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Tank Gun Stabilization Using Mathematical Modeling

by

Usama Ghazi

A thesis submitted in partial fulfillment for the degree of Master of Science in Mechanical Engineering

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To my parents Mr. Muhammad Ramzan Joya Mrs. Kishwar Parveen

And

It is with my deepest gratitude and warmest affection that I dedicate this thesis to my Supervisors "Dr. Khawar Naveed Abbasi" who have been constant source of knowledge and inspiration for me in this whole period.

CERTIFICATE OF APPROVAL

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Abstract

Demands on increasing the battlefield mobility, that is, the ability of tanks to move when in actual or imminent contact with enemy forces, inevitably lead to the requirement of firing on the move, instead of having to stop every time they engage a target. This requirement call, in turn, for gun manage structures which reduce the results of movement on the main armament of tanks and in particular its capability to hit objectives. In modern tanks, there is an independently stabilized gunner's periscope. These periscopes have thermal imaging and day TV CCD imaging cameras over which a very accurately gyro stabilized head mirrors. The RCWS is a study based on a gimbal system as well as controlling the gun mount. An effective approach to the movement control problem for robotic manipulators is the so known as kinematic manage. Gun mount is a 2R serial joint structure which is stabilized using kinematic and dynamic mathematical analysis. For kinematic analysis forward kinematics, Jacobian and inverse kinematic analysis has been done whereas for dynamic analysis Lagrangian approach has been used. Further on these analysis has been verified using MATLAB coding and simulating the tank gun on a track while keeping the target location locked. Analysis and stabilization of the mount in two dimensional space has been done i-e for yow and pitch axis. This thesis is just a launching Pad and can be extended to three dimensional space.

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Abbreviations

ARWS	Advanced Radar Warning System
CCD Imaging	Charged Coupled Device
D-H	Denavit–Hartenberg
ED	Elevation Drive
EMR	Enhanced Marksman Rifle
FG	Free Gyroscope
G	GUN
GC	Gun Controller
ISIS	Integrated Signals Intelligence System
P	Hydraulic Pump
PA	Power Amplifier
PS	Phase Sensor
RCWS	Remote Control Weapon Station
RG	Rate Gyroscope
SCT	Synchro-Control-Transfer
SS	Sight Servo
TV	Television
VA	Voltage Amplifier

Symbols

φ_A	Angular Position
$M(\varphi_k)$	Disturbance Torque
U_{pp}	Orientation signal given by administrator
f_k	No. of degree of freedom of Jth Joint
m	No of links
j	No of Total Joints
DOF	Degree of freedom
N_i	No of teeth
J	Moment of Inertia
T	Torque
T_a	Actuator Torque
T_d	Disturbance Torque
J_g	Inertia of Gun
T_{gf}	Torque due to gun firing
D	Cupola Suspension damping Elevation
K	Cupola Suspension Spring Constant
$\frac{dy}{dx}\theta$	Cupola Pitch rate
K_b	Constant Back Emf
$\frac{d\Theta(t)}{dx}$	Angular Velocity
$M(\Theta)$	Definite Mass Matrix
$V(\Theta, \Theta^{\cdot})$	Vector Which Contains Coriolis and Centripetal Torque
$g(\Theta)$	Vector which Contains Gravitational Torque
$A(:: 1)$	Base to first joint A position Matrix
$A(:: 2)$	First to Second joint A position Matrix

Chapter 1

Introduction

In a battlefield aiming for targets play a vital role in winning a battle. Tanks in such cases lead the army of a country. Target aiming in tanks can only be possible if the gun mount is stabilized. Stabilization of the gun mount is a major concern. With this datum, it was expected that the tank would be more accurate in hitting the target while moving. In recent years multiple researches on tank gun and barrel stabilization have been carried out and revealed that stabilizing the gun and using a remotely controlled weapon station can have accountable significance. This chapter briefly explains how stabilization help in achieving the targets while the tank is in motion. The main aim of the research is also explained in this chapter.

1.1 Background Of Study

The need to enhance maneuverability in the battlefield i.e. the ability of the tank to maneuver when in certain or possible contact with an enemy, surely advancing toward fire while moving, instead of stopping and engaging a target every time. This obligation demand, in turn, for gun structures which reduce the results of movement on the primary weaponry of tanks and especially its capability to hit objectives [1, 2].

The movement of armored weapons can be limited using weapon stabilization systems designed to control the firearms' spatial orientation.

Figure 1.1, demonstrate effect of gun stabilization during tank movement [3]. In figure (a) shows that the tank gun is un-stabilized because of which aim of the gun is in another direction whereas in figure (b) the gun is stabilized and the target is locked because of which the tank movement has no effect on aiming target. Systems that perform this type of motion primarily in a closed-loop servo system that uses a gyroscope to identify the position of the gun in reference to inertial space and uses the velocity or position feedback of the signals they provide [4].

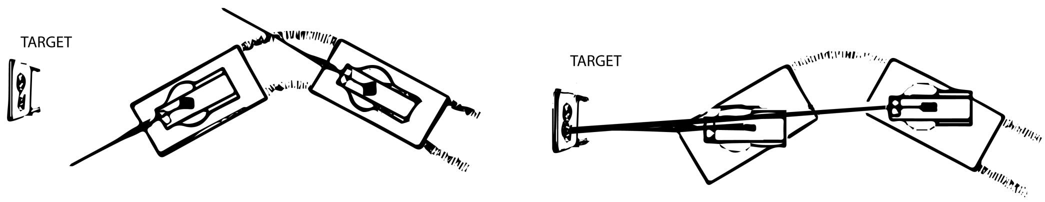

Figure 1.1: Stabilization effect in tank [3]

Basic control systems include two independent closed-loop servo systems for azimuth and elevation. In figure 1.2 input to gyroscope is the signal which the gun releases. This signal initially contains disturbance because of the movement of tank or disturbances due to the firing of a gun, these disturbances are removed by the help of a feedback loop having a gyroscope in it. Gyroscope detects the angular velocity at axis relative to the inertial frame and compensates for error with a Servo control loop (Figure 1.2) [1]. An inertial reference frame is a frame where the movement of a particle not subjected to force is in a straight line at a constant velocity. A servo loop is a feedback system in which the controlled variable is usually a mechanical position. Control system designers must calculate the system transfer function so as to design control systems used for precise control of a mechanical system.

Simple structures manipulated by the gyroscope have proved reasonably effective and if they don't allow gunners to aim correctly while moving, they could however make the best extraordinarily small adjustments when tank forestall to fire. However, the reaction of the primary structures isn't sufficiently low degree while tanks pass at high speed on difficult terrain. which results in having more complicated structures development in the Nineteen Sixties.

These second technology systems include two extra gyroscopes in an open loop that respond to vehicle angular velocities and provide advance instructions to the Azimuths and elevations drive, thus stabilizing the weapon.

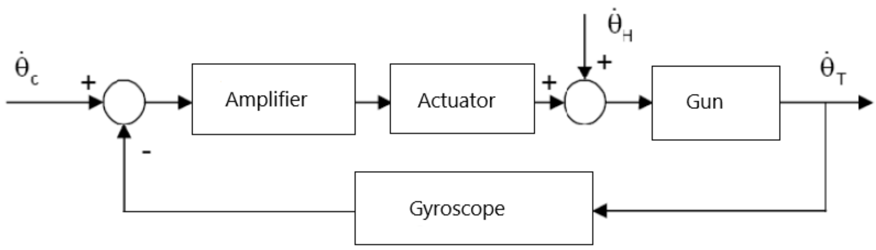

FIGURE 1.2: SERVO Stabilization system [1]

Therefore, an additional gyroscope is attached in the hull to sense the angular rotation of the structure and to develop feed-forward directions for a transverse motor. The second gyroscope is attached to the cupola to detect the angular motion in the elevation plane of the gun and generate forward directions for the motor. Thus, the need for two gyroscopes mounted on the weapon is diminished when adjusting the feed loop errors and the stabilization of the weapon is significantly improved [4]. The result of all the corrections that were united in the structures of the second generation was to reduce the errors of targeting weapons and then the chances of hitting moving targets increase.

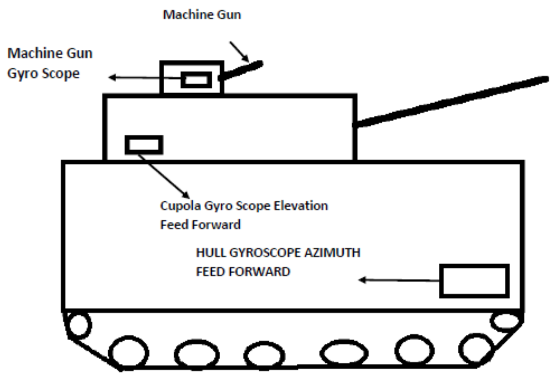

FIGURE 1.3: Second generation stabilization system

In modern tanks, there is an independently stabilized gunner's periscope. Periscope is a device consisting of a tube attached to a set of mirrors, by which an observer can see things that are otherwise out of sight. These periscopes (Figure 1.5) comprise of day TV CCD imaging cameras and thermal imaging cameras (Figure 1.4) mounted on the tank.

FIGURE 1.4: DAY TV CCD CAMERA [4]

The factor of stabilization precision for a head mirror is normally about four-five times greater to that of tank cupola and gun (≤ 0.151 rad mirror accuracy).

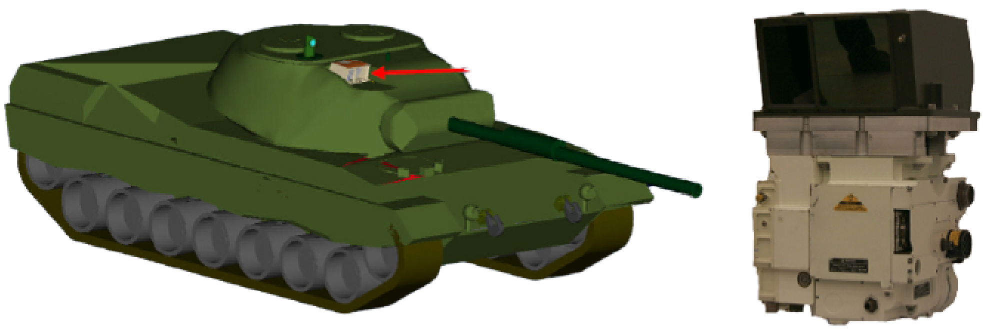

FIGURE 1.5: Gyroscope stabilized head mirror with gunner periscope [4]

The high level of line-of-sight stabilization achieved by independently stabilized scopes increases the quality of the image they manage, giving the shooter more opportunity to quickly identify the target. Accuracy of the line of sight turns it an inertial source for guns and cupolas. In fact, this is achieved every time a stable independent sight is used and then the gun and the cupola are kept insight, which leads to direct type fire control system [1, 4]. Direct type fire means launching a projectile directly on a target within the gunner line-of-sight.

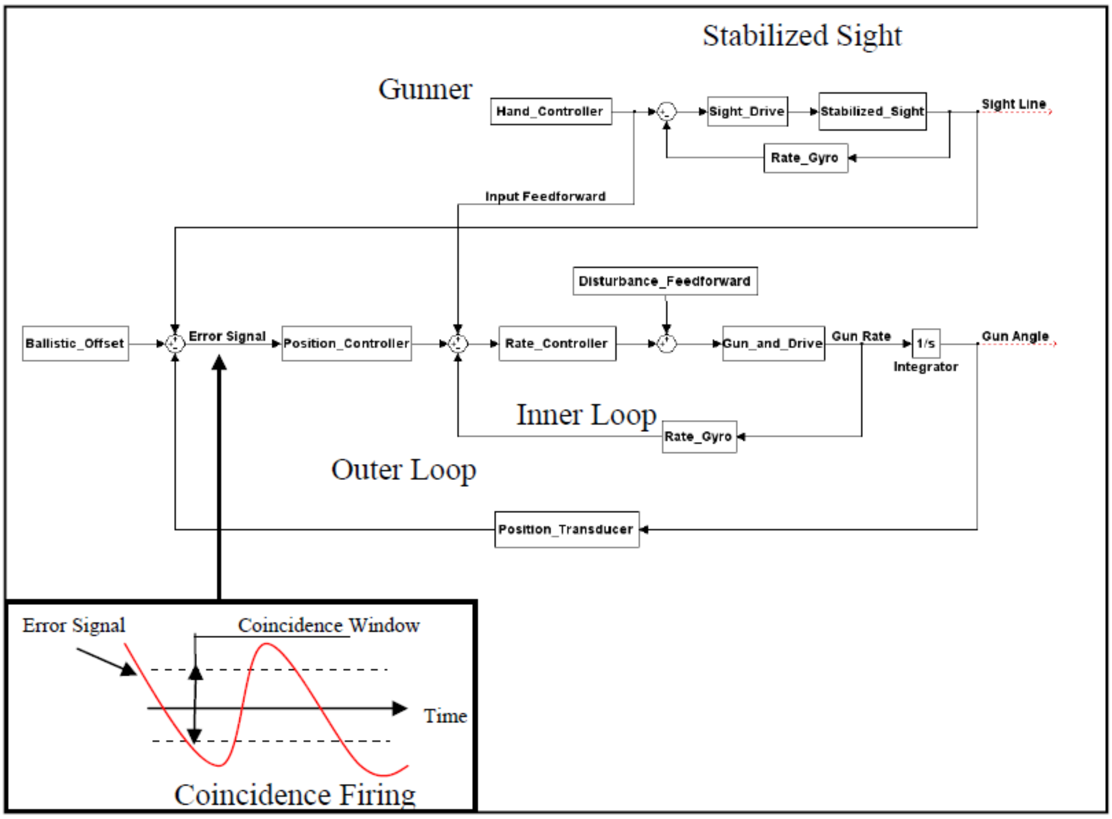

FIGURE 1.6: Direct type fire control system

The basic coincidence control structure is only allowed to be fired by a gun control computer as long as the spontaneous error is within a predetermined value (coincidence window).If the value of the error exceeds the defined value(lies outside the coincidence window), at the point where the shooter presses the shooting button, the fire control computer stops shooting.. Normally, the value of the coincidence window is 0.5 mrad.

The Tank Gun Stabilizer is an electro-hydraulic control system that allows you to track targets, aim targets, and stabilize gun positions. Theses stabilization system contains sensors at every point of Tank and are very accurate in sensing targets.

The functional structure of the system is shown in Fig.1.7. The stabilizer has been divided into following functional parts [5, 6]:

1. Gun-sight (including Gunner Controller (GC), Sight Servo (SS), Free Gyroscope (FG), Synchro- Control-Transformer (SCT))
2. Electronic amplifier (including Voltage Amplifier (VA), Phase Sensor (PS), Power Amplifier (PA))
3. Servo-valve (including Electromagnet (E), Hydraulic pump (P))
4. Elevation drive (including Hydraulic servo-motor (ED))
5. Tank gun (including Gun as a controlled system (G)).
6. Gyro-box (including Rate Gyroscope (RG), Synchro-Control-Transformer (SCT))

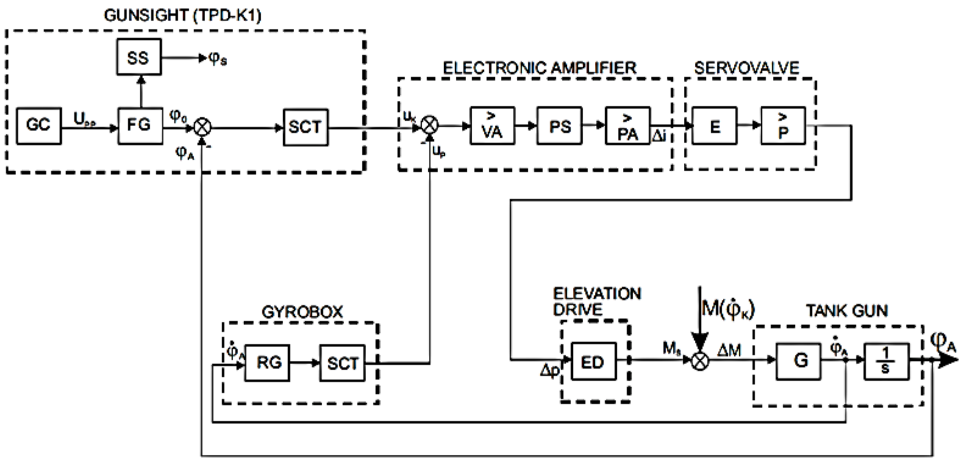

FIGURE 1.7: Functional scheme of tank gun stabilizer [6]

Here in this system the gunner is inside the tank and moving gun using a joystick. Disturbances coming into the system move or tends to rotate the gun from its position which will be sensed by a gyroscope. This signal will then be sent to an electronic amplifier which amplifies the signal and disturbance signal will be nullified using an elevation drive or a motor. Two main functions of stabilizer are as follow:

- Changing the angular position of the gun relative to the mount by means of the gunner controller GC while pointing to the target and tracking the target.

- The gun angular position φ_A is stabilized in the presence of the disturbance torque $M(\varphi_k)$ caused by the movement of tank.

Under operating conditions, the target process is simultaneously monitored and targeted.

The input signals are:

- $U_{pp}-$ Orientation signal given by administrator.
- φ_k-Disturbing signal brought about by the longitudinal vibration of the cupola.

Here, φ_A represents the output signal of gun angular displacement.

The static attributes are mostly linear or almost linear. The mathematical models of a system with strong non linearities should include [7, 8]:

1. Coulomb frictional forces among the gun and the turret
2. Electronic amplifier congestion
3. Congestion of the hydraulic pump
4. Congestion of the hydraulic servo-motor

The system contain two feed backs:

1. Feed-back angular orientation released by free gyroscope and produced by the weapon longitudinal angular displacements
2. Feed-back recognized through rate gyroscope and produced by the weapon longitudinal movement's angular speed

Now by using these feedback loops the gyroscope connected to the system and to the system structure will sense its motion and will provide negative feedback to the electronic amplifier. The Electronic amplifier then provides the signals to the servo valve to rotate the servo motor and which will adjust the motion of the structure and removes all the disturbances making it more stable.

1.2 Problem Statement

Gun mounts which are now being used in Pakistani tank are manually operated and are unsafe for both gunner and the tank. These mounts are made to be stabilized using shock absorber and dampers. This requirement is nowadays replaced by using a gyro stabilized controller attached with the tank body. Stabilization of gun mount using feedback loops and gyroscope such that there are few errors due to firing of gun and movement of the tank.

1.3 Thesis Overview

Research work comprises six chapters. Chapter 1 presented the context of the stabilization of a tank gun. It also includes the aims and objectives of the research. Also, the methodology of research was sentenced to this chapter. Chapter2 contains the literature review where research work is done in previous years has been discussed and also the limitations of these research's has been briefed more over the solution to these problems is discussed. Chapter 3, discusses the detail description of the methodology of the research conducted and also discusses the research design and methods used to conduct this research. Chapter 4 deals with Mathematical modeling of the system and based on this modeling MATLAB formulation has been done to satisfy the results and simulations. Chapter 5 summarizes the results of modeling. These results are compared, discussed and are then concluded in chapter 6 with some future recommendations.

Chapter 2

Literature Review

2.1 Introduction

In the past three decades, there has been extensive research on the Stabilization of gun mounts. This chapter provides a summary of a stabilized gun mount and an un-stabilized gun mount. This chapter is divided into two parts. The first part contain the research work which has been previously done and the second part contains the limitation of this researches and ways to counter these problems.

2.2 Gun Mount System

A gun mount is an assembly used to support a weapon, usually a gun. Gun mounts can be divided into two categories: fixed mounts and non-stationary mounts.

2.2.1 Static Mount

A non-portable weapon support component that can be mounted directly on the ground, on a fortification, or as part of a vehicle.

2.2.1.1 Turret

The gun turret protects the weapon's personnel or mechanism, and at the same time, a weapon can be aimed and fired in multiple directions. The turret is a rotating weapon platform that crosses one's armor accurately, called barbet (ships) or baskets (on tanks), and has a protective structure at the top (gunhouse).

If it has no gunhouse it is a barbette, if it has no barbette (ie, it is mounted to the outside of the vehicle's armor) it is an installation.

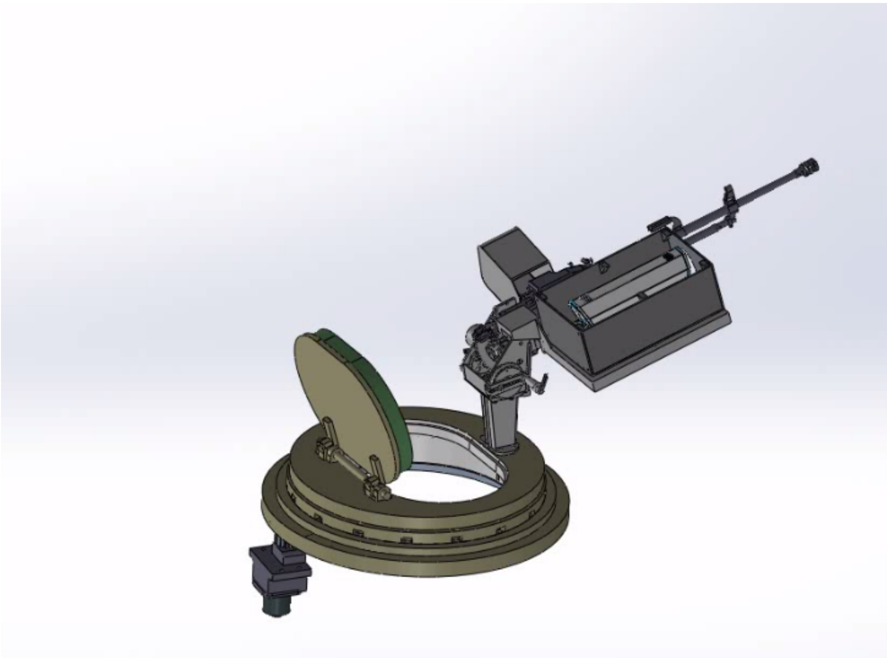

FIGURE 2.1: Turret

Turrets are usually used to mount machine guns, auto-cannons or large caliber guns. They can be manually operated or controlled remotely. A small turret, or sub-turret on a larger one, is called a cupola.

The term cupola also describes rotating turrets that carry no weapons but instead are sighting devices, as in the case of tank commanders. A finial mounted on a cupola turret, is an extremely small sub-turret or sub-sub-turret.

The gun is usually fixed on its horizontal axis and is rotated by the rotation of the turret, with trunnions (A trunnion is a cylindrical protrusion used as a mounting or pivoting point) mounted on a gun which allows it to elevate. Alternatively, an oscillating turret moves the entire top of the turret to lift and suppress the gun.

2.2.1.2 Casemate

The Casemate is a fixed primary armored structure with a traverse gun mount: In general, it is either a gun mounted through a fixed armor plate (usually seen on tank destroyers and assault riffles) or a gun mount consisting of a partial cylinder of armor squeezed between the top and bottom plates (as in sponson guns of the first generation tanks and the secondary weapons of the Dreadnought-era warships).

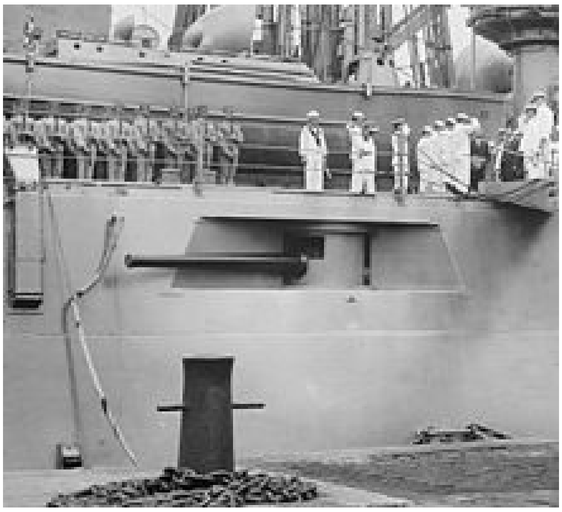

FIGURE 2.2: Casemate

2.2.2 Non-static Gun Mounts

A non static or ground mount is a sub-class of weapon mount that is portable.

2.2.2.1 Baseplate

Generally used by infantry mortars, it is a flat plate that is attached to the weapon directly or by means of a ball and socket joint. The plate is either square, rectangular or circular and is designed to disperse the recoil force of the weapon and prevent it from dropping onto the ground: it is most often, but not always, used with a two-legged support to lift the barrel at the required angle.

2.2.2.2 Monopod

A monopod has one leg and does not offer stability along the coordinate axis of movement. Monopods have the advantage of being light and compact, even though they do not offer sufficient stability for use with large firearms in firing mode. Monopods are generally used for short-barrelled precision firearms. Many sniper rifles have a monopod built into the stock which, combined with a front bipod, acts like a tripod.

2.2.3 Stability of Gun Mount

The resistance of the weapon and its stability while firing depends on the value of the recoil forces. In the case of weapons integration, if the barrel is attached to the mount, the recoil force is fully transmitted to the mount. To prevent the inconvenience of such connections or to reduce the load mount and its intensity through extended firing time, The barrel is attached to the mount which allows the movement of the barrel or the entire weapon during firing along the axis of the barrel [9].

2.3 Remote Control Weapon Station (RCWS)

A remote control weapon system is an armament station that could be lodged on to any form of vehicle or other structure. This device is used in modern military electric engines because as it allows an artilleryman to maintain the relative safety of armed vehicles [9]. However, there are weapons such as joysticks that do not require anyone's presence. The most famous are the rifles used by the Israeli army in Gaza.

On the other hand, the use of remote-controlled weapon systems (RCWS) can add a new dimension to strategies. RCW is becoming an essential part of modern combat platforms, not only in armored vehicles, but also in tanks, planes and naval ships, and even on robot platforms where RCWS improves telepresence with the deadly power of robots. They are taking on more and more combat roles.

The Elbit Systems Remote Weapons Control Station (RCWS) is designed to operate dynamically or statically in two axes and is fully stabilized for high firing speeds. The U.S. military product CS R-400(v)2 is a remote stabilized single weapon station, with the ability to use different calibers to use target video rails and multi-axis stabilization [10]. The innovative and advanced L-3 integrated lighting technology and ARWS and ISIS provide image stabilization with three thermal field of view sensors and a safe laser rangefinder, allowing the gunner to identify the enemy at a distance of about two kilometers [10]. This study uses a mathematical model of the system, which stabilized the assembly using the stabilization method to improve the accuracy of the design.

2.4 Working Principle of RCWS

RCWS is a study based on the Gimbal system and weapon support control. The universal gimbal mechanism, as shown in Figure 2.3, works to rotate the supported body around the first and second orthogonal axes. The support frame is a fixed and rotating mounting surface. The first element is yaw, and it is attached to the second element that controls the tone. The gyro sensor is located on a rotating part that detects errors in the system scale.

There is a long history for the motion control of robotic manipulators, it always presents a broad research area for the researchers and engineers of control system design because of the new advancements in control methodologies. An immense number of model-oriented control techniques have been used for governing the positions of robots e.g. Disturbance observer-based controller [11], Artificial Neural

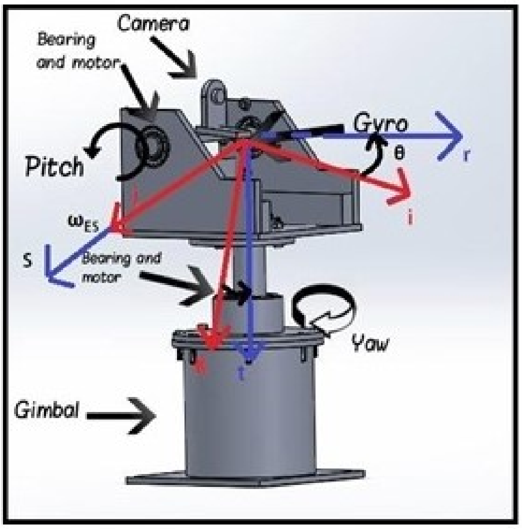

FIGURE 2.3: Gimbal diagram [12]

Networks (ANNs) [12, 13], Fuzzy logic control [14]. These model-oriented techniques require precise mathematical or in other words ideal model for the control of manipulator, which makes them highly complex and computationally more time taking, more importantly for the manipulator with high degrees of freedom. In comparison to this, the control techniques which are not model oriented, do not require the precise information of parameters, neither mechanical nor for the actuation part of the manipulator. These model less techniques make the design process easy [15].

Irrespective of the advancement in the area of control system design, the PID control technique is the commonly used strategy in industries due to the ease of implementation and simpleness of its design [16].

2.5 PID Control System

PID control is a usual model less feedback control methodology, which has been used widely for industrial applications due to its performance reliability, simple design, and implementation both in hardware and software. More importantly, it does not need the ideal mathematical model for the implementation [17].

Similarly, because of the order reduction feature in the PID control method, insensitive behavior for parametric changes and disturbances, the sliding mode approach is a proficient method for the control of higher-order complex dynamics systems. This control approach relies on a law referred to as reaching law. Which has the competency to modify the dynamic properties of the plant in reaching phase, and to control the ranting associated with the control input [14].

A gun stabilization mount is the same as a robotic arm used in different industries commonly known as a kinematic manipulator. There is a large group of robotic production equipment that supplies the desired movement in production processes, like arc spray painting, welding, assembly, selection and placement, milling, drilling, cutting, polishing etc. For this type of equipment, the industrial robot is an increasingly common type of equipment. Various manipulator configurations are now available, such as cylindrical, rectangular, spherical, rotating and horizontal manipulator connections.

2.6 Gun Stabilization Techniques

Redundant kinematic manipulators are preferred over non-redundant robots. A redundant manipulator has innumerable and varied results to the manipulator's joint variables for a provided undertaking. This can dodge hurdles and individualities and is a super contender for development strategies. Several optimization approaches have been implemented to choose the most appropriate route of a manipulator with the use of different standards, which include minimal time, least kinetic power and hurdle dodging.

A manipulation attempt is usually defined according to a specific trajectory of the end effector. As joint servo controls the manipulator, mapping from the target area to the joint space is needed. The movement plan adapts the nature of a preferred route to a route that describes the time structure of the standard configurations of the arm between the starting point and the last vacation point.

Kinematic control is an effective way of solving motion problems by robot manipulators. This is based entirely on the transformation of the inverse kinematics in which the reference value is sent to the joint servos resembling an allotted end-effector route. A vertically revolute RR configuration having two degrees of freedom is usually suitable for the assembly of small components such as electronic parts [16].

Even though the ultimate focus is on robotics, it is often very productive to perform simulations with real robots prior to the investigation [16]. This is due to the fact simulations are less complicated to set up, cheap, fast and handy to practice. The creation of a new robot and the programming of experiments takes many hours.

A simulated Configuration modeling is more economical than real robots, which allows you to better understand the project. Modeling is often faster compared to real robots, and all parameters are simply displayed on the screen [3].

The ability to perform real-time modeling and simulations is essential in the final stages of the design process. The final design can be determined before time-consuming and costly prototyping begins [4].

The lack of precise handling dynamics from a computational point of view has become particularly evident in recent years. Simulation of robotic systems using different programs will help in designing, construction and study of robots in the real world.

Simulation is essential to robot programmers in order to evaluate and visualize the behavior of robots and to validate and optimize the route program [15]. In addition, saving more time and money and playing an important role in the evaluation of production automation [16]. The ability to perform simulations offers a variety of alternatives that help to solve different problems. You can study, design, visualize and verify an object before creating it [17].

Chapter 3

Research Methodology

In this chapter, the stabilization of the gun mount in both the pitch and in yow axis has been considered. As the stabilization system improves the ability to hit targets while the tank is moving, disturbance's effect on the gun are taken into account.

Gun mount is typically a revolute revolute joint which means that there are 2 joints in serial connection making it a 2R serial manipulator. During the motion of a tank or while hitting the targets unbalance forces acts on a gun. The imbalance caused by the assembly of the gun out of the center of mass, the friction in the joint, the hydraulic actuator as the drive unit and the control system are the reasons for using a stabilizer system. The stabilization system, in this case, is comprised of two gyroscopes that controls the motion in pitch and yow axis and a feedback controller which will remove all the disturbances coming into the system. In this study, all these components are mathematically modeled and simulated in a two-dimensional reference framework under certain conditions.

This chapter focuses mainly on the explanation of how the mount has been stabilized using the mathematical modeling and simulation of the gun mount structure in two-dimensional space that has been analyzed using MATLAB. The structure of the thesis is lined up along with the proposition of solutions to the subject. The methods to improvise within the frame of study have been presented.

3.1 Degrees of Freedom

It is defined as the range of Independent parameters that are needed to fully describe a mechanism in its configuration area. For a revolute revolute joint degree of freedom is two. The equation used to decide the range of degrees of freedom of a robotic is as follows [19]:

$$\sum DOF = \lambda(m-1) - \sum_{k=1}^{j}(\lambda - f_k) \tag{3.1}$$

Here m indicates the number of links (including the ground link), j shows the number of total joints, f_k is number of degrees of freedom of the j^{th} joint and λ has value depending upon type of mechanisms. if the degree of freedom for the system increases the system becomes unstable resulting in decreasing the accuracy of the gun.

3.2 Stabilization Controller Design

3.2.1 Pitch and Yow Stabilization Control

The stabilization controller for yow and pitch is the direct-type stabilization with disturbance as input to the system and a gyroscope as feedback. Direct type stabilization is the one in which the gyroscope is placed at the end effector. In this study end effector is a gun muzzle. A detailed analysis of the gun mount structure has been conducted and a controller that will be used to stabilize the gun is optimized. The difference from the current controllers studied in this thesis is the stabilization of the gun assembly during the tank movement, keeping in mind the target position. Classic coincidence algorithms that are used verify the coincidence of the reference position of the direct aim and the position of the weapon. However, the gun position is measured by an optical encoder mounted on the pitch axis of the gun in the mounting frame. During the simulation of tank flexibility of the gun mount is discarded i.e the angle from each joint is taken in a straight line from the axis parallel to the gun.

3.2.2 Simulations

For simulation, MATLAB coding has been used. The whole simulation is divided into two parts first part is for kinematics and the second part contains dynamics of the system.

Kinematic simulation contains Denavit–Hartenberg (DH) method where at each joint its position is considered. As the model contains two revolute joints so the model contains two-position matrices which are base to the first joint and from the first joint to the second joint. For the position matrix of the whole system, both matrices are multiplied.

similarly In the second phase Inverse kinematic modeling has been done where the position of the base axis has been simulated using the end effector angles. In this study end effector is the gun muzzle and the angle which it makes with the target is θ_3.

For the dynamics, the jacobian method has been used. A Jacobian method is an iterative algorithm used to determine the solutions to the diagonally dominant system of linear equations in numerical linear algebra. Then by applying Euler Lagrangian formulation the Coriolis, velocity, and gravitational matrices have been solved. Further on a two-dimensional simulation of the tank has been taken into account.

In these simulations, the tank is in motion whereas the target is stationary. Disturbances at each path has been generated keeping in view the yow angle θ_1 and pitch angle θ_2 there is a change in angle which is the angle of the gun from the target i.e θ_3. This will be the first step before designing the stabilization controller, and the goal is to tune the servo feedback controller to a servo input. The gyroscope will act as a feedback sensor.

3.3 Disturbance Modeling

A tank is a huge vehicle (≈ 60 tons) that runs on its tracks. These tracks are powered by a heavy-duty engine. This generates a lively atmosphere for the tank. The engine itself produces a significant amount of vibration. These tracks are not soft like rubber tires. When the tank is in motion, the tracks generate strong vibrations on both axles, which affects the control system. These vibrations are very complex to simulate [9].

When the weapon fires, the shock wave spreads and retreats. Although compensation for the axis of the weapon causes a large number of perturbations and gun jump, the simulation is out of scope for this study, and ammunition explosion is simulated only as a disturbance that results in pitch and yow of the weapon.

Here in this mathematical model of the gun based on which a controlled will be designed and used with a gyroscope which provides feedback to the controller making the gun mount stabilized. Firstly the angle of the target from the gun has been locked which is θ_3 then the path of the tank is drawn in MATLAB plot in two-dimensional space. This is the path which the tank travels keeping in view its target. Each path is divided into thirteen points as shown in figure 3.1.

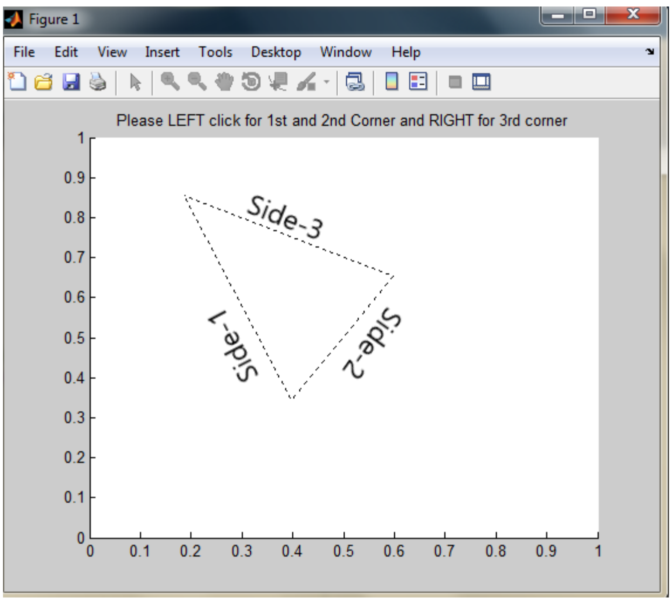

Figure 3.1: Each path division into 13 points

At each point value of yow and pitch angle will change this change in angle is sensed by gyroscope and gyroscope provide feedback to the system which changes the position of the gun. The projectile motion of bullet is also kept here which is used for changing θ_3 new angle has been called θ_3'. Solid works model of the gun is shown in figure 3.2

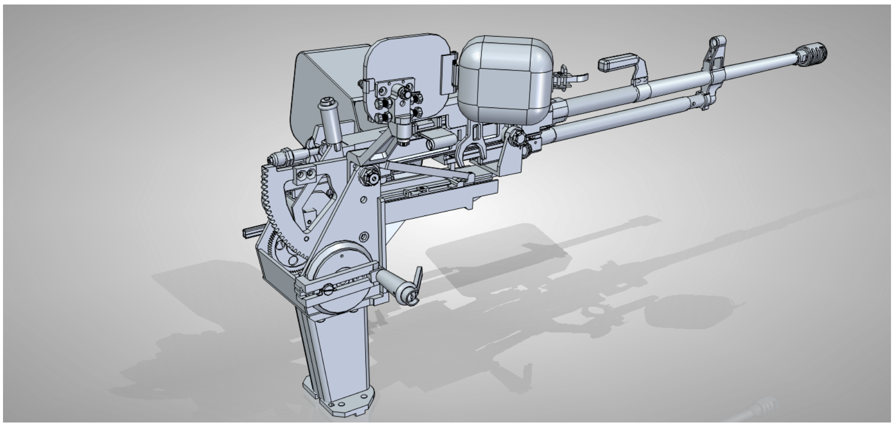

FIGURE 3.2: Solid works model of gun mount with gun

Solid works model contains a gun mount on which the gun is placed and all the firing takes place from there. A lock pin is used to lock the gun at the angle which is required. This lock will help to keep the gun at a single degree while firing or when the tank is moving. Two bullet boxes will help to contain the fired bullet shells and the bullets. Further on this Solid works model is taken into account and mathematical modeling has been performed to stabilize the gun mount.

Stabilization is the brain of the controller. PID controllers are used to control the position and speed of a system. To adjust the controller parameters, one must use the experimental setup or build the mathematical model.

The use of an experimental assembly is problematic and dangerous. In order to avoid these undesirable conditions, such as damage to system elements, operator injuries, etc., a mathematical model is created and the adjustment of controller parameters and system changes are performed with the help of this model. Finally, the entire system or its elements can be used to examine other models and for modernization.

Chapter 4

Mathematical Modeling

Mathematics provides tools to solve problems arising from various human activities through mathematical models that provide approximations to the actual behavior of more or less complicated practical systems. Currently, mathematics is the tool for approximate interpretation of physical or practical behavior. Here Mathematical modeling is divided into different parts. The first Part for the Mathematical Modeling includes Kinematics and the second part contains Dynamics of the system and the second part contains the simulation of tank based on this mathematical modeling.

4.1 Serial Manipulator Kinematics

Kinematics study is a relationship between the dimensions and connectivity of kinematic chains and the position, speed, and acceleration of each link in the robotic system to control motion and calculate actuator forces and moments [20]. A serial manipulator consists of a hard, fast base, a series of links connected by joints, and ends with a disconnected end that no longer supports the device or end effector. There is no closed loop in the evaluation of parallel manipulators. By activating the joints, the end effector can still be aligned and operated in a plane or three-dimensional area to perform the desired tasks with the given end effector. The serial manipulator configurations are defined using the well-known Denavit-Hartenberg (D-H) parameters.

Two well-known problems are modeled, namely forward kinematics and inverse kinematics. Serial manipulator consists of links and joints. The range of joints and links determines the degree of freedom of a manipulator, which in turn determines the capabilities of a serial manipulator.

If all the joints have joints with a certain degree of freedom, then J = Dof. If J <Dof , then one or more of the joints are of multi-degree-of-freedom and this isn't used in mechanical serial manipulators. This is due to the fact that it is difficult to discover and activate one or more degree of freedom joints in the same neighborhood in a serial manipulator.

Variables that describe the position and orientation of a link or end effector are called assigned area variables. The measurement of project space is 6 for 3D movements and 3 for planar movements. Finally, there are usually mechanical links, gears, etc. between actuators and joints. Space of all actuating variables is called the actuating space. If the actuator dimension is greater than 3 for a planar movement and greater than 6 for a 3D movement, the manipulator is said to be redundant. If the actuator space dimension is smaller than the degree of freedom, the manipulator is said to be under-actuated. Kinematics of the Serial Manipulator is divided into Forward and Inverse kinematics Mathematical Modeling

4.1.1 Serial Manipulator Forward Kinematics

The direct kinematics or forward kinematics of a serial manipulator is as follows: given the parameters of the hyperlink and the joint variable, $a_i - 1$, $\alpha_i - 1$, d_i, and θ_i identify the position and orientation of the last link in the fixed or reference coordinate system. Direct kinematics is the most feasible problem in manipulator kinematics and is deduced from the basis of the hyperlink transformation matrix. This kinematics can be mathematical manipulated using Denavit-Hartenberg method or also known as DH- Parameters.

4.1.1.1 Denavit-Hartenberg DH- Parameters

Position matrix is made using the Denavit-Hartenberg DH-Parameters. A convention generally used to select reference frames in robot packets is the Denavit-Hartenberg or D-H convention.

Where the four portions θ_i, a_i, d_i, α_i parameters are related to link i and joint i. The 4 parameters a_i, α_i, d_i and θ_i are usually named as link duration, link rotation, link offset and joint perspective respectively. These names are derived from specific elements of the geometric correlation between the coordinate frames.

Since the matrix A_i is a single-variable function, it appears that three of the previous four variables are constant at a given joint, while the fourth parameter, θ_i for a pivot and a prismatic joint, is the common variable. Any homogeneous transformation matrix can be represented by six numbers, including, for example, three numbers to indicate the fourth column of the matrix and three Euler angles to indicate the 3×3 rotation matrix in the upper left corner. In the D-H matrix, there are 4 parameters for evaluation. How is this feasible?

The solution to that, at the same time as it is necessary to unite the frame rigidly to the hyperlink i, we are free to choose the starting point commonly known as the base of the manipulator and body axis coordinates. For example, it is not mandatory that the base, Oi, of body i be positioned at the end of hyperlink i.

It is not even necessary for the body i to be inside the hyperlink; the body i can be in the free area as long as the body i is rigidly connected to the hyperlink i [22]. By specifying the origin and coordinate axes, it is possible to reduce the number of desired parameters from six to four (or in some cases less).

In this study we have only a revolute revolute joint serial manipulator i-e i = 2. Parameters for the DH table will be calculated by looking at figure 4.1. Final position matrix will then be calculated using this matrix by multiplying the position matrix of every joint with the previous one.

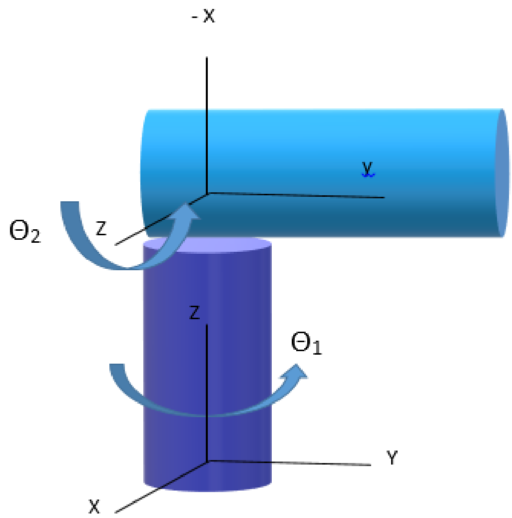

FIGURE 4.1: RR Joint

By looking at the figure the DH Matrix will be written as

TABLE 4.1: DH PARAMETERS

Joint i	α_i	a_i	d_i	Θ_i
1	-90	a_1	d_1	Θ_1
2	0	a_2	d_2	Θ_2

The general way to show the matrix is given as

$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & cos\alpha_i c\theta_i & \sin\alpha_i c\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.1)$$

For the first joint $i = 1$

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & a_1\sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.2}$$

For 2^{nd} joint $i = 2$

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.3}$$

The solid object in the coordinate system has different possibilities because it undergoes more than one rotation around the different axes of the source system. To achieve such rotation, it is necessary to multiply the rotation matrices that represent the individual rotations to obtain the combined matrix rotation. The multiplication of matrices is not commutative, so they must be multiplied in the same order as the system's rotation in space [21].

Now for the whole assembly we have to multiply both matrices

$$A_2^0 = A_1^0 \times A_2^1$$

Putting the initial conditions $d_1 = 0$, $a_2 = 0$. Final matrix will be,

$$A_2^0 = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & -\sin\theta_1 & -d_2\sin\theta_1 + a_1\cos\theta_1 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & \cos\theta_1 & d_2\cos\theta_1 + a_1\sin\theta_1 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.4}$$

$$A_2^0 = \begin{bmatrix} U_X & V_X & W_X & Q_X \\ U_Y & V_X & W_X & Q_Y \\ U_Z & V_Z & W_Z & Q_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This yields us to

$$U_X = \cos\Theta_1 \cos\Theta_2$$

$$U_Y = \sin\Theta_1 \cos\Theta_2$$

$$U_Z = \sin\Theta_2$$

$$V_X = -\cos\Theta_1 \sin\Theta_2$$

$$V_Y = -\sin\Theta_1 \sin\Theta_2$$

$$V_Z = -\cos\Theta_2$$

$$W_X = -\sin\Theta_1$$

$$W_Y = C\Theta_1$$

$$W_Z = 0$$

$$Q_Z = d1$$

$$Q_X = d2\cos\Theta_1 + a1\sin\Theta_1$$

$$Q_Y = -d2\sin\Theta_1 + a1\cos\Theta_1$$

4.1.2 Inverse Kinematics

The problem of inverse manipulator kinematics has been studied for years. The solution to inverse kinematics requires a lot of computation and generally takes a lot of time compared to real-time manipulators. The tasks to be performed by the manipulator are inside Cartesian space and the actuators work purely in spatial coordinates. Cartesian space consists of an orientation matrix and a position vector.

However, the joint space is represented by the angles of the joint. Conversion of position and orientation from the manipulative effect of Cartesian space into the joint space is known as the problem of inverse kinematics. There are two response tactics, geometric and algebraic, which are used to obtain inverse kinematics.

The purpose is to decide the values of joint variables of a manipulator configuration with a purpose to place the end-effector at a favored position and orientation relative to the base, given the manipulator geometry, i.e., link lengths, offsets, twist angles, and the region of the base.

Here in this thesis Analytical model has been used. From A_2^0 equation* 4.5

$$\begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & -s\theta_1 & a_2 c\theta_1 c\theta_2 - d_2 s\theta_1 + a_1 c\theta_1 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & c\theta_1 & a_2 s\theta_1 c\theta_2 + d_2 c\theta_1 + a_1 s\theta_1 \\ -s\theta_2 & -c\theta_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.5}$$

Comparing the matrix with general DH Position matrix.

$$T_W^B = \begin{bmatrix} c_\phi & -s_\phi & 0.0 & x \\ s_\phi & c_\phi & 0.0 & y \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We get

$$X = a_2 \cos\theta_1 \cos\theta_2 - d_2 \sin\theta_1 + a_1 \cos\theta_1 \tag{4.6}$$

$$Y = a_2 \sin\theta_1 \cos\theta_2 + d_2 \cos\theta_1 + a_1 \sin\theta_1 \tag{4.7}$$

Square equation 4.6 and 4.7 and then add,

$$X^2 + Y2 = a_2^2 \cos^2\Theta_2^2 + a_1^2 + d_2^2 + 2a_1a_2\cos\Theta_2$$

Solve for θ_2

$$\Theta_2 = \cos^{-1}[\frac{X^2 + Y^2 - a_1^2 - d_2^2}{2a_1a_2}]$$

Now from above equation

Taking Θ_1 as common from equation 4.6 and 4.7

$$X = \cos_1(a_2\cos\Theta_2 + a_1) - \sin\Theta_1(d_1))$$

$$Y = \sin\Theta_1(a_2\cos\Theta_2 + a_1) + \cos\Theta_1(d_2)$$

$$\sin\Theta_1 = [\frac{Y(a_2\cos\Theta_2 + a_1) - xd_1}{(a_2\cos\Theta_2 + a_1)^2 + d_1^2}]$$

$$\cos\Theta_1 = [\frac{X(a_2\cos\Theta_2 + a_1 + 2d_1^2) + Yd_1(a_2\cos\Theta_2 + a_1)}{(a_2\cos\Theta_2 + a_1)^3 + d_1^2}]$$

As we know that $\tan\Theta$ is$\frac{\sin\Theta}{\cos\Theta}$ Taking inverse of tangent will give value of Θ_1

4.1.3 Jacobian

The table by which Jacobian matrix will be taken into account is as follow

TABLE 4.2: Jacobian general Table

	Prismatic	Revolute
Linear	$R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d^0_n - d^0_{i-1})$
Rotational	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

The upper row is used to determine the linear velocity of the end effector and the second row determines the rotation speed of the end effector. There are also two columns the first column will use any time there is a prismatic joint and the 2nd column will be used for the revolute joint.

The general way to show the jacobian matrix is as follow

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{wx} \\ \dot{wy} \\ \dot{wz} \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

In the B matrix where empty boxes are shown the first three rows are for linear velocities and the bottom three rows are for the rotational velocities. As the no of joints and links increases the size of the matrix also increases.

Now for our case, we have a two Revolute joint which means that there is no linear velocity so the first row will be marked as zero and as there is only rotational part so by looking at the above table jacobian matrix will be written as.

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{wx} \\ \dot{wy} \\ \dot{wz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{4.8}$$

$$R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = R_1^0 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_1^0 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix}$$

Similarly for the 2nd column

$$R_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^1 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \\ 0 \end{bmatrix}$$

Putting the values in equation 4.8

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{wx} \\ \dot{wy} \\ \dot{wz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\sin\theta_1 & \cos\theta_2 & 0 \\ \cos\theta_1 & \sin\theta_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\dot{wx} = (-\sin\theta_1 \times \dot{\theta}_1) + (\cos\theta_2 \times \dot{\theta}_2)$$

$$\dot{wx} = (-\cos\theta_1 \times \dot{\theta}_1) + (\sin\theta_2 \times \dot{\theta}_2)$$

As the degree of freedom is 2 so there is no rotation in the third axis which means $\dot{wz} = 0$

4.1.4 Langrange Formulation

Now coming on to the dynamics the Lagrange formulation is taken into the account. The Langrangian feature, L, for a machine is defined to be the distinction between the kinetic and potential energies expressed as a feature of positions and velocities. In order to make the nomenclature, we shall introduce a shorthand for the complete set of positions in an N-number o0f machine and velocities. Then, Langrangian is defined as follows:

$$L(\Theta, \dot{\Theta}) = k(\Theta, \dot{\Theta}) - u(\Theta) \tag{4.9}$$

In phrases of the Langrangian, the classical equations of motion are given with the aid of the Euler-Lagrange equation:

The equations that result from the software of the Euler-Lagrange equation to a particular Langrangian are known as the equations of movement. The answer to the equations of motion for a given preliminary situation is called a trajectory of the effector. The Euler-Lagrange equation results from what's referred to as a motion principle. The Euler-Lagrange equation is defined as,

$$\tau_i = \frac{d}{dt}\frac{\delta L}{\delta \dot{\theta}_i} - \frac{\delta L}{\delta \dot{\theta}_i} \tag{4.10}$$

The free body diagram for 2R joint will be given as

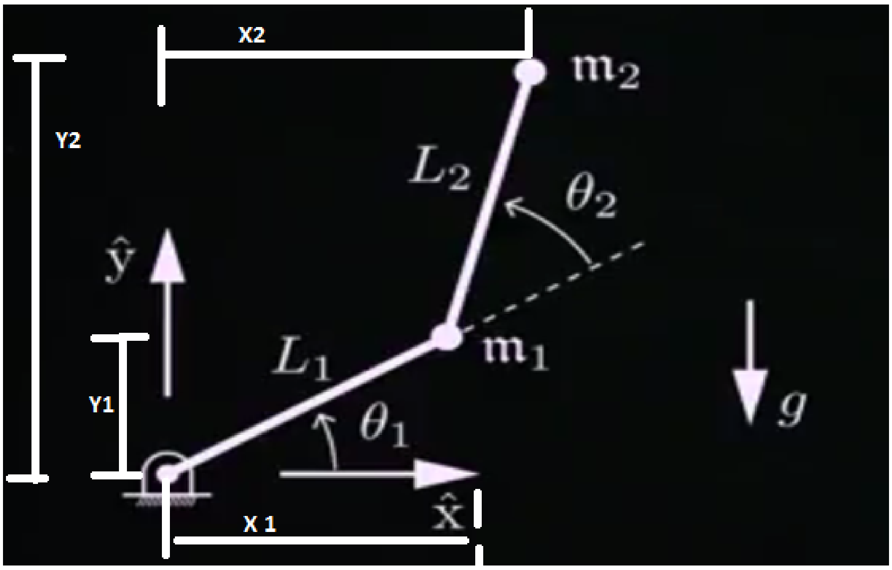

FIGURE 4.2: Free body diagram of RR joint

By looking at the figure 4.2 $X_1 = L_1 \cos \Theta_1$
$\dot{x_1} = (-L_1 \sin \Theta_1)(\dot{\Theta_1})$

And

$Y_1 = L_1 \sin \Theta_1$
$\dot{y_1} = (L_1 \cos \Theta_1)(\dot{\Theta_1})$

$$X_2 = L_1 \cos \Theta_1 + L_2 \cos(\Theta_1 + \Theta_2)$$

$$\dot{x_2} = [-L_1 \sin \Theta_1 - L_2 \sin(\Theta_1 + \Theta_2)[\dot{\Theta_1}]] - [L_2 \sin(\Theta_1 + \Theta_2][\dot{\Theta_2}]]$$

$$Y_2 = L_1 \sin \Theta_1 + L_2 \sin(\Theta_1 + \Theta_2)$$

$$\dot{y_2} = [L_1 \cos\Theta_1 + L_2 \cos(\Theta_1 + \Theta_2)[\dot{\Theta_1}]] + [L_2 \cos(\Theta_1 + \Theta_2][\dot{\Theta_2}]]$$

For Link-1:

$$K.E_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (\dot{x_1} + \dot{y_1})^2$$

Putting values we get

$$K.E_1 = \frac{1}{2} m_1 L_1^2 \dot{\Theta}_1^2 \tag{4.11}$$

For Link-2:

$$K.E_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 (\dot{x_2} + \dot{y_2})^2$$

Putting values we get

$$\begin{aligned} K.E_2 = \frac{1}{2} m_2 [(L_1^2 + 2L_1 L_2 \cos\Theta_2 + L_2^2)(\dot{\Theta}_1^2) \\ +2(L_2^2 + L_1 L_2 \cos\Theta_2)(\dot{\Theta}_1 \dot{\Theta}_2) + (L_2^2 \dot{\Theta}_2^2)] \end{aligned} \tag{4.12}$$

4.1.4.1 Potential Energy

$$P.E = m * g * h$$

LINK-1:

$$P.E_1 = m_1 * g * h_1 = m_1 * g * L_1 \sin\Theta_1$$

LINK-2:

$$P.E_2 = m_2 * g * h_2 = m_2 * g * (L_1 \sin\Theta_1 + L_2 \sin(\Theta_1 + \Theta_2))$$

As we know that

$$Langrange = L = \sum_{1}^{2}(K.E - P.E)$$

Putting the values

$$L = \frac{1}{2}m_1L_1^2\dot{\Theta}_1^2 + \frac{1}{2}m_2L_1^2\dot{\Theta}_1^2 + \frac{1}{2}m_2L_2^2\dot{\Theta}_1^2 + \frac{1}{2}m_2L_2^2\dot{\Theta}_2^2 + m_2L_2L_1\dot{\Theta}_1^2\cos\Theta_2 + m_2L_2^2\dot{\Theta}_1\dot{\Theta}_2$$
$$+m_2L_2L_1\cos\Theta_2\dot{\Theta}_1\dot{\Theta}_2 - m_1gL_1\sin\Theta_1 - m_2gL_1\sin\Theta_1 - m_2gL_2\sin(\Theta_1+\Theta_2) \quad (4.13)$$

Now by the Newton's equation of motion.

$$\tau = \frac{d}{dt}\frac{\delta L}{\delta\dot{\theta}_i} - \frac{\delta L}{\delta\dot{\theta}_i}$$

$$\tau_1 = (m_1L_1^2 + m_2(L_1^2 2L_1L_2\cos\Theta_2 + L_2^2))\ddot{\Theta}_1 + m_2(L_1L_2\cos\Theta_2 + L_2^2)\ddot{\Theta}_2 -$$
$$m_2(L_1L_2\sin\Theta_2(2\dot{\Theta}_1\dot{\Theta}_2 + \dot{\Theta}_2^2)) + (m_1+m_2)L_1g\cos\Theta_1 + m_2gL_2\cos(\Theta_1+\Theta_2)$$

And

$$\tau_2 = m_2(L_1L_2\cos\Theta_2 + L_2^2))\ddot{\Theta}_1 + m_2L_2^2\ddot{\Theta}_2$$
$$+m_2L_1L_2\dot{\Theta}_1^2\sin\Theta_2 + m_2gL_2\cos(\Theta_1+\Theta_2)$$

Now adding these two equations to get the value for τ

As,

$\tau = \tau_1 + \tau_2$

and from Euler equation

$$\tau = M(\Theta)\dot{\Theta} + V(\Theta,\dot{\Theta}) + g(\Theta)$$

Putting all the terms into an equation We get

$$M(\Theta) = \begin{bmatrix} m_1 L_1^2 + m_2(L_1^2 + 2L_1 L_2 \cos\Theta_2 + L_2^2) & m_2(L_1 L_2 \cos\Theta_2 + L_2^2) \\ m_2(L_1 L_2 \cos\Theta_2 + L_2^2) & m_2 L_2^2 \end{bmatrix}$$

$$V(\Theta, \dot{\Theta}) = \begin{bmatrix} -m_2(L_1 L_2 \sin\Theta_2 (2\dot{\Theta}_1 \dot{\Theta}_2 + \dot{\dot{\dot{\Theta}}}_2^2) \\ m_2 L_1 L_2 \dot{\Theta}_1^2 \sin\Theta_2 \end{bmatrix}$$

$$g(\Theta) = \begin{bmatrix} (m_1 + m_2) L_1 g \cos\Theta_1 + m_2 g L_2 \cos(\Theta_1 + \Theta_2) \\ m_2 g L_2 \cos(\Theta_1 + \Theta_2) \end{bmatrix}$$

Where $M(\Theta)$ is definite mass matrix, $V(\Theta, \dot{\Theta})$ is the vector which contains Coriolis and centripetal torques and $g(\Theta)$ is the vector which contains gravitational torques.

4.2 Gears

Gears being one of the main parts in tanks and are used to rotate the turret and hull. Considering the main gears as a driver and a driven, mathematical formulation will be

No. of teeth of 1st gear= N_1

No. of teeth of 2nd gear = N_2

Moment of inertia = J_1

Moment of inertia = J_2

Torque on Gear 1 = $T_1(t)$

Torque on Gear 2 = $T_2(t)$

Distance travelled along gears, figure4.3

$$r_1 \theta_1(t) = r_2 \theta_2(t) \tag{4.14}$$

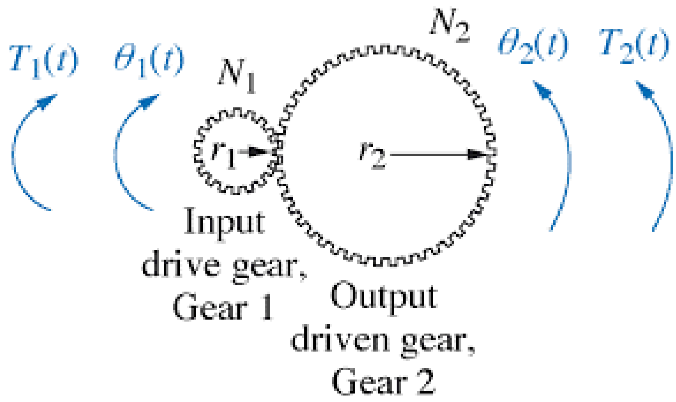

FIGURE 4.3: Gears Rotation

As,

$$\theta_1(t) = \frac{N_2}{N_1}\theta_2(t)$$

and

$$T_1(t)\theta_1(t) = T_2(t)\theta_2(t)$$

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

Mechanical impedance is a measure of the resistance of the assembly to move under the influence of harmonic force. This allows the forces to be correlated with the speeds acting on the mechanical system. The mechanical impedance of a point in a structure is the ratio between the force exerted at a point and the speed achieved at that point [18].

Mechanical Impedance Driven by gears will be,

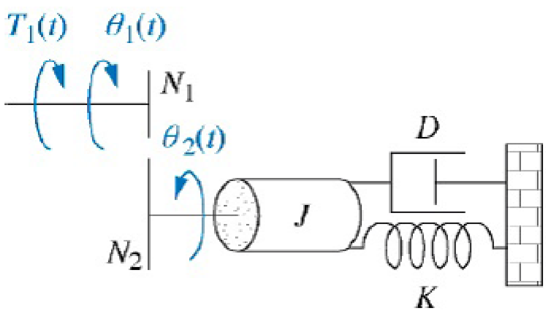

FIGURE 4.4: Mass Spring System

$$(Js^2 + Ds + K)\theta_1(s) = T_1(s)\frac{N_1}{N_2} \tag{4.15}$$

$\theta_1(s) = \frac{N_1}{N_2}\theta_2(s)$

Putting the value of $\theta_1(s)$

$$\{J[\frac{N_1}{N_2}]^2 s^2 + DJ[\frac{N_1}{N_2}]^2 s + K[\frac{N_1}{N_2}]^2\}\theta_2(s) = T_1(s) \tag{4.16}$$

$$G(s) = \frac{\theta_2(s)}{T_1(s)}$$

The torque applied to the gun equals the actuator torque T_a plus the disturbances T_d acting on the gun. This torque results in the angular acceleration of a gun as the mass of the gun is always considered.

Angular acceleration of the gun is given as

$$\frac{d_2 y}{dx^2}(\theta) = \frac{1}{J_g}(T_a + T_d) \tag{4.17}$$

Where J_g is inertia of a gun.

The acceleration with which the copula moves while firing or disturbance is given as

$$\frac{d_2 y}{dx^2}(\theta_{Cupola}) = \frac{1}{J}(T_a + T_{gf} - D\frac{dy}{dx}\theta_{Cupola} + K(\theta_{Cupola})) \tag{4.18}$$

Where T_{gf} is torque due to gun firing

D = cupola suspension damping while pitch

K = cupola suspension spring constant

$\frac{dy}{dx}$ = cupola pitch rate

FIGURE 4.5: Tank gun mount on cupola

4.3 DC Motor

Servo motor is to be used in main battle tanks, as the actuator is a rotary or linear drive that allows precise angular or linear movements. It consists of a motor that is coupled with a sensor. It also requires an advanced controller, a module designed in particular to be used with servomotors. Servomotors are not particularly impressive although the term servomotor is regularly used for use in closed-loop controlled systems.

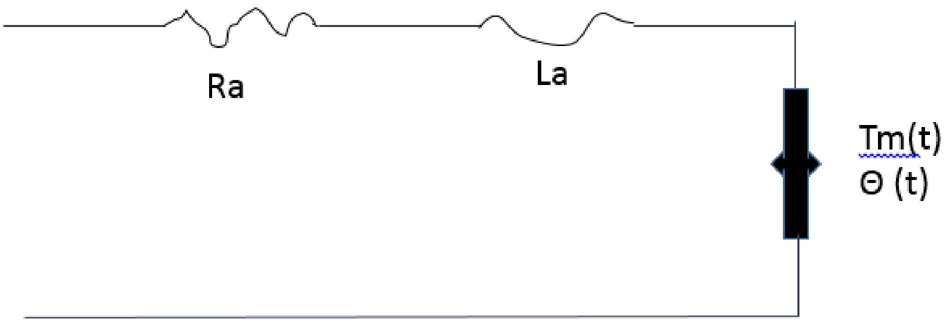

FIGURE 4.6: DC motor

Magnetic force produced by permanent magnet in motor.

$$F = BLI(t)$$

EMF produced by a conductor moving in a magnetic field

$$E = BLV$$

If Armature is rotating in a magnetic field its voltage is proportional to speed. Back Emf force generated in an armature will be given as

$$V_2(t) = K_b \frac{d\Theta(t)}{dt} \tag{4.19}$$

Where V_2 is back emf

K_b = constant back emf force.

$\frac{d\Theta(t)}{dt}$ =angular velocity.

Taking Laplace to solve the differential equation.

$$V_2(s) = K_b * S * \Theta_m(s) \tag{4.20}$$

Here Laplace is taken to switch the equation from a time domain to frequency domain, as the frequency of a DC motor remains constant the Back Emf Generated will then be calculated.

Chapter 5

Results and Discussion

MATLAB codding is separately done for each mathematical formulation. These program codes will then be exported for electrical circuit design for the gyroscope. These codes generate the position points of the gun and also the angles for pitch and yow motion.

5.1 Forward Kinematics Code:

Forward kinematics Coding is done to show the position of the base relative to the gun. The Results of the coding are compared with the manual calculations. This code will let the controller know what is the exact position of the gun based on these positions point the gyroscopes will be sensing the disturbances and will be removing the errors. These errors are basically the disturbances which are coming into the system as a result of firing of tank or because of the movement of the tank. Disturbances into the system will tend to move or tend to rotate the gun from its axis and gyroscope will be sensing this movement and will send this signal to controller which will remove the disturbance and this signal will further be sent to the motor which will help the gun to gain its stabilized position.

```
syms a1 d1 t1 a2 d2 t2
%INPUT DH PARAMETERS
dh= [-90 a1 d1 t1 ; 0 a2 d2 t2]
for i = 1:size(dh,1)
    %I DESCRIBES THE NUMBER OF JOINTS
    alpha = dh(i,1);
    ai = dh(i,2);
    di = dh(i,3);
    t = dh(i,4);
    %GENERAL DH POSITION MATRIX
    A(1,1,i)= cos(t);
    A(1,2,i)= -sin(t)*cos(alpha);
    A(1,3,i)= sin(t)*sin(alpha);
    A(1,4,i)= ai*cos(t);
    A(2,1,i)= sin(t);
    A(2,2,i)= cos(t)*cos(alpha)
    A(2,3,i)= -cos(t)*sin(alpha);
    A(2,4,i)= ai*sin(t);
    A(3,1,i)= 0;
```

```
    A(3,2,i)= sin(alpha);
    A(3,3,i)= cos(alpha)
    A(3,4,i)= di
    A(4,1,i)= 0;
    A(4,2,i)= 0;
    A(4,3,i)= 0;
    A(4,4,i)= 1;

end
T = [1,0,0,0;0,1,0,0;0,0,1,0;0,0,0,1];
for i=1:(size(dh,1))
       T = T * A(:,:,i);
       %T IS THE FINAL MATRIX
end
pitch = asin(-T(3,1));
roll= asin(T(2,1)/cos(pitch));
yaw = acos(T(3,3)/cos(pitch));
disp(T);
```

5.1.1 Results

Here i is used a_1, d_1, t_1, a_2, d_2 and t_2 as a DH variables so that at the end i get the same results as with mathematical modeling with hand. Also note that the values of α here are taken according to Revolute Revloute joint which are -90 and 0 degrees.

```
dh =

[ -90, a1, d1, t1]
[   0, a2, d2, t2]
```

```
A(:,:,1) =

[ cos(t1), -cos(90)*sin(t1), -sin(90)*sin(t1), a1*cos(t1)]
[ sin(t1),  cos(90)*cos(t1),  sin(90)*cos(t1), a1*sin(t1)]
[       0,         -sin(90),          cos(90),         d1]
[       0,                0,                0,          1]


A(:,:,2) =

[ cos(t2), -sin(t2), 0, a2*cos(t2)]
[ sin(t2),  cos(t2), 0, a2*sin(t2)]
[       0,        0, 1,         d2]
[       0,        0, 0,          1]
```

Here $A(:,:,1)$ means A_1^0 matrix and $A(:,:,2)$ means A_2^1 matrix. Matrix A_1^0 shows the position orientation for base to first joint for the manipulator. Whereas A_2^1 shows the position orientation for first joint to the end joint as we got only two joints in this manipulator. Multiplication of both the matrices will give the orientation for the whole system.

Multiplying both matrices and also putting the initial conditions Final matrix will be:

```
[ cos(t1)*cos(t2) - cos(90)*sin(t1)*sin(t2), - cos(t1)*sin(t2) - cos(90)*cos(t2)*sin(t1),
[ cos(t2)*sin(t1) + cos(90)*cos(t1)*sin(t2),   cos(90)*cos(t1)*cos(t2) - sin(t1)*sin(t2),
[                         -sin(90)*sin(t2),                             -sin(90)*cos(t2),
[                                        0,                                            0,

-sin(90)*sin(t1), a1*cos(t1) - d2*sin(90)*sin(t1) + a2*cos(t1)*cos(t2) - a2*cos(90)*sin(t1)*sin(t2)]
 sin(90)*cos(t1), a1*sin(t1) + d2*sin(90)*cos(t1) + a2*cos(t2)*sin(t1) + a2*cos(90)*cos(t1)*sin(t2)]
         cos(90),                                      d1 + d2*cos(90) - a2*sin(90)*sin(t2)]
               0,                                                                         1]
```

Now putting any raw values to check the answer

```
dh =

   -90     0     5    45
     0     0    10    30
```

```
A(:,:,1) =

    0.7071         0   -0.7071         0
    0.7071         0    0.7071         0
         0   -1.0000         0    5.0000
         0         0         0    1.0000


A(:,:,2) =

    0.8660   -0.5000         0         0
    0.5000    0.8660         0         0
         0         0    1.0000   10.0000
         0         0         0    1.0000
```

And The Final Matrix Become

The results are same as that when putted in A_2^0 Matrix is Equation 4.5

```
T =

    0.6124   -0.3536   -0.7071   -7.0711
    0.6124   -0.3536    0.7071    7.0711
   -0.5000   -0.8660         0    5.0000
         0         0         0    1.0000
```

$$A_2^0 = \begin{bmatrix} 0.61 & -0.35 & -0.707 & -7.0711 \\ 0.61 & -0.35 & 0.707 & 7.0711 \\ -0.5 & -0.86 & 0 & 5.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{5.1}$$

These matrices are showing the exact positions of the gun on which it will be stabilized. Comparison of the two calculations show that there is a very little error which is because of movement of tank. Simulation on this basis is done in section 5.4 where the tank has been given a trajectory keeping the target location locked.

5.2 Jacobian

```
syms a1 d1 t1 a2 d2 t2
dh= [-90 a1 d1 t1 ; 0 a2 d2 t2]
J = sym('j',[6,size(dh,1)])
T = eye(4)
R = sym('r',[3,3])
R = eye(3)
o=eye(3);
digits(2);
for i = 1:size(dh,1)
    alpha = dh(i,1);
    ai = dh(i,2);
    di = dh(i,3);
    t = dh(i,4);
    A(1,1,i)= cos(t);
    A(1,2,i)= -sin(t)*cos(alpha);
    A(1,3,i)= sin(t)*sin(alpha);
    A(1,4,i)= ai*cos(t);
    A(2,1,i)= sin(t);
    A(2,2,i)= cos(t)*cos(alpha)
    A(2,3,i)= -cos(t)*sin(alpha);
    A(2,4,i)= ai*sin(t);
    A(3,1,i)= 0;
    A(3,2,i)= sin(alpha);
    A(3,3,i)= cos(alpha);
    A(3,4,i)= di;
    A(4,1,i)= 0;
    A(4,2,i)= 0;
```

```
        A(4,2,i)= 0;
        A(4,3,i)= 0;
        A(4,4,i)= 1;
        %R= vpa(R*A(1:3,1:3,i))
        R = A (1:3,1:3)
       % k = R(1:3,1:3)*[0; 0 ; 1];
        if i==1
            k = R(1:3,1:3)*[0; 0 ; 1]

        end
        if i==2
            k = R(1:3,1:3)*[1; 0 ; 0]

        end
        %if dh(i,4) == dh(i,4)
           J(4,i) = k(1,1,1)
           J(5,i) = k(2,1,1)
           J(6,i) = k(3,1,1)
       % else
            J(1:3,i)=0

        %end


    end
```

```
T = [1,0,0,0;0,1,0,0;0,0,1,0;0,0,0,1];
for i=1:(size(dh,1))
       T = T * A(:,:,i);
end
pitch = asin(-T(3,1));
roll= asin(T(2,1)/cos(pitch));
yaw = acos(T(3,3)/cos(pitch));
disp(T);
q = J;
disp(J);
```

5.2.1 Results

```
R =

[ cos(t1), -cos(90)*sin(t1), -sin(90)*sin(t1)]
[ sin(t1),  cos(90)*cos(t1),  sin(90)*cos(t1)]
[       0,         -sin(90),          cos(90)]
```

```
k =

-sin(90)*sin(t1)
 sin(90)*cos(t1)
         cos(90)
```

```
J =

[                0, j1_2]
[                0, j2_2]
[                0, j3_2]
[ -sin(90)*sin(t1), j4_2]
[  sin(90)*cos(t1), j5_2]
[          cos(90), j6_2]
```

```
[                0,       0]
[                0,       0]
[                0,       0]
[ -sin(90)*sin(t1), cos(t1)]
[  sin(90)*cos(t1), sin(t1)]
[          cos(90),       0]
```

5.3 Inverse Kinematics:

```
clc;clear all;
syms a2 a1 t1 t2 d2 d1 m n x y c(t2)
%SOLVING FOR THETA 2
%________
%________
A=[a2*cos(t2)*cos(t1)-d2*sin(t1)+a1*cos(t1);a2*sin(t1)*cos(t2)+d2*cos(t1)+a1*sin(t1);d1;1]
X = A(1,1)
Y = A(2,1)
X^2 + Y^2
(X^2 + Y^2) - a1^2 - d1^2
X=x
Y=y
cos(t2) = ((X^2 + Y^2) - a1^2 - d2^2)/(a2*cos(t2)+2*a2*a1)
t2=acos(((X^2 + Y^2) - a1^2 - d2^2)/(a2*cos(t2)+2*a2*a1))
%===========================================
%===========================================
%NOW SOLVING FOR THETA 1
%===========================================
M = A(1,1)
N = A(2,1)
M=m
N=n
%cos(t2)= c(t2)
sin(t1) = [n*[a1+a2*cos(t2)]-m*d2]/[(a2*cos(t2)+a1)^2 + d2^2]
cos(t1) = m*[((a1 + a2*cos(t2))^2)-d2^2]+ [n*d2 * ( a1+ a2*cos(t2))]/[(a1+a2*cos(t2))^3 + d2^2]
tan(t1) =sin (t1)/cos(t1)
```

5.3.1 Results

```
cos(t2) =

-(a1^2 + d2^2 - x^2 - y^2)/(2*a1*a2 + a2*cos(t2))


t2 =

pi - acos((a1^2 + d2^2 - x^2 - y^2)/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2))))
```

```
sin(t1) =

-(d2*m - n*(a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 -
y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2))))))/((a1 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^2 + d2^2)
```

So we get values of θ_1 and θ_2 from the link lengths

```
cos(t1) =

m*((a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/
(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^2 - d2^2) +
(d2*n*(a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 -
y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2))))))/((a1 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^3 + d2^2)
```

```
tan(t1) =

-(d2*m - n*(a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 -
y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2))))))/((m*
((a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/
(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^2 - d2^2) +
(d2*n*(a1 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 -
y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2))))))/((a1 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 -
(a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^3 + d2^2))*((a1 - (a2*
(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 - (a2*
(a1^2 + d2^2 - x^2 - y^2))/(2*a1*a2 + a2*cos(t2)))))^2 + d2^2))
```

$$\mathrm{t}_1 = -a\tan((d_2m - n(a_1 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))(2a_1a_2 + a_2\cos(t_2))))))/[(((m((a_1 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1]^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 + a_2\cos(t_2)))))]^2 + d_2^2) + (d_2n(a_1 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_2^2 + [d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 + a_2 cos(t_2))))))/((a_1 - (a_2([a_1]^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 + a_2\cos(t_2)))))]^3 + d_2^2))((a_1 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 - (a_2(a_1^2 + d_2^2 - x^2 - y^2))/(2a_1a_2 + a_2\cos(t_2)))))]^2 + d_2^2)))$$

(5.2)

5.4 Theta Finder

Input the End Effector value i.e. θ_3. This is the angle at which the target will be locked or we can say that the angle of the target from the gun in the horizontal axis. This angle is also demonstrated in figure 5.2 and figure 5.3. The whole motion of the system will be followed using this angle. This motion is in two-dimensional Cartesian space.

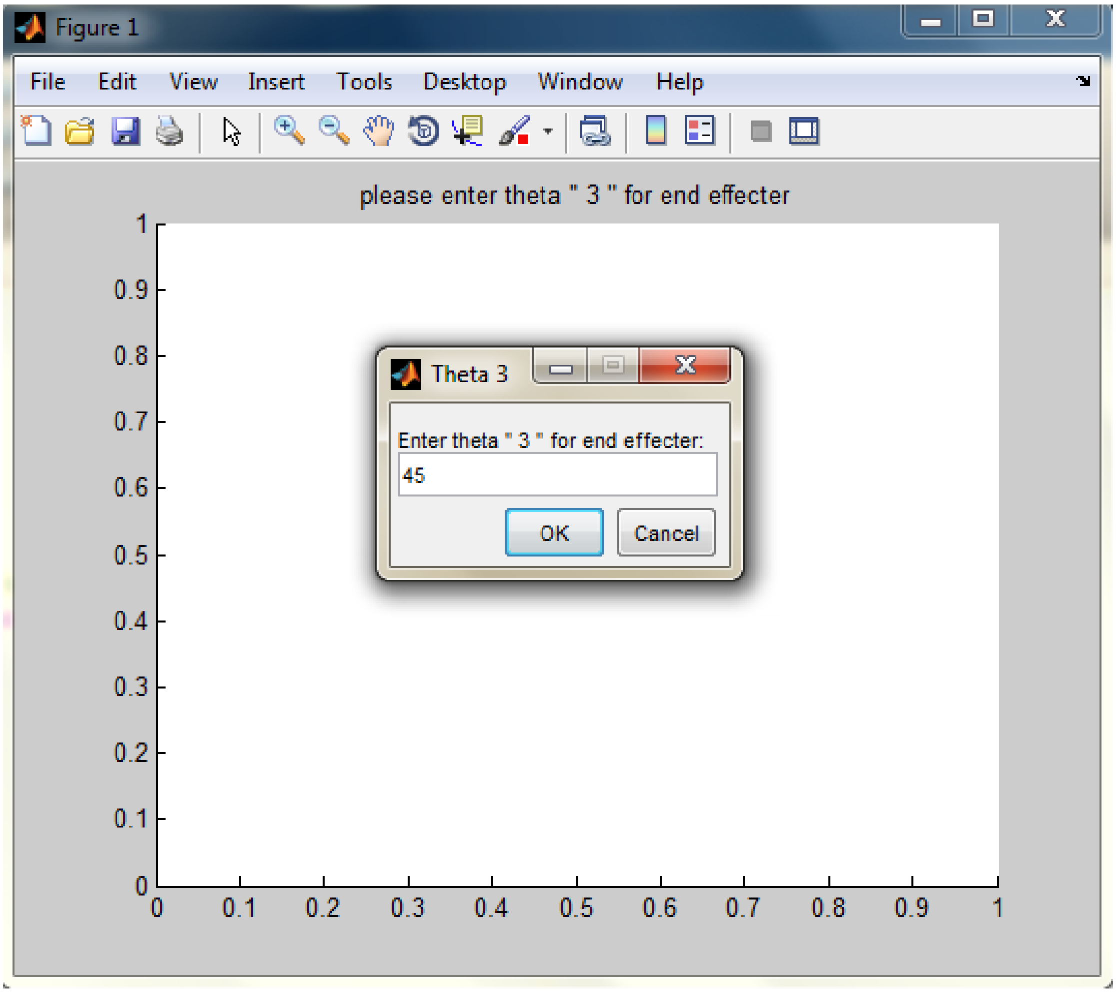

FIGURE 5.1: θ_3 Input angle

After that path for end effector will be drawn into MATLAB Plot as shown in Figure 5.2. This is the path in which the Tank travels as the target is stationary. Every side is divided into thirteen points and on every point, the value of the θ_1 and θ_2 will be changed. θ_1 is the angle of pitch and θ_2 is the angle in yow. These thirteen points are shown as dotted points in figure 5.2. Based on this trajectory the table for disturbances to be countered will be formed.

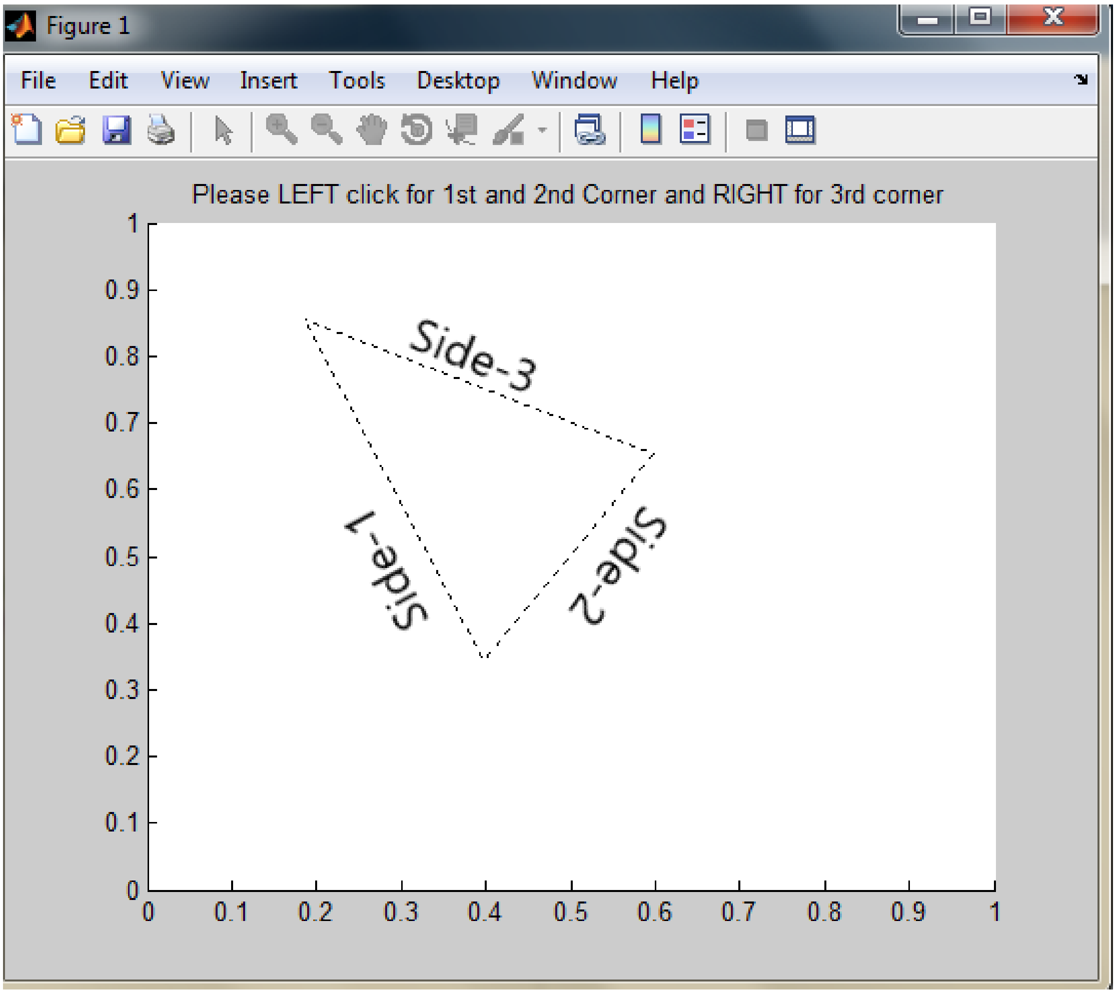

FIGURE 5.2: Tank Travel Path

The path followed is shown below. This is the path which on which the tank is moving keeping in view the target angle i.e θ_3 the change in θ_1 and θ_2 will be because of this path. Every side of the path is divided into 13 sub-points where at each point the gun undergoes the change in angles either in pitch or in a yow axis.

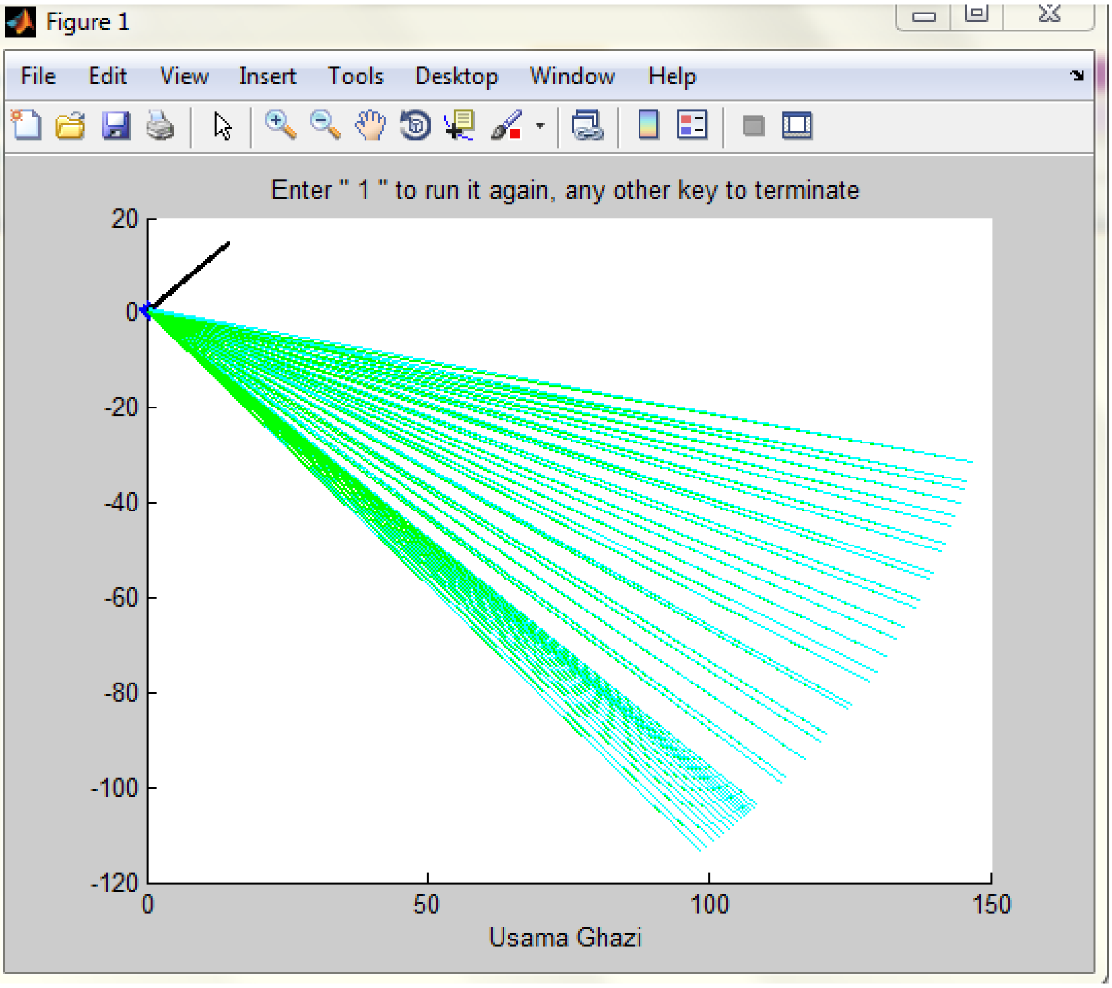

FIGURE 5.3: Computing values for θ_1 and θ_2 at each point.

Values of θ_1 and θ_2 against every value of X and Y of every side will be saved in Microsoft excel file. Each side has units in kilometers. ΔX and ΔY are the disturbances this disturbance is to be measured in meters and $\Delta\theta_1$ is the change in pitch angle and $\Delta\theta_2$ is the change in yow angle. After the computation is complete the code will ask to again run or not if yes we have to put 1 else any other number.

TABLE 5.1: Values of Theta 1, Theta 2 and Errors for Side 1

Side$_1X(km)$	Side$_1Y(km)$	θ_1 S1 (degrees)	θ_2 S1 (degrees)	ΔX	ΔY	$\Delta\theta_1$	$\Delta\theta_2$
1.0497	3.7110	-15.6470	179.7053				
1.1835	3.5955	-18.0752	179.7108	0.1338	-0.1155	-2.5164	0.0049
1.3174	3.4800	-20.5917	179.7157	0.1338	-0.1155	-2.5164	0.0049
1.4512	3.3645	-23.1911	179.7200	0.1338	-0.1155	-2.5994	0.0043
1.5850	3.2490	-25.8664	179.7238	0.1338	-0.1155	-2.6753	0.0037
1.7188	3.1336	-28.6089	179.7269	0.1338	-0.1155	-2.7425	0.0031
1.8526	3.0181	-31.4082	179.7294	0.1338	-0.1155	-2.7992	0.0024
1.9864	2.9026	-34.2524	179.7313	0.1338	-0.1155	-2.8441	0.0018
2.1203	2.7871	-37.1283	179.7324	0.1338	-0.1155	-2.8758	0.0011
2.2541	2.6716	-40.0217	179.7329	0.1338	-0.1155	-2.8934	0.0004
2.3879	2.5561	-42.9181	179.7327	0.1338	-0.1155	-2.8963	-0.0001
2.5217	2.4406	-45.8027	179.7319	0.1338	-0.1155	-2.8845	-0.0008
2.6555	2.3251	-48.6611	179.7303	0.1338	-0.1155	-2.8583	-0.0015

Table 5.2: Values of Theta 1,Theta 2 and Errors for Side 2

$Side_2X(km)$	$Side_2Y(km)$	$\theta_1S2(degree)$	$\theta_2S2(degree)$	ΔX	ΔY	$\Delta\theta_1$	$\Delta\theta_2$
2.7229	2.3946	-48.5319	179.7229				
2.7902	2.4641	-48.4093	179.7156	0.0674	0.0695	0.1226	-0.0073
2.8576	2.5336	-48.2927	179.7082	0.0674	0.0695	0.1166	-0.0073
2.9249	2.6032	-48.1817	179.7008	0.0674	0.0695	0.1110	-0.0073
2.9923	2.6727	-48.0758	179.6934	0.0674	0.0695	0.1058	-0.0073
3.0596	2.7422	-47.9747	179.6861	0.0674	0.0695	0.1010	-0.0073
3.1270	2.8117	-47.8781	179.6787	0.0674	0.0695	0.0966	-0.0073
3.1943	2.8813	-47.7857	179.6713	0.0674	0.0695	0.0924	-0.0073
3.2617	2.9508	-47.6971	179.6639	0.0674	0.0695	0.0885	-0.0073
3.3290	3.0203	-47.6122	179.6566	0.0674	0.0695	0.0848	-0.0073
3.3964	3.0898	-47.5307	179.6492	0.0674	0.0695	0.0814	-0.0073
3.4638	3.1593	-47.4525	179.6418	0.0674	0.0695	0.0782	-0.0073
3.5311	3.2289	-47.3772	179.6344	0.0674	0.0695	0.0752	-0.0073

Table 5.3: Values of Theta 1, Theta 2 and Errors for Side 3

$Side_3X$ (km)	$Side_3Y$ (km)	$\theta_1 S3$ (degrees)	$\theta_2 S3$ (degrees)	ΔX	ΔY	$\Delta\theta_1$	$\Delta\theta_2$
3.3299	3.2748	-45.2995	179.6432				
3.1288	3.3208	-43.1201	179.6514	-0.2012	0.0460	2.1794	0.0082
2.9276	3.3668	-40.8382	179.6591	-0.2012	0.0460	2.2819	0.0077
2.7264	3.4128	-38.4542	179.6663	-0.2012	0.0460	2.3840	0.0071
2.5253	3.4587	-35.9699	179.6728	-0.2012	0.0460	2.4842	0.0065
2.3241	3.5047	-33.3890	179.6787	-0.2012	0.0460	2.5809	0.0058
2.1229	3.5507	-30.7167	179.6839	-0.2012	0.0460	2.6722	0.0052
1.9217	3.5967	-27.9604	179.6884	-0.2012	0.0460	2.7562	0.0045
1.7206	3.6426	-25.1295	179.6922	-0.2012	0.0460	2.8309	0.0037
1.5194	3.6886	-22.2353	179.6952	-0.2012	0.0460	2.8942	0.0029
1.3182	3.7346	-19.2908	179.6974	-0.2012	0.0460	2.9445	0.0022
1.1171	3.7806	-16.3106	179.6988	-0.2012	0.0460	2.9801	0.0013
0.9159	3.8265	-13.3104	179.6994	-0.2012	0.0460	3.0001	0.0005

Now based on these all above values the change in the angles is sensed by the gyroscope. This change is because of the disturbances generated into the system because of the movement of the tank in a two-dimensional space. These disturbances are removed using the feedback of the gyroscope as shown in figure 5.4. But remember that first we have to lock the target angle so that the sensor will have to adjust the movement according to it.

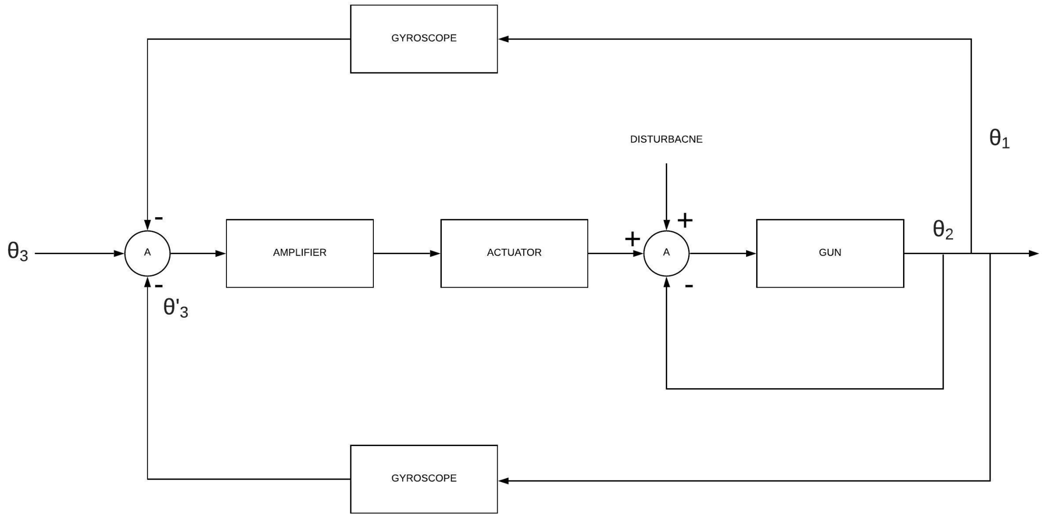

FIGURE 5.4: Disturbance involved in a system

Figure 5.4 shows that the disturbance coming into the system changes the position of the gun which is sensed by gyroscope and is countered such that the gun is stabilized. Disturbance change the angle θ_1, θ_2 at every point. Due to this change, the angle of the gun will also change i.e. θ_3.

This change at every point can be demonstrated as

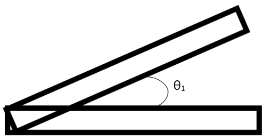

FIGURE 5.5: change in θ_1 due to disturbance

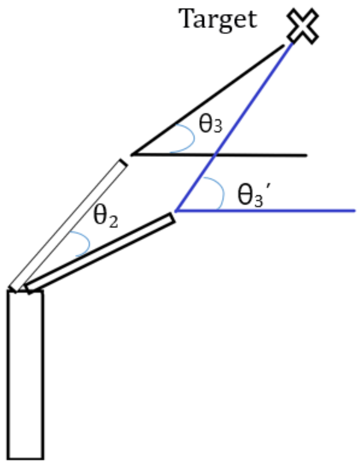

FIGURE 5.6: change in θ_3 due to disturbance

The change in θ_3 can be calculated as:

$$\Delta\theta_3 = \theta_3 - \theta_3' \tag{5.3}$$

Figure 5.5 and figure 5.6 shows that the change in the angle as the tank moves from one point to the next point. As stated in section 3.3 change in angles is because of the projectile motion of the bullet. Firstly the target is locked at some angle but as the tank moves, there will be a disturbance input which changes the θ_1 and θ_2. The change between the two positions of a gun is stated as $\Delta\theta_1$ and $\Delta\theta_2$ in the above figures. The transfer function of the whole system will be the ratio of yow and pitch angles θ_1 and θ_2 and input angle shown as θ_3 Disturbance involved in the system is ΔX and ΔY . These disturbances are shown in table 5.1, table 5.2, and table 5.3. The above system is developed in two-dimensional space but can be extended to three-dimensional space in the future.

Chapter 6

Conclusion and Future Recommendations

In this report, a new approach for stabilizing tank gun mount with the help of mathematical modeling has been developed. The main objective was to stabilize the weapon in the yow and pitch axis of the main battle tank. With this new approach, the firing will be more accurate, and consequently more effective. It may increase the interest of customers, and result in an increase in exports. After the gun mount will be fabricated it can be used in any military vehicle. The yaw and pitch dynamics were modeled based on the gun mounting characteristics. The order for mathematical modeling remained broad enough to cover the scope of the project. The design and development of the gyroscopic gun assembly algorithm have been performed and simulated. The gyroscope is positioned inside the structure to counter disturbances which are subjected to the frame. The gyroscope sensor considers the axis parallel to the tank. The pitch angle can be tilted more effectively from -40 to 75 degrees. In this project, the target is static whereas the tank is in motion. The disturbance is because of the change in location. Two Dimensional location analysis has been simulated in Matlab and the position of the gun is stabilized by removing the disturbances. This project can be used as a launching pad. The gyroscope sensor acts as a measuring device for the tilting perspective and can control the crankshaft clockwise or counterclockwise to level the surrounding disturbances.

Furthermore, this study carried out the stabilization of the platform on three axes and can be extended, with the help of high-speed engines to improve motor reaction time and also yow and pitch time calculation can be done i-e how much time will it take for the gun to pitch or to do the yow motion.

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