CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

Comparative Analysis of MUSIC and Monopulse for Direction of Arrival Estimation

by

Tooba Ijaz

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I dedicate this thesis to my husband, kids, parents and parents in law

CERTIFICATE OF APPROVAL

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by
Tooba Ijaz
(MEE-171018)

THESIS EXAMINING COMMITTEE

S. No.	Examiner	Name	Organization
(a)	External Examiner	Dr. Muhammad Abid	PIEAS, Islamabad
(b)	Internal Examiner	Dr. Imtiaz Ahmad Taj	CUST, Islamabad
(c)	Supervisor	Dr. Aamer Iqbal Bhatti	CUST, Islamabad

Dr. Aamer Iqbal Bhatti
Thesis Supervisor
November, 2019

Dr. Noor Muhammad Khan
Head
Dept. of Electrical Engineering
November, 2019

Dr. Imtiaz Ahmad Taj
Dean
Faculty of Engineering
November, 2019

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(Tooba Ijaz)
Registration No: MEE-171018

Abstract

Estimation of Direction of Arrival (DOA) of a target has always been an important field of study for many engineering applications like sonar, radar and wireless communications. Use of high resolution beamformers is a popular method for this estimation. Monopulse technique can also be used to obtain accurate angle measurements in tracking radars. Based on current studies, Multiple Signal Classification (MUSIC) is found to be better among other beam forming techniques like LCMV, MVDR, ESPRIT used for DOA estimation. But the two popular DOA estimation and target tracking algorithms monopulse and MUSIC are not studied comparatively for two-dimensional (2D) direction-of-arrival (DOA) estimation. So, this thesis presents a comparative study of these two techniques monopulse and Multiple Signal Classification (MUSIC) for (2D) direction-of-arrival (DOA) estimation and target tracking using Uniform rectangular arrays (URA) in phased array radars. Simulations are performed in MATLAB to make comparison on the basis of accuracy of measurements, number of antenna elements, performance under poor Signal to Noise Ratio (SNR) conditions and ability to resolve multiple targets. Results show that in poor SNR conditions, monopulse is capable of giving more accurate measurements than MUSIC. But in better SNR scenario MUSIC will always be a good choice because of its accuracy, correct DOA estimation even with smaller antenna arrays and ability to detect multiple targets.

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Abbreviations

CBF	Conventional Beam Former
CW	Continuous Wave
DOA	Direction of Arrival
ESPIRIT	Estimation of Signal Parameters via Rotational Invariance Techniques
LCMV	Linearly Constrained Minimum Variance
ML	Maximum Likelihood
MUSIC	Multiple Signal Classification
MVDR	Minimum Variance Distortion Less Response
RMSE	Root Mean Square Error
SNR	Signal to Noise Ratio
ULA	Uniform Linear Array
URA	Uniform Rectangular Array

Symbols

Δ_{EL}	Elevation Difference
Δ_{TR}	Traverse Difference
$P_{mu}(\theta)$	Spatial Spectrum
R_x	Covariance Matrix
$s(n)$	Signal Vector
$s_k(t)$	Complex Signal Envelope

Chapter 1

Introduction

Radio Detection and Ranging abbreviated as Radar uses electromagnetic waves properties to detect an object in a volume of space. A radio signal is first propagated in space and when this signal reflects from different objects in space, these reflections also called echoes are collected and different processing techniques are applied on them to obtain information about the objects, which also known as targets [1] as shown in Fig 1.1.

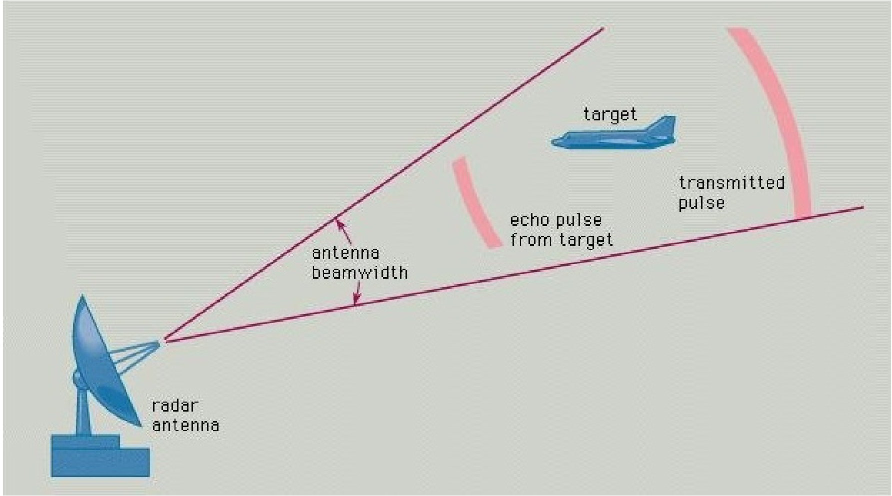


FIGURE 1.1: Basic Radar Function [1]

This information can be range of target, its speed or its direction relative to the receiver. Range of the target is determined by calculating round trip time taken by the pulse [2] as shown in Fig 1.2. The target bearing is determined by the antenna direction measured from antenna-rotation sensor.

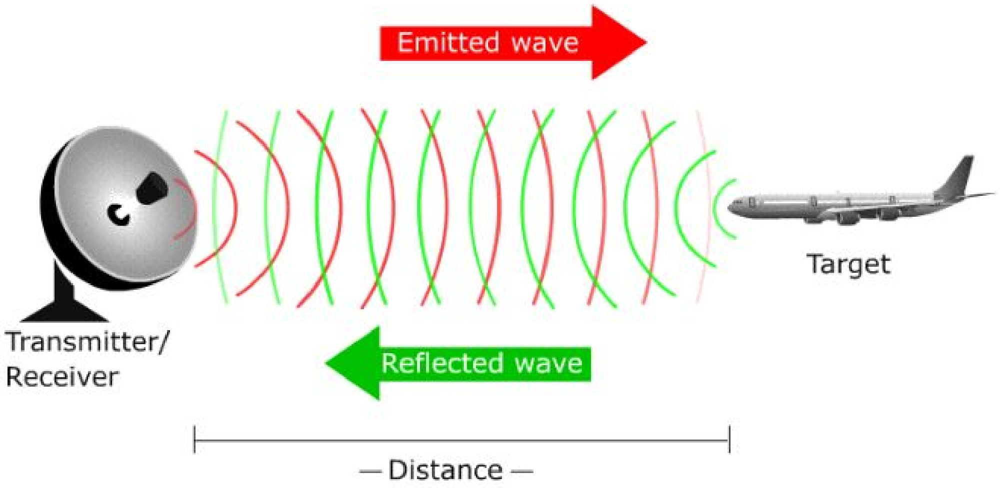


FIGURE 1.2: Target Range Calculation [2]

The radiation source, whose parameters are to be determined can be active or passive. Active source may include any distant antenna, astronomical body or jammer while passive sources include any target reflecting some part of power incident on it. The term target can refer to anything that we need to detect. High tech modern radar systems can extract useful information even in high noise levels by using advanced signal processing techniques.

1.1 Types of Radars

Some of the types of radars are as follows.

1.1.1 Monostatic Radars

In monostatic radars same antenna is used for transmission and reception. Transmission and reception chains are separated from each other through duplexers [3]. Synchronization is not an issue in these radars. Fig 1.3 shows mono-static radar.

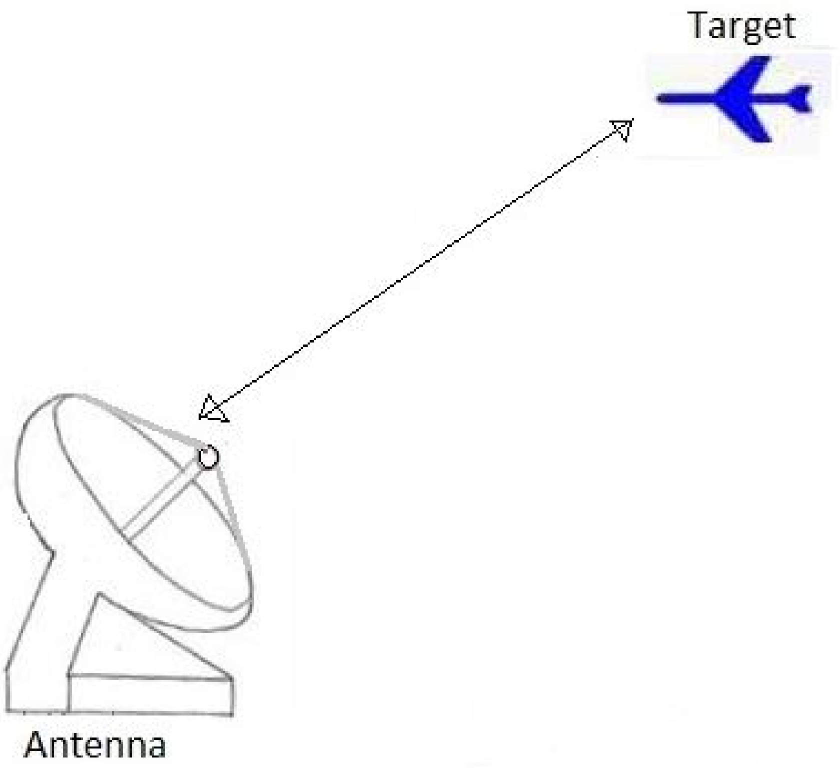


Figure 1.3: Monostatic Radar [3]

1.1.2 Bistatic Radars

In bistatic radars, separate antennas at different locations are used for transmission and reception. Requirement of tight synchronization make deployment of these radars difficult [3]. Fig 1.4 shows bi-static radar. Nowadays special bistatic units are used in which target position is correlated by using multiple receiving sites.

1.1.3 Pulsed Radars

Pulsed radars transmit train of rectangular shaped pulses modulating a sine wave carrier as shown in Fig 1.5. Complicated circuitry makes them expensive but availability of anti-jamming features justifies the cost [4]. They have a high range and performance is not affected in presence of more than one target.

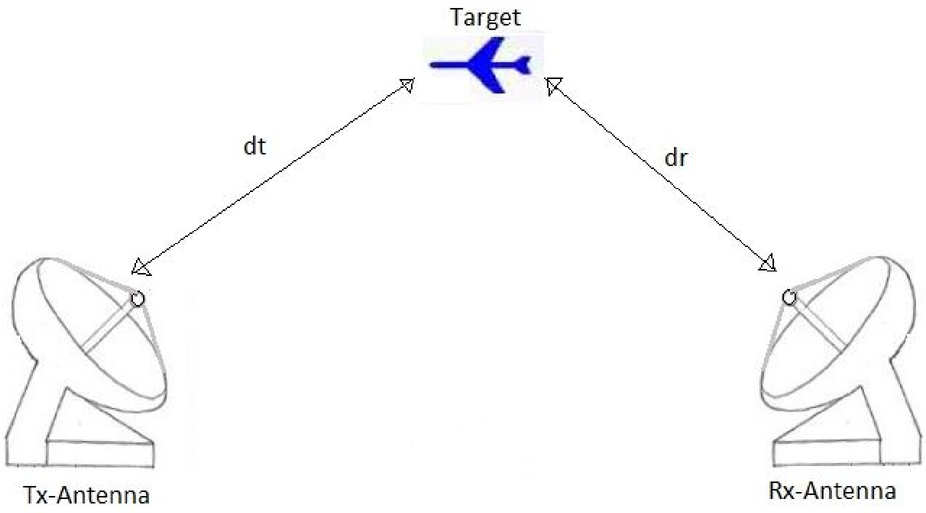


FIGURE 1.4: Bistatic Radar [3]

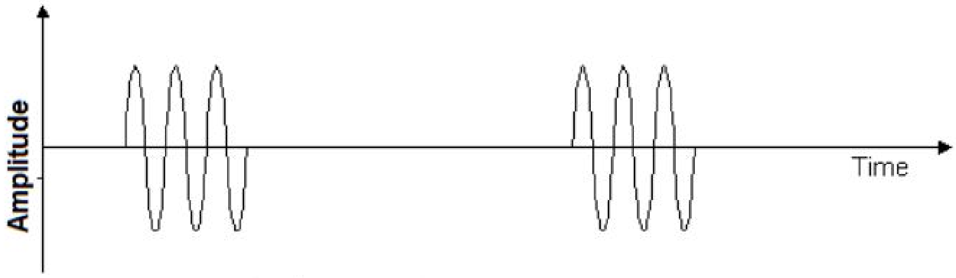


FIGURE 1.5: Pulsed Radar Waveform [4]

1.1.4 Continuous Wave (CW) Radars

These radars transmit low power continuous wave signals. They are easy to manufacture but can easily be jammed. So they are not used in military applications [4]. CW radars waveform is shown in Fig 1.6.

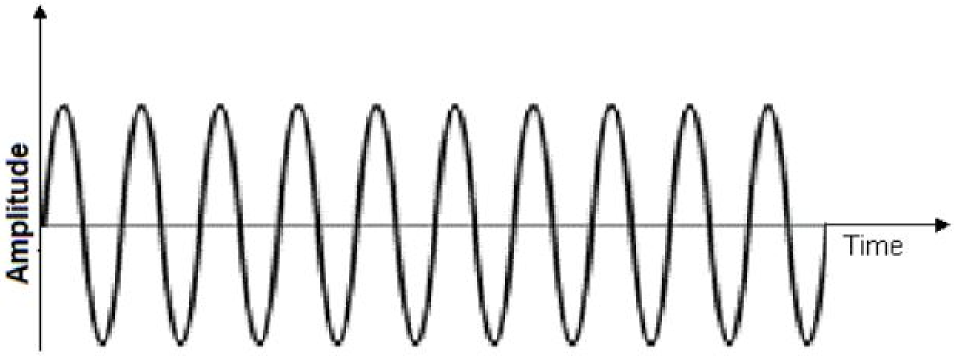


FIGURE 1.6: Continuous Wave Radar Waveform [4]

1.1.5 Weather Radars

Weather conditions like rain, snow, wind speed etc. can be detected through weather radars [1].

1.2 Parts of Radar System

Some very basic parts of a radar system are shown in Fig 1.7 and are described briefly below [1].

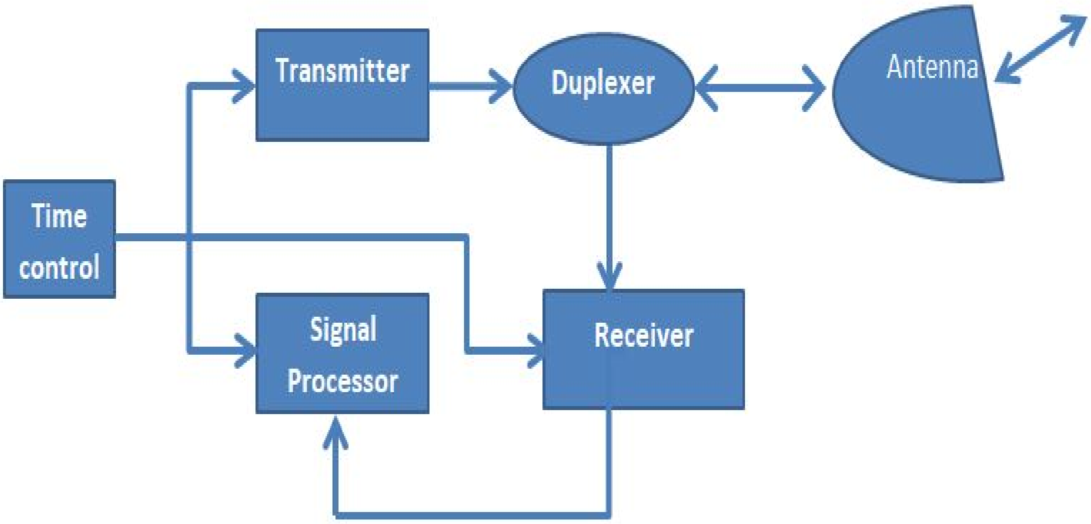


FIGURE 1.7: Block Diagram of Basic Radar System

1.2.1 Transmitter

Short duration high energy pulses are produced by transmitters which are then radiated by antenna into space.

1.2.2 Receiver

Radar receiver demodulates and amplifies the received signal.

1.2.3 Duplexer

Bi-directional communication over a single path, mostly in mono static radars, is made possible through electronic devices called duplexers. Duplexers isolate receiver and transmitter during radar communications which allows them to share a common antenna.

1.2.4 Antenna

Pulses from the transmitter are transferred to space with required distribution and efficiency through antennas. Identical process happens upon reception. To achieve desired resolution and accuracy, narrow radiation pattern is required. This is made sure by antenna that the signal has the required pattern in space. Antennas transfer the returned echoes to receiver with a minimum loss.

1.2.5 Signal Processor

Signal processing is a major part in radar functioning. Targets are separated from clutter on the basis of their Doppler content through different signal processing techniques e.g. pulse compression, (Moving Target Indicator) MTI filtering, Constant False Alarm Rate detection. etc.

1.3 Phased Array Antenna

In a phased array antenna, lot of radiating elements are present. Each element has a phase shifter. Beams are steered in the desired direction through constructive and destructive interference of signals. It is done by shifting the phase of the signals emitted from all elements. The direction of the main beam is always pointed in increasing phase shift. When the radiating signal is obtained through continuous phase shift provided by electronic phase shifters then the beam direction can be adjusted electronically.

The main advantages of phased array radar system are that the beam can jump in few microseconds from one target to the other and if a fault occurs in any of the antenna element, it can affect the sharpness of beam but overall operation of system continues.

1.4 DOA Estimation

Direction from which propagating signals arrive at the sensor array is known as direction of arrival (DOA). DOA estimation is a major task that needs to be performed by Radar systems. It also has its uses in sonar, objects tracking, wireless communication and navigation. Signals received at the sensor reflected from multiple sources are used to determine the direction of these sources. For a same signal to reach different array elements, it has to travel different distances.

Wave-way difference is the term used to describe this difference in distances. There exists a phase difference between elements of the arrival array because of wave-way difference. Signals azimuth can be calculated with the use of this phase difference. This is the basic principle of DOA estimation.

Many different methods exist to estimate Direction-of-Arrival (DOA). Beam forming and monopulse tracking technology are two of the many techniques used for this purpose.

1.4.1 Beamforming

Beamforming is also an important technique to estimate DOA. Signal arriving from a specific direction is estimated in presence of noise through a receive beam former. Linear combination of output of array sensors gives weight vectors to attenuate signals coming from unwanted directions and only allow signals from desired direction to pass. Different beam forming algorithms used for direction of arrival (DOA) estimation are discussed in section 3.1. Main beam forming algorithm of interest in this thesis is Multiple Signal Classification (MUSIC).

1.4.1.1 MUSIC

In Music beam forming algorithm, first we obtain the array output covariance matrix, this matrix is then decomposed into signal subspace and noise subspace. This algorithm implies that signal steering vectors and noise eigen vectors are orthogonal to each other.

These two orthogonal subspaces are used to constitute spectrum function. Then, spectral peak search is performed on this spectrum function to estimate DOA of signal. High resolution and stability makes MUSIC one of the often used algorithm for DOA estimation. Detailed derivation of MUSIC is given in chapter 3.

1.4.2 Monopulse Tracking

In tracking radars, accurate angle measurements can be obtained using monopulse technique. In monopulse system, for the estimation of one angle there are two identical antennas which in case of amplitude comparison monopulse have same phase center or are separated to have different phases in phase comparison monopulse.

In this thesis amplitude comparison monopulse is used. It is the form of monopulse in which the amplitude ratio of the received echoes gives information about how deviated target is from antenna axis. In amplitude comparison monopulse radars, four simultaneous receive beams are formed and then comparison is made on each pulse.

1.5 Thesis Overview

This thesis is divided into 6 chapters which are

- Chapter 1 gives a brief introduction of complete thesis.
- Chapter 2 deals with a brief overview of the past development in direction of arrival (DOA) estimation techniques. First initial development in beam

formers is discussed and then their most advanced techniques are also mentioned. Monopulse technology and its evolution is described. Literature about comparison of all these techniques is also reviewed later in this chapter.

- In chapter 3 different beam forming algorithms and their merits and demerits are discussed. Detailed derivation of MUSIC beam forming algorithm is also given.
- Chapter 4 is a detailed study of monopulse tracking technology. Phased array radar to be used in simulations in next chapter is also designed in this chapter.
- In chapter 5 simulations are performed to compare MUSIC and monopulse performance under different conditions and results are obtained.
- Chapter 6 gives conclusions about performance of these techniques on the basis of results obtained in previous chapters.

Chapter 2

Literature Review

This chapter deals with a brief overview of the past development in direction of arrival (DOA) estimation techniques. The chapter starts with the introduction of initial development in beam formers and then their most advanced techniques are also discussed. Another topic of discussion is monopulse technology and its evolution. Literature about comparison of all these techniques is also reviewed later in this chapter.

2.1 DOA Estimation

The estimation of angle of arrival of a plane wave is known as estimation of direction-of-arrival (DOA) or direction finding. It has always been an important field of study, since the increased use of radars in the middle of 20th century, for many engineering applications like sonar, radar itself and wireless communications.

In early days DOA was estimated using narrow beam antennas which were steered mechanically. Now with the advent of digital signal processing a large number of techniques are available for estimation of DOA [5] but still with a strong theoretical basis and a number of applications, it is an active research topic.

2.2 Techniques for DOA Estimation

Any signal produced by a certain source contains information about that source. Nature of source can be determined through signal wave form and its location can be calculated from signals spatial and temporal characteristics [6]. Various methods exist to estimate the direction-of-arrivals (DOAs) of multiple sources using the signals received at the sensor array.

Beam forming is one of the prominent techniques for DOA estimation. Beam former techniques are generally classified as conventional and adaptive beamformers. The conventional techniques mostly depend on maximum likely hood detection and adaptive techniques are mostly based on subspace decomposition.

In a conventional beam former, each sensor encounters a delay because of path difference. This delay is used as a weight vector. So in order to improve signal reception while suppressing noise, the outputs of the sensors are coherently summed up using that weight. These weights of vectors are independent of incoming data and are calculated beforehand [7]. The structure of a conventional beam former is shown in Fig 2.1.

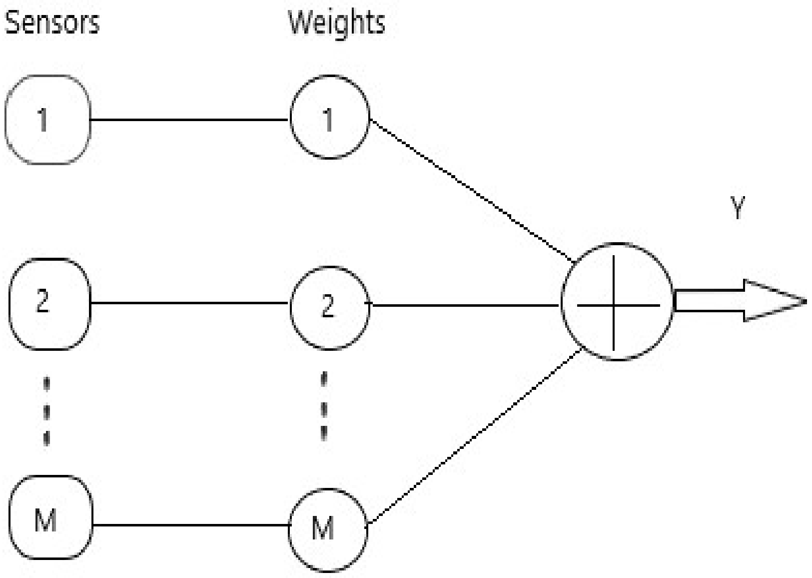


FIGURE 2.1: Conventional Beamformer

In order to enhance our required signal and suppress interference and noise at the same time, we make use of adaptive beam formers. The main objective of adaptive beam forming is location of a directional source by optimization of a bunch of weight vectors. Different methods are used to achieve this optimization [7]. The basic structure of adaptive beam former is shown in Fig 2.2.

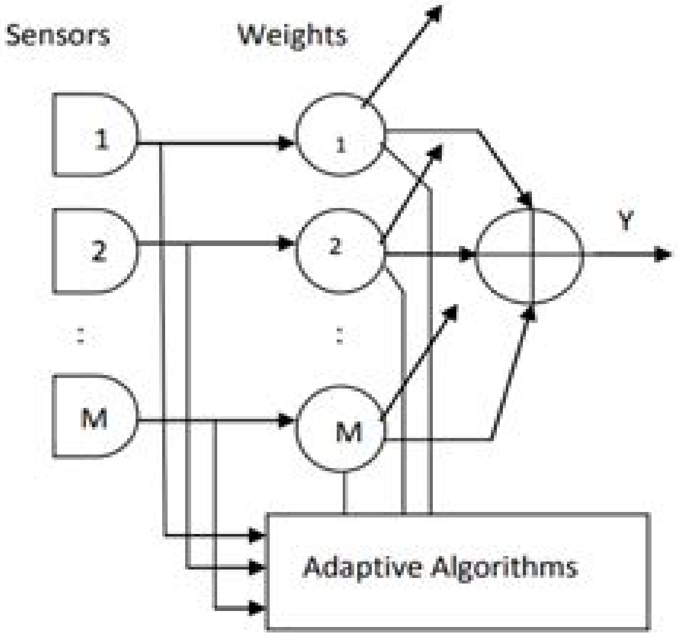


Figure 2.2: Adaptive Beamformer [7]

Another major technique for DOA estimation is Monopulse. In tracking radars, accurate angle measurements can be obtained using monopulse technique. In monopulse system, the signals received simultaneously in two or more antennas are compared to get information about the angular location of the target. For the estimation of one angle there are two identical antennas, sum and difference beams are produced from outputs of these antennas. The monopulse ratio of difference beam to sum beam contains the information about angle.

Literature Survey of both Beam formers and monopulse is discussed in the following paragraphs.

2.3 Beamformers

2.3.1 Beam Scan

Beam scan algorithm was the first used conventional beam forming technique. Its major principle is that all array sensors are used to estimate one certain direction and the output power is measured.

The direction with maximum power is the desired DOA [8]. Its one of the major short coming is that beam in one desired direction is formed by use of all degrees of freedom of an array so in case of multiple signal sources, the height of beam width limits this method as a result it has very poor resolution.

2.3.2 MVDR

Capon introduced Minimum Variance Distortion less Response (MVDR) in 1969 [9]. This technique improves the limitations encountered in conventional beam formers. MVDR minimizes the total output power to reduce the effect of interference. It has much better resolution as compared to beam scan.

This method also has some short comings; like if any other signal is close to signal of interest, MVDR is likely to produce errors.

2.3.3 LCMV

In [10], another method was proposed for DOA estimation in the year 1972. It is known as linearly Constrained Minimum Variance. This method has even better resolution than MVDR and has increased robustness as it performs well even for closely placed targets. Using this method, multiple constraints can be put along target direction (steering vector). As a result, the chance of signal suppression arriving at different from desired direction is reduced. Its disadvantage is strong degradation in poor SNR conditions.

2.3.4 MUSIC

In 1979, Shcmidt proposed Multiple Signal Classification (MUSIC) method for DOA estimation [11] . It works by Eigen vector decomposition of array covariance matrix into signal sub space and noise subspace.

2.3.5 Root MUSIC

In 1983, a variation in MUSIC was proposed in [12], known as root Music. It works by calculation of zeros of a polynomial which gives a direct estimate of DOA and as a result there is no need to search for maxima which is a requirement for MUSIC. But its disadvantage is its limitation to uniformly spaced linear antennas only [7].

2.3.6 ESPRIT

In 1989, Estimation of Signal Parameters via Rotational Invariance Techniques, ESPIRIT was proposed in [13]. It is based on the rotational invariance property of the signal space to estimate DOA and the pseudo-spectrum calculation is not required like MUSIC.

2.4 Monopulse

To understand monopulse, a basic knowledge of conical scanning is required. In a conical scan system, signal is sent out slightly off antennas boresight. The lobe is then rotated around antennas boresight by rotation of feed horn. A simple conical scan radar is shown in Fig 2.3 . A strong return is provided by target located on the boresight as it is always illuminated by the lobe. The target which is not centered on the boresight and is located to one side will only be illuminated when lobe points to that particular side and a maximum will be obtained only then [14].

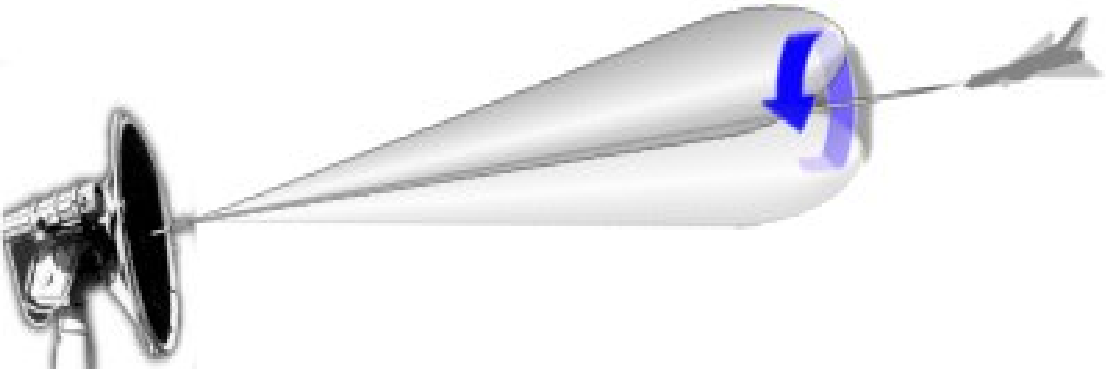


FIGURE 2.3: Conical Scan Radar [15]

One problem with this technique is that the target return signal can be affected by a number of factors which have nothing to do with beam position like rain clouds or change in target heading. These changes can affect the performance of the system regarding the position of the target as the system is designed only relevant to strengthening and weakening of signal due to targets position relative to the beam position.

Monopulse systems are very similar to conical scanning but here the beam is split into two parts and then the two signals are transmitted in slightly different directions [16]. The received signals are separately amplified and then compared with each other.

This comparison indicates the direction of target by observing the direction with stronger return. Changes in target heading do not affect this comparison as it is carried out during one pulse only which is a very small time [14]. Basic monopulse functioning is shown in Fig 2.4.

Monopulse radar was first introduced by Robert M. Page in 1943 .it was a very expensive, labor-intensive technique at that time due to complexity, and less reliability. It was only used in need of extreme accuracy that can justify the high cost.

After 1970s, the complexity and cost of implementation of monopulse systems is greatly reduced because of availability of advanced digital signal processing techniques and now this technique is extensively used in modern radar systems.

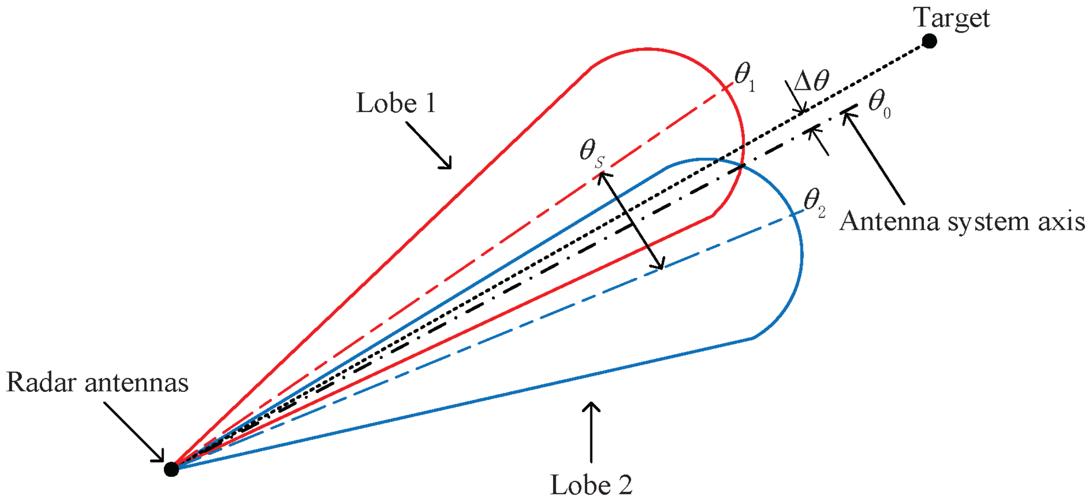


FIGURE 2.4: Monopulse Functioning [17]

2.5 Comparison of DOA Estimation Techniques

In the following section literature about comparison of different DOA estimation techniques is reviewed.

In [7], a survey about different beam forming techniques for DOA estimation is carried out. The techniques discussed in this survey are linearly Constrained Minimum Variance (LCMV) beam former, Minimum Variance Distortion less Response (MVDR) beam former, Multiple Signal Classification (MUSIC) beam former, Root Music and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) beam former. Merits and demerits of all these techniques are discussed and it is found that MUSIC has a higher resolution and accuracy compared to other techniques. Root MUSIC and ESPRIT also have high resolution but they require specific array structures which is a drawback.

In [18], comparison is carried out between different DOA estimation techniques, based on beam forming and subspace, for room reflections. The techniques compared are Eigen beam EB- MUSIC, EB-MVDR and EB-ESPRIT. After detailed study it was concluded in this paper that RB-MVDR and EB-MUSIC can localize multiple reflections but EB-ESPRIT was not found suitable for multiple reflections.

In [6], again a comparison is made between conventional delay and sum beamformer (CBF), MVDR, MUSIC and ESPRIT. Resolution and computational complexities of all these techniques are discussed. The results obtained show that resolution of both MUSIC and ESPRIT is better among all techniques. Computational complexity of ESPRIT is also least of all but its requirement of specific array structure gives MUSIC precedence over ESPRIT.

In [19], different classical and modern methods of DOA estimation are discussed and compared on the basis of their mean square error (MSE), required array geometry and computational complexity. The results deduced show that techniques which are array geometry dependent are not suitable for many applications while techniques that can be applied to arbitrary array geometries are in great demand.

In [20], array signal processing techniques for DOA estimation are studied. The techniques in [20] are divided into three categories which are conventional beam forming, sub-space based beam forming and maximum likelihood (ML) detection methods. Among conventional methods MVDR is considered better as it keeps the gain fixed towards the look direction and minimizes the interference and noise effects.

Main disadvantage of these techniques is found to be the lack of angular resolution and requirement of large number of sensors in order to obtain a higher resolution. In sub-space based category, MUSIC and ESPRIT are discussed and it is concluded that both techniques require small sample size and provide high resolution at low SNR. Moving towards the ML detection methods, it is concluded that these methods provide higher accuracy than conventional and subspace-based methods but they are computationally very complex.

In [21], search free DOA estimation techniques are studied. These techniques are applicable to arbitrary array geometries. Interpolated root-MUSIC technique and popular manifold separation (MS) technique are also studied by analyzing asymptotic first order performance. Attractive balance between computational complexity and DOA estimation performance is provided through a proposed two rooting-based DOA estimators for arbitrary arrays.

In [22], two high resolution methods MUSIC and ESPRIT are compared on the basis of snap shots, SNR and number of antennas. After several simulations it is concluded that MUSIC is more stable, accurate and better in resolution than ESPRIT.

In [23], Monopulse, MUSIC and Root MUSIC are studied in regard to angular resolution of these methods in case of two targets for uniform linear array (ULA).

2.6 Gap Analysis

Considering above studies MUSIC is found to be better among other beam forming techniques for DOA estimation. But all these studies are based on uniform linear arrays (ULAs). Due to relatively simple implementation, Uniform Linear Array (ULA) is a popular geometry for array signal processing. Despite this advantage, it does not have a uniform performance in all directions and Angle of Arrival (AOA) estimation performance degrades considerably in the angles close to endfire also ULA cannot analyse more than one dimension.

In many instances a target must be located in two dimensions, i.e. azimuth and elevation angle or height above sea level for the performance of a particular system mission. Tracking systems also require estimation of both elevation and azimuth angles in order to accurately track targets that are moving in three dimensions. Keeping these requirements in view it is observed that two popular DOA estimation and target tracking algorithms monopulse and MUSIC are not studied comparatively for two-dimensional (2D) direction-of-arrival (DOA) estimation.

2.7 Problem Statement

Comparison should be done among these two techniques to know which one is better for target tracking and to figure out the circumstances under which they can be alternatively used. 2D direction-of-arrival (DOA) estimation can be done by using Uniform Rectangular Arrays (URAs) in phased array radars. So, in this thesis we compare monopulse and MUSIC for two-dimensional (2D) direction-of-arrival (DOA) estimation and target tracking on the basis of different parameters like accuracy of estimates, performance under SNR degradation, effect of number of antenna elements and ability to detect multiple targets. After looking at the results, it can be decided which technique is better for certain circumstances.

Chapter 3

Direction of Arrival Estimation and Beamforming

In many engineering applications like radar, sonar, objects tracking, wireless communication, navigation and array signal processing, estimation of direction of arrival DOA is a major research prospect. Another important notion in the field of array signal processing is spatial spectrum. It tells about signals distribution in space.

So, direction of arrival (DOA) can be estimated with the knowledge of signals spatial spectrum available. For this estimation, different beam forming algorithms can be used. This chapter briefly explains different beam formimg algorithms and their merits and de-merits. Multiple signal classification (MUSIC) algorithm which is used for further analysis and comparisons in this thesis is also derived and explained in detail.

In modern smart antennas main focus is on development of algorithms for DOA estimation. Use of a fixed antenna for DOA estimation comes with many limitations. Physical shape of an antenna affects the beam width of antennas main lobe. So, one has to increase the physical aperture of receiving radar antenna in order to increase the accuracy of measurements. This is not an ideal solution for many systems like aircraft antenna or missile seekers which have limited space on board.

Their receiving antenna size cannot be increased beyond a limit; as a result they have a wide main lobe beam width and poor resolution. It is difficult for them to distinguish between multiple signals falling in main lobe of the antenna.

In order to solve this problem and improve the resolution of these systems, use of antenna array with advanced signal processing techniques is an excellent option. Performance of an array of sensors for signal reception and parameter estimation is much better than a single sensor [24].

3.1 Basic Structure for DOA Estimation

As discussed earlier, signals spatial spectrum estimation can also be referred as DOA estimation. This entire space can be divided into three stages as shown in Fig 3.1[24].

1. Target stage.
2. Observation stage.
3. Estimation stage.

Target stage consists of parameters of signal source and a complex environment. Observation stage receives signals from target space. The received data contains some signal characteristics, environment characteristics and some characteristics of array elements. Estimation stage is basically reconstruction of target stage where different spatial spectrum estimation techniques are used to extract signal from complex environment.

One major question still to be addressed is what basically direction of arrival is. Angle of arrival (AOA) or direction of arrival (DOA) is the angle between the plane wave's direction vector and array normal.

For a same signal to reach different array elements, it has to travel different distances as shown in Fig 3.2, this difference in distances is known as wave-way

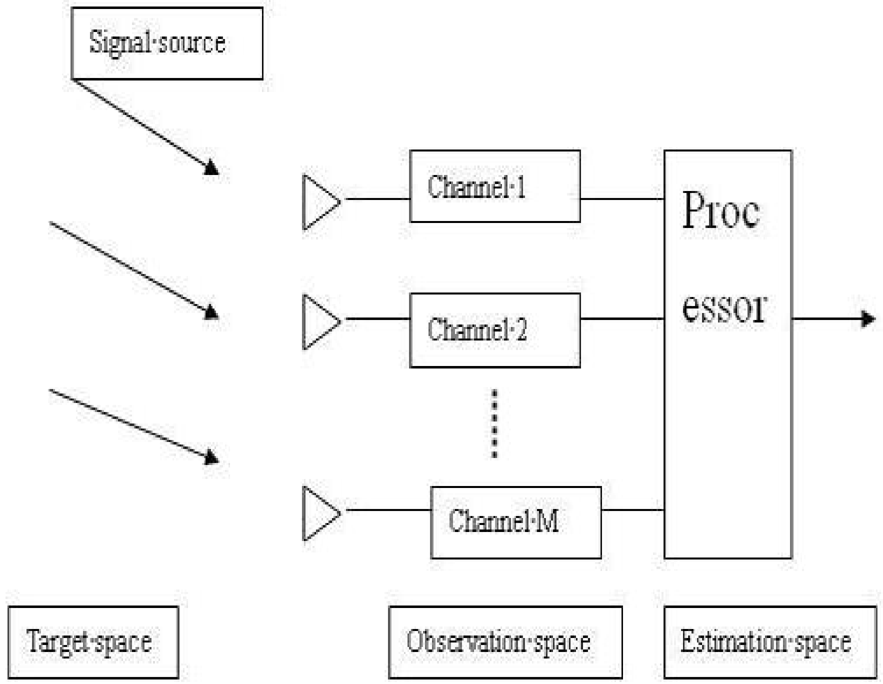


Figure 3.1: Stages for DOA Estimation [24]

difference. As a result of this difference, there exists a phase difference between elements of the arrival array. Signals azimuth can be calculated with the use of this phase difference. This is the basic principle of DOA estimation.

3.2 Beamforming

Beamforming is one of the prominent techniques for DOA estimation. It is a digital technique that is used to focus transmitter or receiver of radar in one particular direction. Radar can be focused over elevation and azimuth both using beam forming [25]. Signal arriving from a specific direction is estimated in presence of noise through a receive beam former.

Linear combination of output of array sensors gives weight vectors as shown in Fig 3.3. These weight vectors are then used to attenuate signals coming from unwanted

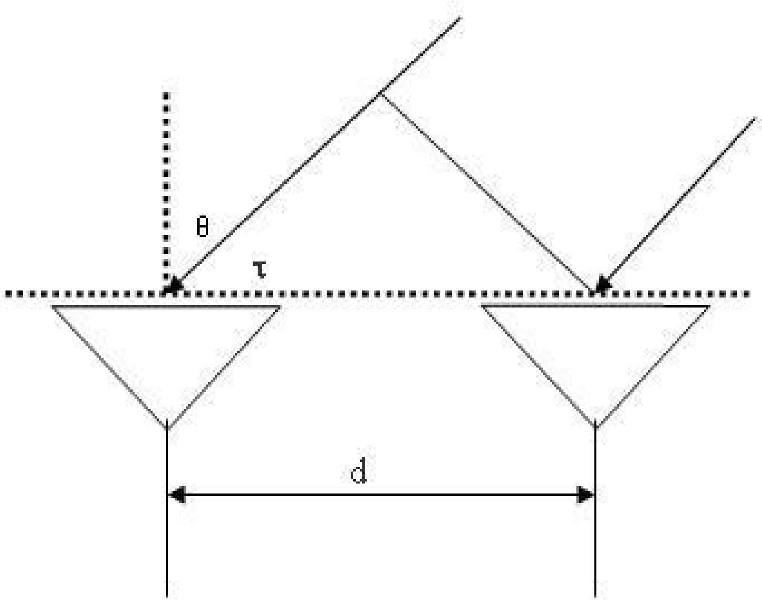


Figure 3.2: Basic Array Model for DOA Estimation [24]

directions and allow signals from desired direction to pass only [7]. Beam forming techniques have two major classifications conventional and adaptive beam formers or maximum likelihood and subspace based approaches.

In conventional beamformers, we have pre-calculated weight vectors which depend only on array response and are independent of received data. Path difference results in delay in each sensor which is then used as a weight so that in order to improve signal reception in noise, the outputs of these sensors can be coherently summed up.

An adaptive beamformer is a system that performs adaptive spatial signal processing with an array of transmitters or receivers. Adaptive beamformers are data dependent since incoming data plays an important role in weight vectors calculation. Optimization of weight vectors for localization of directional source is main focus of adaptive beamformers which can be achieved by application of linear constraints to weight vectors [7].

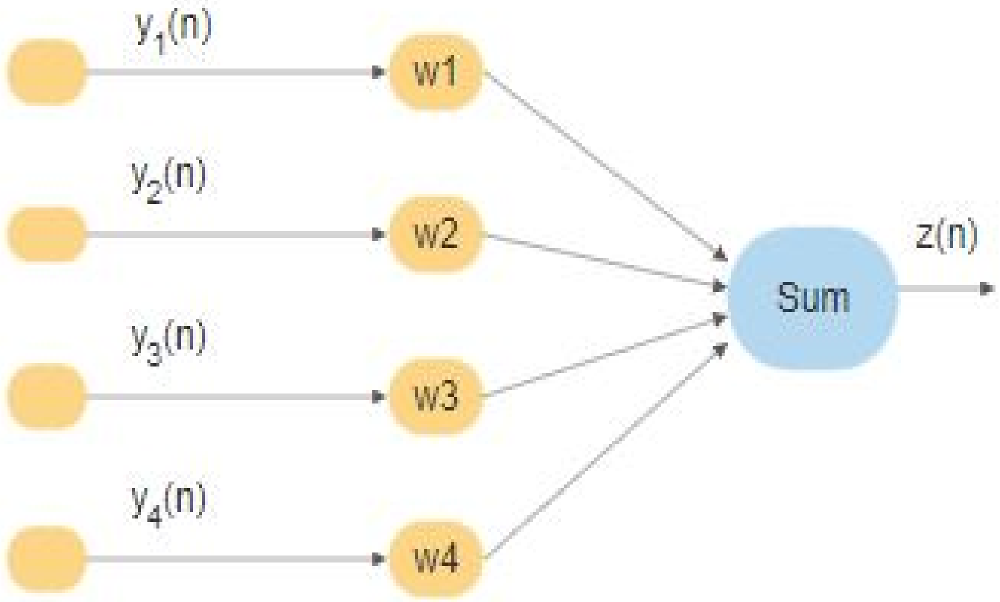


FIGURE 3.3: Basic Beamformer Structure

3.3 Beamforming Algorithms

In the following section basic function and performance of different beamforming algorithms will be briefly discussed. For this discussion we make use of uniform linear array (ULA). In order to consider waves coming from sources to ULA as plane waves, the sources or targets are assumed to be in the far field.

The data coming from the ULA is

$$y(k) = S(\phi)S(k) + N(k) \tag{3.1}$$

Here, $S(\phi)$ is the steering matrix of order $(A \times B)$ where B is the number of sources and A is the number of sensors. N(k) represents additive white Gaussian noise and the signals vector is represented by S(k). M (k=1, 2, 3,.,M) represents number of snapshots.

In most of conventional beam formers, spectrum like function of DOA is formed. Highest peaks indicate DOA estimates. In order to obtain these peaks, array is steered in all directions simultaneously measuring output power. The locations with maximum power are DOA estimates [6]. The output power of beam former is

$$P(w) = w^H R w \tag{3.2}$$

Here, R is sample correlation matrix of order (AxA) calculated as

$$R = \frac{1}{M} \sum_{n=1}^{M} x(n) x^H(n) \tag{3.3}$$

3.3.1 Conventional Delay and Sum Beamformer

In this type of beamformer, the beamformer's output power is maximized for a certain input signal. Here the weight vector is considered to be the steering vector. Considering equations (3.1) to (3.3), the output power for different angles is measured as

$$P(\phi) = v^H(\phi) R v(\phi) \tag{3.4}$$

Here $v(\phi)$ is the steering vector. As mentioned earlier the locations with maximum power are DOA estimates.

A limitation of this beamformer is that 2 targets within the beam width cannot be resolved. One solution to this problem is to reduce the beam width with increase in number of sensors but it will increase the beam formers cost.

3.3.2 Minimum Variance Distortion Less Response (MVDR) Beamformer

To reduce the limitations of delay and sum beam former like resolution of two closely spaced targets, this method also based on maximum likelihood detection was proposed. In this technique signal arriving along one desired direction is preserved and signals from all other directions are suppressed. The power spectrum is given by

$$P(\phi) = \frac{1}{(v^H(\phi)R^{-1}v(\phi))} \tag{3.5}$$

In many cases interference cannot be separated from the target signal and therefore weights are to be calculated using data that includes the target signal. In such condition, if signal is received from slightly different than desired direction, MVDR tends to suppress it. This phenomenon occurs because for MVDR beam former all signals are undesired interferences except the one along the desired direction. This problem is known as signal self-nulling in MVDR.

3.3.3 LCMV Beamformer

To prevent the problem of signal self-nulling, an LCMV beamformer can be used. Using this method, multiple constraints can be put along target direction (steering vector). As a result, the chance of signal suppression arriving at different from desired direction is reduced.

The output of LCMV beam former is given as

$$z(i) = w^H r(i) \tag{3.6}$$

Here r(i) is the receive vector and w is the beam forming vector to be designed.

$$w = [w_1, ., w_M]^T \tag{3.7}$$

It contains complex weights that are multiplied with signals at each sensor. If weight vector is adapted using LCMV criterion, it means we are minimizing the output variance while keeping the gain of the array into a desired signal direction θ_1 simultaneously fixed.

The optimal solution for the calculation of the weights is obtained by solving the minimization problem by using the method of Lagrange multipliers.

$$w_{opt} = \frac{\gamma R^{-1} v(\theta_1)}{v^H(\theta_1) R^{-1} a(\theta_1)} \tag{3.8}$$

3.3.4 Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT)

ESPRIT is a subspace based algorithm. It is one of computationally most effective technique as DOA is not estimated through search of all possible steering vectors. This algorithm achieves this reduction in computational complexity with the structure of the sensor array being imposed with a constraint on it. It exploits translational invariance in structure of sensors which means sensors occur in matched pairs with identical displacement vectors [13]. ESPRIT uses pair of sub arrays as shown in Fig 3.4.

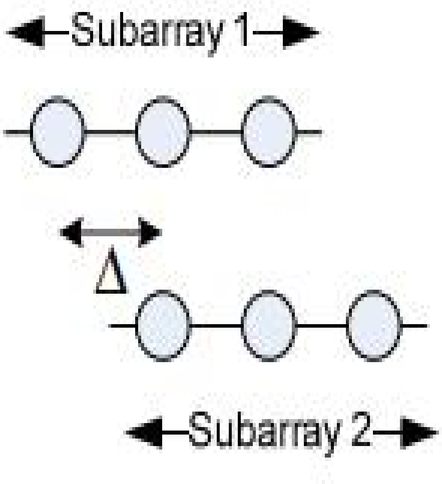


Figure 3.4: Sub Arrays Used in ESPRIT [6]

Sensor array geometry for multiple sources is shown in Fig 3.5. The elements in each doublet are separated translationally by a known constant displacement vector and have same sensitivity patterns. Estimates of DOA are given by following formula.

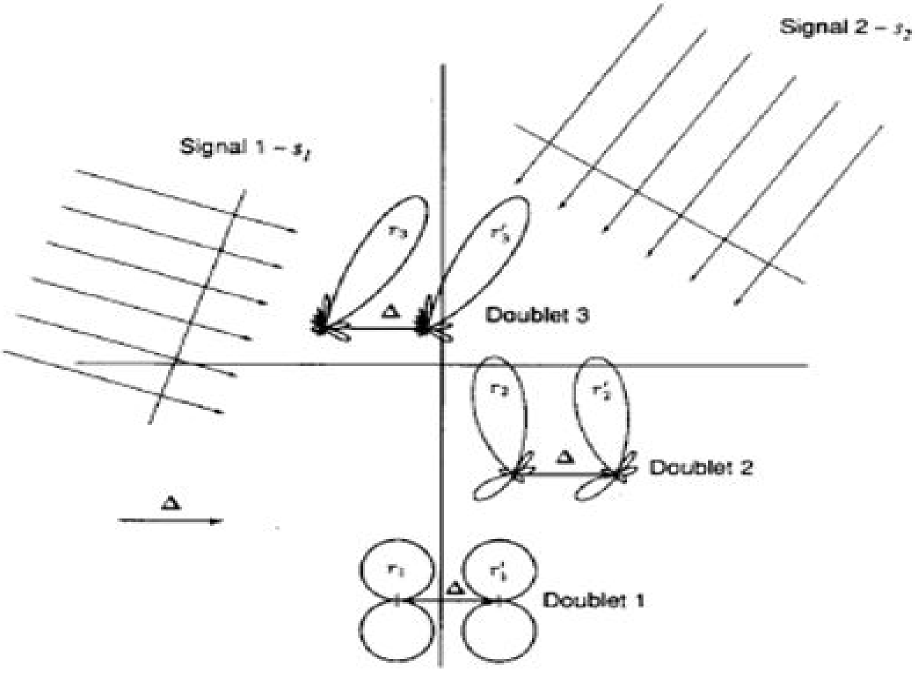


Figure 3.5: Array Geometry for Multiple Source DOA Estiamtion Using ESPRIT [13]

$$\phi_k = sin^{-1}\left[\frac{arg(\psi_k)}{\beta d}\right] \tag{3.9}$$

Where ϕ_k is the DOA estimate, $\beta = \frac{2\pi}{\lambda}$ is the subspace rotational operator and ψ_k is its kth Eigen value, wavelength is λ and the spacing between the sensors is d.

Table 3.1 represents the computational complexity of different beam forming algorithms in a decreasing trend.

Computational Complexity
LCMV
MVDR
CBF
MUSIC
ESPRIT

Table 3.1: Computational Complexity of Different Beamforming Algorithms

Table 3.2 summarizes the resolution capability and merits and demerits of different beam forming algorithms.

Algorithm	**Resolution**	**Merits**	**Demerits**
CBF	Poor	Easy Implementation	Two targets within beam width cannot be resolved
MVDR	Good	Distortionless performance in direction of interest	Self-nulling
LCMV	Good	Improved robustness than MVDR	Strong degradation of the output SINR
MUSIC	Very Good	High level of orthogonality between signals. Higher resolution and accuracy	Gives the Pseudo spectrum only
ESPRIT	Excellent	No need of searching the maxima in Pseudo spectrum.	Constraint on array structure

Table 3.2: Merits and Demerits of Different Beamforming Algorithms

Studying all above mentioned techniques and their advantages and disadvantages, it is to be stated there is still one more algorithm that provides a high resolution and has no constraint on array geometry. This algorithm is Multiple Signal Classification (MUSIC). In the following section this algorithm will be studied in detail. It will be used for comparison with monopulse technology in further chapters.

3.3.5 Multiple Signal Classification (MUSIC)

Determination of multiple wave fronts parameters arriving at an array of antenna through different experimental and theoretical techniques is described with the term multiple signal classification (MUSIC). The basic concept behind this algorithm is subspace decomposition of array output covariance matrix to obtain two subspaces namely signal subspace and noise subspace.

This algorithm implies that all noise eigen vectors and signal steering vectors are orthogonal to each other. Spectrum function is constituted with these two orthogonal subspaces and estimation of DOA of signal is then performed through spectral peak search. MUSIC method high resolution and stability makes it useful in many applications.

3.3.5.1 Mathematical Derivation of MUSIC

For mathematical derivation of MUSIC algorithm [24], first assume following conditions also shown in Fig 3.6.

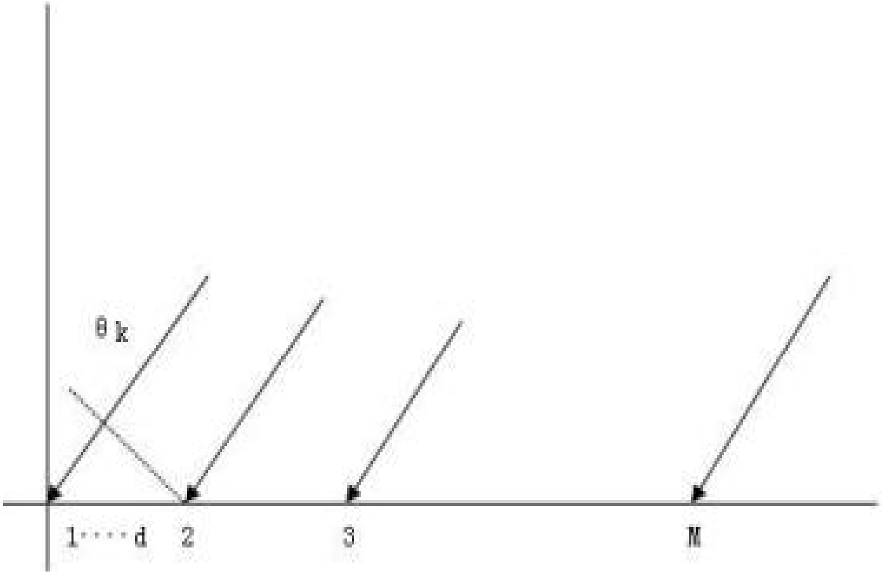


Figure 3.6: Model for MUSIC DOA Estimation [24]

1. The number of testing signal sources is D and each source has the same centre frequency w_0.
2. Antenna array has M (M >D) elements and all elements have same characteristics.

3. Spacing between array elements is d.

Let there be k signal sources (k=1, 2, 3,...,D), the wave front signal will then be $S_k(t)$ which can be expressed as

$$S_k(t) = s_k(t)exp\{j\omega_k(t)\} \tag{3.10}$$

$S_k(t)$ is the signal and $s_k(t)$ is its complex envelope and $\omega_k(t)$ is its angular frequency. Now consider the signal coming as output of m array element.

$$y_m(t) = \sum_{k=1}^{D} v_m(\theta_k)s_k(t) + n_m(t) \tag{3.11}$$

where $s_k(t)$ is signal strength of signal source k.

We can use matrices to describe this expression.

$$Y = VS + N \tag{3.12}$$

$$Y = [y_1(t), y_2(t), ..., y_M(t)]^T \tag{3.13}$$

$$S = [S_1(t), S_2(t), ..., S_D(t)]^T \tag{3.14}$$

$$V = [v(\theta_1), v(\theta_2), ..., v(\theta_D)]^T \tag{3.15}$$

$$\begin{bmatrix} 1 & 1 & ... & 1 \\ e^{-j\phi_1} & e^{-j\phi_2} & ... & e^{-j\phi_D} \\ e^{-j(M-1)\phi_1} & e^{-j(M-1)\phi_2} & ... & e^{-j(M-1)\phi_D} \end{bmatrix}$$

with

$$\phi_k = \frac{2\pi dsin\theta_k}{\lambda} \tag{3.16}$$

$$N = [n_1(t), n_2(t), ..., n_M(t)]^T \tag{3.17}$$

Now, we have to obtain covariance matrix of array output which is given as follows,

$$R_y = E[YY^H] \tag{3.18}$$

Here H represents the conjugate transpose matrix. Substituting (3.12) in (3.18), we get,

$$R_y = E[(VS+N)(VS+N)^H] \tag{3.19}$$

$$= VE[SS^H]V^H + E[NN^H] \tag{3.20}$$

$$= VR_sV^H + RN \tag{3.21}$$

Above equation has two parts,

$$R_s = E[SS^H] \tag{3.22}$$

is signal correlation matrix, and

$$RN = \sigma^2 I \tag{3.23}$$

is noise correlation matrix. We get,

$$R_y = VR_sV^H + \sigma^2 I \tag{3.24}$$

M positive real eigen values $\lambda_1, \lambda_2, ..., \lambda_M$ belong to $R_y$which correspond to $w_1, w_2, ..., w_M$ M eigen vectors but only D eigen values are of relevance for the signal.

For practical purposes, we use sample covariance $\tilde{R_y}$ as R_y cannot be obtained directly.

$$\tilde{R_y} = \frac{1}{N}\sum\nolimits_{i=1}^{N} y(i)y^H(i) \tag{3.25}$$

Next perform the Eigen decomposition of array covariance matrix, these Eigen values are sorted with respect to their size. The larger D Eigen values (Eigen vectors) belong to signal and the smaller M-D Eigen values (Eigen vectors) belong to noise.

$$\lambda_1 \geq \lambda_2 \geq, ..., \geq \lambda_M > 0 \tag{3.26}$$

Now we use noise characteristics as columns to create noise matrix En.

$$E_n = [W_{D+1}, W_{D+2}, ..., W_M] \tag{3.27}$$

Using this matrix, we define spatial spectrum $P_{mu}(\theta)$.

$$P_{mu}(\theta) = \frac{1}{v^H(\theta) E_n E_n^H v(\theta)} \tag{3.28}$$

$$= \frac{1}{||E_n^H v(\theta)||^2} \tag{3.29}$$

It can be observed that the denominator of the above formula is an inner product of noise matrix and signal vector.The value of this denominator should be zero as $v(\theta)$ is orthogonal with every column of E_n but it is a minimum value because of noise.

$P_{mu}(\theta)$ has a peak. In the above formula this peak is found by changing θ. Value of θ which gives the peak is estimate of angle of arrival.

Chapter 4

Direction of Arrival Estimation with Monopulse Tracking Radar Technology

Automatic tracking of targets is tracking radar primary function. Contrary to search radars, tracking radars beam responds to targets motion and follows it as a part of automatic tracking loop. Tracking radar provides data for estimation of targets path and future position by measuring targets current coordinates. When radar first enters in the vicinity of a target it searches small volumes of space in a predetermined pattern and is in acquisition mode. Once target is acquired it enters tracking phase and is said to be locked on the target.

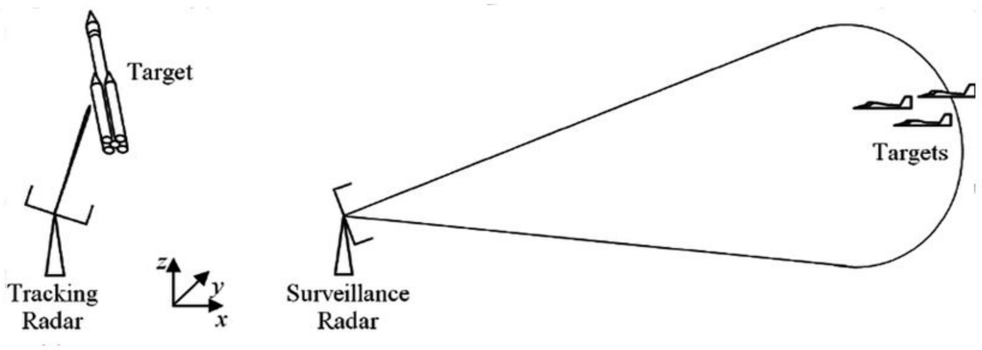


FIGURE 4.1: Figure Showing Difference Between Tracking and Surveillance Radar [26]

Most of the applications of tracking radars require high precision and accurate prediction of future position. This is also referred to as estimation of direction of arrival (DOA). The classical four quadrant array configuration as shown in Fig 4.2 can be used for accurate DOA estimation using monopulse method [27]. The radiation source can be active or passive.

Active source may include any distant antenna, astronomical body or jammer while passive sources include any target reflecting some part of power incident on it. The term target can refer to anything that we need to detect. Monopulse method also provides robustness against interference and jamming.

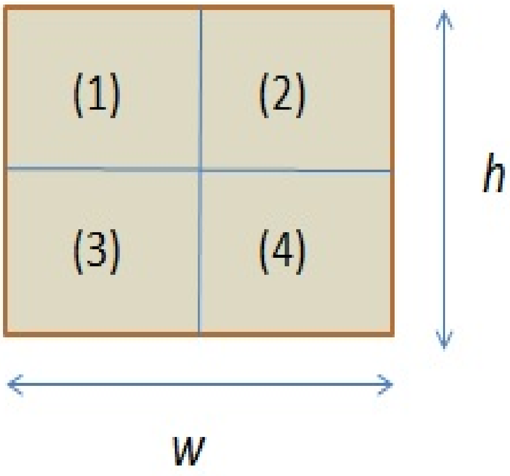


FIGURE 4.2: Four Quadrant Array Configuration [27]

In early times, the most common methods used to estimate DOA of a signal were sequential lobing and conical scanning. In sequential lobing, the radar beam points slightly to both sides of the target one by one, rapidly alternating, instead of directly pointing at the target. Sequential lobing in elevation is shown in Fig 4.3.

The echoes from the target are unequal if it is above or below the axis. Complete angle tracking can be obtained by interleaving the same operation in both coordinates. Four successive beam positions are required for this purpose as shown in Fig 4.4.

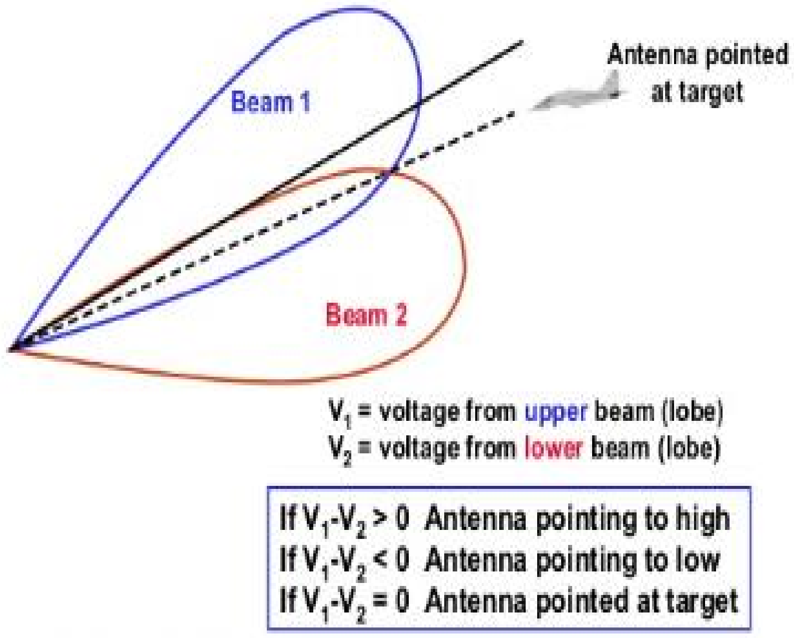


FIGURE 4.3: Beams for Sequential Lobing [14]

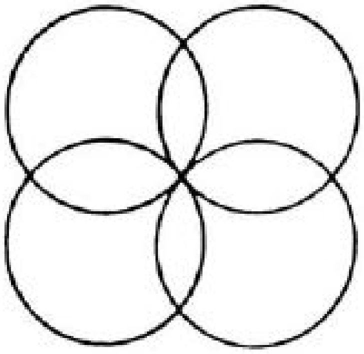


FIGURE 4.4: Sequential Lobing in Traverse and Elevation [14]

The position of the beam can be changed electronically or mechanically. If the feed is rotated mechanically around the axis, the beam should preferably be rotated in continuous circular path around the crossover axis instead of four discrete steps. This results in a conical scan as shown in Fig 4.5.

In these methods, the amplitude modulation of the received signals contains the angular information of target. Random variations in the effective scattering cross

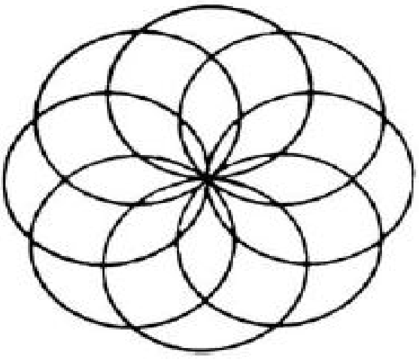


FIGURE 4.5: Conical Scanning [14]

section of the target can cause amplitude fluctuations in the received signals. These fluctuations can result in erroneous angle indications.

The sensitivity of these techniques to echo amplitude fluctuations led to development of radar that simultaneously provides all lobes for angle error sensing [28]. Effect of time change in amplitude of echo signal is eliminated in this technique as lobes are simultaneously compared on a single pulse. Initially this technique was called simultaneous lobing which later changed to monopulse as it is capable to obtain complete angle error information on just a single pulse.

Initially it was a very expensive, labor-intensive technique due to complexity, and less reliability. It was only used in need of extreme accuracy that can justify the cost. The complexity and cost of implementation of monopulse systems is greatly reduced because of availability of advanced digital signal processing techniques and now this technique is extensively used in modern radar systems.

4.1 Methods of Implementation of Monopulse Technique

In monopulse radars, the signals received simultaneously in two or more antennas are compared to get information about the angular location of the target. There are two basic types of mono pulse radars.

1. Amplitude Comparison Monopulse.
2. Phase Comparison Monopulse.

We will be using amplitude comparison monopulse in our further simulations but brief description of phase comparison monopulse is also given later in this chapter.

4.1.1 Amplitude Comparison Monopulse

This mode of implementation is very much similar to lobe switching, but here four receiving beams are simultaneously formed instead of obtaining target echoes in four sequential beam positions and then comparison is made on each pulse [14]. Feed horn assembly is shown in Fig 4.6.

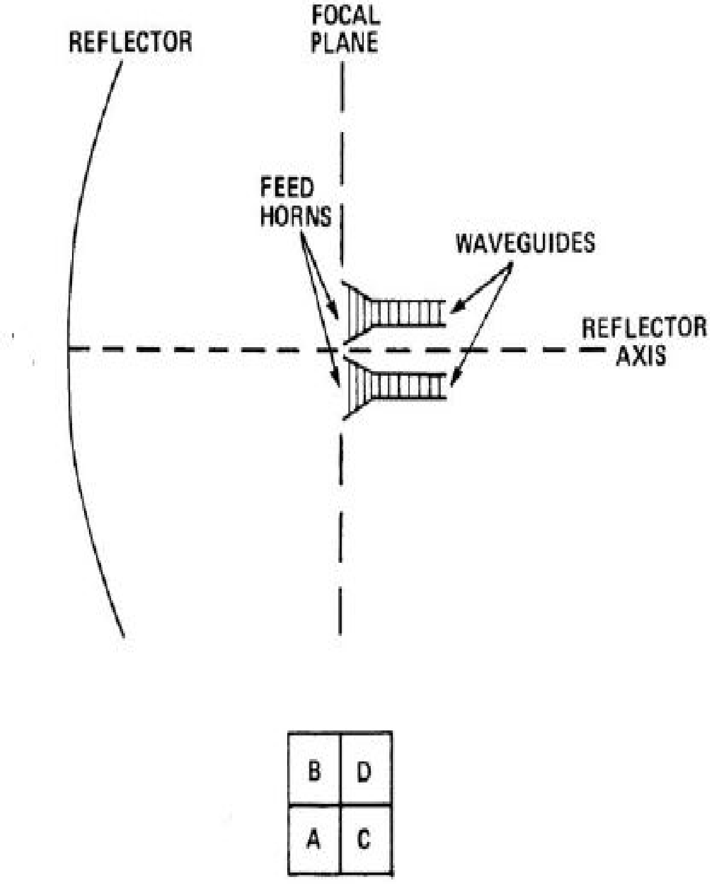


Figure 4.6: Figure Showing Feedhorn Assembly in Monopulse [14]

Four squinted beams are produced by four feed horns as shown in Fig 4.7.

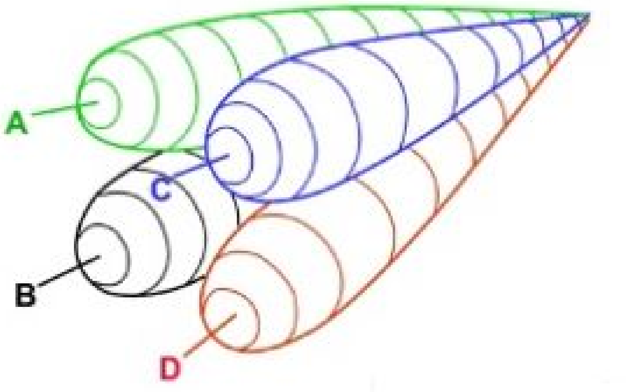


Figure 4.7: Four Squinted Beams Obtained in Amplitude Comparison Monopulse [14]

Lower beams are produced by upper horns. If the outputs of these beams are connected to four separate, identical receivers, their responses would differ in amplitude according to DOA of incident plane wave but their phases will be same as shown in Fig 4.8.

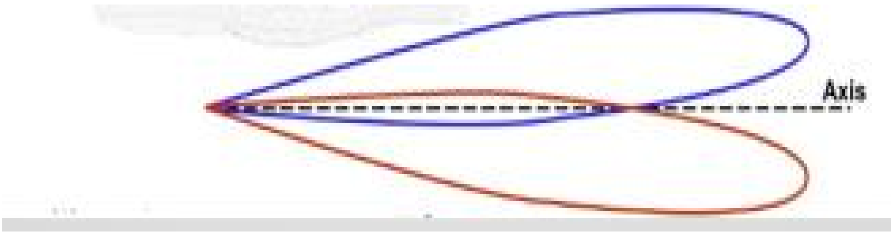


Figure 4.8: Basic Beam Formation in Amplitude Comparison Monopulse [29]

Equal amplitudes are obtained in the four beams only for a target located on the axis of symmetry of the antenna assembly. If the target is displaced from the axis, the two angular components of the target direction relative to the axis can be determined from the ratios of these amplitudes.

4.1.1.1 Processing Steps for Angle Measurements

The basic functional diagram of amplitude comparison monopulse is given in Fig 4.9.

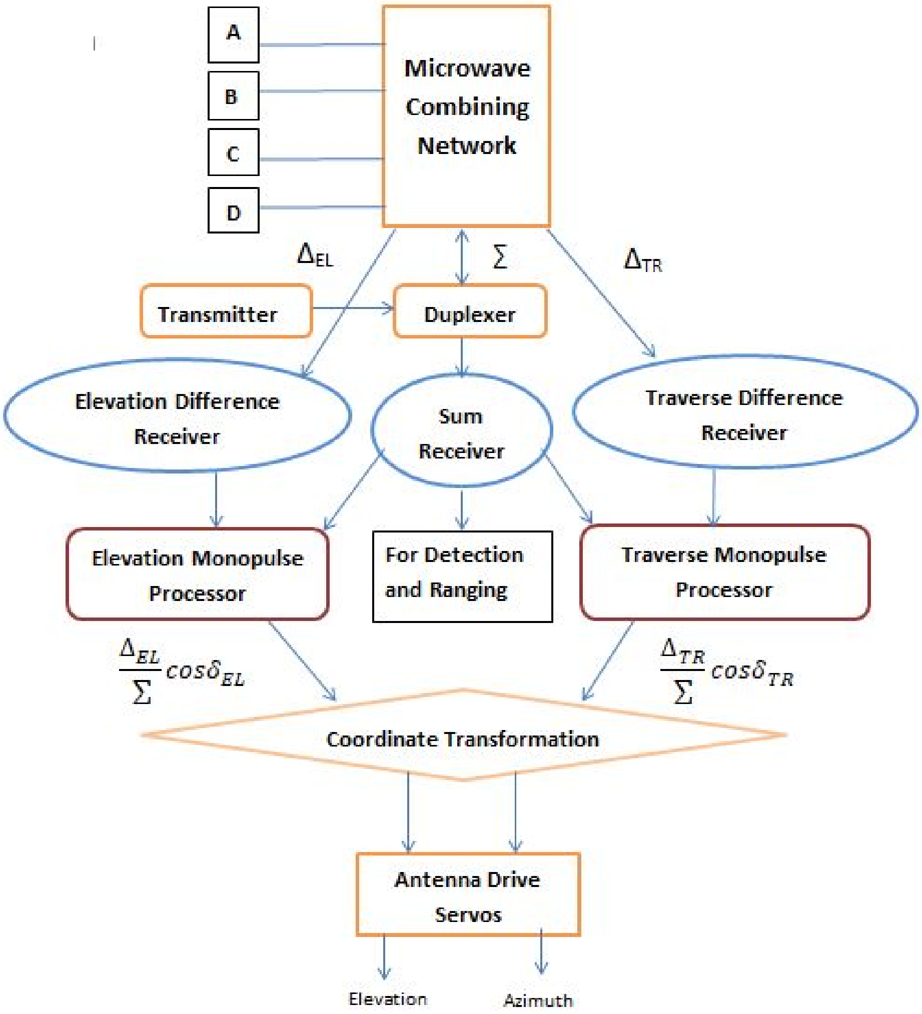


Figure 4.9: Basic Functional Diagram of Amplitude Comparison Monopulse

Corresponding received voltages in four beams are represented by A, B, C and D. Theoretically, four identical receivers should be connected at the output of the four horns, the output of these receivers is then compared. But practically even if the four receivers are initially adjusted for equal gain and phase, their output would unequally vary as a function of radio frequency, time and environmental conditions. As a result large drifts occur in the measurement of off-axis target angles. In order to solve this problem, we form the sum, an elevation difference, and a traverse difference at the outputs of these horns before connecting to receivers as shown in Fig 4.10.

This is done with the help of hybrid junctions; they have much lesser drift than receivers active circuitry, so we get a more stable null axis. The comparator

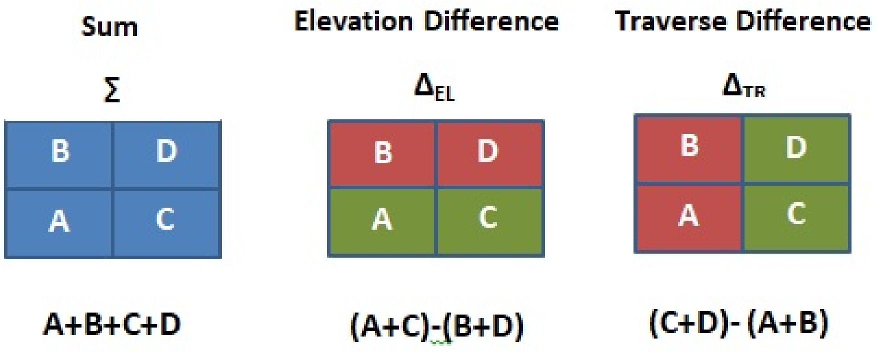


Figure 4.10: Sum, Elevation and Traverse Difference Formed in Monopulse [30]

circuitry used for four horn monopulse system is shown in Fig 4.11.

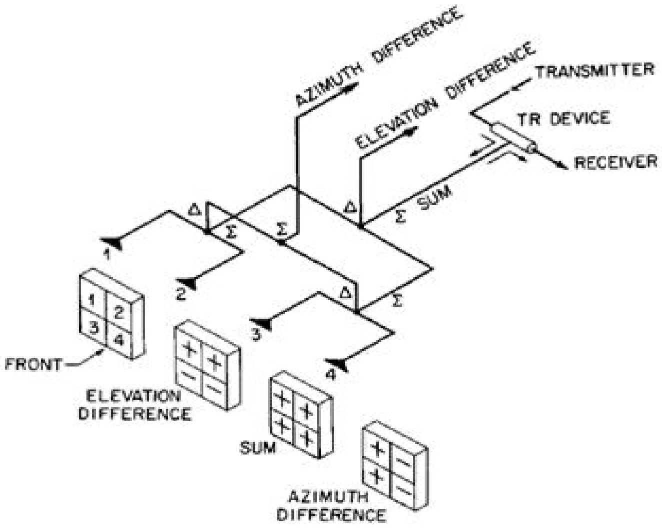


Figure 4.11: Comparator Circuitary Used with Four Horn Monopulse [31]

$$Sum = \Sigma = \frac{A+B+C+D}{2} \tag{4.1}$$

$$Elevation - difference = \Delta_{EL} = \frac{(A+C)-(B+D)}{2} \tag{4.2}$$

$$Traverse - difference = \Delta_{TR} = \frac{(C+D)-(A+B)}{2} \tag{4.3}$$

Now, the RF voltages are converted to intermediate frequency IF through local oscillators, these IF voltages are then amplified and filtered. Next, we obtain the output by multiplying the ratio of difference voltage amplitude to the sum voltage amplitude with cosine of the relative phase angle between the two voltages. The relative phase is denoted by δ_{TR} in traverse and δ_{EL} in elevation.

$$Output1 = \frac{\Delta_{EL}}{\Sigma} cos\delta_{EL} \tag{4.4}$$

$$Output2 = \frac{\Delta_{TR}}{\Sigma} cos\delta_{TR} \tag{4.5}$$

Coordinates of these outputs from the monopulse processor are transformed to get required azimuth and elevation angles.

4.1.1.2 Monopulse Signal Processing

In conventional implementation of monopulse parallel and identical receivers are used for Σ, Δ_{EL} and Δ_{AZ}. Normalization of difference signals to obtain error signal is done with a common AGC [32]. The error signal is given as,

$$E_{\Delta} = \frac{|\Delta|}{|\Sigma|} cos\phi \tag{4.6}$$

ϕ is the phase angle between Σ and Δ. The purpose of AGC is to keep loop gain in error signal constant as target signals amplitude keeps on changing with target fluctuations or range. So characteristics of AGC loop are very critical to the performance of mono pulse radar.

4.1.2 Phase Comparison Monopulse

In this mode of implementation sum and difference patterns are formed but receiving beams with different phase centers are employed. These beams can be obtained from side-by-side antennas or separate portions of an array. Relative phase between signals gives information about how deviated target is from antenna axis. Basic beam structure for phase comparison mono pulse is shown in Fig 4.12. The beams are parallel and identical; the signals produced by target will have same amplitudes but different phases on basis of which target direction will be obtained.

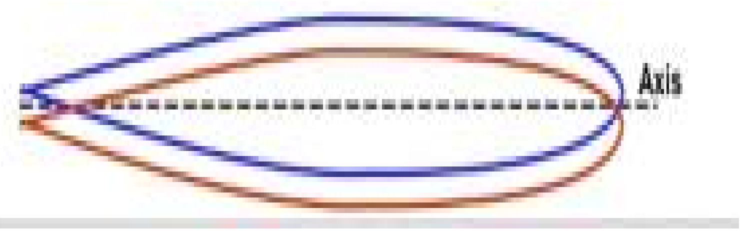


FIGURE 4.12: Basic Beam Formation in Phase Comparison Monopulse [29]

4.2 Phased Array Antenna Design

Desired antenna pattern can sometimes not be achieved with single antenna so instead antenna array is used. Antenna array is a combination of group of antenna elements to achieve desired directional characteristics.

In an array antenna, the radiation pattern can be changed electronically by controlling the phase and amplitude given to each antenna element. A phased array is usually electronically scanned array of antennas which can be steered electronically without moving the antenna to point in different directions.

4.2.1 Types of Antenna Elements

Following are two basic types of antenna elements used in phased array antennas.

1. Isotropic Antenna Element.
2. Cosine Antenna Element.

4.2.1.1 Isotropic Antenna Element

An isotropic antenna element radiates equal power in all directions. For a single isotropic element, the polar plot is shown in following Fig 4.13 . It can be seen that this element is radiating equal power in all directions.

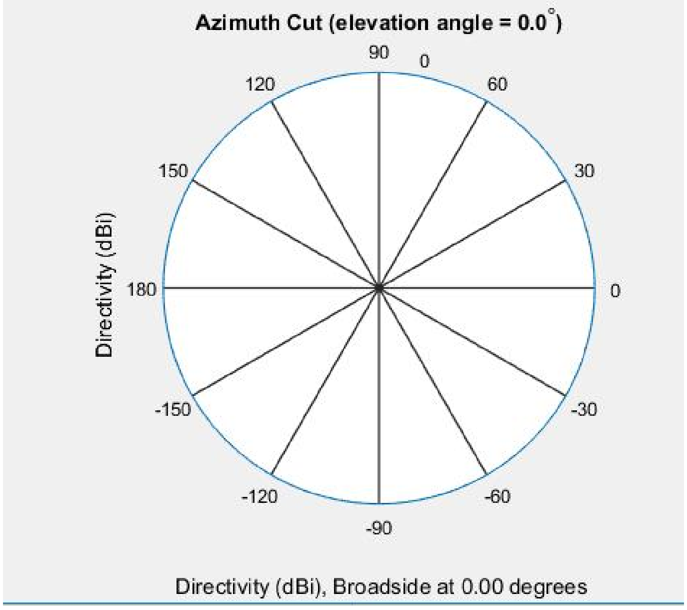


FIGURE 4.13: Antenna Pattern for Single Isotropic Element

4.2.1.2 Cosine Antenna Element

When considering realistic environment, tapered illumination is preferred as it has reduced side lobes. Also for mono pulse radar it is more desirable to transmit radiations in specified directions. Cosine antenna element is directional antenna with more power radiated in one direction as shown in Fig 4.14.

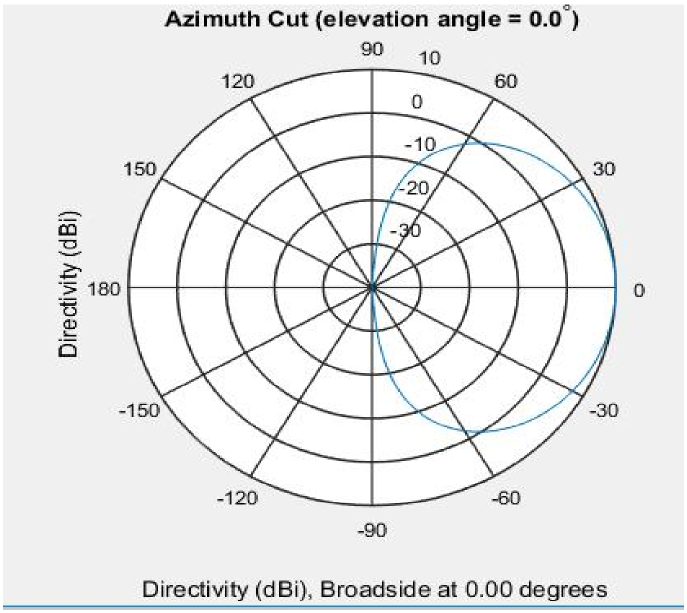


FIGURE 4.14: Antenna Pattern for Single Cosine Element

4.2.2 Uniform Rectangular Array Design

Antenna array of 900 cosine antenna elements will be designed in two ways

1. One uniform rectangular array (URA) of size 30x30 will be designed.
2. Four sub arrays of size 15x15 will be designed.

4.2.2.1 Uniform Rectangular Array of Size 30x30

To design array of size 30x30 following design parameters are considered for simulations.

Parameters	**Values**	**Units**
Bandwidth	X-band	GHz
Array size	30x30	-
Antenna Element	cosine	-
Lambda	0.03	meters
Spacing between elements	Lambda/2	meters
Length of each element	0.01	meters
eta	0.7	-

Table 4.1: Design Parameters for 30x30 Array

Using above parameters we can calculate the 3 dB beam width for this array.

Beamwidth Calculation

Using above parameters we can calculate the 3 dB beam width for this array.

- Length of each element in array =length=0.01 meters
- Spacing between elements =spacing=lambda/2
- No of elements along horizontal direction=E=30

- Horizontal length of array =D=(E*length)+(29*spacing)
- eta=0.7
- Effective horizontal length of array=D_eff=$eta * D$
- Lambda=wave speed/frequency

$$\theta_3 = \frac{\lambda}{(D_eff)} \tag{4.7}$$

The calculated 3dB beam width is 3.34 degrees.

Using Matlab , we can view geometry of this array shown in Fig 4.15.

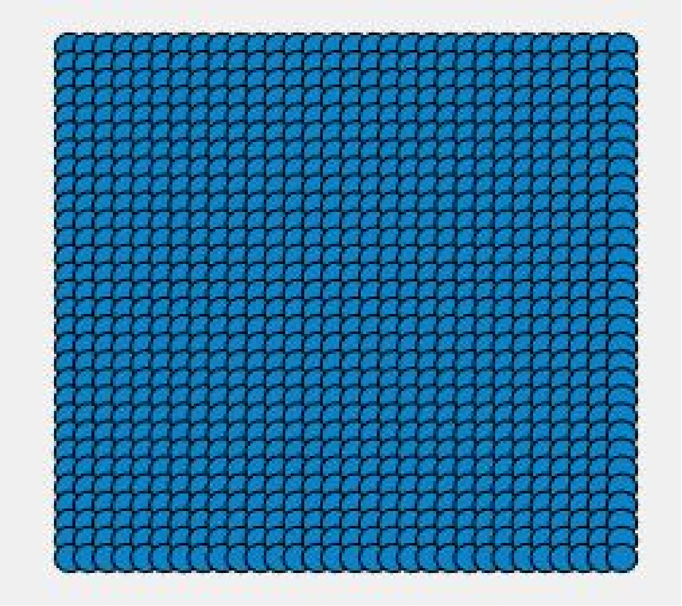
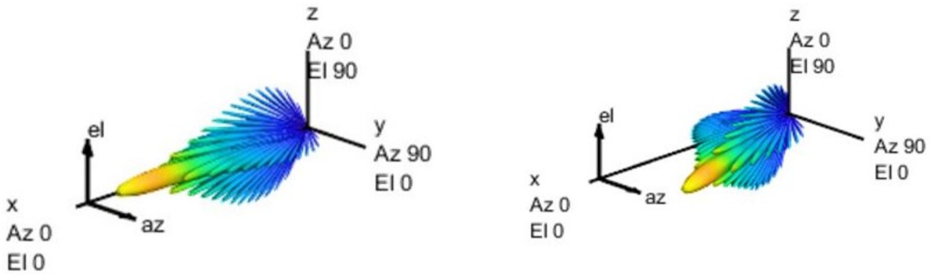
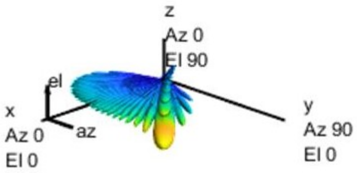


Figure 4.15: Array(30x30) Geometry

Next we will get 3D polar plot for this array when it is unsteered, and then steered at angles of 20 and 45 degrees as shown in Fig 4.16 (a), (b) and (c) respectively.



(a) Unsteered 30x30 array (b) 30x30 array steered at 20 degrees



(c) 30x30 array steered at 45 degrees

Figure 4.16: 3D Polar Plots of 30x30 Array Steered at Different Angles

4.2.2.2 Four Subarrays of Size 15x15 Each

To design sub arrays of size 15x15 following design parameters are considered for simulations for each sub array.

Parameters	**Values**	**Units**
Bandwidth	X-band	GHz
Array size	15x15	-
Antenna Element	cosine	-
Lambda	0.03	meters
Spacing between elements	Lambda/2	meters
Length of each element	0.01	meters
eta	0.7	-

Table 4.2: Design Parameters for 15x15 Subarrays

Beamwidth Calculation

Using above parameters we can calculate the 3 dB beam width for each sub array.

- Length of each element in array =length=0.01 meters
- Spacing between elements =spacing=lambda/2
- No of elements along horizontal direction=E=15
- Horizontal length of array =D=(E*length)+(14*spacing)
- eta=0.7
- Effective horizontal length of array=D_eff=$eta * D$
- Lambda=wave speed/frequency

$$\theta_3 = \frac{\lambda}{(D_eff)} \tag{4.8}$$

The calculated 3dB beam width is 6.82 degrees.

Using Matlab , we can view geometry of this array including 4 sub arrays shown in Fig 4.17.

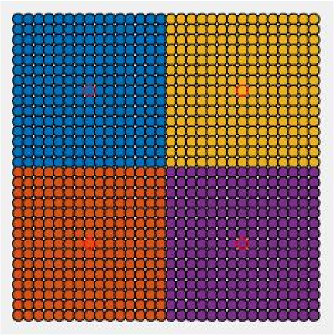
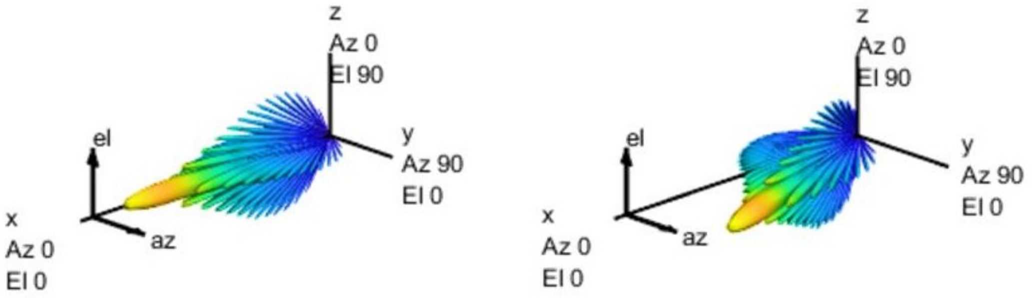


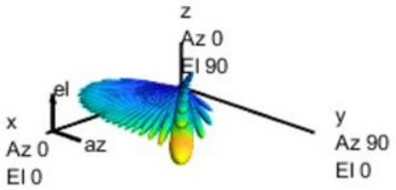
Figure 4.17: Array Geometry for Four Subarrays

Next we will again get 3D polar plot for these subarrays when unsteered, steered at angles of 20 degrees and 45 degrees as shown in Fig 4.18 (a), (b) and (c) respectively.



(a) Unsteered subarrays

(b) Subarrays steered at 20 degrees



(c) Subarrays steered at 45 degrees

Figure 4.18: 3D Polar Plots of Four Subarrays Steered at Different Angles

Chapter 5

Simulations and Results

In previous chapters, two major techniques Multiple Signal Classification (MUSIC) and monopulse that are used for direction of arrival (DOA) estimation are discussed. In this chapter comparison is made between these two techniques on the basis of different parameters.

Simulations are performed in MATLAB to compare the performance of these techniques. After looking at the results, it can be decided which technique is better for DOA estimation and target tracking. The parameters under which comparison is made are as follows.

1. Accuracy.
2. No. of Antenna Elements.
3. Performance under SNR degradation.
4. Multiple Target Detection.

Phased array radar designed in chapter 4 is used for simulations.

5.1 Accuracy

Accuracy is a measure of how close the measured value is to the true value. To compare accuracy of DOA estimation of monopulse and MUSIC , target moving towards the radar with initial position of [11000, 2000, 3000] and initial velocity of [-100; 30 ; 70] is taken. The azimuth and elevation angles of moving target are continuously estimated for 50 target positions through monopulse and MUSIC estimator and are then plotted against their true (actual values).

First, azimuth angle values estimated by monopulse and MUSIC are plotted along with actual azimuth angle of target as shown in Fig 5.1.

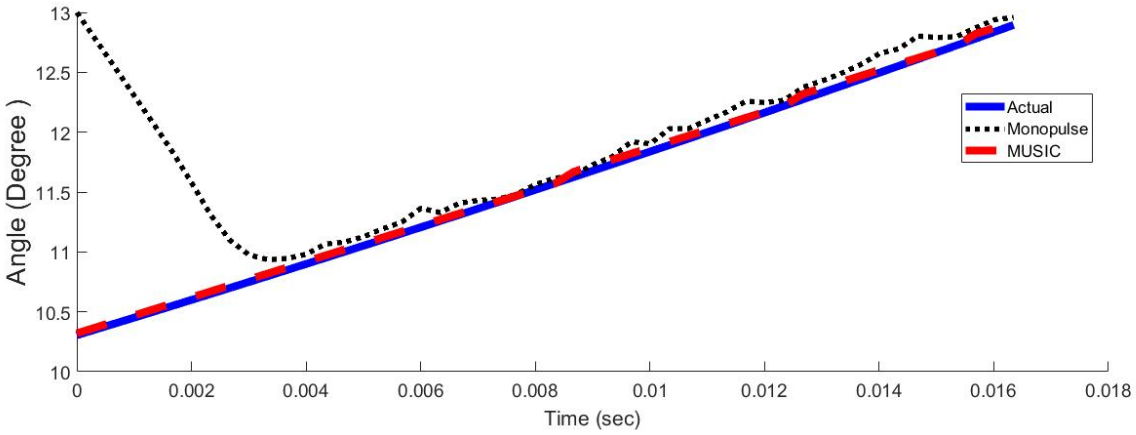


Figure 5.1: Azimuth Angle for Approaching Target

Comparing the estimates provided by both techniques, it can be seen that azimuth angle values estimated by MUSIC are more close to the actual values of moving target.

Secondly, elevation angles estimated by both techniques are plotted along with actual elevation angles of target as shown in Fig 5.2.

It is obvious from this figure also that elevation angle values estimated by MUSIC are more accurate in comparison to monopulse estimates.

So comparison on the basis of accuracy shows that for both azimuth and elevation angles, estimates provided by MUSIC are more accurate than monopulse.

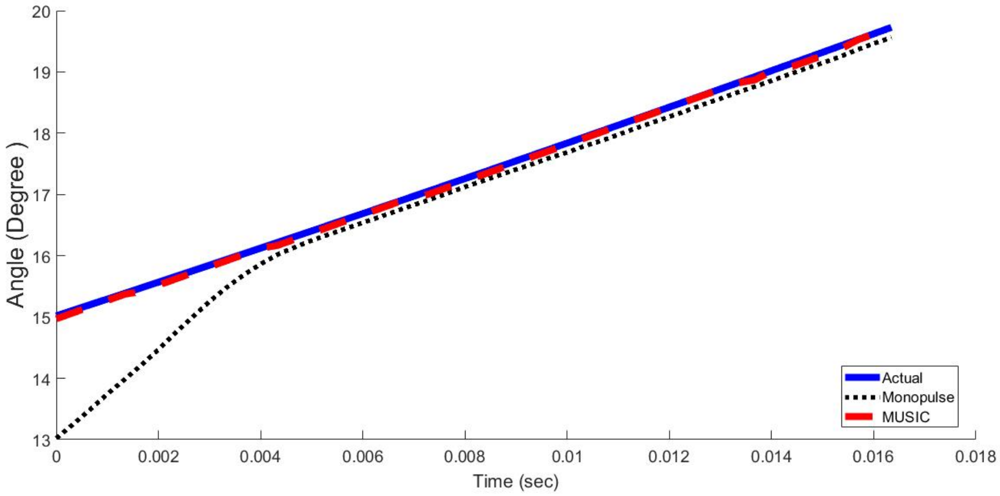
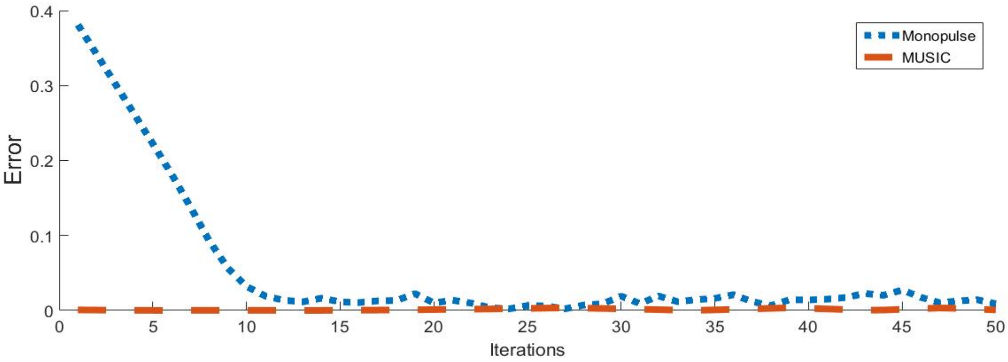


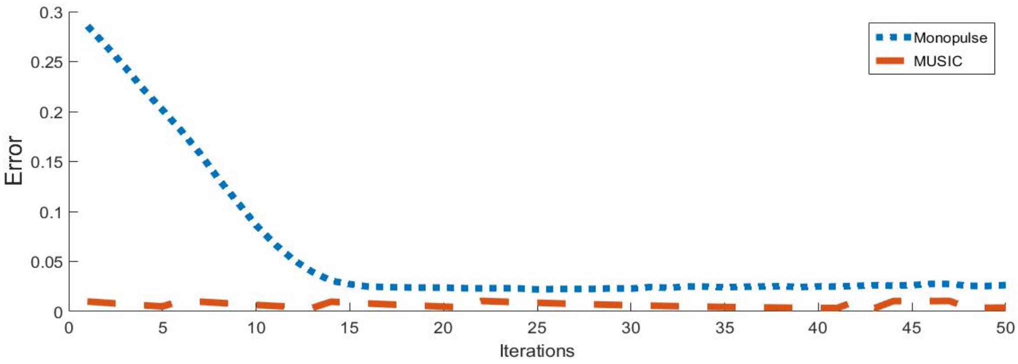
FIGURE 5.2: Elevation Angle for Approaching Target

5.1.1 Estimation Error

Estimation error of both techniques can also be plotted to elaborate this observation.



(a) Error in Azimuth Angle Estimation



(b) Error in Elevation Angle Estimation

FIGURE 5.3: Estimation Error

Estimation error of monopulse is higher than MUSIC, so MUSIC proves to be more accurate target tracker.

5.2 Number of Antenna Elements

In this section, target tracking of monopulse and MUSIC estimators is compared on the basis of number of antenna elements present in phased array radar. First azimuth angle and then elevation angle is measured and plotted. Azimuth tells you what direction to face and Elevation tells you how high up in the sky to look. Both are measured in degrees.Following array sizes will be considered.

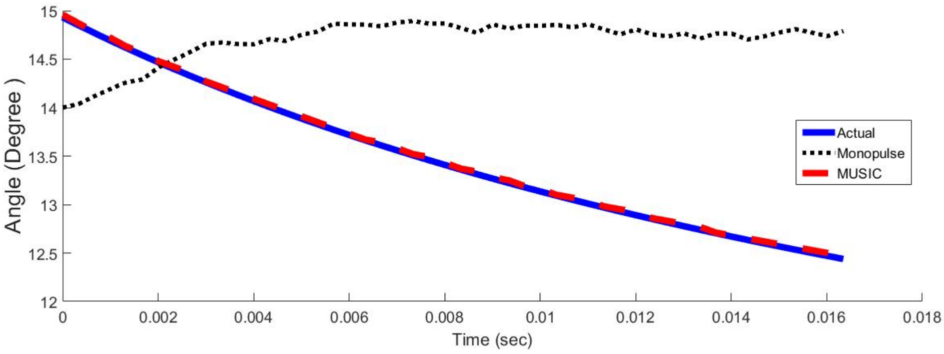
1. 10x10 Elements Array.
2. 20x20 Elements Array.
3. 30x30 Elements Array.

Parameters	**Values**	**Units**
Frequency	10	GHz
Initial Target Position	[3000,800,800]	meters
Initial Target Velocity	[200,30,70]	meters per second

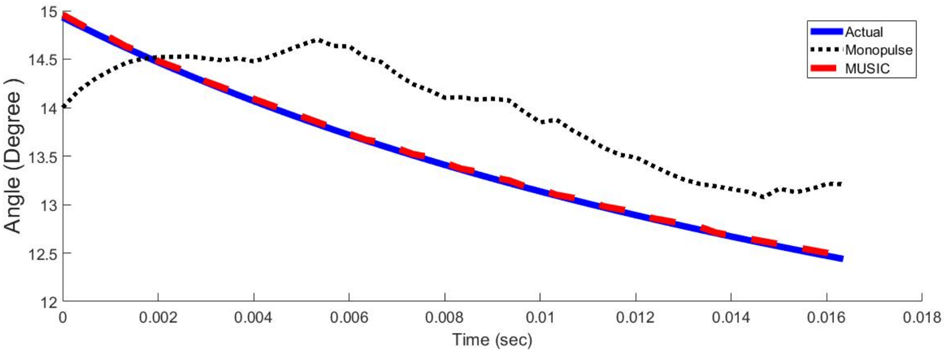
Table 5.1: Basic Parameters for Comparisons

5.2.1 Azimuth Angle Estimation

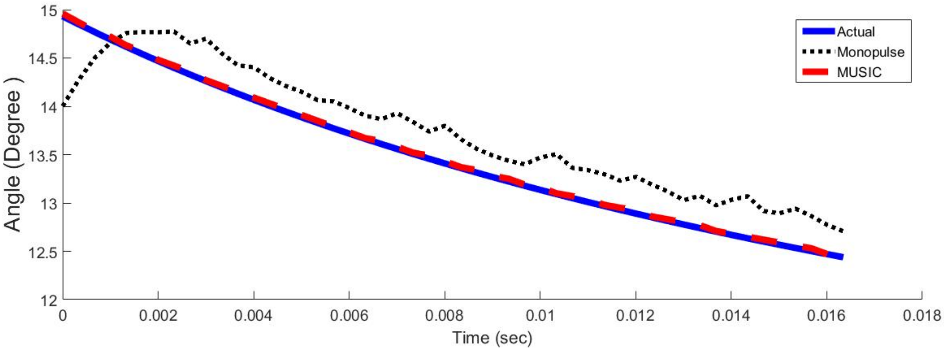
First azimuth angle is estimated for all three array sizes. Basic parameters are shown in Table 5.1. The azimuth angle estimates of both techniques obtained by using uniform rectangular antenna (URA) of size 10x10, 20x20 and 30x30 are shown in Fig 5.4 (a), (b) and (c) respectively.



(a) 10x10 Array



(b) 20x20 Array

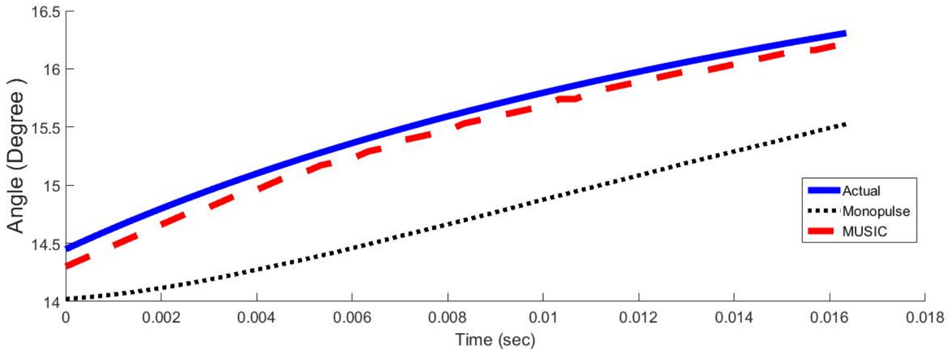


(b) 30x30 Array

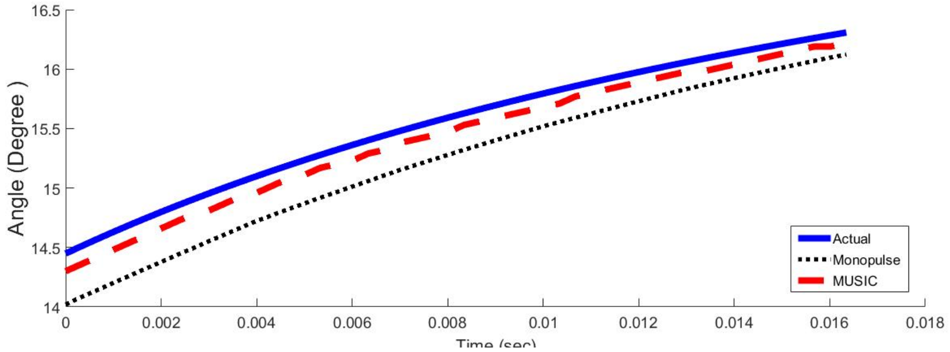
FIGURE 5.4: Azimuth Angle Estimation

5.2.2 Elevation Angle Estimation

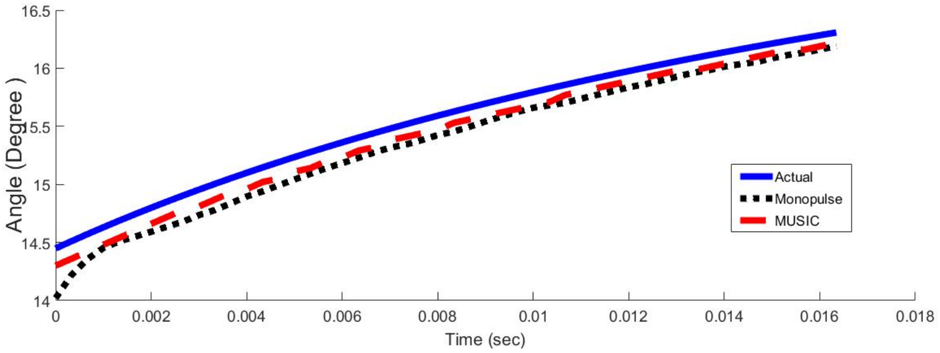
Now, the elevation angle estimates of both techniques obtained by using uniform rectangular antenna (URA) of size 10x10, 20x20 and 30x30 are shown in Fig 5.5 (a), (b) and (c) respectively.



(a) 10x10 Array



(b) 20x20 Array



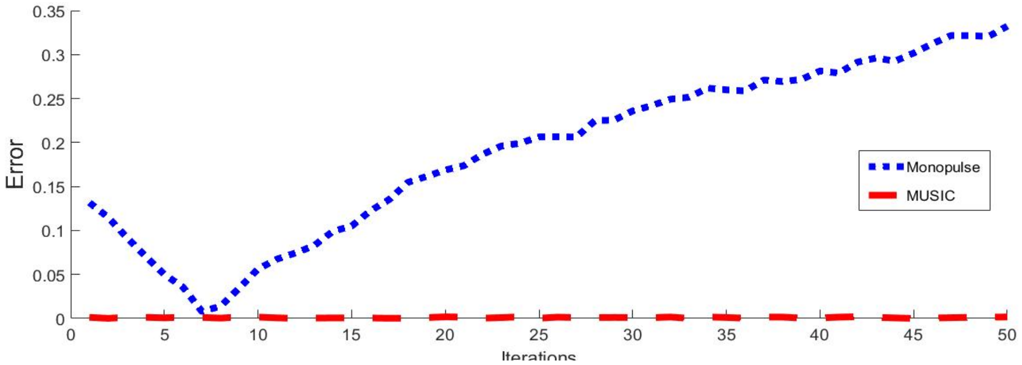
(b) 30x30 Array

FIGURE 5.5: Elevation Angle Estimation

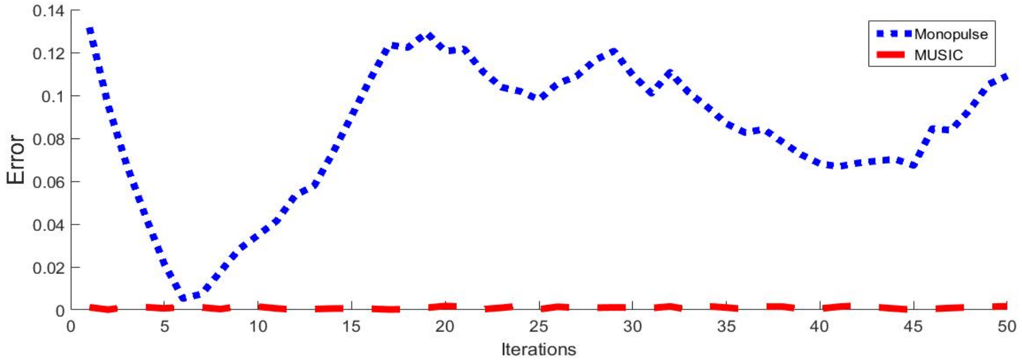
For azimuth angles compare Fig 5.4 (a), (b), (c) and for elevation angles compare Fig 5.5 (a), (b) and (c). In both figures, monopulse can be seen showing poor performance with less number of antenna elements. As beam width has inverse relation with area of antenna array so with increase in array size beams become more pointed and angle estimation of monopulse improves. In contrast to this, performance of MUSIC is not affected by array size. MUSIC correctly estimates direction of arrival even with small number of antenna elements.

5.2.3 Estimation Error

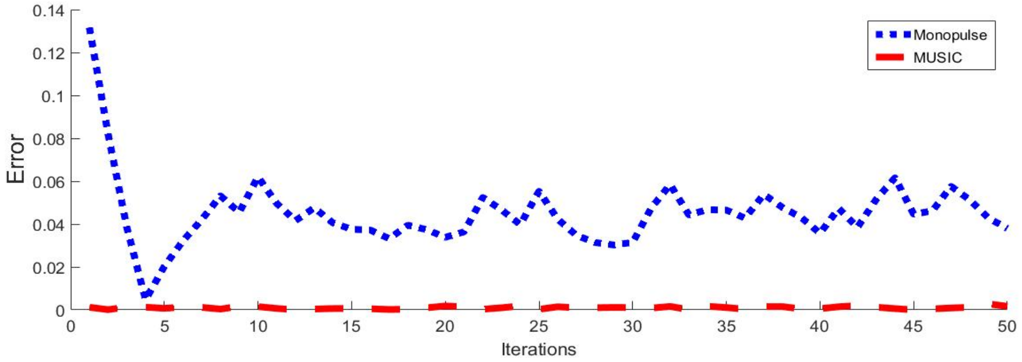
To confirm our conclusions above, estimation error for azimuth and elevation angles is plotted for all the three categories discussed.



(a) 10x10 Elements

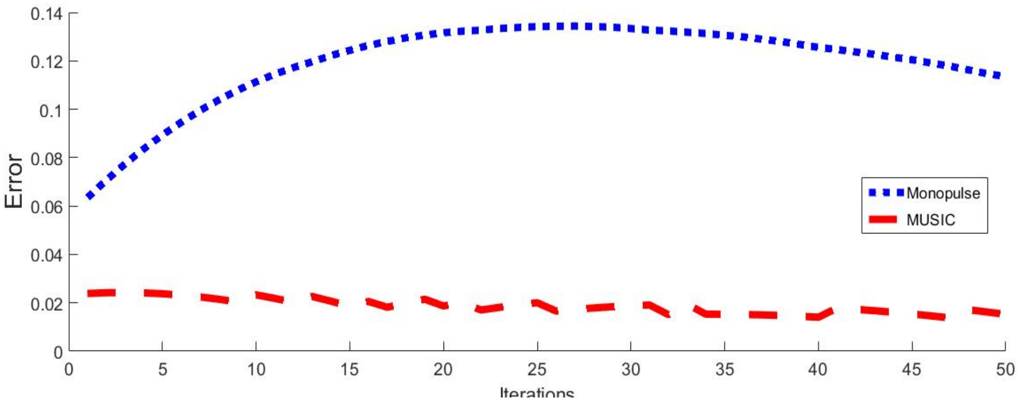


(b) 20x20 Elements

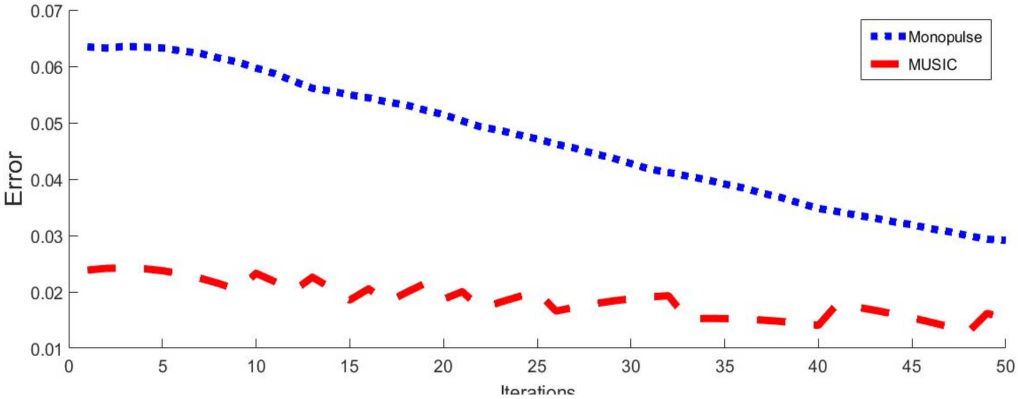


(c) 30x30 Elements

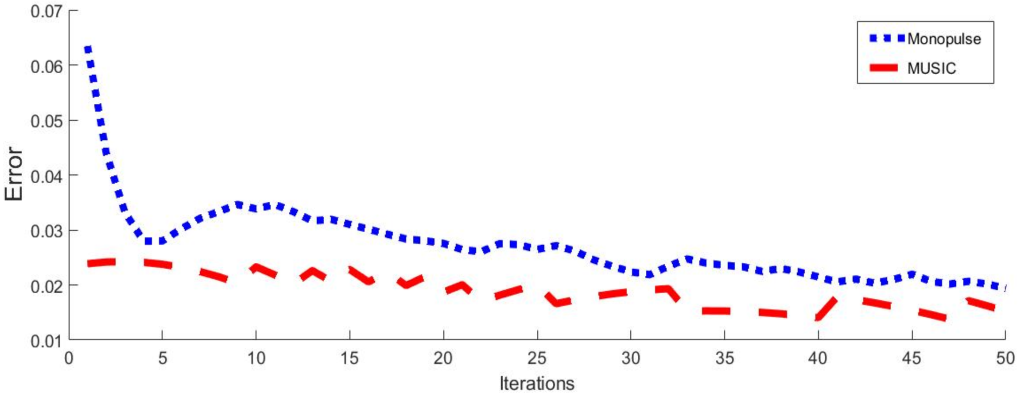
FIGURE 5.6: Error in Azimuth Angles for Different Number of Antenna Elements



(a) 10x10 Elements



(b) 20x20 Elements

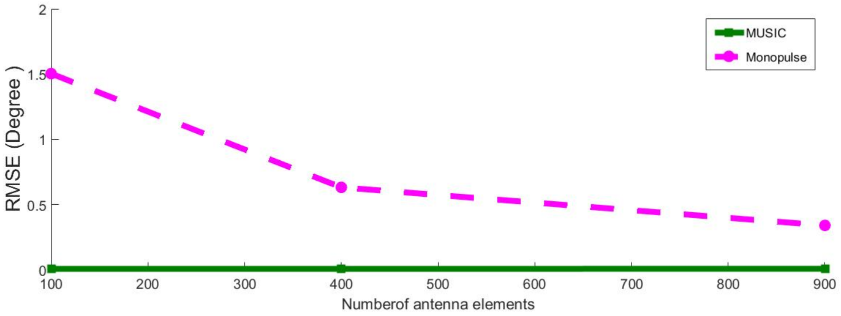


(c) 30x30 Elements

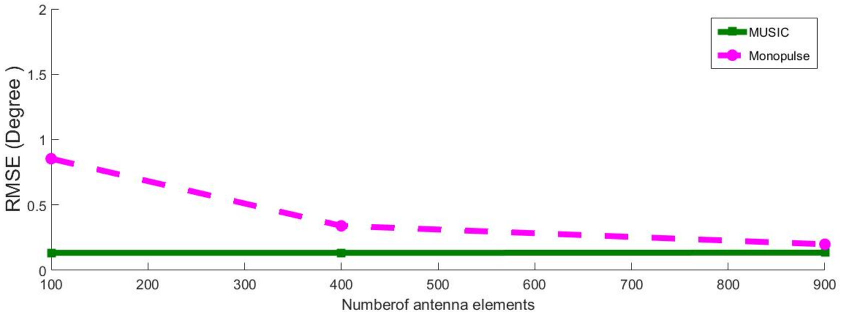
FIGURE 5.7: Error in Elevation Angles for Different Number of Antenna Elements

Comparing Fig 5.6 (a) (b) and (c) and Fig 5.8 (a) (b) and (c), it can be seen that as number of elements has increased estimation error of monopulse for both azimuth and elevation angles has significantly reduced while error in MUSIC estimates is not much effected.

Root mean square error can also be plotted against array size to get a clearer picture.



(a) Azimuth Angle



(b) Elevation Angle

FIGURE 5.8: Root Mean Square Error vs Number of Antenna Elements

So evaluating overall performance, it can be inferred that if phased array radar with less number of antenna elements is available then MUSIC should be preferred as it accurately tracks target even with small arrays but for sufficiently large arrays there is little difference between performance of both techniques so either one can be used. Expressing the results in tabular form we get Table 5.2.

No.of Antenna Elements	**Preferred Technique**
100	MUSIC
400	MUSIC
900	MUSIC, Monopulse

TABLE 5.2: Results for Comparison on Basis of Number of Antenna Elements

5.3 Performance Under SNR Degradation

In the next section, performance of both techniques is evaluated under degrading SNR conditions. For this evaluation target with parameters shown in Table 5.1 is taken. Azimuth and elevation angles are measured using both techniques for different values of Signal to Noise Ratio (SNR). RMSE obtained in each case is given in Table 5.3.

SNR (dB)	**Azimuth RMSE (deg)**		**Elevation RMSE (deg)**		**Preferred**
	MUSIC	**Monopulse**	**MUSIC**	**Monopulse**	
-30	38.05	0.3074	24.0121	0.2312	Monopulse
-25	28.3138	0.2359	19.6024	0.2336	Monopulse
-23	23.8699	0.2223	19.6624	0.2390	Monopulse
-22	10.3795	0.2550	1.9214	0.2390	Monopulse
-20	0.1020	0.2198	0.1402	0.2408	MUSIC, Monopulse
0	0.0156	0.23	0.0219	0.2383	MUSIC

TABLE 5.3: Root mean Square Values with Respect to Changing SNR

Fig 5.9 shows RMSE vs SNR plot for azimuth angles and Fig 5.10 shows RMSE vs SNR plot for elevation angles. Since RMSE values are very small in degrees so it is plotted in logarithmic scale to make the results clearer.

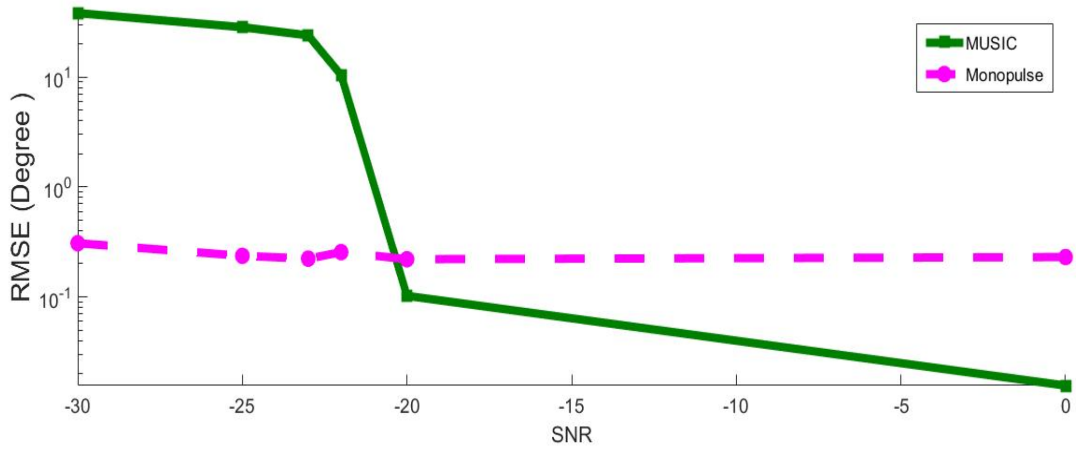


FIGURE 5.9: RMSE vs SNR Plot for Azimuth Angle Estimation

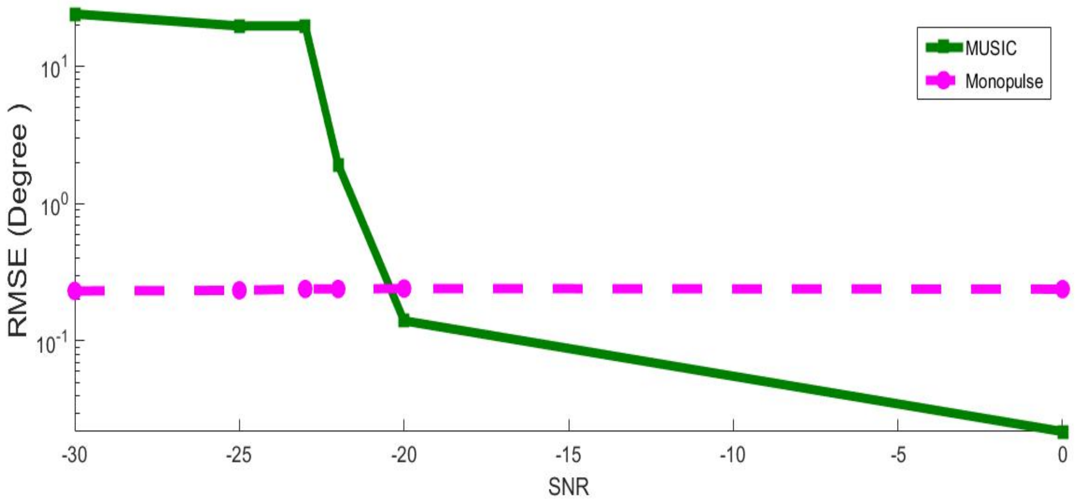


FIGURE 5.10: RMSE vs SNR Plot for Elevation Angle Estimation

Observing these figures, it can be deduced that performance of MUSIC algorithm is severely affected in low SNR and there is a large error in estimations and target tracking. As the SNR improves its estimation also improves and even gets better than monopulse when SNR is good. On the other hand low SNR is not having much effect on monopulse angle estimation and it is performing well even in poor SNR conditions.

5.4 Multiple Target Detection

For early warning radar systems, multiple targets separation in main beam is the key problem. For multiple targets flying in a formation, separating these targets in range domain is difficult as they may be in the same range cells. As these targets have very close radial velocities so it is also difficult to separate them in Doppler domain.

From literature survey, it is concluded that classical monopulse technique cannot be used for accurate measurement of targets angle as it is a pre requisite that a radar resolution cell has only one target in it [23, 33]. If there is more than one target in a radar range resolution cell, the angle estimated by monopulse can be very different from the actual angular separation of these targets. Such merge of measurements can cause problems in track filtering and data association [34].

Some typical solutions to this multiple target resolution problem are widening of beam width, narrowing of antenna beam width or dwell time extension but all these solutions increase the hardware cost of radar system. So we consider another solution i.e use of super resolution method like MUSIC. MUSIC can easily resolve closely spaced targets. To verify multiple target resolution capability of MUSIC we take four targets as shown in Table 5.4.

Target Parameters	Units	Target 1	Target 2	Target 3	Target 4
Range	Km	20.9	15.9	2.83	7.11
Azimuth Angle	degrees	17.22	18.43	5.71	8.13
Elevation Angle	degrees	0.27	6.85	44.8	6.45

TABLE 5.4: Actual Parameters of Target

Using the phased array radar with 30x30 elements described above, we apply 2D MUSIC estimator on these four targets and get the results shown in Table 5.5. It can be also be seen in Fig 5.11 that how MUSIC 2D estimator has correctly resolved four targets giving values very near to true values in azimuth and elevation.

Target Parameters measured by MUSIC	Units	Target 1	Target 2	Target 3	Target 4
Azimuth Angle	degrees	17	18.5	5.5	8
Elevation Angle	degrees	0	7	44.5	6.5

TABLE 5.5: Angular Parameters of Targets Measured by MUSIC

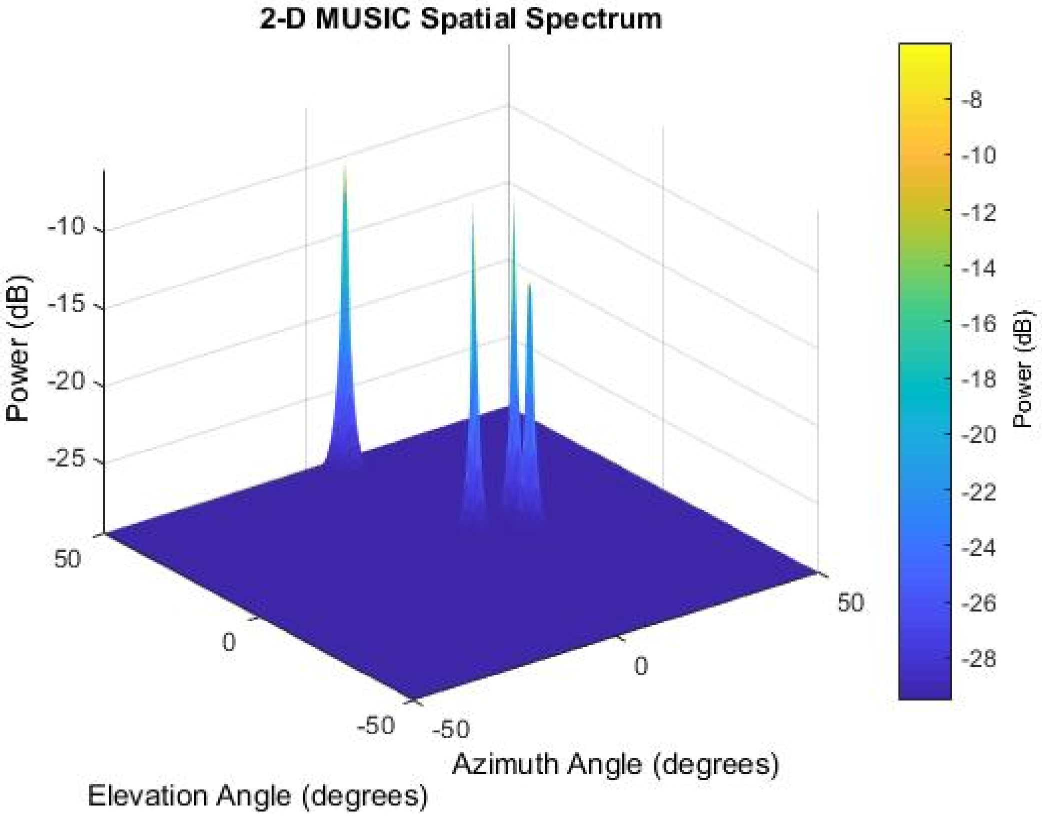


FIGURE 5.11: Spectrum Showing Four Targets Resolved by MUSIC

Next, we make use of phased array radar designed with four sub arrays of size 15x15 in previous chapter and apply MUSIC estimator to separate four targets given in Table 5.4 in azimuth and elevation.The obtained results are shown in Table 5.6.

Target Parameters measured by MUSIC	Units	Target 1	Target 2	Target 3	Target 4
Azimuth Angle	degrees	17	18.5	5.5	8
Elevation Angle	degrees	0	7	44.5	6.5

TABLE 5.6: Angular Parameters of Targets Measured by MUSIC with Four Sub Arrays

It can be also be seen in Fig 5.12 that how MUSIC 2D estimator with four sub-arrays has correctly resolved four targets giving values very near to true values in azimuth and elevation and better power compared to single array.

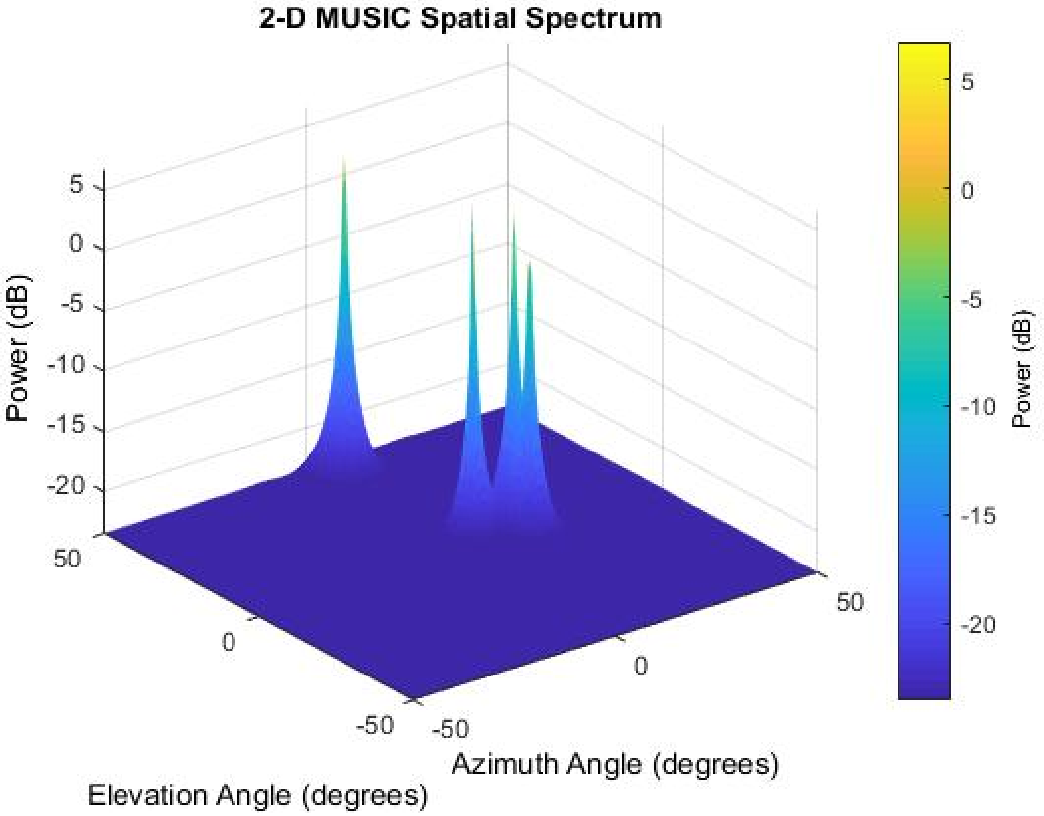


FIGURE 5.12: Spectrum Showing Four Targets Resolved by MUSIC with Four Sub Arrays

Chapter 6

Conclusions

One of the problems of interest in a number of fields like sonar and radar systems, acoustic signal processing and wireless communication is the estimation of Direction of Arrival (DOA) of propagating waves. In recent times, fruitful results are provided by variety of techniques and algorithms.

6.1 Conclusions

In this thesis, first an understanding of Direction of Arrival (DOA) estimation was established. Based on literature survey it was concluded that among beamforming algorithms for DOA estiamtion, MUSIC performs well. This work aims to provide a comparative study between two major angular estimation techniques, monopulse and MUSIC for two dimensional target tracking. Detailed study of these two techniques is provided. In order to compare these techniques, simulations were performed in MATLAB.

First the comparison is done on basis of accuracy of measured azimuth and elevation angles for a target moving towards a phased array radar. Simulations clearly show that angular parameters values measured by MUSIC are close to the actual values as the target is moving in comparison to the estimates provided by monopulse making MUSIC more accurate in target tracking.

Second comparison is done on the basis of number of antenna elements present in phased array radar. On basis of angle values estimated by both techniques and root mean square error (RMSE) plots, it is concluded that for small phased array radar with less antenna elements, MUSIC gives better estimates but for large arrays either monopulse or MUSIC can be used for tracking.

Third comparison is on basis of performance under degrading SNR conditions, It is clear from the simulations that even in poor SNR conditions, monopulse still correctly tracks target in azimuth and elevation but MUSIC response is severely affected. So,it can be said that in poor SNR conditions, monopulse performs better than MUSIC.

Lastly, both techniques are compared for multiple target detection. Literature study proved that for multiple targets classical monopulse technique cannot be used for accurate measurement of targets angle as it is a pre requisite that a radar resolution cell has only one target in it so simulations were performed for MUSIC with four targets and it clearly resolved all four targets in azimuth and elevation.

Based on the overall results obtained from simulations, it may be concluded that in poor SNR conditions, it is advantageous to use monopulse rather than MUSIC. Otherwise in better SNR scenario MUSIC will always be a better target tracker because of its accuracy, correct DOA estimation even with smaller antenna arrays and ability to detect multiple targets.

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