and the MDD were studied together by Kim (2011), he also discussed the problem of selection of investment fund in context of MDD. Following Chekhlov et al. (2000) definition, Goldberg and Mahmoud (2014) show that the Conditional Expected Drawdown (CED) is not a coherent risk measure but a convex measure, and hence can be used as an optimizer. The most commonly used fund management industry indicator for risk evaluation is the maximum drawdown i.e. the maximum loss from peak to trough. in context of measure of risk, it is not that developed. Therefore, maximum drawdown distributions for tail mean or conditional expected drawdown(CED) is formalized as drawdown risk. CED is described as degree one positive homogenous risk measure and therefore characterized linearly to factors and convex in order to be used as quantitative optimization. Risk measures based on CED, expected shortfall (ES) and volatility are tested empirically. CED is extremely vulnerable to serial correlation. A study conducted to US equity and US bonds that fits AR (1) models, resulted in higher correlations between CED and autoregressive parameter as compared to volatility or ES.

Portfolio optimization using drawdowns has also been considered in Chekhlov et al. (2000). Other related studies include Grossman and Zhou (1993), Harding, Harding et al. (2003), Leal and Mendes (2005), Hayes (2006), Vecer (2007), and Gray and Vogel (2013). Alexander and Baptista (2006) again discussed portfolio selection problem and introduced a drawdown constraint. In practice, the MDD measure is extensively used by hedge funds managers and commodity trading advisors (CTA). Menchero and Poduri (2008) developed a standard framework based on the marginal risk contributions for the analysis of portfolio risk. The impact of trade on portfolios overall drawdown risk can be assessed by the investors, through integration of this framework and drawdown risk.

Krokhmal et al (2003) describe the drawdown as a measure which adopts a conservative approach to quantify the nancial losses, on the basis of the fact, that drawdown uses the most unfavourable past investment instance for the comparison to the current instance. Some investors prefer to define their acceptable losses in terms of the specific percentage of the initial investment, and this phenomenon is well depicted in this approach. If the drawdown is large, it would create a panic situation for the investor in terms of capital, although earlier he was accepting the smaller drawdowns. In this scenario, investor gets the indication that fund is in danger zone and now this is the time to move on and invest the amount somewhere else with healthy signs for future returns. This detailed discussion makes the conclusion that MDD is not only able to measure the loss for series of time but also record the sequential activity, which is a unique feature of MDD, not held by any other risk measures. That is why MDD is declared as a loss measure with memory.

Chapter 3

Data and Methodology

3.1 Data

The data used in this study consists of the daily stock prices of 15 commercial banks listed at PSX. The time period covered for each bank and size of sample is mentioned in the table below,

Table 3.1: Details of Sample

Banks	**Abbr.**	**Sample Period**	**Size of Data**
Allied Bank Limited	ABL	2005-17	3054
Askari Bank Limited	ACBL	2000-17	4439
Bank AlFalah Limited	BAF	2004-17	3336
Bank Islami Pakistan Limited	BIPL	2006-17	2895
Bank of Khyber	BoK	2006-17	2916
Bank of Punjab	BoP	2000-17	4439
Faysal Bank Limited	FBL	2000-17	4439
JS Bank Limited	JSB	2007-17	2676
MCB Bank Limited	MCB	2000-17	4439
Meezan Bank Limited	MBL	2002-17	3892
Samba Bank Limited	SMB	2004-17	3383
Silk Bank Limited	SLK	2001-17	4000
Soneri Bank Limited	SNB	2000-17	4439

Banks	Abbr.	Sample Period	Size of Data
Standard Chartered Bank	SCB	2007-17	2653
Summit Bank Limited	SBL	2008-17	2449

3.2 Methodologies for Estimating the $\mathbf{MDaR}_\alpha$

Variance estimation or returns and model parameters estimation is done through a rolling window based method. This study has a long time series of data, of length T, available for estimation, where T is much larger than t, the number of observations used to form the estimations in constructing efficient portfolios. The below mentioned rolling window method is used in this study as benchmark, that provides consistent estimators of the 1-month out-of-sample parameters and variables. For each time step, the window moves for one month in the estimation process.

Future distribution of the MDD is calculated through two different classes of models, which are parametric and non-parametric.

3.3 Econometric Models (Parametric)

The simplest approach is to work out the desired quantile by fitting a parametric distribution to the data. Extreme Value Theory is appropriate in this regard as it suggests distributions for MDD which are able to capture the tail characteristics by modeling extremes.

Following models are used for simulation of returns curves:

1. GARCH (1,1) with normal innovations.
2. ARMA (1,1)-GARCH (1,1) with normal innovations.
3. EGARCH (1,1) with normal innovations
4. ARMA (1,1)-EGARCH (1,1) with normal innovations.

5. GJR-GARCH (1,1) with normal innovations

6. ARMA (1,1)-GJR-GARCH (1,1) with normal innovations.

Most probably the ARCH/GARCH models are important, because they recognize the fact that historical data can be used to estimate the volatility and traditional econometric techniques are helpful in highlighting the useless models in this regard. GARCH and ARMA approaches can be applied through Eviews and number of other statistical soft wares.

The GARCH model was proposed by (Bollerslev, 1986), to take the variance into account. It captures the volatility dynamics estimates. In this case, volatility is not constant and modeled it GARCH (1,1) model.

The daily returns are calculated by

$$\mathrm{R}_t = \ln(\mathrm{P}_t/\mathrm{P}_{t-1}) \; {*100}$$

Where R_t is the daily return at time t. The equation ln $(\mathrm{P}_t/\mathrm{P}_{t-1})$ is the natural logarithm of today's share price at time t divided by yesterday's share price at time t-1. The series of return can further be divided into two parts:

$$\mathrm{R}_t = \mathrm{E}(\mathrm{R}_t/\mathrm{I}_{t-1}) + \varepsilon_t$$

The first part comprises of the conditional mean return $\mathrm{E}(\mathrm{R}_t/\mathrm{I}_{t-1})$, and is considered to be an Autoregressive process, that uses the available information for the time period up to and including t-1 and calculates the expected return at time t. Following equation defines the second part ε_t which is supposed to be unpredictable.

$$\varepsilon_t = \mathrm{z}_t\sigma_t$$

the conditional standard deviation of ε_t is denoted by σ_t while the sequence of z_t is an iid with unit variance and zero mean (Angelidis et al 2004).

3.3.1 GARCH

Bollerslev and Taylor expanded the ARCH model in separate studies and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was developed (Bollerslev 2009). The conditional variance equation carries p lags of conditional variance in GARCH (p, q).

$$\mathrm{R}_t = \beta_o + \beta_1 \mathrm{R}_{t-1} + \beta_2 \mathrm{e}_{t-1} + \beta_3 {\sigma_t}^2$$

$${\sigma^2}_t = \gamma_o + \gamma_1 {\mu_{t-1}}^2 + \gamma_2 {\sigma_{t-1}}^2$$

The coefficient γ captures market news. Table 3.2 depicts two potential outcomes of the values of the coefficient. The number 1 exhibits a situation having spiky and sharp volatility efficiently responding to market movements, represents by a high value of γ. Whereas a low value of the μ shows inconsistent market volatility in the long run. We can put it in the way that the degree of consistency in the market news is the focus of μ. The second outcome in the table exhibits an opposite situation. γ has a lower value showing that the movements in the market are not well responded by the coefficient, while the high value of μ indicates a large influence of market news in the long run (Dowd 2010).

Table 3.2: Possible values of coefficient

Coefficients	γ	μ
High Values	1	2
Low Values	2	1

The fulfillment of following condition indicates the stationarity of GARCH (1,1) process,

$$\gamma_1 + \mu_1 < 1$$

As a result, the convergence of the conditional variance toward the unconditional variance can be anticipated in the long run.

The conventional realities of financial returns like weak and short memory, second order stationarity, asymmetries and strong linear autocorrelation among squared lagged returns etc. are captured through advanced conditional models.

3.3.2 E-GARCH

Exponential-GARCH model (EGARCH (p.q)), is the expanded and modified form of GARCH model, it converts the dependent variable into its natural logarithmic value before using and hence predicts a positive value. Market news, whether positive or negative, impacts the variance and creates an asymmetric effect. E-GARCH model is capable enough to capture that effect also. Nelson (1991) explained the asymmetry in detail and gave the assessment that volatility is affected more from negative shocks in comparison to the positive shocks, which means volatility will be increased more due to bad news in comparison to the good news of same size - financial time series of stock prices and exchange rate experience it normally.

3.3.3 GJR-GARCH

The third model is the Glosten, Jagannathan and Runkle-GARCH model. Unlike original GARCH model, it only emphasizes on the magnitude of the shock and does not give any assumption about the positivity or negativity of the shock and that would be independent to the response variable. It only behaves as a function based on the size of the shock (Glosten et al 1993).

3.3.4 ARMA Models

For standard GARCH model, the skewness and leptokurtosis of the financial time series cannot be fully captured by the normal innovation distribution, here arises the need to use an ARMA-GARCH model by involving another innovation assumption on z-distribution. Having said this, the ARMA model may be considered as an appropriate method for the assessment and understanding the dependence and the causal structure and to better find the predictions of the future values in each time series. The serial dependence in the mean and variance is created through ARMA combined with the Generalized Autoregressive Conditionally Heteroskedastic (GARCH) model (Bollerslev, 1986). GARCH model has the drawback that it is unable to differentiate positive and negative returns and treats them equally. The EGARCH model of Nelson (1991) and the GJR-GARCH (Glosten et al., 1993) introduce a leverage term to adjust the variance whenever there are stressful market reactions to bad news. All models are estimated by maximum likelihood. In the second step we simulate the returns series by setting the window size H to calculate the MDaRα.

A significant change in the behavior of expected MDD can be observed with the changing scenario of a portfolio. It depends on the current state of portfolio like the portfolio is losing money, on break even or profitable. The below mentioned formulas capture this transition effect by showing asymptotic behavior. The asymptotic behavior is important from the perspective of trading desks, which prefer smaller drawdowns and larger returns. In short the long time survival of systems is preferable. The expected MDD can be calculated through:

$$E(MDD) = \begin{cases} \frac{2\sigma^2}{\mu} Q_p\left(\frac{\mu^2 T}{2\sigma^2}\right) \overset{T\to\infty}{\longrightarrow} \frac{\sigma^2}{\mu}\left(0.63519 + 0.5\log T + \log\frac{\mu}{\sigma}\right) & \text{if } \mu > 0 \\ 1.2533\sigma\sqrt{T} & \text{if } \mu = 0 \\ \frac{-2\sigma^2}{\mu} Q_n\left(\frac{\mu^2 T}{2\sigma^2}\right) \overset{T\to\infty}{\longrightarrow} -\mu T - \frac{\sigma^2}{\mu} & \text{if } \mu < 0 \end{cases}$$

The transition of the expected MDD is evident, with phase shift of T from T to $\surd$ T as μ converts itself from negative to zero ,and then from $\surd$ T to log T, as μ converts from zero to positive. This transitional behavior can be used as a hypothesis test in order to define the standing of a portfolio, that whether it is losing prof, on breakeven or making money.

3.4 Historical Simulation (Nonparametric)

Historical simulation method is used to estimate the unconditional underlying distribution by applying the empirical distribution without making any assumptions on data generating process. The historical MDaR_α for a period of H days is the empirical percentile of the MDD series.

The First step is parametric i.e. to fit an econometric model to the data, and the second step is nonparametric in which simulations are used to estimate the risk measure. This methodology has an additional attribute that it is able to describe the whole distribution of risk measure. In this study we use the combination of parametric and non-parametric approaches to calculate the MDaR_α i.e., the semiparametric approach.

3.5 Maximum Drawdown

The risk of a portfolio is measured in just one number through a risk measure, in order to assess tail risk one needs risk measures such as the MDaR derived from extreme statistics. Let $\mathrm{P}_t = \ln(\mathrm{P}_t)$ be the logarithm of the asset price P_t at time t, t $\in \{1, , \mathrm{H}\}$. We can define MDD for this period as:

$$MDD = \underset{1 \leq k < H}{\max} \quad \underset{< k < j \leq H}{\max} \{pk - pj, 0\}$$

(1.1)

In this way a non-negative random variable is yielded, whose duration D, $1 \leq \mathrm{D} = \mathrm{j} - \mathrm{k} < \mathrm{H}$, the length of the sequence of log-prices, is also a random variable. When D = 1, the MDD coincides with the worst single (one-period) loss within the window, the Maximum Loss. Alternatively, the MDD may be defined in percentage terms

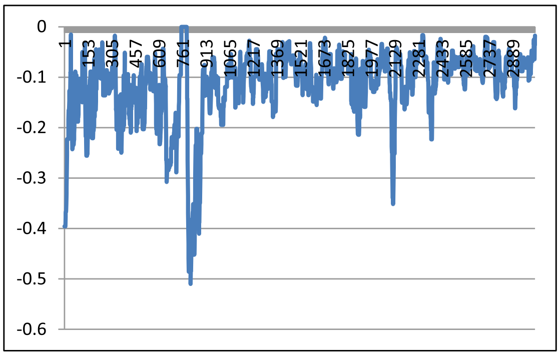
$$MDD = \underset{1 \leq k < j \leq H}{\max} \left(\frac{pk - pj}{pk} \right)$$

(1.2)

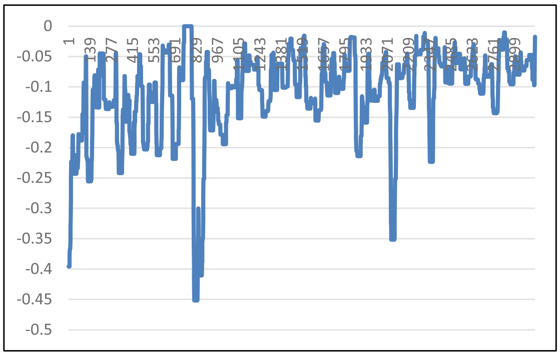
1 = k < j = H, where H is the window size, or zero if all Pk = Pj. The MDD being defined on a sequence of prices is affected by the strength of serial dependence shown by the returns, and its magnitude is sensitive to the crucial choice of H.

3.5.1 Graphical Demonstrations of MDD

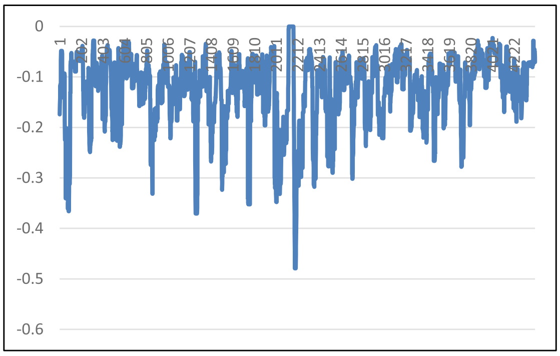
The below graphs show the time series plot of the MDD (through both the formulas mentioned above) based on a daily shifted window with H = 22 days and validate the authenticity of both the formulas.



MDD (%) of ABL

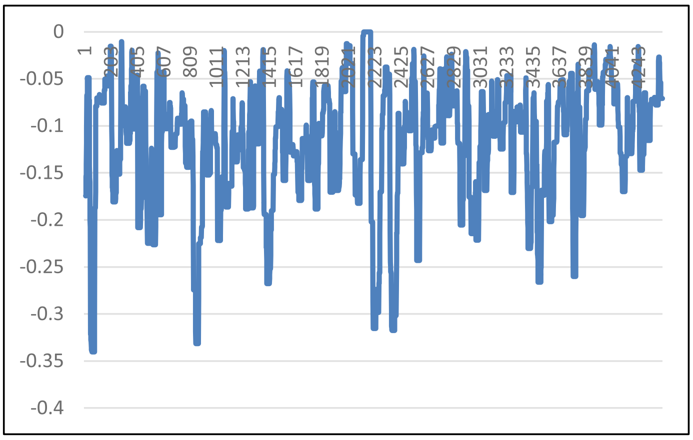


Relative MDD (%) of ABL

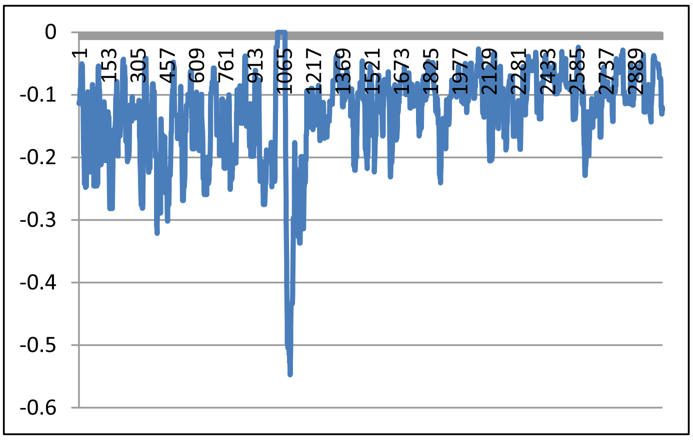


MDD (%) of ACBL

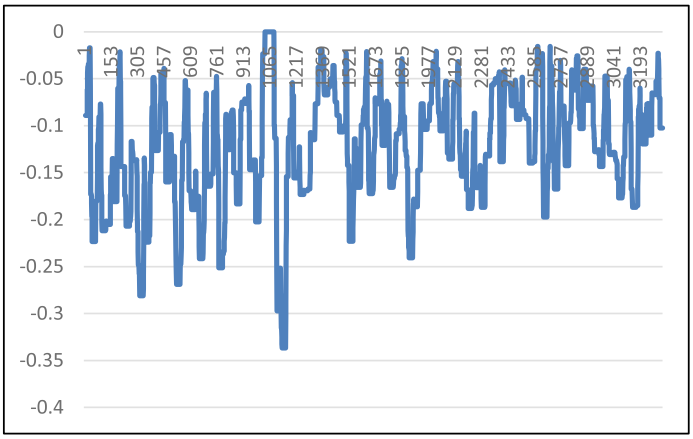
Time series plot of MDD based on a daily shifted window with H=22 days.



Relative MDD (%) of ACBL

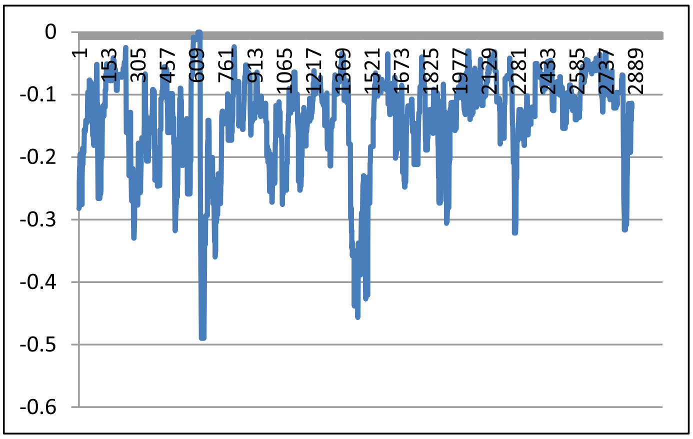


MDD (%) of BAF

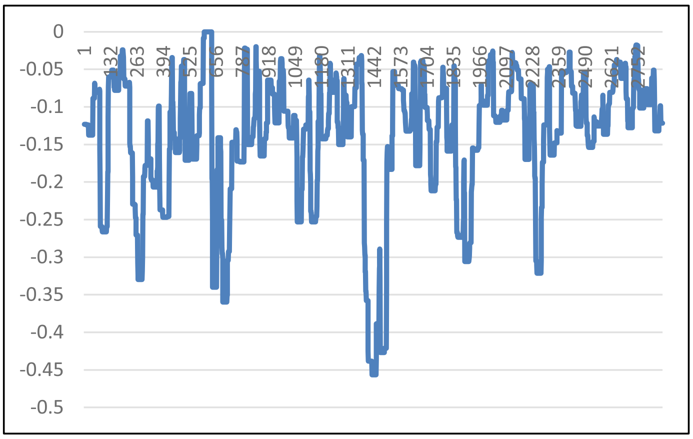


Relative MDD (%) of BAF

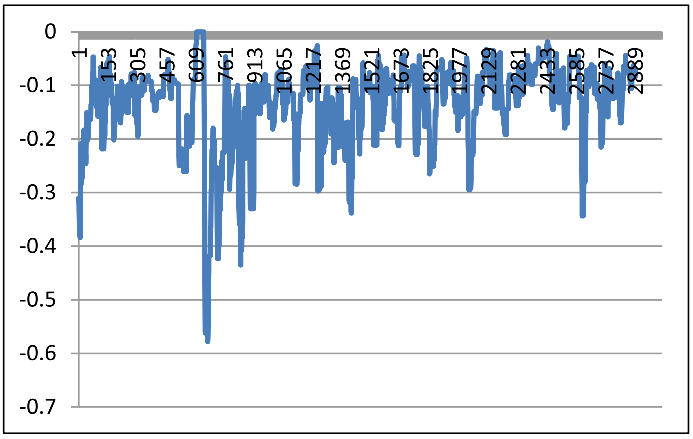
Time series plot of MDD based on a daily shifted window with H=22 days.



MDD (%) of BIPL

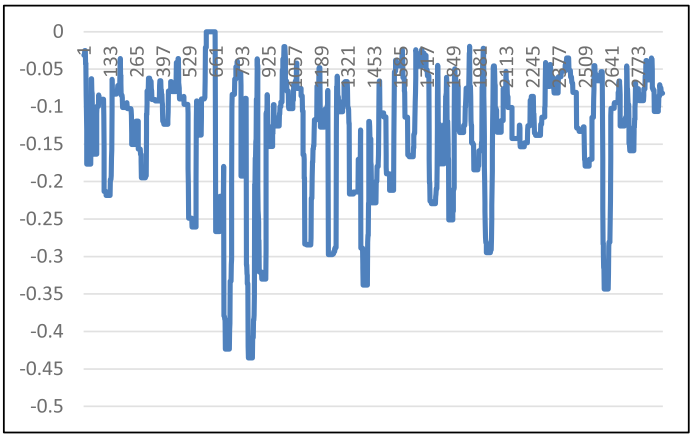


Relative MDD (%) of BIPL

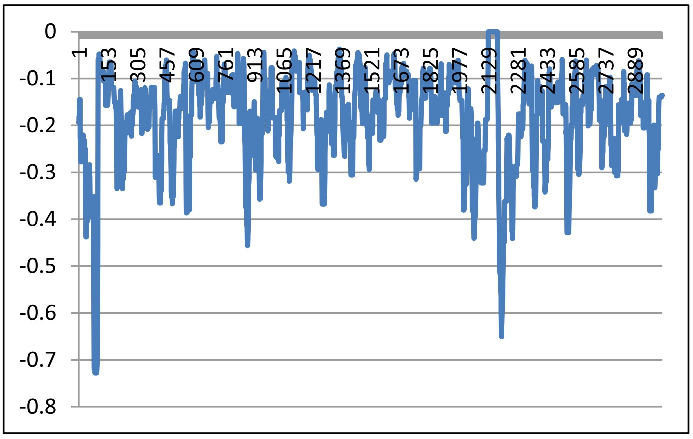


MDD (%) of BoK

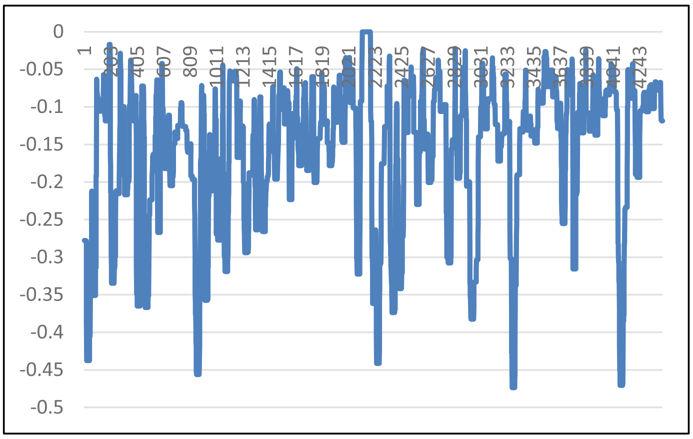
Time series plot of MDD based on a daily shifted window with H=22 days.



Relative MDD (%) of BoK

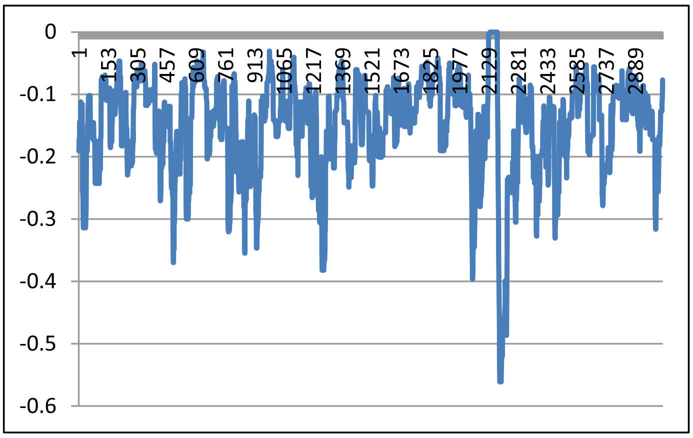


MDD (%) of BOP

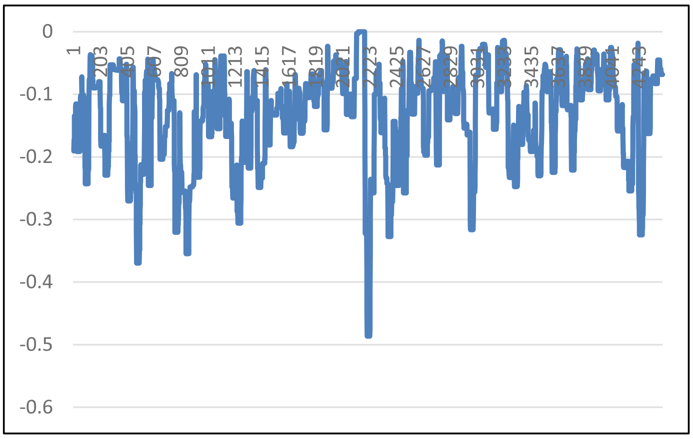


Relative MDD (%) of BOP

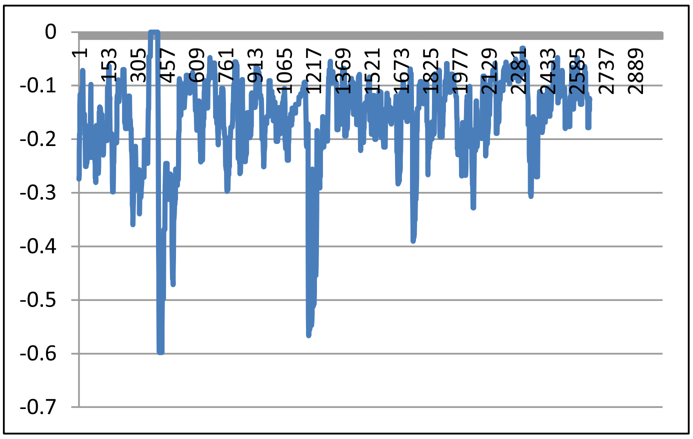
Time series plot of MDD based on a daily shifted window with H=22 days.



MDD (%) of FBL

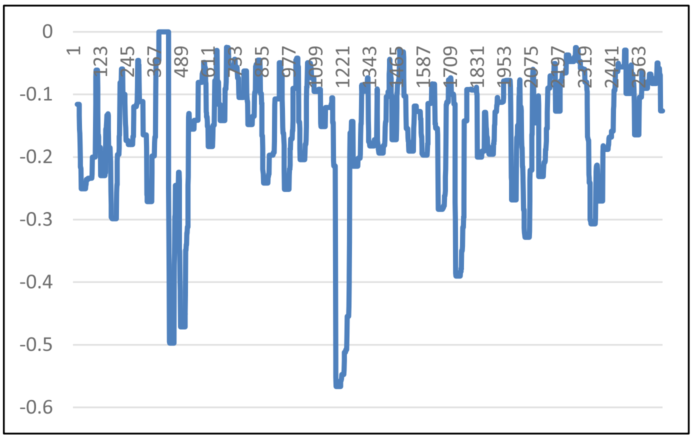


Relative MDD (%) of FBL

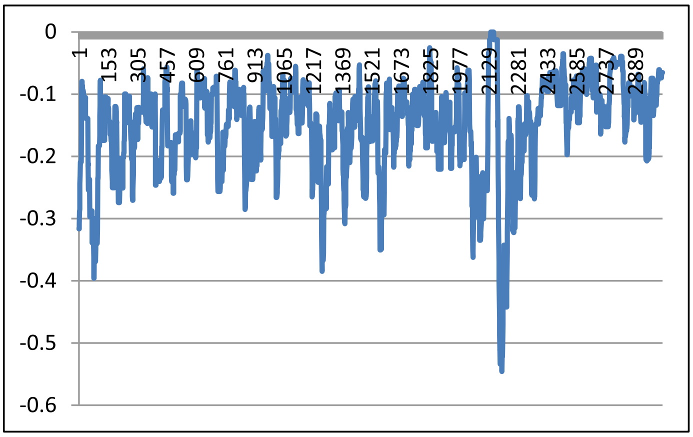


MDD (%) of JSB

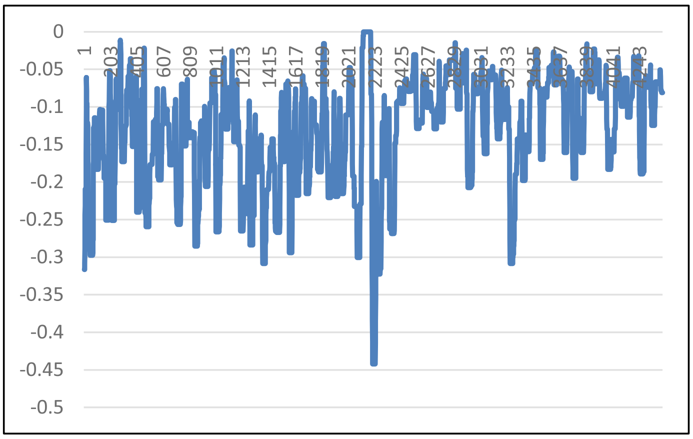
Time series plot of MDD based on a daily shifted window with H=22 days.



Relative MDD (%) of JSB

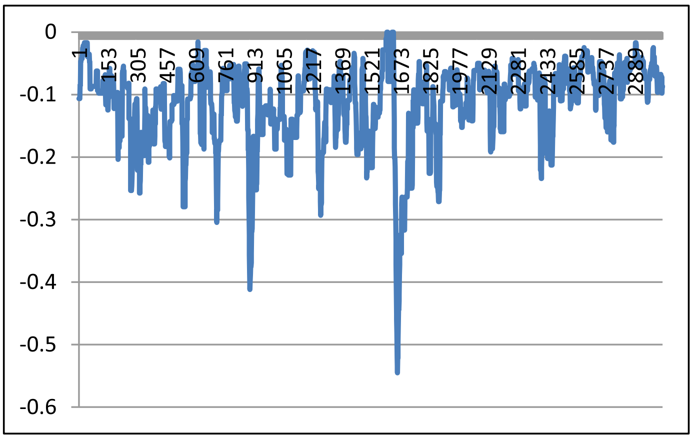


MDD (%) of MCB

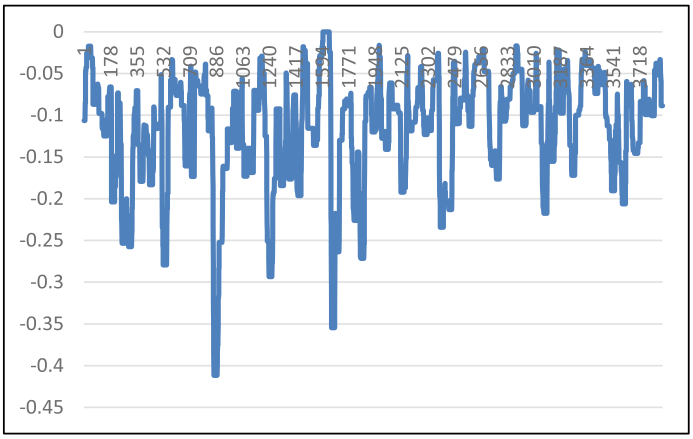


Relative MDD (%) of MCB

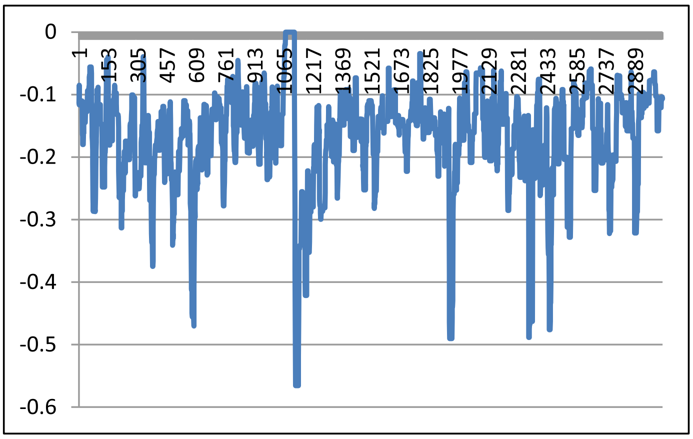
Time series plot of MDD based on a daily shifted window with H=22 days.



MDD (%) of MBL

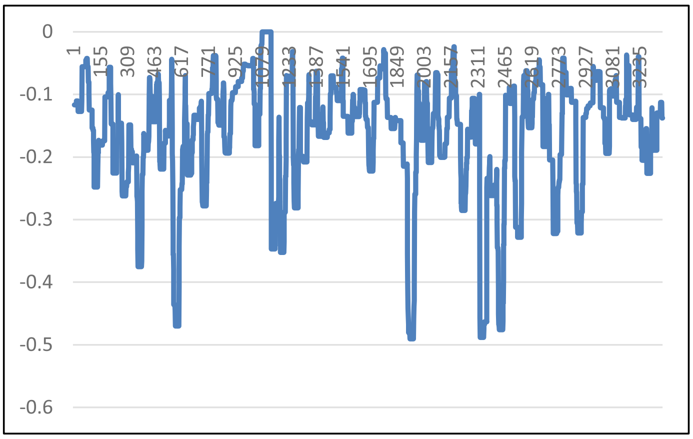


Relative MDD (%) of MBL

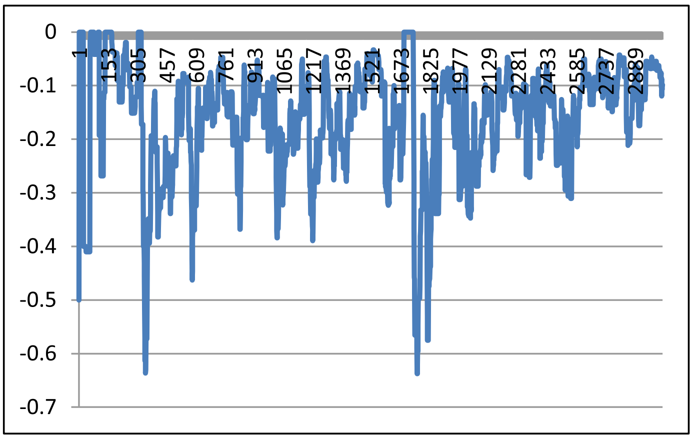


MDD (%) of SMB

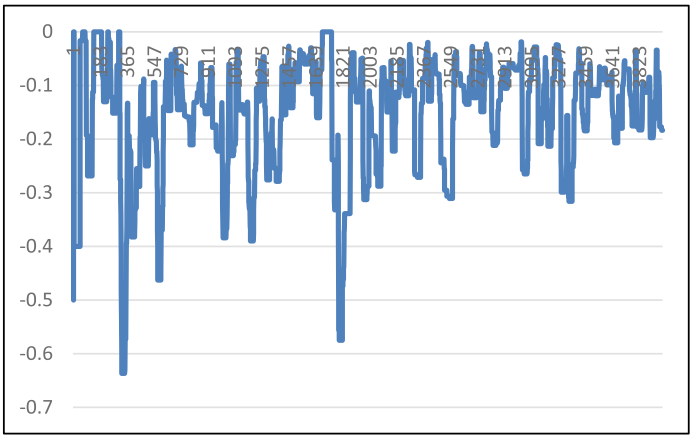
Time series plot of MDD based on a daily shifted window with H=22 days.



Relative MDD (%) of SMB

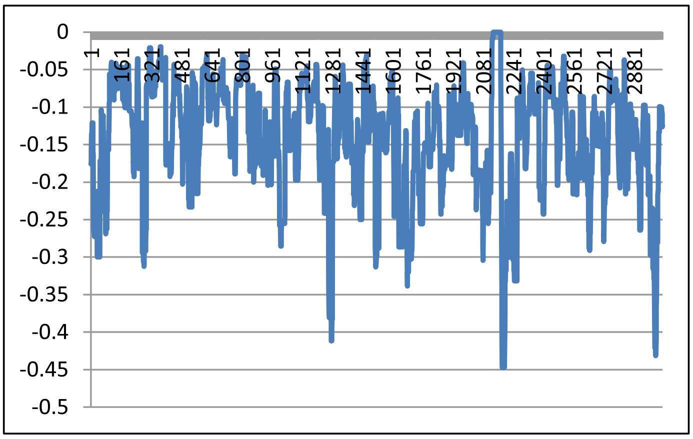


MDD (%) of SLK

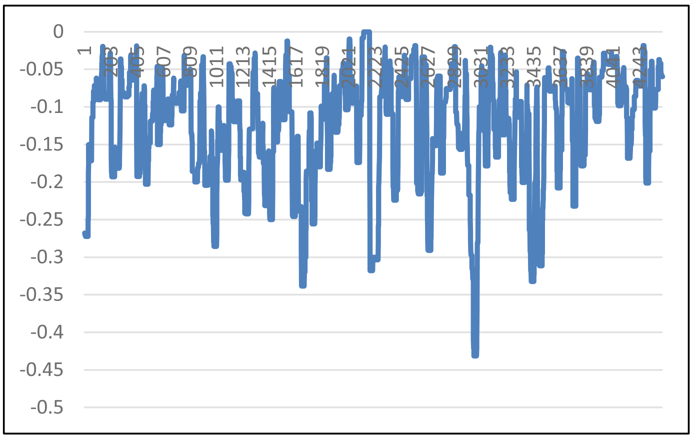


Relative MDD (%) of SLK

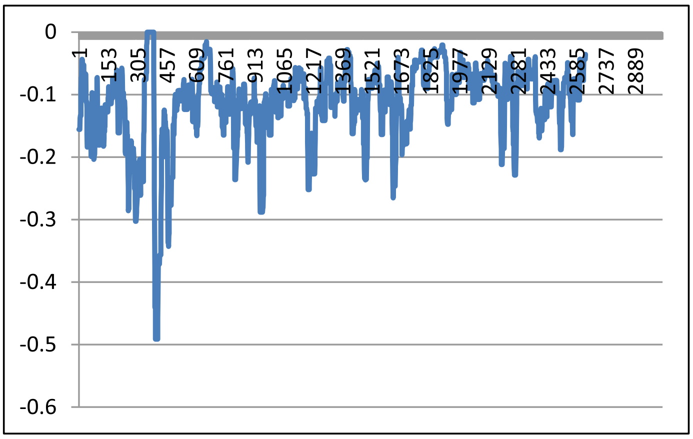
Time series plot of MDD based on a daily shifted window with H=22 days.



MDD (%) of SNB

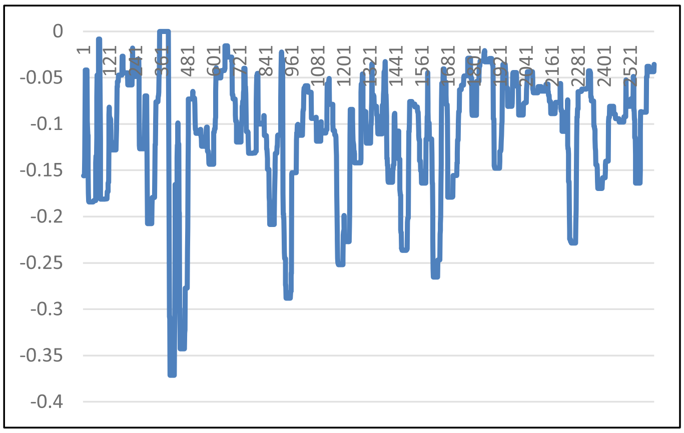


Relative MDD (%) of SNB

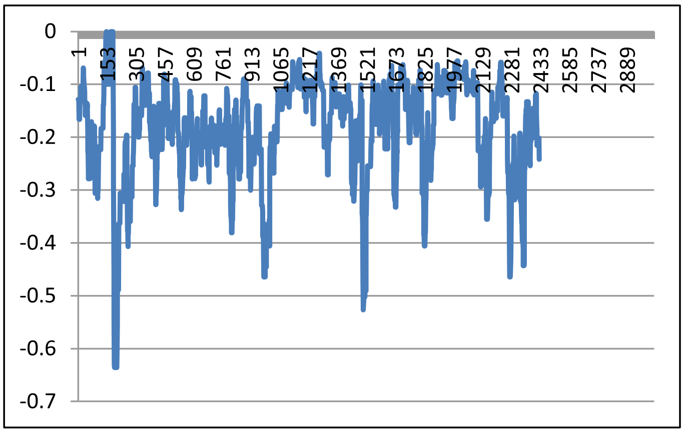


MDD (%) of SCB

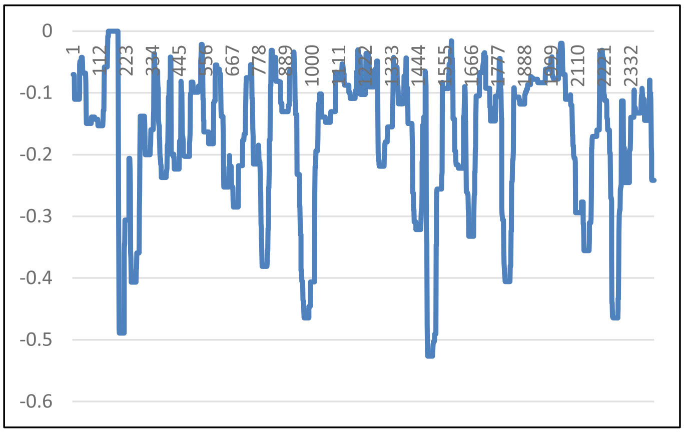
Time series plot of MDD based on a daily shifted window with H=22 days.



Relative MDD (%) of SCB



MDD (%) of SMT



Relative MDD (%) of SMT

FIGURE 3.1: Time series plot of MDD based on a daily shifted window with H=22 days.

The graphs of MDD are constructed through formula 1.1 and graphs of relative MDD represent formula 1.2. Normally people are not able to identify the difference between levels of Maximum Drawdown. First is called the drawdown period and it is from the peak to the bottom; second level is called the recovery period and it is from that bottom point to the original level of the previous peak. The underwater period is the time period between first peak and the second peak. Mostly people are unable to differentiate the underwater period from the drawdown period and refers the period between two peaks also as drawdown.

3.6 Violation in Actual MDD

Table 3.3: Violation in actual MDD

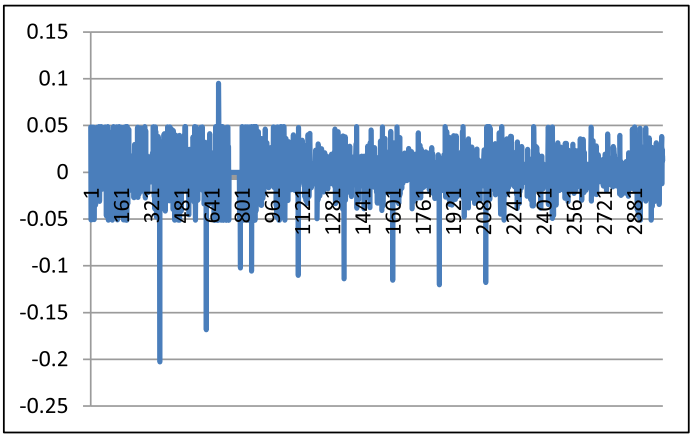
Bank	**Violation in %**
Allied Bank Limited	0.07
Askari Bank Limited	0.1
Bank AlFalah Limited	0.2
Bank Islami Pakistan Limited	0.1
Bank of Khyber	0.1
Bank of Punjab	0.2
Faysal Bank Limited	0.2
JS Bank Limited	0.1
MCB Bank Limited	0.1
Meezan Bank Limited	0.2
Samba Bank Limited	0.2
Silk Bank Limited	0.2
Soneri Bank Limited	0.1
Standard Chartered Bank	0.1
Summit Bank Limited	0.1

Above mentioned table shows the difference in actual returns associated with the stocks and MDD calculated for these stocks. The nominal and negligible difference indicates the significance of maximum drawdown as the risk assessment tool.

3.7 MDD & Log Returns

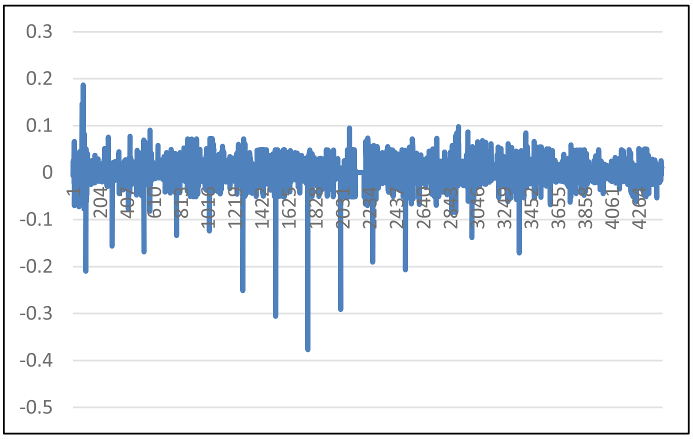
The Maximum Drawdown at Risk α (MDaR_α) is defined as the $(1-\alpha)$ quantile of the MDD distribution. The difference between VaR_α and MDaR_α is based on time period, the first measure is normally computed for short-time period, which can be of one or five days, and the latter being used for longer horizons i.e., at least 10 days. The prices are taken on daily basis and are used to calculate total return indices. The advantage to use logarithmic returns is the infinite upside potential of log returns in normal distribution, and also that the losses in this case cannot go beyond 100%. This study is based on risk measure's performance and the analysis of the risk and return relationship, so the returns are used in percentage form keeping in mind the percentage return's natural correspondence with the market price.

3.7.1 Graphs of Log Returns

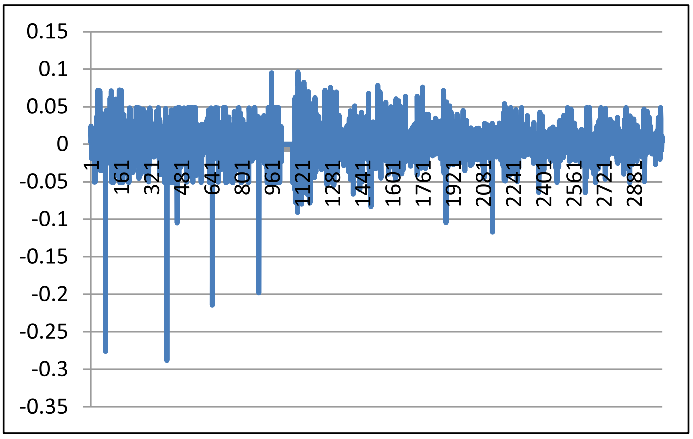


Log-returns (%) of ABL

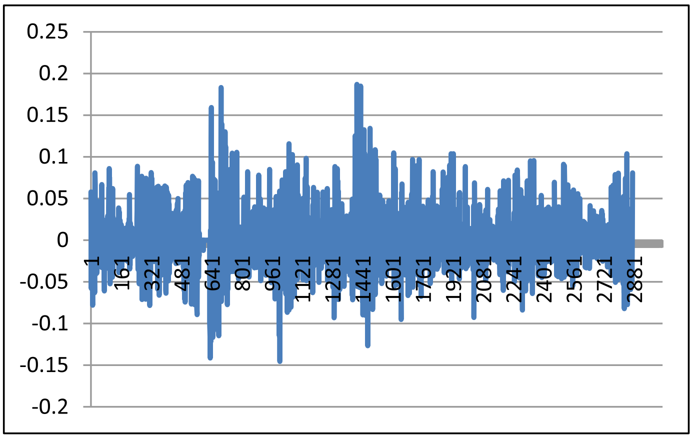
Log-Returns from the daily PSX 100 data.



Log-returns (%) of ACBL

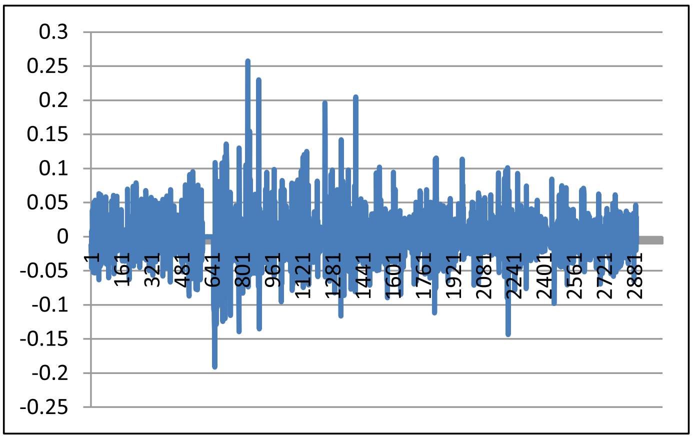


Log-returns (%) of BAF

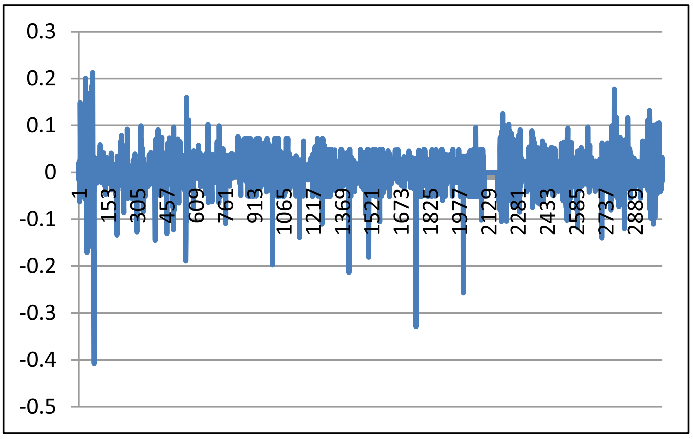


Log-returns (%) of BIPL

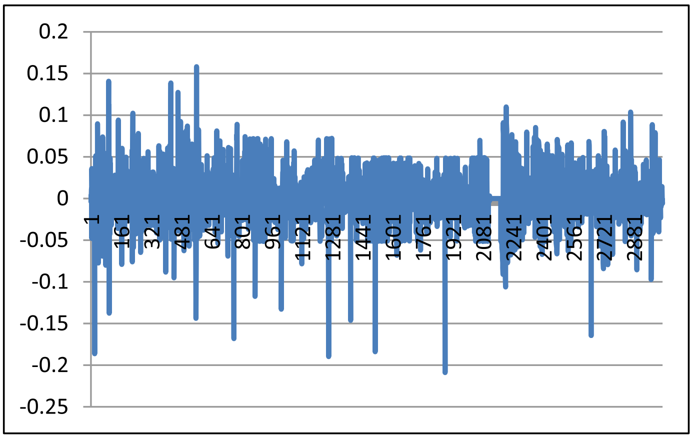
Log-Returns from the daily PSX 100 data.



Log-returns (%) of BoK

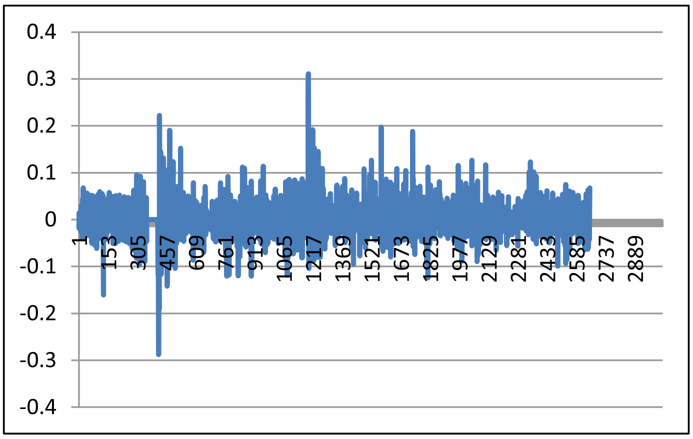


Log-returns (%) of BoP

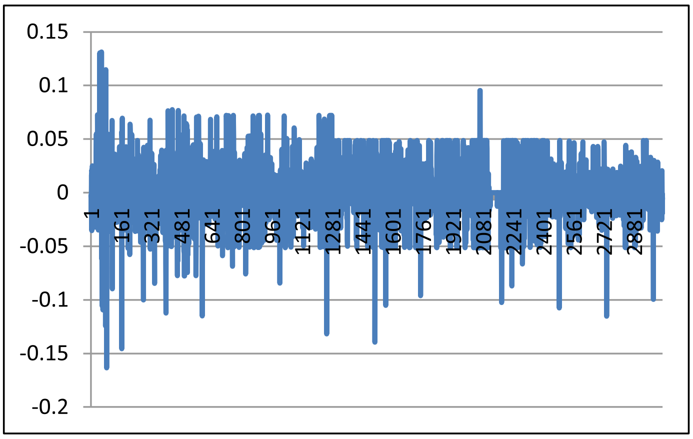


Log-returns (%) of FBL

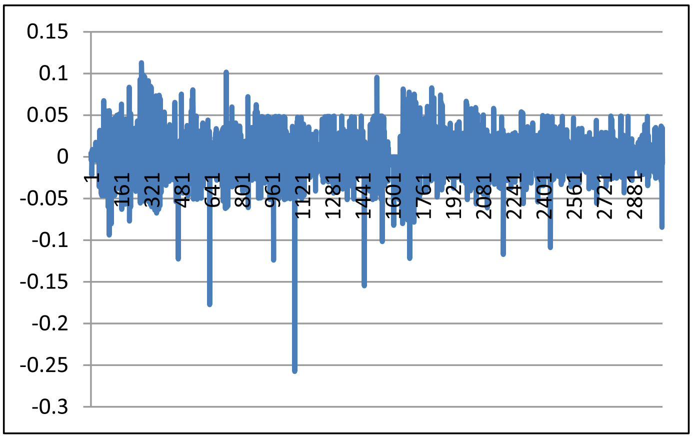
Log-Returns from the daily PSX 100 data.



Log-returns (%) of JSB

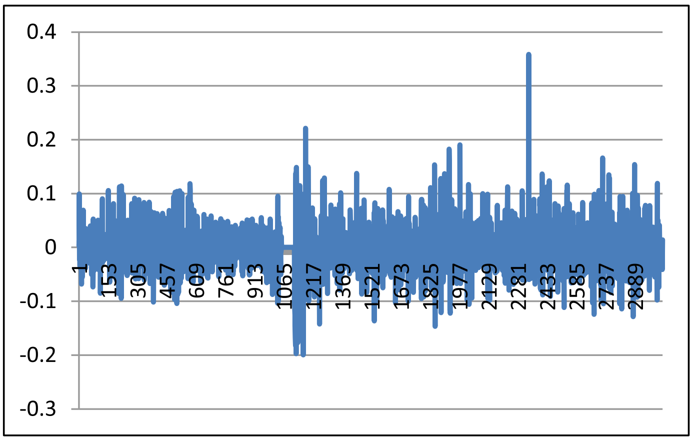


Log-returns (%) of MCB

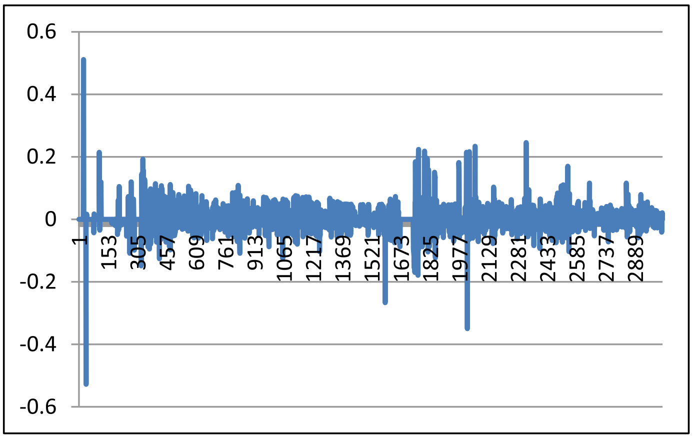


Log-returns (%) of MBL

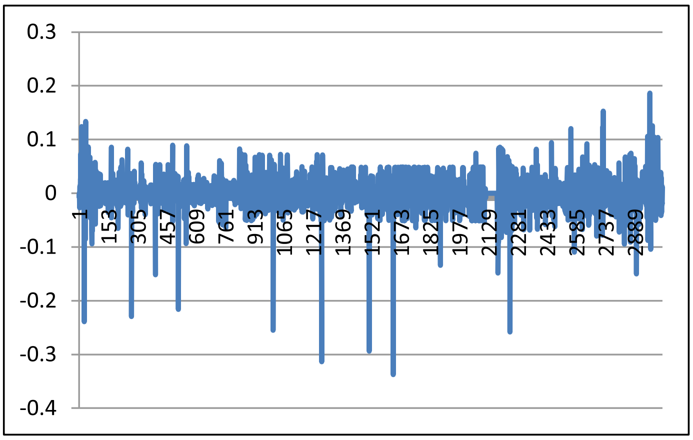
Log-Returns from the daily PSX 100 data.



Log-returns (%) of SMB

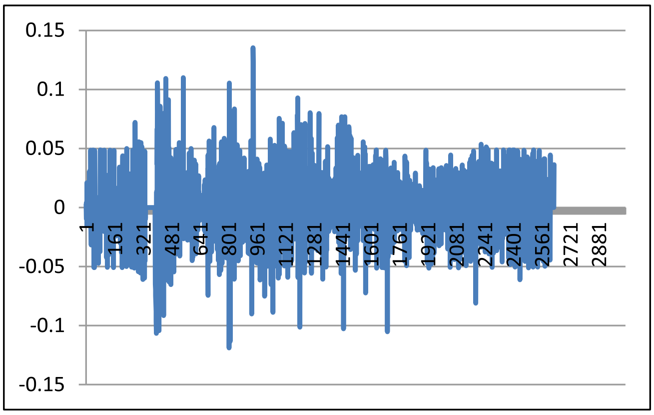


Log-returns (%) of SLK

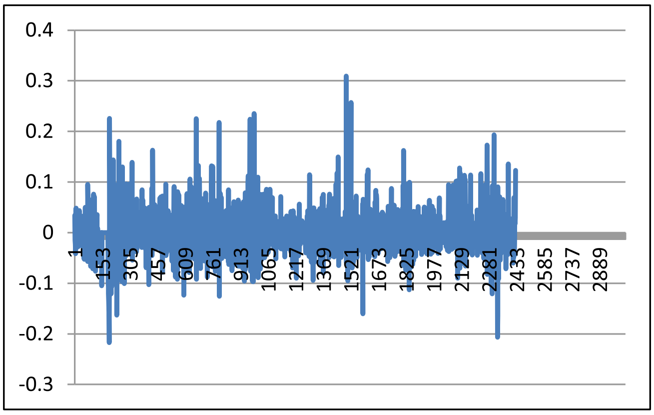


Log-returns (%) of SNB

Log-Returns from the daily PSX 100 data.



Log-returns (%) of SCB



Log-returns (%) of SMT

FIGURE 3.2: Log-Returns from the daily PSX 100 data.

Financial crises of 2008-2009 is the main reason of volatility shown in the graphs. The volatility cluster in the other parts of the graphs is linked to the uncertain conditions of Pakistani financial markets. Overall the returns show a static behavior but distinct volatility clusters are also demonstrated. Hence, proves the fact that the variance exists in the data and is not constant, so GARCH models are an appropriate choice for estimation.

3.8 Backtesting

In finance, the term "backtesting" is used in number of different aspects. Normally backtesting is used in following two ways:

1. In order to measure the past performance of different trading strategies.

2. In order to assess the dependability of the financial risk model. And it is done by forecasting through historical data and then the forecasted returns are compared with the observed returns (Christoffersen, 2009). The risk assessment models need to be validated to check their reliability. This validation is done through backtesting. So in assessment of a risk model, backtesting is the most crucial stage. The adequacy of the model used for risk assessment is checked through backtesting and it is done by using quantitative and statistical methods.

Normally backtesting process is used to achieve three different objectives. The first objective is to compare the assessed values to the actual values and determine the level of difference in both. In this way it can be found that such assessments are compatible or not to the outputs statistically. The statistical testing of hypothesis is used to confirm the acceptability of assessment models. The second objective of the backtesting process is pertain to risk managers. It facilitates them to diagnose the problems in risk models and helps in the improvisation of these methods. The third objective of backtesting is to compare the different risk models and arrange them in ranking on the basis of their performances as risk assessment models. A good risk model should fulfill all of the three mentioned criteria.

The violations in MDD are calculated through backtesting method. The difference in actual and forecasted MDD is compared with the total number of observations to calculate the violations.

$$Violations\,(\%) = \frac{No.ofViolations}{Totalnumberofobservations} \times 100$$

Chapter 4

Results and Analysis

4.1 Descriptive Statistics

4.1.1 Descriptive Analysis of Returns

Mean describes the sample through a single value that is usually also the central value of the data. Standard deviation is a statistical measurement that highlights the historical volatility.

Mean of all the banks is almost equals to 0 with the standard deviation varies from 0.02 to 0.04. Standard deviation in case of 8 out of 15 banks is 0.02 i.e., in ABL, ACBL, BAF, FBL, MCB, MBL, SNB and SCB. Standard deviation from mean in six banks i.e. BIPL, BoP, JSB, SMB, SLK and SBL is 0.03 and BoK has standard deviation of 0.0006, which shows that Bok has least spread out data. Almost 50% of the banks have almost equal spread in stock returns.

In normality, the third moment is the skewness or asymmetry of a density function around the mean. Skewness is a measure of symmetry, or in more precise way it is the lack of symmetry. If skewness value is different from zero, it means that the return series is not normal and deviation exists in it. Negative skewness indicates the concentrated mass of distribution on the right side and hence a left tailed distribution.

Table 4.1: Descriptive Analysis of Log Returns of Stocks

Banks	**Mean**	**Standard Deviation**	**Skewness**	**Kurtosis**	**Minimum**	**Maximum**
Allied Bank Limited	0.0002	0.0211	-0.6472	6.3480	-0.2027	0.0952
Askari Bank Limited	0	0.0248	-2.1541	29.9121	-0.3769	0.1866
Bank AlFalah Limited	0	0.0242	-1.2647	15.0054	-0.2881	0.0963
Bank Islami Pakistan Limited	-0.0003	0.0312	0.6138	3.7538	-0.1455	0.1869
Bank of Khyber	-0.0003	0.0006	0.4649	7.4068	-0.2348	0.2577
Bank of Punjab	-0.0002	0.03296	-1.0050	14.6258	-0.4080	0.2126
Faysal Bank Limited	0.0001	0.027	-0.2888	5.6996	-0.2087	0.1582
JS Bank Limited	-0.0003	0.0361	0.7678	7.9598	-0.2877	0.3111
MCB Bank Limited	0.0005	0.0243	-0.3008	3.3209	-0.1634	0.1313
Meezan Bank Limited	0.0005	0.0231	-0.4380	7.6179	-0.2574	0.1127
Samba Bank Limited	-0.0002	0.0397	0.5311	5.1729	-0.1995	0.3584
Silk Bank Limited	-0.0002	0.0371	1.9698	56.9473	-0.5276	0.6931
Soneri Bank Limited	0	0.0274	-1.5164	20.8557	-0.3374	0.1864
Standard Chartered Bank	-0.0003	0.0252	0.0146	2.5847	-0.1188	0.1353
Summit Bank Limited	-0.001	0.0392	1.0702	7.4083	-0.2171	0.3093

Conversely, positive skewness indicates the concentrated mass of distribution on the left side and hence a right tailed distribution. Almost half of the banks have negative skewness ranging from -0.2888(FBL) to -2.1541 (ACBL) and half are positively skewed ranging from 0.0146 (SCB) to 1.969 8 (SLK).

Kurtosis is the fourth moment of a distribution. Kurtosis describes the nature of the peaks of the distribution. The kurtosis is used to explain the degree of concentration of the stock returns around the mean. The excess kurtosis in returns and the centered mean of zero indicate the leptokurtic characteristics of stocks and indices. The value of kurtosis for returns vary from 2.5847 (SCB) to 29.9121 (ACBL). One bank has very high kurtosis i.e., 56.9473 (SLK). This extremely high value shows that data for this bank has quite heavy tail. Extreme values in the tails have the ability to distort not only the mean and standard deviation, but can also effect the skewness and kurtosis measures. The leptokurtic behavior of stocks is evident from the excess kurtosis in returns.

The minimum and maximum are also useful in understanding the data. Minimum is simply the lowest observation and maximum is the highest observation. Minimum and maximum observations depict the variation in data and highlight if any abnormally high or low observation exists in the data. Mean of the returns is almost zero in all the banks and minimum values range from -0.1188 (SCB) to -0.5276 (SLK). The maximum values range from 0.0952 (ABL) to 0.6931(SLK), which shows that Silk bank has the highest variation in the values.

4.1.2 Descriptive Analysis of MDD

Table 4.2: Descriptive Analysis of MDD

Banks	Mean	Standard Deviation	Skewness	Kurtosis	Minimum	Maximum
Allied Bank Limited	-0.1151	0.0725	-2.0418	6.0380	-0.5099	0
Askari Bank Limited	-0.1271	0.0726	-1.3118	2.1416	-0.4793	0
Bank AlFalah Limited	-0.1288	0.0708	-1.6038	5.0440	-0.5474	0
Bank Islami Pakistan Limited	-0.1421	0.0823	-1.3370	2.1299	-0.4896	0
Bank of Khyber	-0.1425	0.0941	-2.5642	9.9460	-0.6900	0
Bank of Punjab	-0.1683	0.0987	-1.6344	4.5648	-0.7277	0
Faysal Bank Limited	-0.1409	0.0763	-1.4552	3.6794	-0.5608	0
JS Bank Limited	-0.1639	0.0899	-1.8327	5.5042	-0.5980	0
MCB Bank Limited	-0.1369	0.0736	-1.3732	3.3776	-0.5456	0
Meezan Bank Limited	-0.1116	0.0629	-1.8302	5.9906	-0.5449	0
Samba Bank Limited	-0.1691	0.0820	-1.5791	4.4616	-0.5656	0
Silk Bank Limited	-0.1561	0.0982	-1.3568	2.9419	-0.6377	0
Soneri Bank Limited	-0.1325	0.0744	-1.0548	1.3370	-0.4469	0
Standard Chartered Bank	-0.1151	0.0667	-1.8070	5.8913	-0.4912	0
Summit Bank Limited	-0.185	0.0965	-1.4388	3.1876	-0.6359	0

This table shows the statistical analysis of the actual maximum drawdowns of all the banks. Mean of MDD varies from -0.1116 (MBL) to -0.185 (SBL). Standard deviation for all the banks is on higher side showing greater spread in MDDs. Lowest standard deviation is 0.0629 for MBL and highest deviation is 0.0987 for

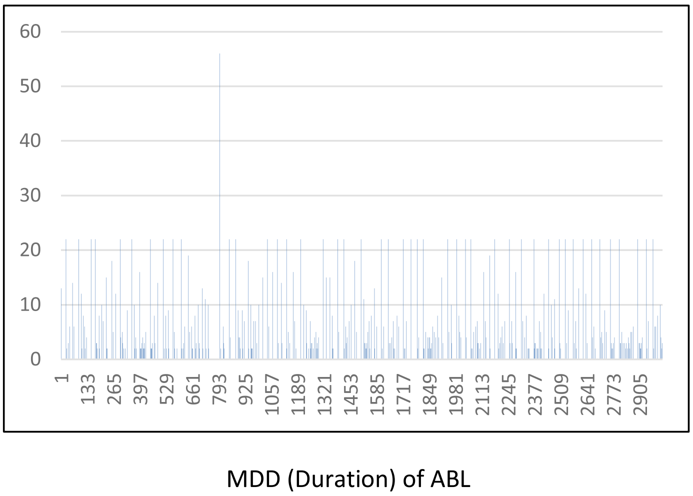
BoP. Skewness is a measure of symmetry, or more precisely, the lack of symmetry. Skewness values for all the banks are negative ranging from -1.0548 (SNB) to -2.5642 (BoK). Negative skewness indicates that right side has the concentrated mass of distribution and the distribution is left tailed. The degree of concentration of MDDs around mean is the kurtosis. The value of kurtosis for MDDs vary from 1.3370 (SNB) to 9.9460 (BoK). Minimum values range from -0.4469 (SNB) to -0.7277 (BoP) and maximum is 0 in all the banks.

4.2 Size & Duration

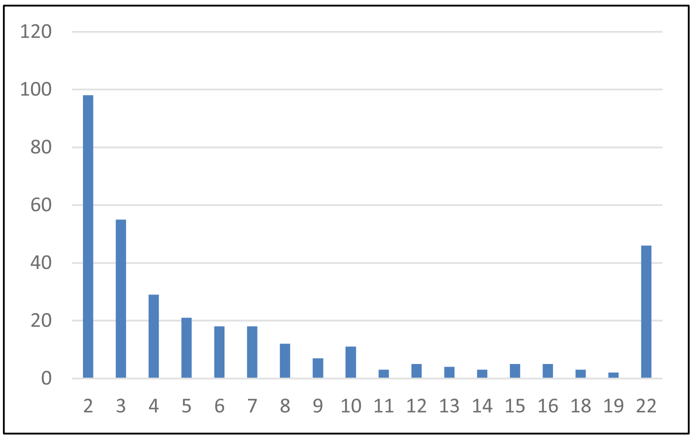
The Max Drawdown Duration is the time stocks take from the beginning of the retrenchment to the new high. It can be described as the investment's worst amount of time period between peaks.

The empirical distributions of the MDD size and duration are described through graphs and table below. Among the 4348 MDDs there are 831 impressive long durations: seven hundred twenty-two of 22 days, thirty-two of 23 days, thirty-six of 21 days and forty-one MDDs lasting for 20 days.

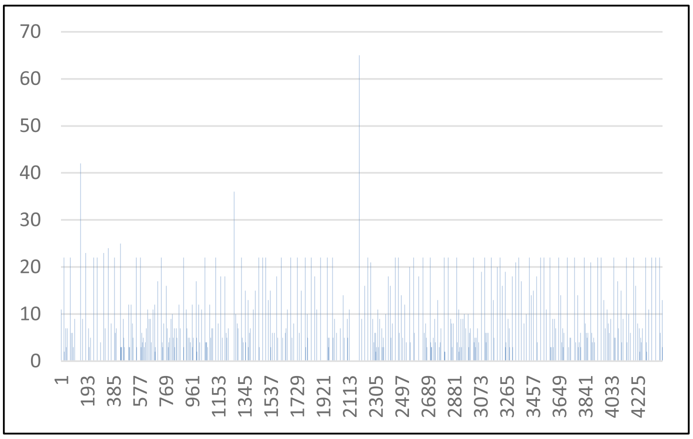
4.2.1 Graphs of Durations



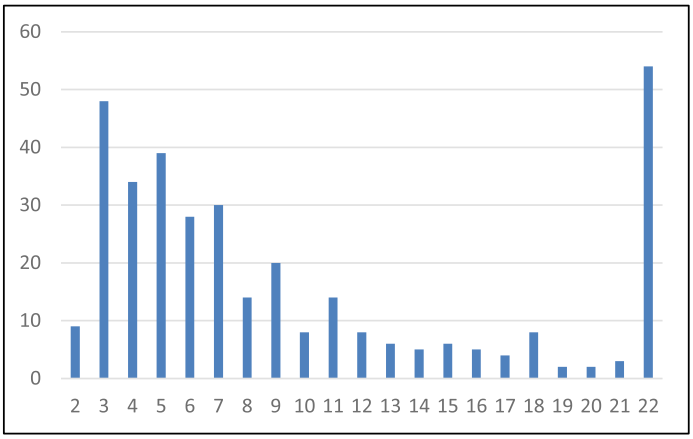
MDD duration empirical distributions.



MDD (Duration) of ABL

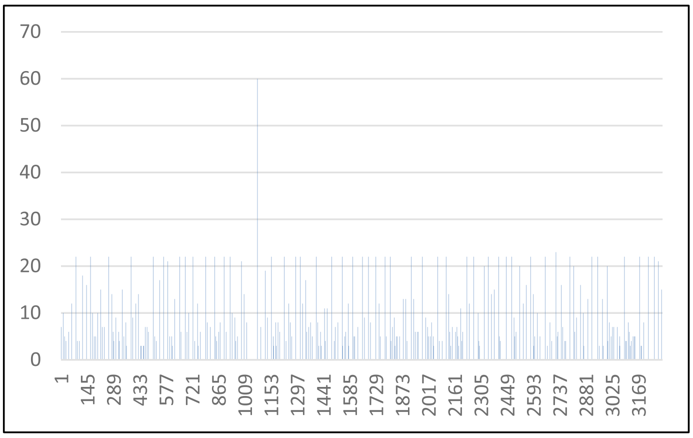


MDD (Duration) of ACBL

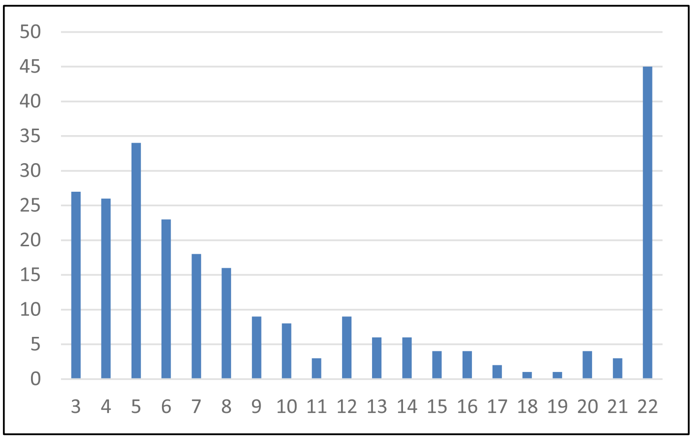


MDD (Duration) of ACBL

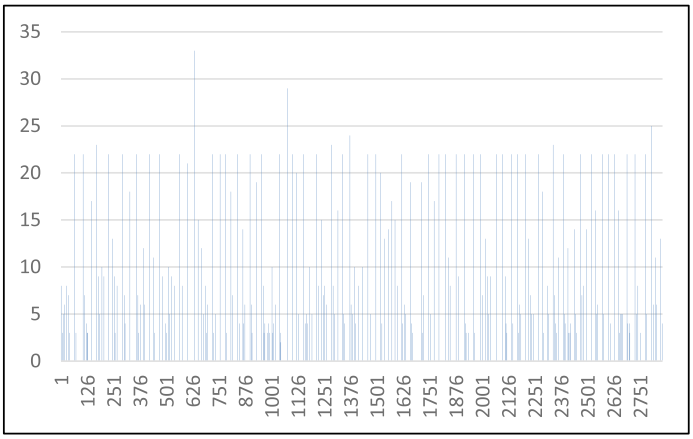
MDD duration empirical distributions.



MDD (Duration) of BAF

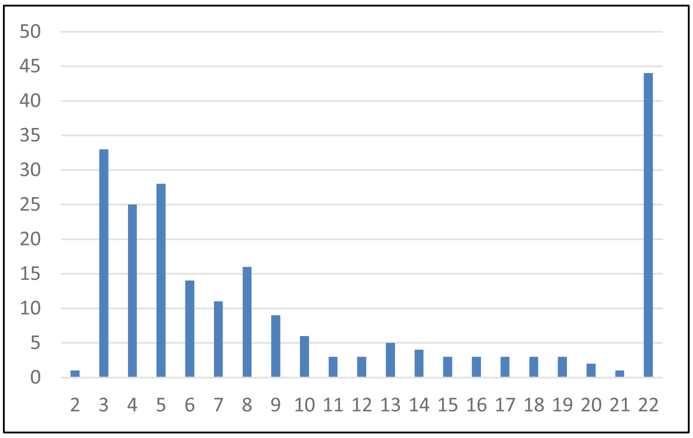


MDD (Duration) of BAF

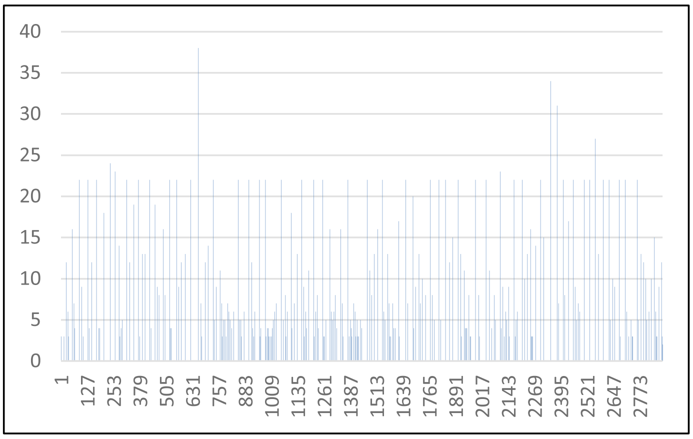


MDD (Duration) of BIPL

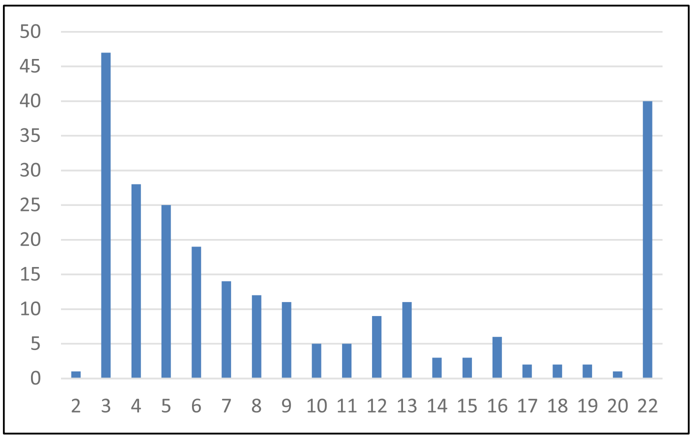
MDD duration empirical distributions.



MDD (Duration) of BIPL

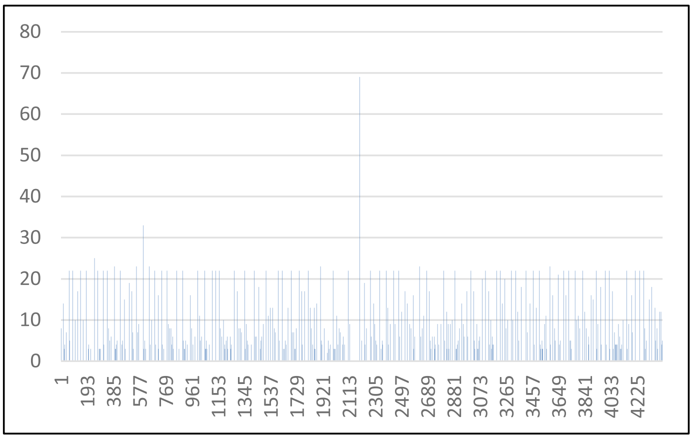


MDD (Duration) of BoK

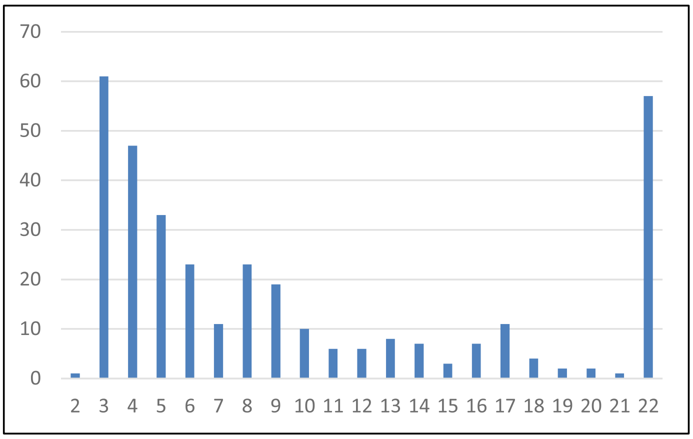


MDD (Duration) of BoK

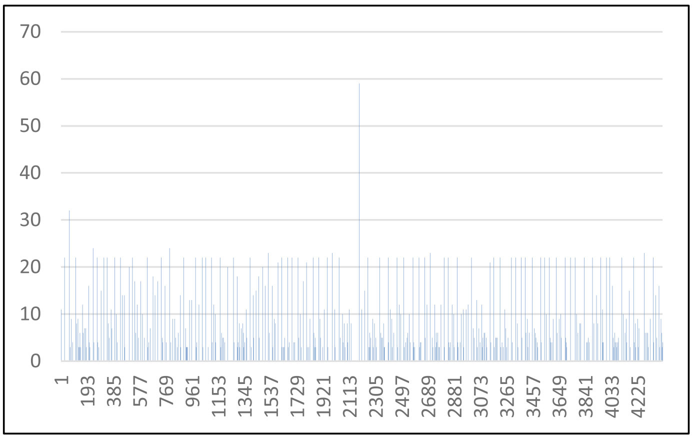
MDD duration empirical distributions.



MDD (Duration) of BOP

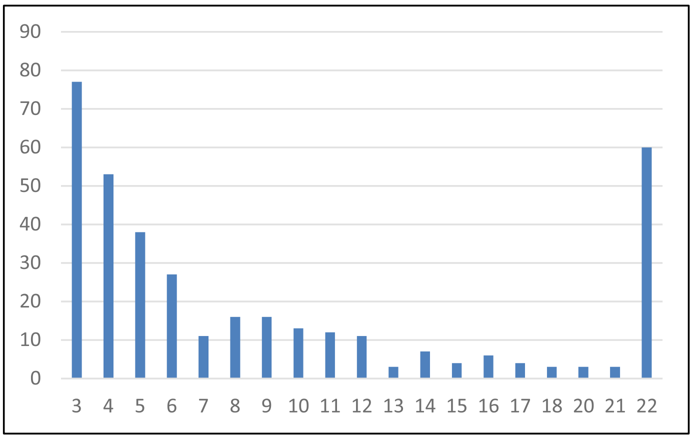


MDD (Duration) of BOP

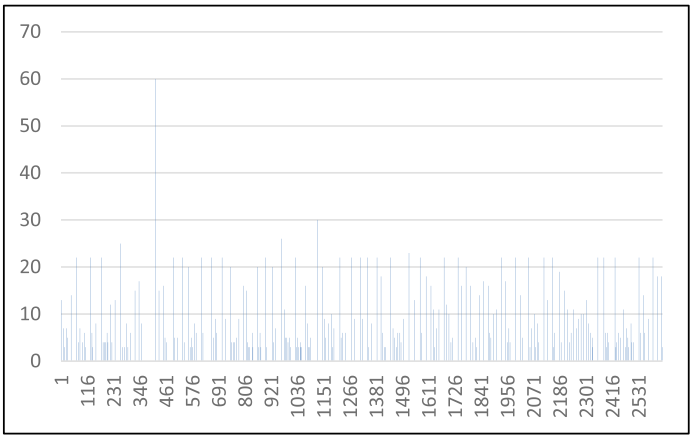


MDD (Duration) of FBL

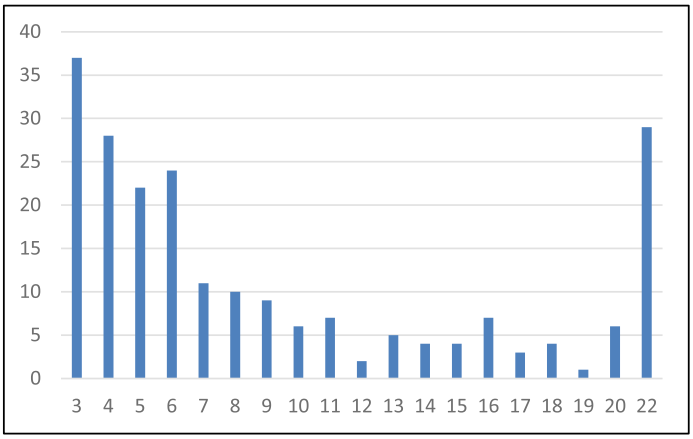
MDD duration empirical distributions.



MDD (Duration) of FBL

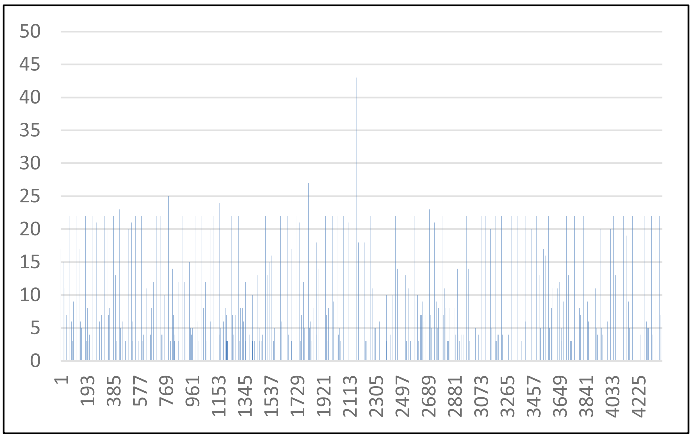


MDD (Duration) of JSB

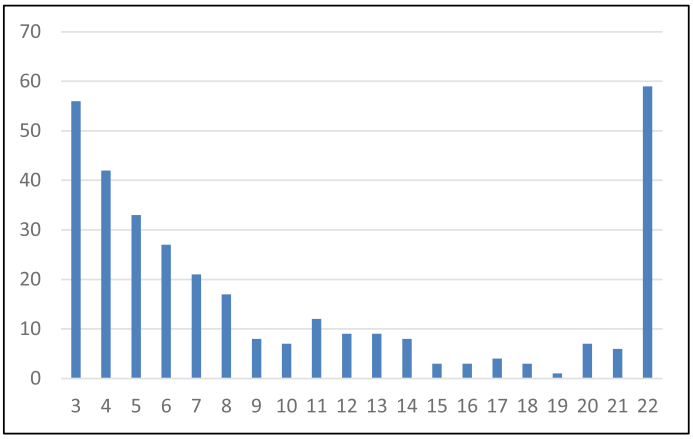


MDD (Duration) of JSB

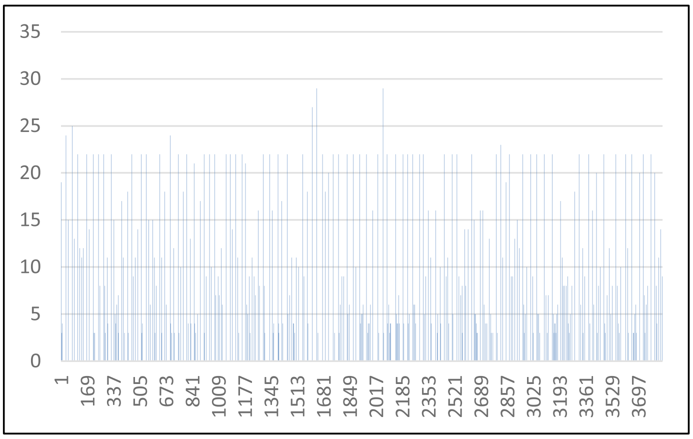
MDD duration empirical distributions.



MDD (Duration) of MCB

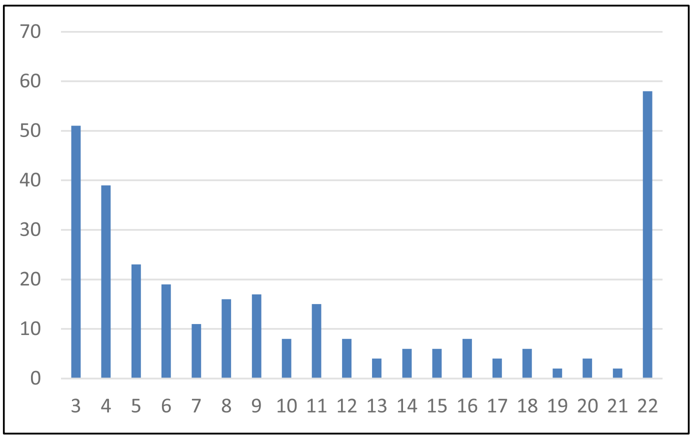


MDD (Duration) of MCB

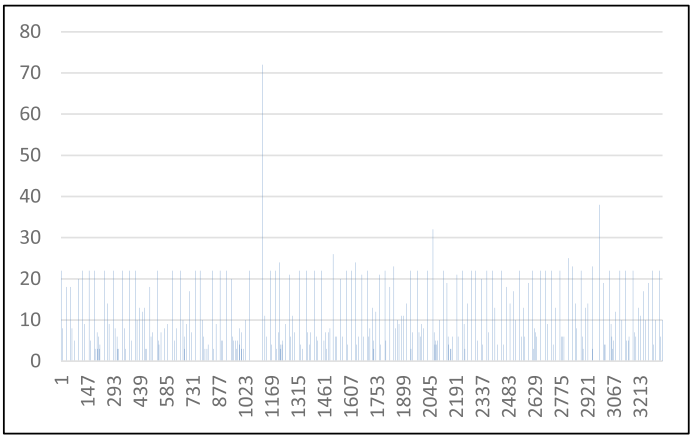


MDD (Duration) of MBL

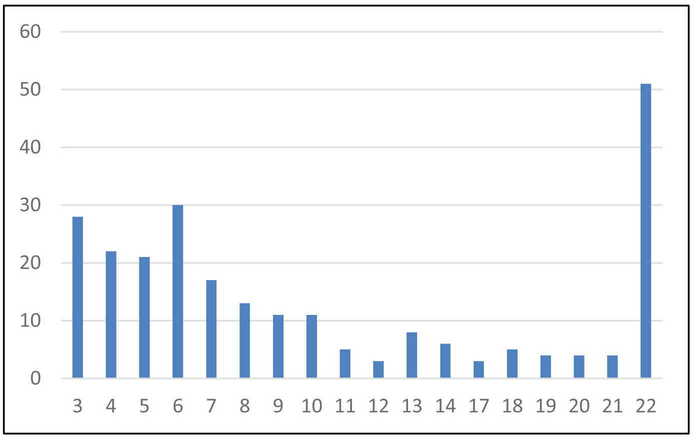
MDD duration empirical distributions.



MDD (Duration) of MBL

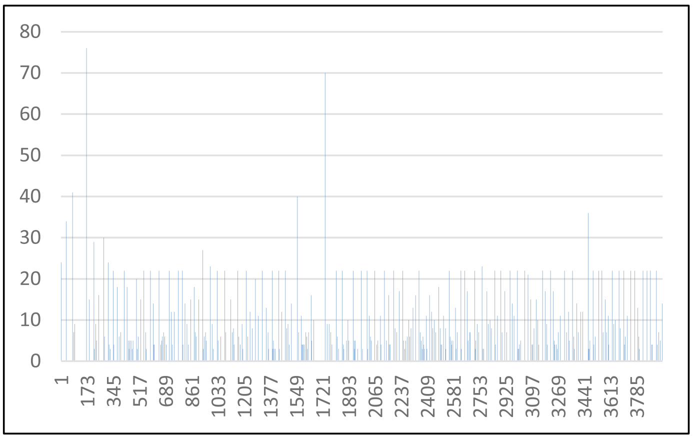


MDD (Duration) of SMB

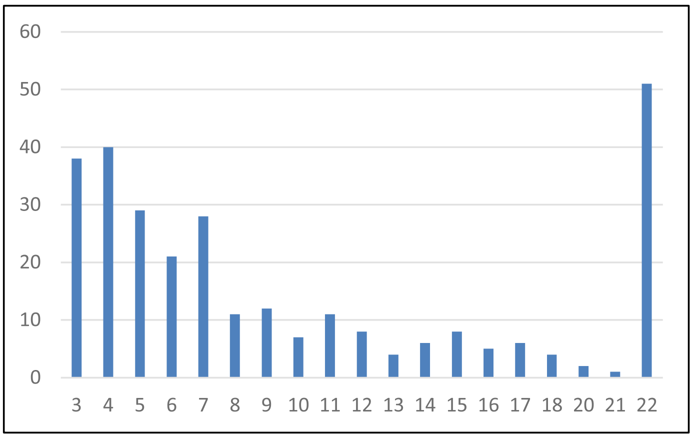


MDD (Duration) of SMB

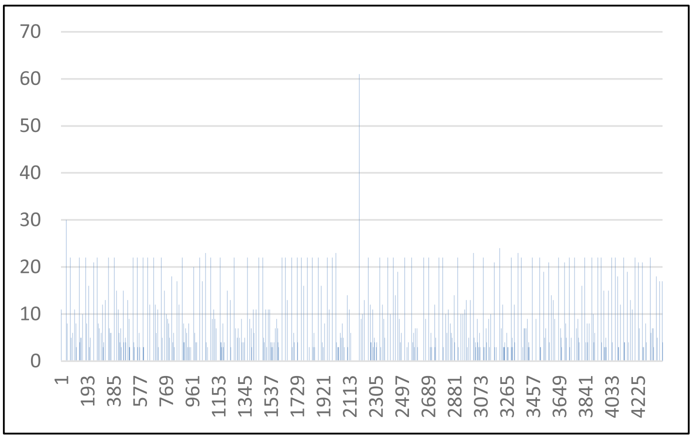
MDD duration empirical distributions.



MDD (Duration) of SLK

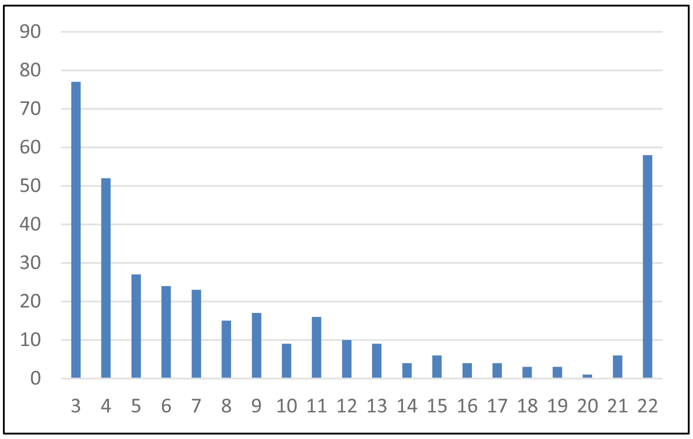


MDD (Duration) of SLK

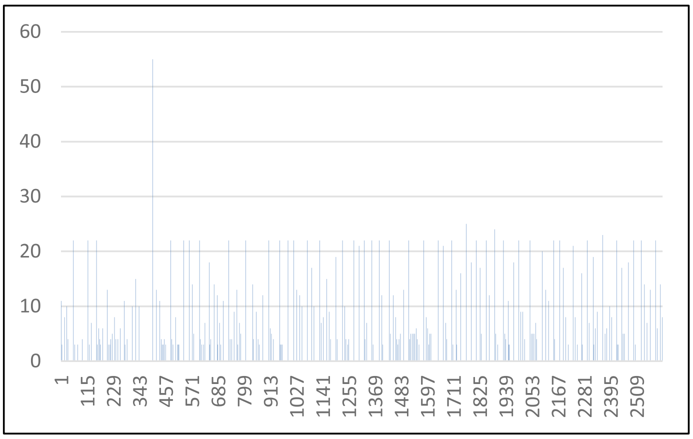


MDD (Duration) of SNB

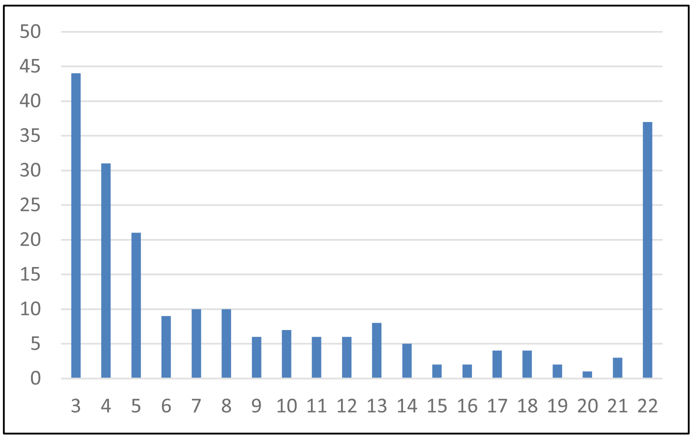
MDD duration empirical distributions.



MDD (Duration) of SNB

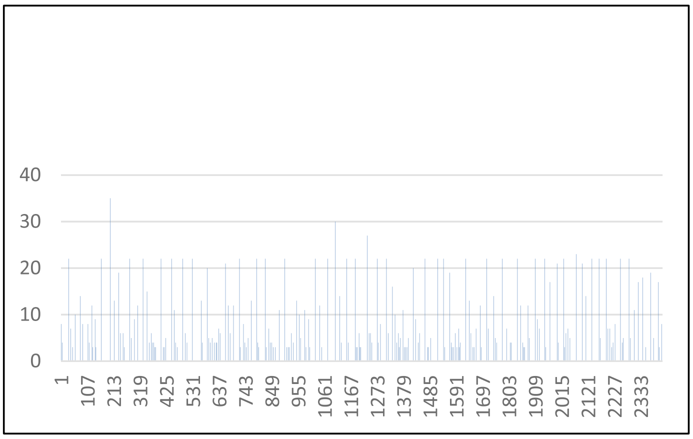


MDD (Duration) of SCB

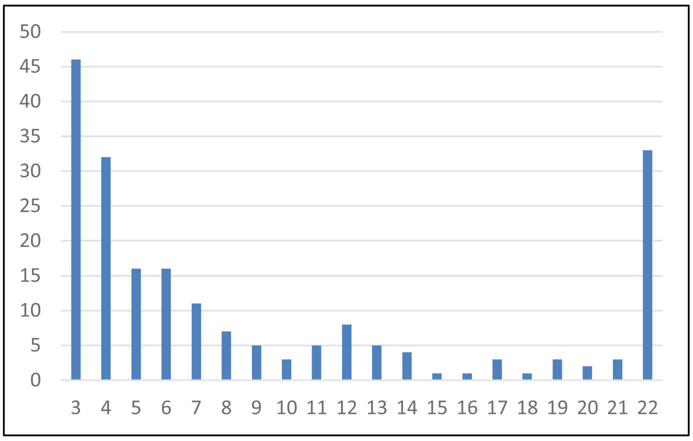


MDD (Duration) of SCB

MDD duration empirical distributions.



MDD (Duration) of SMT



MDD (Duration) of SMT

Figure 4.1: MDD duration empirical distributions.

4.2.2 Table of Durations of MDD of Stocks

Table 4.3 reports the duration of MDD for each stock of the sample taken from banking sector.

Table 4.3: Duration of MDD of stocks

Banks	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**
ABL	98	55	29	31	18	18	12	7	11	3	5	4	3	5	5	0	3	2	0	0	46	0
ACBL	9	48	34	39	28	30	14	20	8	14	8	6	5	6	5	4	8	2	2	3	54	2
BAF	0	27	26	34	23	18	16	9	8	3	9	6	6	4	4	2	1	1	4	3	45	0
BIPL	1	33	25	28	14	11	16	9	6	3	3	5	4	3	3	3	3	3	2	1	44	3
BoK	1	47	28	25	19	14	12	11	5	5	9	11	3	3	6	2	2	2	1	0	40	2
BoP	1	61	47	33	23	11	23	19	10	6	6	8	7	3	7	11	4	2	2	1	57	6
FBL	0	77	53	38	27	11	16	16	13	12	11	3	7	4	6	4	3	0	3	3	60	4
JSB	0	37	28	22	24	11	10	9	6	7	2	5	4	4	7	3	4	1	6	0	29	0
MCB	0	56	42	33	27	21	17	8	7	12	9	9	8	3	3	4	3	1	7	6	59	3
MBL	0	51	39	23	19	11	16	17	8	15	8	4	6	6	8	4	6	2	4	2	58	1
SMB	0	28	22	21	30	17	13	11	11	5	3	8	6	0	0	3	5	4	4	4	51	3
SLK	0	38	40	29	21	28	11	12	7	11	8	4	6	8	5	6	4	0	2	1	51	2
SNB	0	77	52	27	24	23	15	17	9	16	10	9	4	6	4	4	3	3	1	6	58	4
SCB	0	44	31	21	9	10	10	6	7	6	6	8	5	2	2	4	4	2	1	3	37	1
SBL	0	46	32	16	16	11	7	5	3	5	8	5	4	1	1	3	1	3	2	3	33	1
Total	110	725	528	420	322	245	208	176	119	123	105	95	78	58	66	57	54	28	41	36	722	32

The table shows that long durations are mostly existing for less than 10 days like seven hundred twenty-five durations of 3 days, five hundred and twenty-eight durations of 4 days, four hundred and twenty durations of 5 days, three hundred and twenty-two durations of 6 days, two hundred and forty-five durations of 7 days and two hundred and eight durations of 8 days. For the period of more than 10 days, durations are showing a shorter span of life like seventy-eight durations of 14 days, fifty-eight durations of 15 days, sixty-six durations of 16 days, fifty-seven durations of 17 days, fifty-four durations of 18 days and twenty-eight durations of 19 days. The one extraordinary long duration in all the graphs is depicting the effect of stock exchange crises of 2008-2009, when stock prices were declared frozen for specific time period. Table 4.4 is revealing two extraordinary long durations of seven hundred twenty-five and seven hundred twenty-two of 3 and 22 days respectively.

4.3 Estimation Through Non Parametric Model

Estimation through Non parametric approach is done through historical simulations. Historical simulation uses the actual distribution of risk factors. This means, estimation of the actual distribution of changes in the risk factors is not required. In historical simulation, historical data of returns or any random variable is used to simulate the expected outcomes. This method uses the actual data or variables experienced in past with assumption that the future performance gets the direction from the performance in the past. Each simulation involves factoring in a specific value of a random variable and calculating the value of the project or asset.

Table 4.4: MDD through Historical Simulation Method

Banks	α = **5%**	α = **2.5%**	α =**1%**
Allied Bank Limited	-0.25	-0.333	-0.4007
Askari Bank Limited	-0.278	-0.3154	-0.3525
Bank AlFalah Limited	-0.2477	-0.2816	-0.3686

Banks	α = **5%**	α = **2.5%**	α =**1%**
Bank Islami Pakistan Limited	-0.2988	-0.3597	-0.4211
Bank of Khyber	-0.3105	-0.3843	-0.5624
Bank of Punjab	-0.3609	-0.3908	-0.4676
Faysal Bank Limited	-0.28	-0.3841	-0.3937
JS Bank Limited	-0.3118	-0.439	-0.5407
MCB Bank Limited	-0.2673	-0.3154	-0.3672
Meezan Bank Limited	-0.2289	-0.2637	-0.3318
Samba Bank Limited	-0.3136	-0.417	-0.4785
Silk Bank Limited	-0.3386	-0.4	-0.4968
Soneri Bank Limited	-0.2857	-0.3036	-0.3752
Standard Chartered Bank	-0.2392	-0.2805	-0.3741
Summit Bank Limited	-0.3653	-0.443	-0.5038

At 95% confidence level, the Historical simulation method reports the highest risk of 36.5% and 36.1% in SBL & BoP respectively. It means that there are 95% chances that the loss will not exceed 36.5% and 36.1%. Historical simulation reports that MBL & SCB have the lowest risk of 22.9% and 23.9%. The potential loss for one day to the investor is lower in these stocks. It means that SBL and BoP are the riskiest banks in the portfolio and MBL and SCB are the least risky banks.

At 97.5% confidence level, the Historical simulation method reports the highest risk of 44.3% for SBL and 43.9% for JSB. It means that there is a 97.5% chance that the loss will not exceed 44.3% & 43.9% respectively. Historical simulation reports that MBL and SCB have the lowest risks of 26.4% and 28.1%. The potential loss for one day to the investor is lower in these stocks. It means that SBL and JSB are the riskiest banks in the portfolio and MBL and SCB are the least risky banks at the 97.5% level of confidence.

At 99% confidence level, the Historical simulation method reports the highest risk of 56.2% at BoK and 54.1% at JSB. It means that there is a 99% chance that the loss will not exceed 56.2% and 54.1% respectively. Historical simulation reports that MBL has the lowest risk of 33.2%. The potential loss for one day to the investor is lower in this stock. It means that SBL is the riskiest bank in the portfolio and MBL is the least risky bank at the 33.2% level of confidence. The Historical simulation method reports that the level of risk increases as the level of confidence increases.

4.4 Forecasting Through Parametric Models

In order to find the best model, which predicts accurately about the future risk, it is really important to evaluate the models correctly, as all the models have more or less same scope. In such scenario, the strengths and weaknesses of the models can be quite helpful in choosing the correct model.

4.4.1 GARCH and ARMA GARCH Models

If the returns of the stocks are having large number of observations i.e. in thousands, then the volatility can be most appropriately evaluated through GARCH model.

TABLE 4.5: Violation Ratio in MDD-GARCH & ARMA GARCH Models.

	GARCH		**ARMA GARCH**	
	% Viol.	Avg.Error	% Viol.	Avg.Error
Allied Bank Limited	2.8	12.1	2.8	12.1
Askari Bank Limited	2.7	5.6	2.7	5.6
Bank AlFalah Limited	0	0	0	0
Bank Islami Pakistan Limited	1	13.2	1	13.2
Bank of Khyber	3.2	10.7	3.2	10.7
Bank of Punjab	5.5	6.8	5.5	6.8
Faysal Bank Limited	1.9	12.2	1.9	12.3
JS Bank Limited	3.5	11.5	3.5	11.6
MCB Bank Limited	2.1	19.6	2.1	10.9
Meezan Bank Limited	40	30.6	40	22.5

	GARCH		ARMA GARCH	
Samba Bank Limited	4.7	0.1	4.7	0.1
Silk Bank Limited	0.9	0.4	2.3	2.4
Soneri Bank Limited	55.7	13.1	55.7	72
Standard Chartered Bank	17.1	9.4	17.1	9.4
Summit Bank Limited	3	87.2	3	87

The highest violation in forecasting the MDD through GARCH model is reported in SNB (55.7%) and lowest violation is reported in BAF (0%).It means that the 55.7% of forecasted values of MDD through GARCH model are lower than the actual MDD values in SNB, and no value is forecasted less than actual MDD in case of BAF.

The violation range through GARCH model varies from 0.9% (SLK) to 5.5% (BoP) with the exception of MBL (40%) and SNB (55.7%), which can be due to asymmetric data.

4.4.2 GJR GARCH and E-GARCH

GARCH and ARMA GARCH models are unable to capture the effect of good or bad market news so we moved forward to EGARCH and GJR GARCH models. Positive and negative news of market cause an asymmetric effect on the variance, and the model gets able to capture that effect through EGARCH by creating a dummy. GJR GARCH is the function of the size of the shock.

Table 4.6: Violation Ratio in MDD-GJR GARCH & E GARCH Models

	GJR GARCH		E GARCH	
	% Viol.	Avg.Error	% Viol.	Avg.Error
Allied Bank Limited	3.3	4.2	2.2	11.8
Askari Bank Limited	0	0	0	12.6
Bank AlFalah Limited	2.5	12	0	12.4
Bank Islami Pakistan Limited	1.2	19.2	0	14.1
Bank of Khyber	1	13	2.2	9.4

	GJR GARCH		E GARCH	
Bank of Punjab	5.4	30.5	100	70
Faysal Bank Limited	2.1	11.8	0	14
JS Bank Limited	3.7	8.7		
MCB Bank Limited	2.2	21.3	0	
Meezan Bank Limited	33.8	68.6		
Samba Bank Limited	5.1	2.8		
Silk Bank Limited	0.7	15.1	0.3	14.7
Soneri Bank Limited	48.2	13.3		
Standard Chartered Bank	17.6	5		
Summit Bank Limited	3.1	2.5		

The violation ratio in GJR GARCH model ranges from 0% in ACBL to 17.6% in SCB.MBL and SNB still have abnormally high violation rates of 33.8% and 48.2% respectively even through GJR GARCH model.

EGARCH model is unable to capture the variations in the data as most of the outcomes are not in congruence with the required forecasting of MDD. The violations ratio captured through EGARCH ranges from 0%(ACBL, BAF, BIPL) to 100% in case of Bop, which are not acceptable.

4.4.3 ARMA GJR-GARCH and ARMA E-GARCH

The serial dependence in the mean and variance is created through ARMA combined with the GARCH models, so then we applied EGARCH and GJR GARCH with ARMA model to capture the effect of unexplained shock of previous period.

Table 4.7: Violation Ratio in MDD-ARMA GJR-GARCH & ARMA-E-GARCH Models

	ARMA GJR GARCH		ARMA E GARCH	
	% Viol.	Avg.Error	% Viol.	Avg.Error
Allied Bank Limited	3.3	4.2	1.7	49.2
Askari Bank Limited	4.8	11.4	15.5	12.3
Bank AlFalah Limited	1	10.4	1.6	11.9

	ARMA	GJR GARCH	ARMA	E GARCH
Bank Islami Pakistan Limited	1	19.1	2.2	13.4
Bank of Khyber	5.1	3.5	1.7	9.4
Bank of Punjab	5.4	30.5	5.6	35.2
Faysal Bank Limited	2.1	11.8	2	8.6
JS Bank Limited	3.7	8.7	2.3	14.8
MCB Bank Limited	2.2	21.3	2.4	16.4
Meezan Bank Limited	33.8	68	42.6	16
Samba Bank Limited	5.1	2.8	3.4	3.7
Silk Bank Limited	0.7	15.1	12.3	10.7
Soneri Bank Limited	48.2	13.3	2.9	13.2
Standard Chartered Bank	17.6	5	19.4	7.5
Summit Bank Limited	3.1	2.5	41	20.8

The highest violation in forecasting the MDD through ARMA GJR GARCH model is reported in SNB (48.2%) and lowest violation is reported in SLK (0.7%).It means that the 48.2% of forecasted values of MDD through ARMA GJR GARCH model are less than the actual MDD values in SNB, and 0.7%values are forecasted less than actual MDD in case of SLK. The violation range through ARMA GJR GARCH model varies from 0.7% (SLK) to 17.6% (SCB) with the exception of MBL (33.8%) and SNB (48.2%).

The violation ratio in ARMA EGARCH model ranges from 1.6% in BAF to 19.4% in SCB.MBL and SBL have high violation rates of 42.6% and 41% respectively through ARMA EGARCH model.

4.5 Comparison of GARCH Models & ARMA Models

The table reveals that the Exponential-GARCH model is unable to predict the valid results in case of Pakistani financial markets, ARMA E-GARCH model underestimates the risk, providing the worst results. The overall winner is the GJR-GARCH- model, followed closely by ARMA GJR-GARCH and GARCH models.

TABLE 4.8: Comparison of Violation Ratios Across GARCH Based Models

	GARCH	**ARMA GARCH**	**GJR GARCH**	**ARMA GJR GARCH**	**E GARCH**	**ARMA E GARCH**
	% Viol.	% Viol.	% Viol.	% Viol.	% Viol.	% Viol.
Allied Bank Limited	2.8	2.8	3.3	3.3	2.2	1.7
Askari Bank Limited	2.7	2.7	0	4.8	0	15.5
Bank AlFalah Limited	0	0	2.5	1	0	1.6
Bank Islami Pakistan Limited	1	1	1.2	1	0	2.2
Bank of Khyber	3.2	3.2	1	5.1	2.2	1.7
Bank of Punjab	5.5	5.5	5.4	5.4	100	5.6
Faysal Bank Limited	1.9	1.9	2.1	2.1	0	2
JS Bank Limited	3.5	3.5	3.7	3.7		2.3
MCB Bank Limited	2.1	2.1	2.2	2.2	0	2.4
Meezan Bank Limited	40	40	33.8	33.8		42.6
Samba Bank Limited	4.7	4.7	5.1	5.1		3.4
Silk Bank Limited	0.9	2.3	0.7	0.7	0.3	12.3
Soneri Bank Limited	55.7	55.7	48.2	48.2		2.9
Standard Chartered Bank	17.1	17.1	17.6	17.6		19.4
Summit Bank Limited	3	3	3.1	3.1		41
AVERAGE	**9.61**	**9.7**	**8.66**	**9.14**	**11.63**	**10.44**

Chapter 5

Conclusion and Recommendations

5.1 Conclusion

The maximum drawdown holds an important spot in risk measures. Analytical properties of MDD have to be understood properly in order to use it more effectively. Investors use MDD as one of the most commonly used risk indicator as it captures very specific risk features of an asset. In this study we have estimated MDD by developing and testing a simulation based methodology.

The practitioners at the front office trading systems and back office risk management get a lot of help from the GARCH forecasting techniques, which especially included the flexibility and accuracy. These techniques are used to forecast drawdown measure, which, we believe, is useful for practical portfolio management. The main aim of this study has been to evaluate MDD estimates produced by various ARCH/GARCH forecasts, made under different error distributions. Overall, no model is clearly superior, however it is found that to some extent, the serial dependence of returns on mean and volatility is imitated by the GJR-GARCH based simulations methodology, which is able to estimate MDD accurately and changes in volatility level are responded in quick manner.

Some really long durations are exhibiting in the results, but mostly for less than 10 days and for more than 10 days' period, durations normally have a shorter span with a single exception of seven hundred twenty-two durations for 22 days.

A deep analysis of the outcomes depicts the ability of the model to be well adjusted with the economy shifts, with very small excesses beyond the threshold. The perception of the market about future expected crises can be well indicated through behavior of market participants with regard to contracts linked to the MDD. Lastly, it is found that forecasting can be improvised by using leptokurtic distributions with symmetrical models but this study could not found an overall dominating distribution fit for all the models.

5.2 Recommendation

The maximum drawdown method with GJR-GARCH based simulations is exhibiting less violations and better results in comparison to other models, so investors can opt for GJR-GARCH model to capture the risk.

5.3 Directions for Future Research

The MDD needs to be explored in detail as it is able to offer some advantages in comparison to traditional measures.

1. Different stages of investments can be investigated through more comprehensive studies in multivariate setting.

2. Further research can be done with the focus on different distribution, like skewed t-distribution can be tested instead of the models and distributions used in this study.

3. The number of models can also be added up to get different and better results.

4. Another interesting investigation can be done about the improvement in the forecasting by doing alterations in the estimation window length. Prediction windows longer than one year and estimation periods of different length, whether longer or shorter, can be focused in future research including different time periods and different risk measures.

5. Furthermore, risk measure definitions can be tested by using other time periods (e.g. one month rather than one day).

6. The reality and magnitude of effect of heteroscedasticity on the accuracy of predictions need some detailed research. Further research is needed in order to compare different models on the basis of their advantages, in terms of quality of results and cost inculcated while application of these models. At times models focus on attaining high level of quality, which is not even required, and during this process consume a lot of unnecessary time and resources. Further research in this area can develop a new benchmark for the models, in terms of creating a well-balanced composite of quality and amount of resources consumed.

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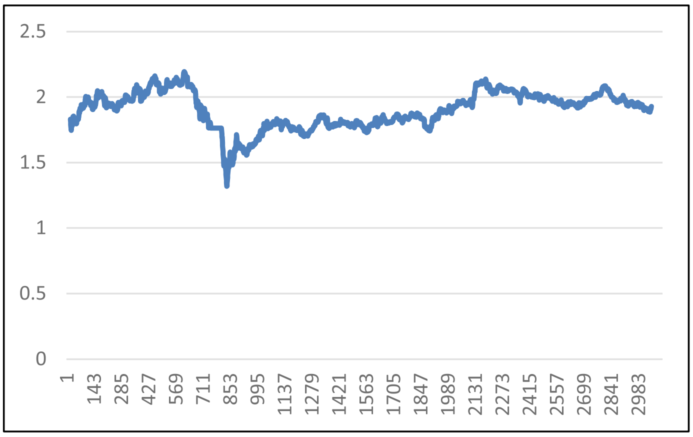
Appendices

The appendix includes graphs of log of daily prices of the stocks used in this study.



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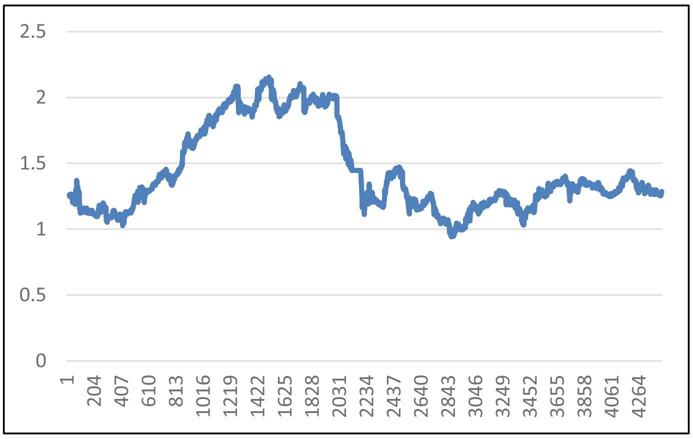

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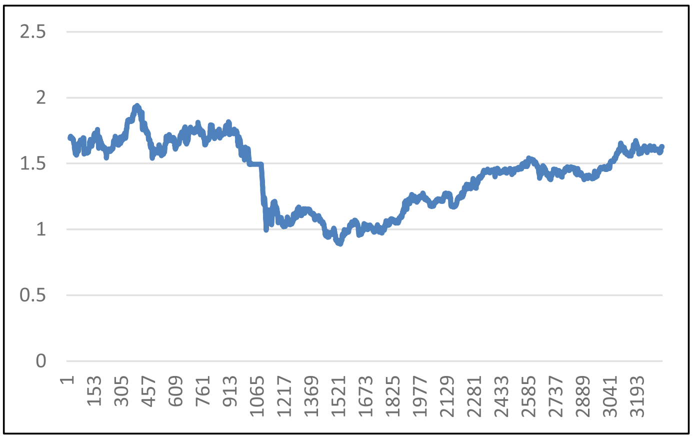

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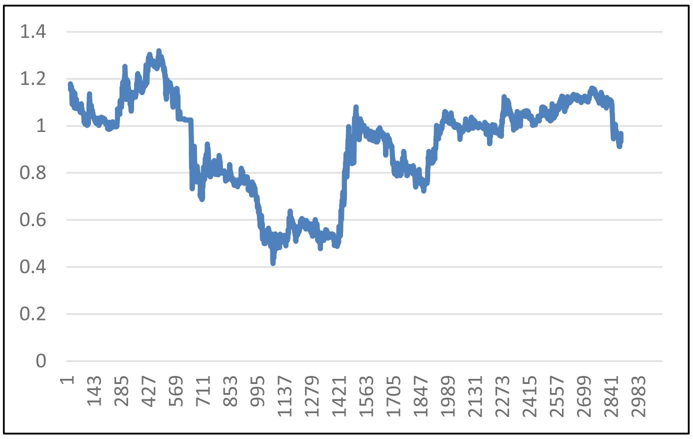

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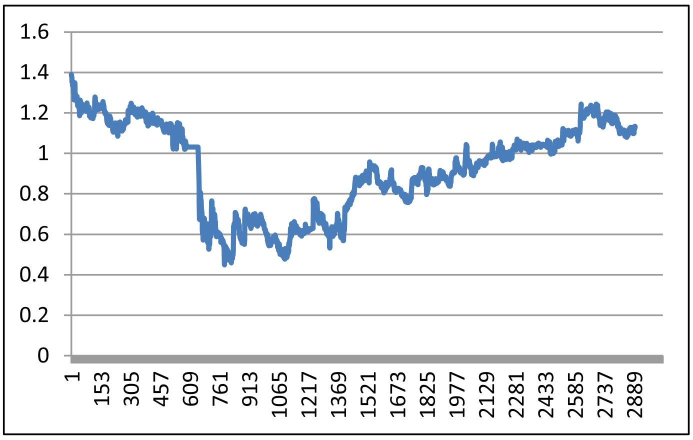
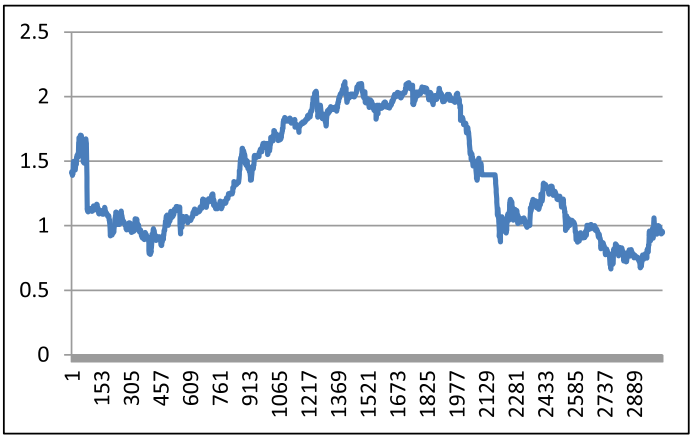


Log-prices of BoK



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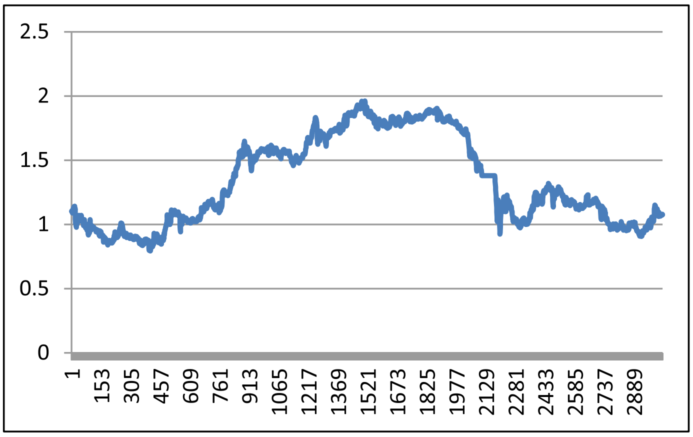
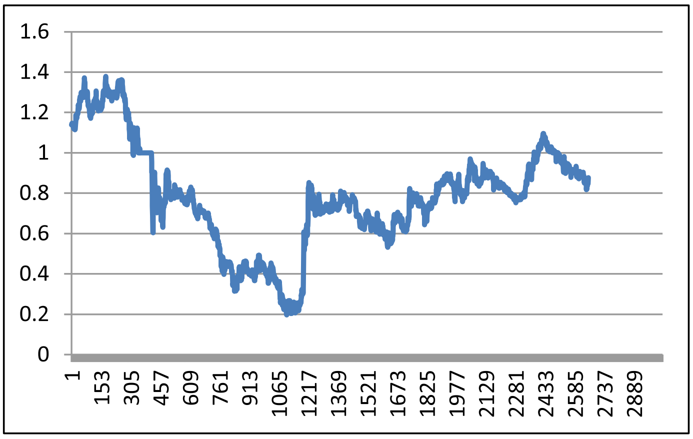


Log-prices of FBL



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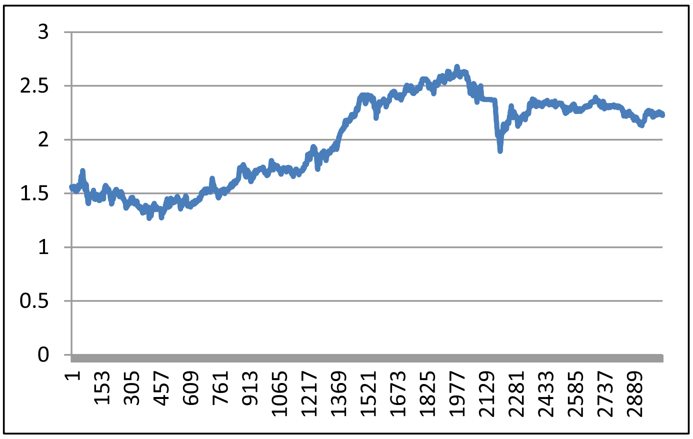
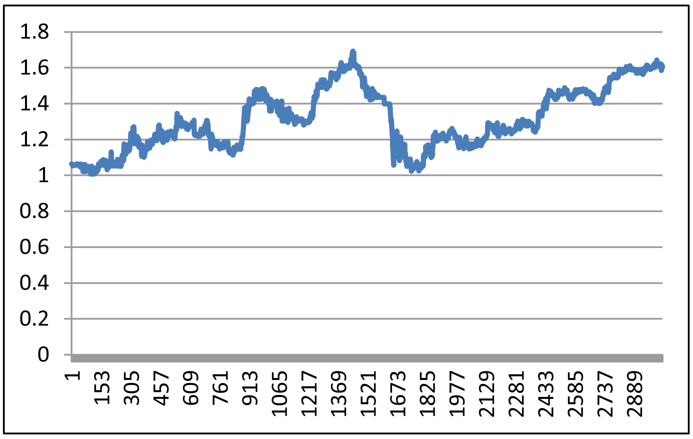
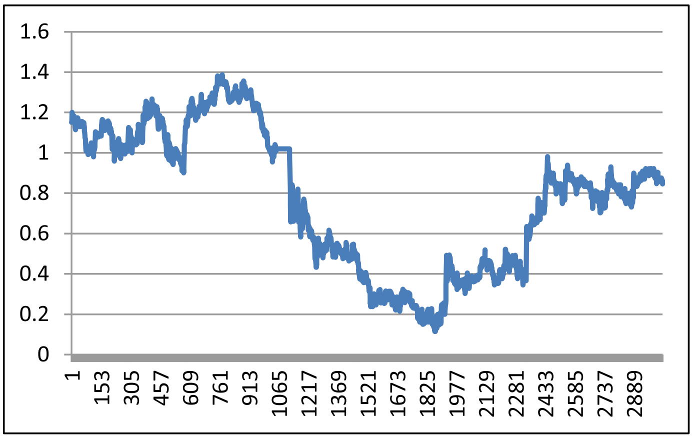


Log-prices of MCB



Log-prices of MBL



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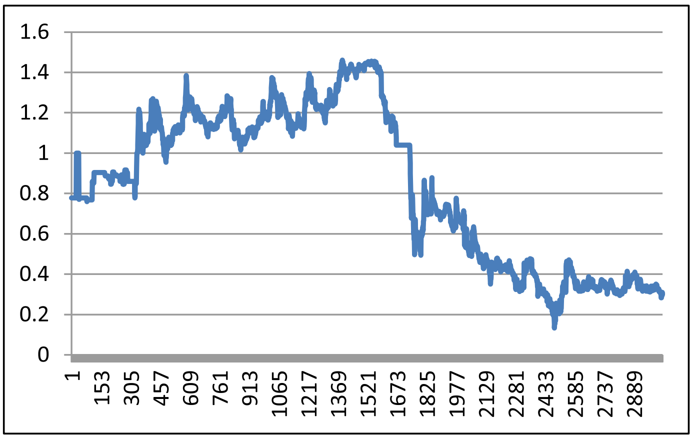
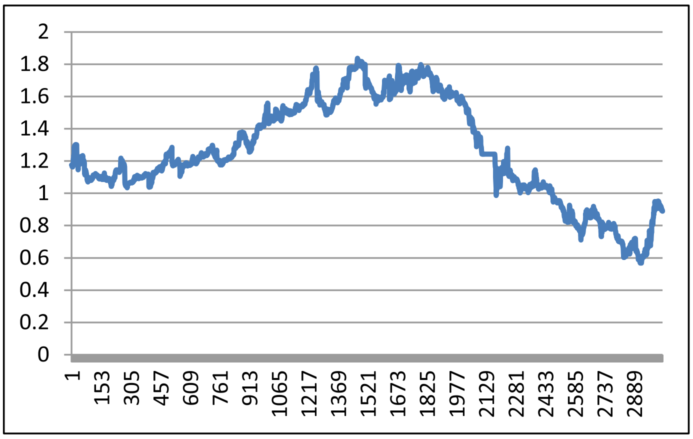
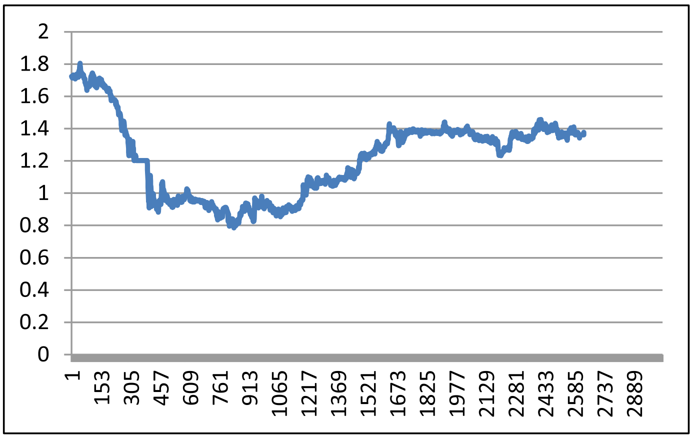


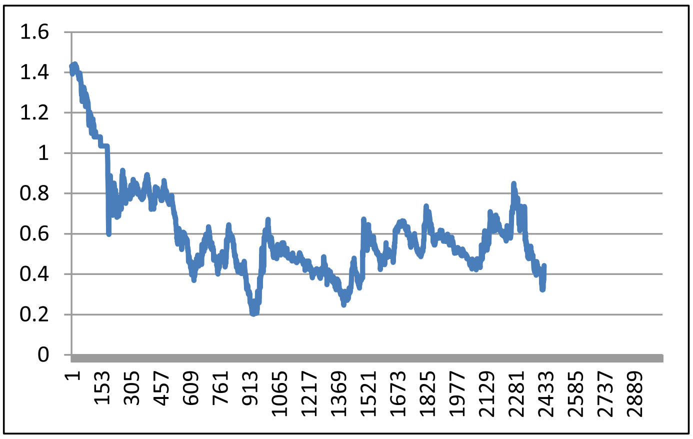
Log-prices of SLK



Log-prices of SNB



Log-prices of SCB



Log-prices of SMT