CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

The Study of Heat and Mass Transfer of Upper Convected Maxwell Fluid Between Two Parallel Plates

by

Farhan Ilyas

A thesis submitted in partial fulfillment for the degree of Master of Philosophy

in the

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Dedicated to my beloved Parents, Siblings and dignified Teachers, who have taught me to work diligently for the things that I aspire to achieve.

CERTIFICATE OF APPROVAL

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Abstract

The main objective of this thesis is to examine the heat and mass transfer in the unsteady squeezing flow of UCM fluid between two parallel plates in the presence of inclined magnetic field. The non-linear partial differential equations describing the proposed flow problem are reduced to a set of ordinary differential equations through appropriate similarity transformations. The shooting method has been used to obtain the numerical results with the help of the computational software MATLAB. The effects of different parameters on the non-dimensional velocity, temperature and concentration profiles are presented by graphs. The numerical values of the skin friction, Nusselt and Sherwood number are also presented and analyzed through tables. The temperature profile is observed to decay for the increasing values of the squeezing parameter, whereas the same is found to rise for a rise in the Eckert number.

Contents

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Abbreviations

BVP	Boundary value problem
IVP	Initial value problem
MHD	Magneto-hydrodynamic
ODEs	Ordinary differential equations
PDEs	Partial differential equations

Symbols

u	Velocity in x-direction
v	Velocity in y-direction
C_f	Skin Friction Coefficient
Nu	Nusselt Number
Sh	Sherwood Number
Pr	Prandtl Number
E_c	Eckert Number
E_{cx}	Local Eckert Number
S	Squeezing Number
S_c	Schmidt Number
γ	Chemical Reaction Parameter
T	Temperature
p	Pressure
C	Concentration
ρ	Fluid Density
ν	Kinematic Viscosity
μ	Fluid Viscosity
k	Thermal Conductivity
C_p	Specific Heat
D	Diffusion Coefficient
$K_1(t)$	Time Dependent Reaction Rate
$h(t)$	Distance Between two Plates
T_H	Temperature of the Upper Wall

θ	Dimensionless Temperature
ϕ	Dimensionless Concentration
B_0	Magnetic Flux Density
λ_r	Viscoelastic Parameter
σ	Electrical Conductivity
ω	Inclination Angle

Chapter 1

Introduction

The interest in the research of mass and heat transfer with different physical parameters impact has been increased over the last few decades. This phenomenon is used in many fields such as, hydrodynamic machines, chemical processing devices, lubricant system, polymer processing, preventing crops by freezing, formation and dispersion of fog and food processing. Duwairi et al. [1] reported the effect of extrusion and squeezing parameters on the rate of heat conduction of the squeezed viscous fluid between two parallel plates. They observed the rate of heat conduction increases and the local coefficient friction decelerates by increasing the squeezing parameter, whereas the heat transfer rate decreases and the skin friction coefficient enhancing by an increment in extrusion parameter. Kai-Long Hsiao [2] has explained the physical characteristic of the energy conversion conjugate mass and heat conduction with radiative thermal effects for an incompressible mixed convection 2D flow of Maxwell fluid over a stretching surface. He clearly noted that the Maxwell fluid free convection energy conversion is better than forced convection. Heat conduction and unsteady flow of nanofluid by a smoothly moving plate was analyzed by Ahmadi et al. [3]. It has been discussed that the unsteady parameter plays an important role on the velocity profile which means that velocity profile is enhanced by increasing the unsteady parameter. Afify [4] discussed the mass transfer and convective, incompressible, electrically conducting flow of viscous fluid in the direction of stretching surface along with the magnetic effect and chemical reaction. It has found that the skin friction coefficient increases whereas the Nusselt number and Sherwood decreased by an increment in the magnetic parameter.

Squeezing flow explains the motion of a droplet of material. Squeezing flow has various applications in science and engineering such as hot plate welding, rheological testing, composite material joining etc. Bhatta et al. [5] observed the unsteady squeezing nanofluid flow based on water between two disks held parallel to each other in the presence of slip impact. It was observed that an increase in Lewis number decelerate the nanoparticle concentration. Adesanya et al. [6] analyzed the unsteady MHD squeezing Eyring-Powell fluid flow over an infinite channel. They concluded that concentration profile decreases with respect to chemical reaction parameter. It was also concluded that the rate of heat transfer increases by expanding the thermal radiation, channel walls and internal heat generation parameters whereas the heat transfer rate decreases with the heat absorption and channel wall compression. Farooq et al. [7] presented the melting heat transfer effect in the squeezing flow of nanofluid over a Darcy porous medium. They analyzed that the temperature distribution increases for the dominating values of thermophresis parameter. Hayat et al. [8] examined the similarity solution of an incompressible squeezing flow of micropolar fluid between two disks held parallel to eachothe along with magnetic effect. They found that in case of suction the angular velocity $h(\eta)$ decreases for the increases values of micropolar parameter K whereas the magnitude of $h(\eta)$ increases with the rise of micropolar parameter K in the case of blowing ($S < 0$). The heat transfer and flow pat over a sensor surface which is immersed in an squeezed channel was examined by Mahmood et al. [9] and found that an increment in the suction through sensor surface increases the heat transfer and skin friction coefficient while on the other hand rise in injection develops the opposite effects on skin friction and heat transfer coefficient. The MHD flow of a squeezing fluid between two disks held parallel to each other such that one of them is impermeable and other is porous was analyzed by Mohyud-Din et al. [10]. The entropy generation under the effect of magnetic field on an unsteady incompressible 2D squeezing flow and mass transfer of Casson fluid between two disks held parallel to eachother was explained by Ojjela et al. [11]. They examined that the temperature of the fluid is enhanced and concentration of the fluid is decreased with Prandtl number and Hartmann number respectively. Sheikholeslami et al. [12] used the Adomian decomposition method (ADM), and calculated an analytical solution of an unsteady flow of nanofluid which is squeezed between two parallel sheets. It was concluded that the Nusselt number has direct relation with nanoparticle volume fraction and Eckert number otherwise it has opposite relation with squeezing number when the two plates move together.

Magnetohydrodynamics deals with the magnetic properties and behaviour of electrically conducted fluids, such as salt water, liquid metals, electrolytes and plasmas. Gholinia et al. [13] analyzed the different physical impacts such as slip flow and magnetic field on Eyring-Powell fluid along with the homogenous-heterogenous reactions due to rotating disk and conclude that temperature profile is decreased with increasing Pr increased with increasing Nt. Hayat et al. [14] observed the magnetic effect on an unsteady 2D second-degree fluid flow between two parallel disks. Jha and Aina [15] computed an approximate solution for MHD flow of an incompressibl fluid which is viscous and electrically conducting in a vertical micro-porous channel developed by electrically non-conducting vertical plates held parallel to eachother in the presence of induced magnetic effect. They noted that fluid velocity and slip velocity is enhanced with the effect of suction/injection parameter. It was also observed that the volume flow rate is reduced with an increase in magnetic parameter and Hartmann number. Khan et al. [16] examined the heat transfer in the nanofluid flow between two plates held parallel to eachother along with the magnetic effect. They noted that the velocity of fluid is not effected by the shape factor. It was also concluded that the nanoparticles of higher shape factor will increase the temperature and decrease the heat transfer rate. Mabood et al. [17] conferred the numerical solution of MHD stagnation point flow of nanofluid based on water (Cu and Al_2O_3) over a porous surface under the influence of volume fraction of nanoparticles, radiations, chemical reactions and viscous dissipation. Siddiqui et al. [18] computed the solution by using Homotopy perturbation method of an unsteady 2D squeezing MHD fluid flow between two plates held parallel to each other. They noted that the velocity profile increases monotonically for fixed value of R and for different values of magnetic parameter$M$.

The MHD flow of UCM fluid past over a porous medium under the influence of slip condition at the boundaries was presented by Abbasi et al. [19]. They found that increase in the Hartmann number associates with the decrement in the velocity profile. Choi et al. [20] examined the combined effect of inertia and viscoelasticity on the two dimensional, incompressible and steady flow past overa a porous surface channel. Hayat and Abbas [21] examined the 2D boundary layer flow of UCM fluid over a porous channel with chemical reaction. They concluded that for the increasing values of Reynolds number Re the velocity has opposite behaviour in viscoelastic fluid ($De \neq 0$) but for the variation of De it has the same behaviour. Mukhopadhyay and Vajravelu [22] analyzed the similarity solutions for the unsteady boundary layer flow and transfer of heat in a Maxwell fluid past over a porous stretching sheet in the presence of heat source/sink. They noted a reduction in the velocity field for the increasing values of Maxwell fluid and magnetic field. Prasad et al. [23] examined the effects of thermal conductivity and internal heat source/sink on MHD flow and heat conduction in the stretching surface of the UCM fluid. It was concluded that the effect of increase in the magnetic parameter and Maxwell parameter decreases thevelocity profile throughout the boundary layer in the presence of temperature dependent thermophysical properties. They also noted that the effect of thermal conductivity and heat source/sink enhanced the temperature in the flow region. Sadeghy et al. [24] investigated the flow of upper convected Maxwell fluid past by a steadily moving rigid plate.

1.1 Thesis Contribution

In this thesis, a detailed review of [25] is conducted and the results have been imitated by considering the additional effects of inclined magnetic field and UCM fluid. In this work, through an appropriate transformation, the governing PDEs are converted into the dimensionless ODEs. The numerical results are calculated by using the shooting technique. Using the tables and graphs, different physical parameter's effects on the flow and heat conduction are also explained.

1.2 Thesis Outline

A brief overview of the content of the thesis is provided as:
In **Chapter 2**, we explain few basic definitions and terminologies. Furthermore some basic laws and dimensionless physical parameters are also included.

Chapter 3 contains the complete review of [25] which considers 2-dimensional unsteady flow of an incompressible viscous fluid which is squeezed between plates held parallel to each other.

In **Chapter 4**, we extend the model given in [25] by considering the additional impact of UCM and inclined magnetic field in the momentum equation. The obtained system of dimensionless ODEs is solved numerically by shooting method. The behaviour of different physical parameters is explained through tables and graphs.

In **Chapter 5**, we recapitulate the thesis and give the conclusion from the whole work and a proposal for the future work.

All the references used in this research work are listed in **Bibliography**.

Chapter 2

Preliminaries

In the current chapter, some definitions, basic laws, terminologies and basic concepts for solving non-linear differential equations are described, which would be used in the next chapters.

2.1 Some Basic Definitions [26–32]

In this section, few basic definitions, laws and terminologies have been presented that are beneficial for the further discussion.

Definition 2.1.1. (Fluid)

"A fluid is a substance that deforms continuously under the application of a shear stress, no matter how small the stress may be. Thus fluids comprise of the liquid and gas (or vapour) phases of the physical forms in which matter exists."

Definition 2.1.2. (Fluid Mechanics)

"Fluid mechanics deals with the behaviour of fluids at rest or in motion."

Definition 2.1.3. (Fluid Dynamics)

"The branch of fluid mechanics that covers the properties of fluid in the state of progression from one place to another is called fluid dynamics."

Definition 2.1.4. (Fluid Statics)

"Fluid static is the part of fluid mechanics, that deals with a fluid and its characteristics at the constant position."

Definition 2.1.5. (Magnetohydrodynamics)

"It is also referred to as the magneto fluid dynamics or hydromagnetics. It deals with the study of magnetic properties of electrically conducting fluids. It is denoted by MHD."

2.2 Physical Properties of the Fluid

Definition 2.2.1. (Viscosity)

"It is the property of the fluid that resists the fluid flow. In other words, a fluid viscosity is that characteristic which measures the amount of resistance to the shear stress. It is denoted by μ and mathematically, it can be written as:

$$\text{Viscosity} = \mu = \frac{\text{shear stress}}{\text{rate of shear strain}}."$$

Definition 2.2.2. (Kinematic Viscosity)

"The ratio of the dynamic viscosity to the density of fluid is said to be kinematic viscosity. Symbolically, it can be written as ν and mathematically, it is expressed as:

$$\nu = \frac{\mu}{\rho},$$

where μ and ρ denote the dynamic viscosity and the density respectively. The dimension of kinematic viscosity is given by $[\frac{L^2}{T}]$. "

Definition 2.2.3. (Stress)

"Stress is a force acted upon a material per unit of its area and is denoted by τ. Mathematically, it can be written as:

$$\tau = \frac{F}{A},$$

where F denotes the force and A represents the area. ”

Definition 2.2.4. (Shear Stress)

“It is a type of stress in which the force vector acts parallel to the material surface or the cross section of a material. ”

Definition 2.2.5. (Normal Stress)

“It is a type of stress in which the force vector acts perpendicular to the surface of material or the cross section of a material.”

2.3 Types of Fluid Flow

Definition 2.3.1. (Flow)

“It is the deformation of the material under the influence of different forces. If the deformation increase is continuous without any limit, then the process is known as flow.”

Definition 2.3.2. (Compressible and Incompressible Flows)

“Flow in which variations in density are negligible is termed as incompressible otherwise it is called compressible. The most common example of compressible flow is the flow of gases, while the flow of liquids may frequently be treated as incompressible.

Mathematically, for compressible flow:

$$\frac{D\rho}{Dt} = 0,$$

where ρ denotes the fluid density and $\frac{D}{Dt}$ is the material derivative given by;

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V}.\nabla,$$

where $\mathbf{V}$ denotes the velocity of flow and ∇ is the differential operator. In Cartesian coordinate system, ∇ is given as:

$$\nabla = \frac{\partial}{\partial x}\widehat{i} + \frac{\partial}{\partial y}\widehat{j} + \frac{\partial}{\partial z}\widehat{k}.\text{''}$$

Definition 2.3.3. (Uniform and non-Uniform Flows)

"The flow is said to be uniform if the magnitude and direction of flow velocity are the same at every point and the flow is said to be non-uniform if the velocity is not the same at each point of flow, at a given instant."

Definition 2.3.4. (Steady and Unsteady Flows)

"A flow is said to be steady flow in which the fluid properties do not change with time at a specific point,

$$\frac{\partial \lambda}{\partial t} = 0,$$

where λ is any fluid property. A flow is said to be unsteady in which the fluid properties change with time, i.e

$$\frac{\partial \lambda}{\partial t} \neq 0.\text{''}$$

Definition 2.3.5. (Laminar and Turbulent Flows)

"A flow is laminar in which the fluid particles move in smooth layers or laminar and a turbulent in which the fluid particles rapidly mix as they move along due to random three-dimensional velocity fluctuations. "

2.4 Types of Fluids

Definition 2.4.1. (Ideal Fluid)

"A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid."

Definition 2.4.2. (Real Fluid)

"A compressible fluid which experience some resistance during the flow is characterized as a real or viscid fluid."

Definition 2.4.3. (Newtons Law of Viscosity)

"It is a relationship in which shear stress is directly and linearly proportional to the velocity gradient. Mathematically, it can be written as:

$$\tau_{yx} \propto \left(\tfrac{du}{dy}\right),$$

$$\tau_{yx} = \mu \left(\tfrac{du}{dy}\right).$$

In the above expression, τ_{yx} is the shear stress applied to the velocity component u of fluid and μ is the viscosity proportionality constant."

Definition 2.4.4. (Newtonian and non-Newtonian Fluids)

"Newtonian fluids fulfill Newtons law of viscosity which can be written mathematically as:

$$\tau_{xy} = \mu\left(\frac{du}{dy}\right),$$

where

μ = Dynamic viscosity
τ_{xy} = Shear stress = $\frac{F}{A}$
$\frac{du}{dy}$ = Rate of shear deformation

The most common examples of Newtonian fluid are water, alcohol, glycerol.
The fluids, which do not obey the Newtons law of viscosity are known as non-Newtonian fluids. For such fluids,

$$\tau_{xy} = k\left(\frac{du}{dy}\right)^n,$$

where

k is the flow consistency index
$\frac{du}{dy}$ is shear rate
n is flow behaviour index.

For $n = 1$ with $k = \mu$, the above equation reduces to the Newtons's law of viscosity. Paints, blood, biological fluids and polymer melts are good examples of non-Newtonian fluids."

2.5 Heat Transfer Mechanism and Properties

Definition 2.5.1. (Heat Transfer)

"It is the energy transfer due to temperature difference. At the point when there is a temperature contrast in a medium or between media, heat transfer must take place. Heat transfer is normally conducted from a high temperature region to a low temperature. For example, heat is transferred from stove to the cooking pan."

Definition 2.5.2. (Mass Transfer)

"Mass exchange is the total movement of mass from one place to another."

Definition 2.5.3. (Conduction)

"Conduction is the process in which heat is transferred through the material between the objects that are in physical contact. For example,

- picking up a hot cup of tea,
- after a car is turned on, the engine becomes hot,
- a radiator is a good example of conduction."

Definition 2.5.4. (Convection)

"Convection is a mechanism in which heat is transferred through fluids (gases or liquids) from a hot place to a cool place. For example,

- macaroni rising and falling in a pot of boiling water,
- streaming cup of hot tea. The steam is showing heat transferred into the air. "

Definition 2.5.5. (Thermal Radiation)

"The process by which heat is transferred from a body by virtue of its temperature, without the aid of any intervening medium is called thermal radiation. For example, toasters use thermal radiations emitted by its element to toast bread."

Definition 2.5.6. (Thermal Conductivity)

"Thermal conductivity k is the property of a material related to its ability to transfer heat. Mathematically,

$$k = \frac{q \bigtriangledown l}{S \bigtriangledown T},$$

where q is the heat passing through a surface area S and the effect of a temperature difference $\bigtriangledown T$ over a distance is $\bigtriangledown l$. Here l, S and $\bigtriangledown T$ all are assumed to be of unit measurement. In SI unit of thermal conductivity is $\frac{W}{m.k}$ and its dimension is $[MLT^{-1}\theta^{-1}]$."

Definition 2.5.7. (Thermal Diffusivity)

"Thermal diffusivity is material's property which identifies the unsteady heat conduction. Mathematically, it can be written as:

$$\alpha = \frac{k}{\rho C_p},$$

where k, ρ and C_p represent the thermal conductivity of material, the density and the specific heat capacity. In SI system unit and dimension of thermal diffusivity are $m^2 s^{-1}$ and $[LT^{-1}]$ respectively."

Definition 2.5.8. (Viscous Dissipation)

"The process in which the work done by fluid is converted into heat is called viscous dissipation."

2.6 Laws of Conservation and Basic Equation

"There are three laws of conservation which are used to model the problems of fluid dynamics, and may be written in integral or differential form. Integral formulations of these laws consider the change of mass, momentum or energy within the control volume."

2.7 Continuity Equation

"The conservation of mass of fluid entering and leaving the control volume, the resulting mass balance is called the equation of continuity. This equation reflects the fact that mass is conserved. For any fluid, conservation of mass is expressed by the scaler equation:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = \mathbf{0}. \tag{2.1}$$

For the steady flow above Eq. (2.1) can be written as:

$$\nabla.(\rho \mathbf{V}) = \mathbf{0}. \tag{2.2}$$

For incompressible flow, Eq. (2.2) becomes:

$$\nabla.\mathbf{V} = \mathbf{0}. \tag{2.3}$$

For incompressible and irrotational flow, the Eq. (2.3) is transformed in terms of velocity potential ψ, which is given by:

$$\nabla^2 \psi = 0. \tag{2.4}$$

Eq. (2.4) is known as Laplace equation."

2.8 Momentum Equation

"The product of the mass and the velocity of a body is called the linear momentum. Newtons second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body. Therefore, the momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved. This is known as the conservation of momentum principal. For any fluid, the momentum equation is:

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla.[(\rho \mathbf{V})\mathbf{V}] - \nabla.T - \rho g = 0. \tag{2.5}$$

Since $T = -p\mathbf{I} + \tau$, the momentum equation takes the form:

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}.\nabla\mathbf{V}\right) = \nabla.(-p\mathbf{I} + \tau) + \rho g. \tag{2.6}$$

Eq. (2.6) is a vector equation and can be decomposed further into three scalar components by taking the scalar product with the basis vectors of an appropriate orthogonal coordinate system. By setting $g = g\nabla z$, where z is the distance from an arbitrary reference elevation in the direction of gravity, Eq. **??** can be also expressed as:

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}.\nabla\mathbf{V}\right) = \nabla. + \nabla(-\rho g z). \tag{2.7}$$

The momentum equation then states that the acceleration of a particle following the motion is the result of a net force, expressed by the gradient of pressure,viscous and gravity forces."

2.9 Energy Equation

"One of the most fundamental laws in nature is the first law of thermodynamics, also known as the conservation of energy principle. It states that energy can be neither created nor destroyed during a process; it can only change forms. In two dimensional system the energy equation for base fluid can be expressed as:"

$$\mathbf{v}.\nabla T = \alpha \nabla^2 T \tag{2.8}$$

2.10 Dimensionless Parameters

Definition 2.10.1. (Reynolds Number)
"This number expresses the ratio of the fluid inertial force to that of molecular friction (viscosity). It determines the character of the flow (laminar, turbulent and transient flows). Mathematically, it can be write as:

$$R_e = \frac{u_0 H}{\nu},$$

where H is characteristic length, u_0 the flow velocity and ν is the kinematic viscosity."

Definition 2.10.2. (Eckert Number)
"It is the dimensionless number used in continuum mechanics. It describes the relation between flows and the boundary layer enthalpy difference and it is used for characterized heat dissipation. Mathematically,

$$E_c = \frac{u^2}{C_p \bigtriangledown T},$$

where u is local flow velocity, C_p is the specific heat and $\bigtriangledown T$ is the difference between wall temperature."

Definition 2.10.3. (Prandtl Number)
"This number expresses the ratio of the momentum diffusivity (viscosity) to the thermal diffusivity. Mathematically, it can be written as:

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho C_p} = \frac{\mu C_p}{k},$$

where ν represents the kinematic viscosity and α denotes the thermal diffusivity. It characterizes the physical properties of a fluid with convective and diffusive transfers."

Definition 2.10.4. (Schmidt Number)

"It is the ratio between viscosity ν and molecular diffusion D. It is denoted by S_c and mathematically we can write it as:

$$S_c = \frac{\nu}{D},$$

where ν is the kinematic viscosity and D is the mass diffusivity."

Definition 2.10.5. (Skin Friction Coefficient)

"skin friction co-efficient occurs between the fluid and solid surface which leads to slow down the motion of fluid. The skin friction co-efficient is defined as:

$$C_f = \frac{2\tau_w}{\rho U_w^2},$$

where τ_w denotes the wall shear stress, ρ the density and U_w the free-stream velocity."

Definition 2.10.6. (Nusselt Number)

"It is the ratio of the convective to the conductive heat heat transfer to the boundary. Mathematically,

$$Nu = \frac{hL}{k},$$

where h stands for convective heat transfer, L stands for the characteristic length and k stands for the thermal conductivity."

Definition 2.10.7. (Sherwood Number)

"It is the non-dimensional quantity which shows the ratio of the mass transport by convection to the transfer of mass by diffusion. Mathematically,

$$Sh = \frac{kL}{D},$$

where L is characteristics length, D is the mass diffusivity and k is the mass transfer co-efficient."

Chapter 3

The Effect of Heat and Mass Transfer on the Unsteady Squeezing Flow Passing Through Parallel Plates

3.1 Introduction

The numerical study of squeezing flow with mass and heat exchange of an incompressible, 2-D viscous fluid between two plates held parallel to each other has been taken under consideration. The governing PDEs are transformed into the dimensionless ODEs by an appropriate transformation. In order to solve ODEs, we used the shooting technique implemented in MATLAB. The results obtained are compared to the articles previously published. By changing the values of different physical parameters, we observed the nature of velocity, temperature and the concentration distributions. At the end, the graphs and tables are shown which are obtained under this investigation. This chapter gives the complete review of [25].

3.2 Mathematical Modeling

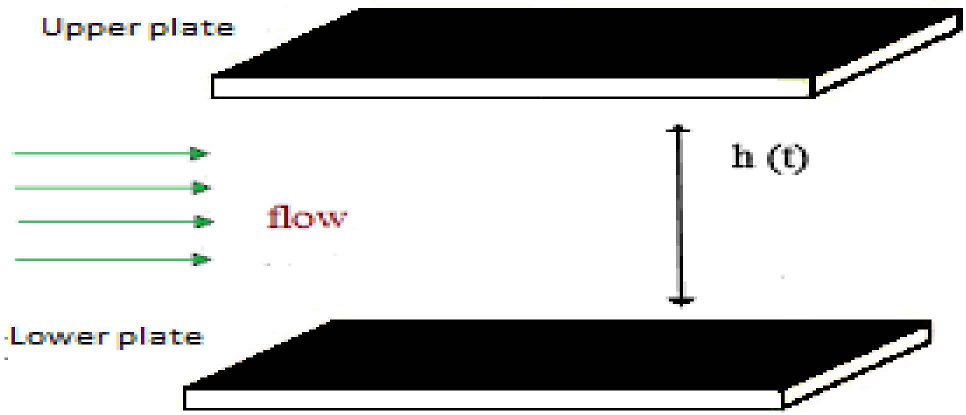

FIGURE 3.1: Physical model of the problem

An incompressible, 2-D unsteady viscous fluid flow, which is squeezed between two plates held parallel to each other has been considered. The distance between plates is $y = l\,(1 - \alpha\, t)^{\frac{1}{2}} = h(t)$. The viscous dissipation effect, the generation of heat due to friction caused by shear in the flow, is retained. Moreover the flow has been considered as symmetric.

The flow is explained by considering the two dimensional governing equations containing the continuity, equation of momentum, energy and the concentration equation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{3.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3.3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right], \tag{3.4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - K_1(t)\,C. \tag{3.5}$$

The conditions associated with the boundary are as:

$$\left.\begin{aligned} &y = h(t) : u = 0,\ v = v_w = \frac{dh}{dt},\ T = T_H,\ C = C_H, \\ &y = 0 :\ v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0. \end{aligned}\right\} \tag{3.6}$$

Here, ρ is the fluid density, u an v are the velocity components in the the direction of x and y respectively, T denotes the temperature, C represents the concentration, p denotes the pressure, ν represents the kinematic viscosity, C_p is specific heat, D denotes the coefficient of diffusion and $K_1(t) = \dfrac{k_1}{(1-\alpha t)}$ defines the rate of reaction which depends on time.

For the conversion of the mathematical model (3.1)-(3.5) into the dimensionless form, the following similarity transformations have been introduced.

$$\left.\begin{aligned} &u = \frac{\alpha\, x}{2\,(1\,-\alpha\, t)} f'(\eta),\ \eta = \frac{y}{[l(1-\alpha t)^{\frac{1}{2}}]},\ v = -\frac{\alpha l}{[2(1-\alpha t)^{\frac{1}{2}}]} f(\eta), \\ &\theta = \frac{T}{T_H}, \phi = \frac{C}{C_H} \end{aligned}\right\} \tag{3.7}$$

Following are some important derivatives necessary for further derivation.

- $\eta = \dfrac{y}{[l(1-\alpha t)^{\frac{1}{2}}]}$
- $\dfrac{\partial \eta}{\partial x} = 0.$
- $\dfrac{\partial \eta}{\partial y} = \dfrac{1}{l\,\sqrt{1-\alpha\, t}}.$
- $\dfrac{\partial \eta}{\partial t} = \dfrac{\alpha \eta}{2(1-\alpha t)}.$
- $u = \dfrac{\alpha x}{2\,(1-\alpha\, t)}\, f'(\eta)$
- $\dfrac{\partial u}{\partial x} = \dfrac{\alpha}{2(1-\alpha t)} f'(\eta).$
- $v = -\dfrac{\alpha\, l}{2\,\sqrt{1-\alpha\, t}}\, f(\eta).$
- $\dfrac{\partial v}{\partial y} = \dfrac{\partial}{\partial y}\left(-\dfrac{\alpha\, l}{2\,\sqrt{1-\alpha\, t}}\, f(\eta) \right)$

$$= -\frac{\alpha\, l}{2\,\sqrt{1-\alpha\, t}}\, f'(\eta)\, \frac{1}{l\sqrt{1-\alpha t}}.$$
$$= -\frac{\alpha}{2(1-\alpha t)} f'(\eta).$$

The equation of continuity (3.1) is satisfied as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\alpha}{2\,(1-\alpha\, t)}\, f'(\eta) - \frac{\alpha}{2\,(1-\alpha\, t)}\, f'(\eta) = 0.$$

For the Equations (3.2)-(3.3) we proceed as follows:

- $$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\alpha\, x}{2\,(1-\alpha\, t)}\, f'(\eta)\right)$$
  $$= \frac{\alpha^2 x}{2\,(1-\alpha\, t)^2}\left(f'(\eta) + \frac{\eta}{2}\, f''(\eta)\right).$$
- $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\alpha x}{2\,(1-\alpha\, t)}\, f'(\eta)\right)$$
  $$= \frac{\alpha}{2\,(1-\alpha\, t)}\, f'(\eta).$$
- $$u\frac{\partial u}{\partial x} = \left(\frac{\alpha\, x}{2\,(1-\alpha\, t)}\, f'(\eta)\right)\left(\frac{\alpha}{2(1-\alpha t)} f'(\eta)\right)$$
  $$= \frac{\alpha^2 x}{4(1-\alpha t)^2}(f'(\eta))^2.$$
- $$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\alpha x}{2(1-\alpha t)} f'(\eta)\right)$$
  $$= \frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}} f''(\eta).$$
- $$v\frac{\partial u}{\partial y} = \left(-\frac{\alpha\, l}{[2\,(1-\alpha\, t)^{\frac{1}{2}}]}\, f(\eta)\right)\left(\frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}}\, f''(\eta)\right)$$
  $$= -\frac{\alpha^2 x}{4(1-\alpha t)^2} f(\eta) f''(\eta).$$
- $$\frac{\partial^2 u}{\partial x^2} = 0.$$
- $$\frac{\partial^2 u}{\partial y^2} = \frac{\alpha x}{2l^2(1-\alpha t)^2} f'''(\eta).$$

The dimensionless form of (3.2) is as follows:

$$
\begin{aligned}
&\frac{\alpha^2\, x}{2(1-\alpha t)^2}\,\left(f'(\eta)\, + \frac{\eta}{2}\, f''(\eta)\right) + \frac{\alpha^2 x}{4(1-\alpha t)^2}(f'(\eta))^2 \\
&- \frac{\alpha^2 x}{4\,(\,1-\alpha t)^2}\, f(\eta)\, f''(\eta) = -\,\frac{1}{\rho}\,\frac{\partial p}{\partial x} + \frac{\alpha\,\nu\,x}{2l^2(1\,-\alpha t)^2}\, f'''(\eta) \\
\Rightarrow\quad &\frac{\alpha^2 x}{2(1-\alpha t)^2}\left(f'(\eta)\, + \frac{\eta}{2}\, f''(\eta) + \frac{1}{2}\,\left(f'(\eta)\right)^2 - \frac{1}{2} f(\eta)\, f''(\eta)\right) \\
&= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\alpha\nu x}{2l^2(1-\alpha t)^2} f'''(\eta).
\end{aligned}
\tag{3.8}
$$

Differentiating (3.8) w.r.t y, we get,

$$
\begin{aligned}
&\frac{\alpha^2 x}{2(1-\alpha t)^2}\left(f''(\eta)\, + \frac{\eta}{2}\, f'''(\eta) + \frac{1}{2}\, f''(\eta)\, + f'(\eta)\, f''(\eta)\, - \frac{1}{2}\, f(\eta)\, f'''(\eta)\right. \\
&\left. -\,\frac{1}{2}\, f'(\eta)\, f''(\eta)\right)\frac{\partial \eta}{\partial y} = -\frac{1}{\rho}\,\frac{\partial^2 p}{\partial x \partial y} + \frac{\alpha\,\nu\,x}{2l^2(1\,-\alpha t)^2}\, f^{(iv)}(\eta)\frac{\partial \eta}{\partial y} \\
\Rightarrow\quad &\frac{\alpha^2 x}{2\,(1-\alpha t)^2}\left(f''(\eta)\, + \frac{\eta}{2}\, f'''(\eta)\, + \frac{1}{2}\, f''(\eta)\, - \frac{1}{2}\, f(\eta)\, f'''(\eta) + \frac{1}{2}\, f'(\eta)\right. \\
&\left. f''(\eta)\right)\frac{\partial \eta}{\partial y} = -\frac{1}{\rho}\frac{\partial^2 p}{\partial x \partial y} + \frac{\alpha\nu x}{2l^2(1-\alpha t)^2} f^{(iv)}(\eta)\frac{\partial \eta}{\partial y}.
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\quad &\frac{\alpha^2 x}{2(1-\alpha t)^2}\left(\frac{3}{2}\, f''(\eta) + \frac{\eta}{2}\, f'''(\eta)\, - \frac{1}{2}\, f(\eta) f'''(\eta) + \frac{1}{2}\, f'(\eta) f''(\eta)\right)\,\frac{\partial \eta}{\partial y} \\
&= -\,\frac{1}{\rho}\,\frac{\partial^2 p}{\partial x\,\partial y} + \frac{\alpha\,\nu\,x}{2l^2(1-\alpha t)^2} f^{(iv)}(\eta)\frac{\partial \eta}{\partial y}.
\end{aligned}
\tag{3.9}
$$

Now, for the momentum equation (3.3), the following derivatives are required:

- $$
\begin{aligned}
\frac{\partial v}{\partial t} &= \frac{\partial}{\partial t}\left(-\frac{\alpha l}{2\sqrt{1-\alpha t}} f(\eta)\right) \\
&= -\frac{\alpha^2\, l}{4\,(1\,-\alpha\, t)^{\frac{3}{2}}}\,\left(f(\eta)\, + \eta f'(\eta)\right).
\end{aligned}
$$

- $$
\begin{aligned}
\frac{\partial v}{\partial x} &= \frac{\partial}{\partial x}\left(-\,\frac{\alpha\, l}{2\sqrt{1-\alpha t}}\, f(\eta)\right). \\
&= 0 \qquad\qquad \left(\because\, \frac{\partial \eta}{\partial x} = 0\right)
\end{aligned}
$$

- $$
\begin{aligned}
u\,\frac{\partial v}{\partial x} &= \left(\frac{\alpha\, x}{[\,2\,(1-\alpha t)\,]}\, f'(\eta)\right).0 \\
&= 0.
\end{aligned}
$$

- $$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{\alpha l}{2\sqrt{1-\alpha t}} f(\eta)\right)$$
$$= -\frac{\alpha}{2\,(1\ -\alpha\, t)}\ f'(\eta).$$
- $$v\,\frac{\partial v}{\partial y} = \left(-\frac{\alpha\, l}{2\,(\sqrt{1-\alpha t}}\ f(\eta)\right)\ \left(-\frac{\alpha}{2\,(1\ -\alpha t)} f'(\eta)\right)$$
$$= \frac{\alpha^2 l}{4\,(1\ -\alpha t)^{\frac{3}{2}}}\ f(\eta)\ f'(\eta).$$
- $$\frac{\partial^2 v}{\partial x^2} = 0.$$
- $$\frac{\partial^2 v}{\partial y^2} = -\frac{\alpha}{2l(1-\alpha t)^{\frac{3}{2}}} f''(\eta).$$

The Eq. (3.3) can be written as:

$$-\frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\left(f(\eta)+\eta f'(\eta)\right) + 0 + \frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\ f(\eta)\ f'(\eta)$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\alpha\nu}{2l\ (1\ -\alpha t)^{\frac{3}{2}}}\ f''(\eta).$$
$$\Rightarrow\ \frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\left(f(\eta)\ f'(\eta)\ - f(\eta)\ -\eta\ f'(\eta)\right)$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\alpha\nu}{2l(1-\alpha t)^{\frac{3}{2}}} f''(\eta). \tag{3.10}$$

Differentiating (3.10) w.r.t x, we get:

$$\frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\left(f(\eta)\ f''(\eta\ + (f'(\eta))^2\ - f'(\eta)\ -\eta\ f''(\eta)\ - f'(\eta)\right)\frac{\partial\eta}{\partial x}$$
$$= -\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y} - \left(\frac{\alpha\nu}{2l(1-\alpha t)^{\frac{3}{2}}}\ f'''(\eta)\right)\frac{\partial\eta}{\partial x}$$
$$\Rightarrow\ \frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y} = 0. \qquad \left(\because \frac{\partial\eta}{\partial x} = 0\right) \tag{3.11}$$

Using (3.11) in (3.9) and we get:

$$\frac{\alpha^2 x}{2(1-\alpha t)^2}\left(\frac{3}{2}\ f''(\eta)\ + \frac{\eta}{2}\ f'''(\eta)\ - \frac{1}{2}\ f(\eta)\ f'''(\eta)\ + \frac{1}{2}\ f'(\eta)\ f''(\eta)\right)\frac{\partial\eta}{\partial y}$$
$$= 0 + \frac{\alpha\,\nu\,x}{2l^2(\ 1-\alpha t\ )^2}\ f^{(iv)}(\eta)\ \frac{\partial\eta}{\partial y}.$$

$$\Rightarrow \quad \frac{\alpha^2 x}{2(1-\alpha t)^2}\left(\frac{3}{2}\, f''(\eta) \,+ \frac{\eta}{2}\, f'''(\eta) \,- \frac{1}{2}\, f(\eta)\, f'''(\eta) \,+ \frac{1}{2}\, f'(\eta)\, f''(\eta)\right) \frac{\partial \eta}{\partial y}$$
$$= \frac{\alpha \nu x}{2l^2(1-\alpha t)^2}\; f^{(iv)}(\eta)\, \frac{\partial \eta}{\partial y}.$$
$$\Rightarrow \quad \frac{\alpha}{2}\Big(3\, f''(\eta) \,+ \eta\, f'''(\eta) \,+ f'(\eta)\, f''(\eta) \,- f(\eta)\, f'''(\eta)\Big) = \frac{\nu}{l^2}\, f^{(iv)}(\eta).$$
$$\Rightarrow \quad \frac{\alpha l^2}{2\nu}\Big(3\, f''(\eta) \,+ \eta\, f'''(\eta) \,+ f'(\eta)\, f''(\eta) \,- f(\eta)\, f'''(\eta)\Big) = \; f^{(iv)}(\eta).$$

Hence the dimensionless form of (3.2)and (3.3) is given by:

$$f^{(iv)}(\eta) \,- S\, \Big(3\, f''(\eta) \,+ \eta\, f'''(\eta) \,+ f'(\eta)\, f''(\eta) \,- f(\eta)\, f'''(\eta)\Big) = 0. \qquad (3.12)$$

To convert the temperature equation into an ordinary differential equation, we first calculate the following derivatives:

- $\theta(\eta) = \dfrac{T}{T_H}.$
- $T = T_H\, \theta(\eta).$
- $\dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial t}\,(T_H\, \theta(\eta))$
  $= \dfrac{\alpha \eta T_H}{2(1-\alpha t)} \theta'(\eta).$
- $\dfrac{\partial T}{\partial x} = \dfrac{\partial}{\partial x}\,(T_H\, \theta(\eta))$
  $= 0. \qquad \left(\because\; \dfrac{\partial \eta}{\partial x} = 0.\right)$
- $u \dfrac{\partial T}{\partial x} = \left(\dfrac{\alpha x}{2(1-\alpha t)} f'(\eta)\right) .0$
  $= 0.$
- $\dfrac{\partial T}{\partial y} = \dfrac{\partial}{\partial y}\,(T_H \theta(\eta))$
  $= \dfrac{T_H}{l\sqrt{1-\alpha t}} \theta'(\eta).$
- $v \dfrac{\partial T}{\partial y} = -\,\dfrac{\alpha l}{[\,2(1\,-\alpha t)^{\frac{1}{2}}\,]}\; f(\eta)\; \dfrac{T_H}{l\,\sqrt{1\,-\alpha t}}\; \theta'(\eta)$
  $= -\dfrac{\alpha T_H}{2(1-\alpha t)} f(\eta)\theta'(\eta).$
- $\dfrac{\partial^2 T}{\partial x^2} = 0.$

- $$\frac{\partial^2 T}{\partial y^2} = \frac{T_H}{l^2(1-\alpha t)}\theta''(\eta).$$

The right side of the equation (3.4) as follows:

$$\frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] = \frac{\nu}{C_p}\left[4\left(\frac{\alpha}{2(1-\alpha t)}f'(\eta)\right)^2 + \left(\frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}}f''(\eta) + 0\right)^2\right].$$

$$\Rightarrow \frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] = \frac{\nu}{C_p}\left[4\left(\frac{\alpha}{2(1-\alpha t)}f'(\eta)\right)^2 + \left(\frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}}f''(\eta)\right)^2\right]. \tag{3.13}$$

Substituting above derivatives and equation (3.13) in (3.4), we get:

$$\frac{\alpha\eta T_H}{2(1-\alpha t)}\theta' + 0 - \frac{\alpha T_H}{2(1-\alpha t)}f(\eta)\,\theta'(\eta) = \frac{kT_H}{\rho C_p l^2(1-\alpha t)}\theta''(\eta) + \frac{\nu}{C_p}\left[4\left(\frac{\alpha}{2\,(1-\alpha\, t)}f'(\eta)\right)^2 + \left(\frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}}f''(\eta) +\right)^2\right]. \tag{3.14}$$

Multiplying (3.14) by $\dfrac{\rho C_p l^2(1-\alpha t)}{kT_H}$ we get:

$$\frac{\alpha\rho C_p l^2}{2k}\Big(\eta\theta'(\eta) - f(\eta)\theta'(\eta)\Big) = \theta'' + \frac{\alpha^2\nu\rho l^2}{kT_H(1-\alpha t)}\Big(f'(\eta)\Big)^2 + \frac{\rho\nu\alpha^2 x^2}{4kT_H(1-\alpha t)^2}\Big(f''(\eta)\Big)^2.$$

$$\Rightarrow \theta'' + \frac{\alpha\rho C_p l^2}{2k}\Big(f(\eta)\,\theta'(\eta) - \eta\theta'(\eta)\Big) + \frac{\alpha^2\nu\rho l^2}{kT_H(1-\alpha t)}\Big(f'(\eta)\Big)^2 + \frac{\rho\nu\alpha^2 x^2}{4kT_H(1-\alpha t)^2}\Big(f''(\eta)\Big)^2 = 0.$$

$$\Rightarrow \theta'' + Pr\,S\Big(f(\eta)\,\theta'(\eta) - \eta\,\theta'(\eta)\Big) + \frac{\alpha^2\,\mu\,l^2}{k\,T_H(1-\alpha t)}\Big(f'(\eta)\Big)^2 + \frac{\mu\alpha^2 x^2}{4kT_H(1-\alpha t)^2}\Big(f''(\eta)\Big)^2 = 0.$$

$$\Rightarrow \quad \theta'' + Pr\,S\left(f(\eta)\,\theta'(\eta)\; - \eta\,\theta'(\eta)\right) + \frac{\alpha^2\mu C_p l^2}{kC_pT_H(1-\alpha t)}\left(f'(\eta)\right)^2$$
$$+\frac{\mu C_p\alpha^2x^2}{4kC_pT_H(1-\alpha t)^2}\left(f''(\eta)\right)^2 = 0.$$
$$\Rightarrow \quad \theta'' + Pr\,S\left(f(\eta)\,\theta'(\eta)\; - \eta\,\theta'(\eta)\right) + PrE_c\left(f'(\eta)\right)^2$$
$$+ Pr\,E_{cx}\left(f''(\eta)\right)^2 = 0. \tag{3.15}$$

Similarly the following procedure elaborates the conversion of concentration equation into the dimensionless form:

- $$\phi(\eta) = \frac{C}{C_H}.$$
  $$\Rightarrow \; C = C_H\,\phi(\eta).$$
- $$\frac{\partial C}{\partial t} = \frac{\partial}{\partial t}\left(C_H\phi(\eta)\right)$$
  $$= \frac{\alpha\eta C_H}{2(1-\alpha t)}\phi'(\eta).$$
- $$\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(C_H\,\phi(\eta)\right)$$
  $$= \; 0.$$
- $$u\,\frac{\partial C}{\partial x} = 0.$$
- $$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(C_H\phi(\eta)\right)$$
  $$= \frac{C_H}{l\sqrt{1-\alpha t}}\phi'(\eta).$$
- $$v\frac{\partial C}{\partial y} = \left(-\frac{\alpha l}{[2(1-\alpha t)^{\frac{1}{2}}]}f(\eta)\right)\left(\frac{C_H}{l\sqrt{1-\alpha t}}\phi'(\eta)\right)$$
  $$= -\frac{\alpha C_H}{2(1-\alpha t)}f(\eta)\phi'(\eta).$$
- $$\frac{\partial^2 C}{\partial x^2} = 0.$$
- $$\frac{\partial^2 C}{\partial y^2} = \frac{C_H}{l^2(1-\alpha t)}\phi''(\eta).$$

Now, the equation (3.5) can be reformed as:

$$\begin{aligned}
&\frac{\alpha\eta C_H}{2(1-\alpha t)}\,\phi'(\eta)\;+0\;-\frac{\alpha\, C_H}{2\,(1-\alpha t)}\,f(\eta)\,\phi'(\eta)=\frac{DC_H}{l^2(1-\alpha t)}\Big(\phi''(\eta)\Big)\\
&\qquad\qquad -K_1(t)\phi(\eta)C_H\\
\Rightarrow\;&\frac{\alpha C_H}{2(1-\alpha t)}\Big(\eta\phi'(\eta)-f(\eta)\phi'(\eta)\Big)=\frac{DC_H}{l^2(1-\alpha t)}\Big(\phi''(\eta)\Big)\\
&\qquad\qquad -K_1(t)\phi(\eta)C_H.
\end{aligned}\tag{3.16}$$

Multiplying equation (3.17) by $\dfrac{l^2(1-\alpha t)}{DC_H}$,

$$\begin{aligned}
&\frac{\alpha l^2}{2D}\Big(\eta\,\phi'(\eta)\;-f(\eta)\,\phi'(\eta)\Big)=\phi''(\eta)\;-\frac{l^2\,(1\;-\alpha t)}{D}\,K_1(t)\,\phi(\eta)\\
\Rightarrow\;&\phi''(\eta)-\frac{\alpha l^2}{2D}\Big(\eta\,\phi'(\eta)-f(\eta)\,\phi'(\eta)\Big)-\frac{l^2\,(1\;-\alpha t)}{D}\,K_1(t)\,\phi(\eta)=\;0.\\
\Rightarrow\;&\phi''(\eta)+\frac{\alpha l^2}{2D}\Big(f(\eta)\,\phi'(\eta)\;-\eta\,\phi'(\eta)\Big)-\frac{l^2\,(1-\alpha t)}{D}\,K_1(t)\,\phi(\eta)=\;0.\\
\Rightarrow\;&\phi''(\eta)+\frac{\nu S}{D}\Big(f(\eta)\,\phi'(\eta)\;-\eta\,\phi'(\eta)\Big)-\frac{k_1 l^2}{D}\,\phi(\eta)=0.\\
\Rightarrow\;&\phi''(\eta)\;+S_c\,S\,\Big(f(\eta)\,\phi'(\eta)\;-\eta\,\phi'(\eta)\Big)\;-S_c\,\gamma\,\phi(\eta)=0.
\end{aligned}\tag{3.17}$$

Dimensionless form of (3.6) is given as follows:

- $$\begin{aligned}&\eta=\frac{y}{l\sqrt{1-\alpha t}}\\ \Rightarrow\;&\eta=\frac{0}{l\sqrt{1-\alpha t}}.\\ \Rightarrow\;&\eta=\;0. \qquad\qquad at\;\; y=0\end{aligned}$$

- $$\begin{aligned}&v(x,y,t)=0 \qquad\qquad at\;\; y=0\\ \Rightarrow\;&-\frac{\alpha l}{2\sqrt{1-\alpha t}}f(\eta)=0. \qquad\qquad at\;\; \eta=0\\ \Rightarrow\;&f(0)=0.\end{aligned}$$

- $$\begin{aligned}&\frac{\partial u}{\partial y}=0 \qquad\qquad at\;\; y=0\\ \Rightarrow\;&\frac{\alpha x}{2l(1-\alpha t)^{\frac{3}{2}}}f''(\eta)=0. \qquad\qquad at\;\; \eta=0\\ \Rightarrow\;&f''(0)=0.\end{aligned}$$

- $$\frac{\partial T}{\partial y} = 0 \qquad at \;\; y = 0$$
  $$\Rightarrow \frac{T_H}{l\sqrt{1-\alpha t}}\theta'(\eta) = 0. \qquad at \;\; \eta = 0$$
  $$\Rightarrow \theta'(0) = 0.$$
- $$\frac{\partial C}{\partial y} = 0 \qquad at \;\; y = 0$$
  $$\Rightarrow \frac{C_H}{l\sqrt{1-\alpha t}}\phi'(\eta) = 0. \qquad at \;\; \eta = 0$$
  $$\Rightarrow \phi'(0) = 0.$$
- $$\eta = \frac{y}{l\sqrt{1-\alpha t}}$$
  $$\Rightarrow \eta = \frac{h(t)}{l\sqrt{1-\alpha t}}. \qquad at \;\; y = h(t)$$
  $$\Rightarrow \eta = \frac{l\sqrt{1-\alpha t}}{l\sqrt{1-\alpha t}}. \qquad at \;\; y = h(t)$$
  $$\Rightarrow \eta = 1.$$
- $$u = 0 \qquad at \;\; y = h(t)$$
  $$\Rightarrow \frac{\alpha x}{2(1-\alpha t)} f'(\eta) = 0. \qquad at \;\; \eta = 1$$
  $$\Rightarrow f'(1) = 0.$$
- $$v = v_w = \frac{dh}{dt} \qquad at \;\; y = h(t)$$
  $$\Rightarrow -\frac{\alpha l}{2\sqrt{1-\alpha t}} f(\eta) = -\frac{\alpha l}{2\sqrt{1-\alpha t}}. \qquad at \;\; \eta = 1$$
  $$\Rightarrow f(1) = 1.$$
- $$T = T_H \qquad at \;\; y = h(t)$$
  $$\Rightarrow T_H\theta(\eta) = T_H. \qquad at \;\; \eta = 1$$
  $$\Rightarrow \theta(1) = 1.$$
- $$C = C_H \qquad at \;\; y = h(t)$$
  $$\Rightarrow C_H\phi(\eta) = C_H. \qquad at \;\; \eta = 1$$
  $$\Rightarrow \phi(1) = 1.$$

Finally, the following ordinary differential equation system is obtained:

$$f^{(iv)}(\eta) - S\left(3\,f''(\eta) + \eta\,f'''(\eta)\; + f'(\eta)\,f''(\eta)\; - f(\eta)\,f'''(\eta)\right) = 0. \tag{3.18}$$

$$\theta'' + Pr\,S\left(f(\eta)\,\theta'(\eta)\; - \eta\,\theta'(\eta)\right) + Pr\,E_c\left(f'(\eta)\right)^2 + Pr\,E_{cx}\left(f''(\eta)\right)^2 = 0. \tag{3.19}$$

$$\phi''(\eta)\; + S_c\,S\left(f(\eta)\,\phi'(\eta)\; - \eta\,\phi'(\eta)\right) - S_c\,\gamma\,\phi(\eta) = 0. \tag{3.20}$$

The dimensionless conditions associated with the boundary are as follows:

$$\left.\begin{array}{l} f(0) = 0,\ f''(0) = 0,\ \theta'(0) = 0,\ \phi'(0) = 0, \\ f(1) = 1,\ f'(1) = 0,\ \theta(1)\ = \phi(1)\ = 1. \end{array}\right\} \tag{3.21}$$

Different parameters used in the above equations have the following formulation:

$$\left.\begin{array}{l} Pr = \dfrac{\mu C_p}{k},\ S = \dfrac{\alpha l^2}{2\nu}, S_c = \dfrac{\nu}{D},\ \gamma = \dfrac{k_1 l^2}{\nu}, \\ E_c = \dfrac{\alpha^2 l^2}{T_H C_p(1-\alpha t)},\ E_{cx} = \dfrac{\alpha^2 x^2}{4C_p T_H (1-\alpha t)^2}. \end{array}\right\} \tag{3.22}$$

The skin friction coefficient, is given as follows:

- $$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)|_{y=h(t)}}{\rho v_w^2}$$

$$\Rightarrow\ C_f = \frac{\mu\alpha x}{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}} f''(\eta)|_{h(t)}. \tag{3.23}$$

$$\eta = \frac{y}{l(1-\alpha t)^{\frac{1}{2}}}$$

$$\Rightarrow\quad \eta = \frac{l(1-\alpha t)^{\frac{1}{2}}}{l(1-\alpha t)^{\frac{1}{2}}}. \qquad \because\quad y = h(t) = l(1-\alpha t)^{\frac{1}{2}}$$

$$\Rightarrow\quad \eta = 1 \qquad at\ \ y = h(t) \tag{3.24}$$

Using (3.24) in equation (3.23), we get the following form:

$$C_f = \frac{\mu\alpha x}{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}} f''(1)$$

$$\Rightarrow \frac{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}}{\mu\alpha x}C_f = f''(1)$$
$$\Rightarrow \left(\frac{2\rho v_w^2 x\sqrt{1-\alpha t}}{\alpha l\mu}\right)\left(\frac{l^2(1-\alpha t)}{x^2}\right)C_f = f''(1)$$
$$\Rightarrow \frac{l^2}{x^2}(1-\alpha t)R_{ex}C_f = f''(1). \tag{3.25}$$

where,

$$R_{ex} = \frac{2\rho v_w^2 x\sqrt{1-\alpha t}}{\alpha l\mu}.$$

Local Nusselt number is defined as the follow:

- $$Nu = -\frac{lk\left(\frac{\partial T}{\partial y}\right)|_{y=h(t)}}{kT_H}.$$

$$\Rightarrow Nu = -\frac{lkT_H}{lkT_H\sqrt{1-\alpha t}}\theta'(\eta)|_{y=h(t)}.$$
$$\Rightarrow Nu = -\frac{1}{\sqrt{1-\alpha t}}\theta'(1)$$
$$\Rightarrow \sqrt{1-\alpha t}Nu = -\theta'(1). \tag{3.26}$$

The local sherwood number is defined as:

- $$Sh = -\frac{lD\left(\frac{\partial C}{\partial y}\right)|_{y=h(t)}}{DC_H}$$

$$\Rightarrow Sh = -\frac{lDC_H}{lDC_H\sqrt{1-\alpha t}}\phi'(\eta)|_{y=h(t)}$$
$$\Rightarrow Sh = -\frac{1}{\sqrt{1-\alpha t}}\phi'(1)$$
$$\Rightarrow \sqrt{1-\alpha t}Sh = -\phi'(1). \tag{3.27}$$

3.3 Solution Methodology

The shooting method has been used to solve the ordinary differential equation system (3.18)-(3.20). Equation (3.18) is numerically solved and then its solution is used in equations (3.19)-(3.20). To solve the equation (3.18) independently by using the shooting method, the following notations have been considered:

$$\begin{aligned} f &= y_1, \\ f' &= y_1' = y_2, \\ f'' &= y_1'' = y_2' = y_3, \\ f''' &= y_1''' = y_2'' = y_3' = y_4. \end{aligned}$$

By using the above notations in equation (3.18), the following system of ODEs is obtained:

$$\begin{aligned} y_1' &= y_2, & y_1(0) &= 0, \\ y_2' &= y_3, & y_2(0) &= r, \\ y_3' &= y_4, & y_3(0) &= 0, \\ y_4' &= S\left(\eta y_4 + 3y_3 + y_2 y_3 - y_1 y_4\right), & y_4(0) &= s. \end{aligned}$$

The above initial value problem will be numerically solved by Runge-Kutta technique of order four. In the above initial value problem, the missing conditions r and s are to be chosen such that:

$$y_1(\eta, r, s)_{\eta=1} - 1 = 0, \qquad y_2(\eta, r, s,)_{\eta=1} = 0. \tag{3.28}$$

To solve the above system of algebraic equation (3.28), we use the Newton's method which has the following iterative scheme:

$$\begin{pmatrix} r^{(k+1)} \\ s^{(k+1)} \end{pmatrix} = \begin{pmatrix} r^{(k)} \\ s^{(k)} \end{pmatrix} - \begin{pmatrix} \frac{\partial y_1}{\partial r} & \frac{\partial y_2}{\partial r} \\ \frac{\partial y_1}{\partial s} & \frac{\partial y_2}{\partial s} \end{pmatrix}^{-1}_{\eta=1} \begin{pmatrix} y_1(\eta, r^{(k)}, s^{(k)}) - 1 \\ y_2(\eta, r^{(k)}, s^{(k)}) \end{pmatrix}_{\eta=1}.$$

To incorporate Newton's method, we further use the following notations:

$$\begin{aligned} &\frac{\partial y_1}{\partial r} = y_5, \quad \frac{\partial y_2}{\partial r} = y_6, \quad \frac{\partial y_3}{\partial r} = y_7, \quad \frac{\partial y_4}{\partial r} = y_8, \\ &\frac{\partial y_1}{\partial s} = y_9, \quad \frac{\partial y_2}{\partial s} = y_{10}, \quad \frac{\partial y_3}{\partial s} = y_{11}, \quad \frac{\partial y_4}{\partial s} = y_{12}. \end{aligned}$$

As a result of these new notations, the Newton's iterative scheme gets the form:

$$\begin{pmatrix} r^{(k+1)} \\ s^{(k+1)} \end{pmatrix} = \begin{pmatrix} r^{(k)} \\ s^{(k)} \end{pmatrix} - \begin{pmatrix} y_5 & y_6 \\ y_9 & y_{10} \end{pmatrix}^{-1}_{\eta=1} \begin{pmatrix} y_1(\eta,\ r^{(k)},\ s^{(k)}) - 1 \\ y_2(\eta, r^{(k)}, s^{(k)}) \end{pmatrix}_{\eta=1}. \tag{3.29}$$

Here k is the number of iterations $(k = 0, 1, 2, 3...)$. Now differentiating the above system of four first order ODEs with respect to r and s, we get another system of eight ODEs. Writing all these twelve ODEs together, we have the following initial value problem (IVP):

$$\begin{aligned}
y_1' &= y_2, & y_1(0) &= 0,\\
y_2' &= y_3, & y_2(0) &= r,\\
y_3' &= y_4, & y_3(0) &= 0,\\
y_4' &= S\left(\eta y_4 + 3y_3 + y_2 y_3 - y_1 y_4\right), & y_4(0) &= s,\\
y_5' &= y_6, & y_5(0) &= 0,\\
y_6' &= y_7, & y_6(0) &= 1,\\
y_7' &= y_8, & y_7(0) &= 0,\\
y_8' &= S\left(\eta y_8 + 3y_7 + y_6 y_3 + y_2 y_7 - y_5 y_4 - y_1 y_8\right), & y_8(0) &= 0,\\
y_9' &= y_{10}, & y_9(0) &= 0,\\
y_{10}' &= y_{11}, & y_{10}(0) &= 0,\\
y_{11}' &= y_{12}, & y_{11}(0) &= 0,\\
y_{12}' &= S\left(\eta y_{12} + 3y_{11} + y_{10} y_3 + y_2 y_{11} - y_9 y_4 - y_1 y_{12}\right), & y_{12}(0) &= 1.
\end{aligned}$$

The RK-4 method has been used to solve the IVP consisting of the above twelve ODEs for some suitable choices of r and s. The missing conditions r and s are updated by using Newtons's scheme (3.29). The iterative procedure is stopped when the following condition is met:

$$\max\{| r^{(k+1)} - r^{(k)} |,\ | s^{(k+1)} - s^{(k)} |\} < \epsilon,$$

for an arbitrarily small positive value of ϵ. Throughout this chapter ϵ has been taken as $(10)^{-10}$.

Now equation (3.19) will be treated similarly by considering f as a known function. For this, let us denote θ by y_{13} and $\theta' = y'_{13}$ by y_{14}.

By using the above notations in equation (3.19), we get the system of equations:

$$\begin{aligned} y'_{13} =& y_{14}, & y_{13}(0) = m, \\ y'_{14} =& PrS\left(\eta y_{14} - y_1 y_{14}\right) - PrE_c y_2^2 - PrE_{cx} y_3^2, & y_{14}(0) = 0. \end{aligned}$$

The above initial value problem will be numerically solved by the fourth order Runge-Kutta technique. In the above initial value problem, the missing condition m is to be chosen such that:

$$y_{13}(\eta, m)_{\eta=1} - 1 = 0, \tag{3.30}$$

To solve the above algebraic equation (3.30), we use the Newton's method which has the following iterative scheme:

$$m^{(k+1)} = m^{(k)} - \left(\frac{\partial y_{13}}{\partial m}\right)^{-1} \left(y_{13}(\eta, m^{(k)})_{\eta=1} - 1\right).$$

To incorporate Newton's method, we further use the following notations:

$$\frac{\partial y_{13}}{\partial m} = y_{15}, \qquad \frac{\partial y_{14}}{\partial m} = y_{16}.$$

As a result of these new notations, the Newton's iterative scheme gets form:

$$m^{(k+1)} = m^{(k)} - \left(y_{15}(\eta, m^{(k)})_{\eta=1}\right)^{-1} \left(y_{13}(\eta, m^{(k)})_{\eta=1} - 1\right). \tag{3.31}$$

Here k is the number of iterations ($k = 0, 1, 2, 3...$). Now differentiating the above system of two first order ODEs with respect to m, we get another system of four ODEs. Writing all these four ODEs together, we have the following initial value problem (IVP):

$$
\begin{aligned}
y'_{13} &= y_{14}, & y_{13}(0) &= m,\\
y'_{14} &= PrS\left(\eta y_{14} - y_1 y_{14}\right) - PrE_c y_2^2 - PrE_{cx} y_3^2, & y_{14}(0) &= 0,\\
y'_{15} &= y_{16}, & y_{15}(0) &= 1,\\
y'_{16} &= PrS\left(\eta y_{16} - y_1 y_{14}\right), & y_{16}(0) &= 0.
\end{aligned}
$$

The RK-4 method has been used to solve the IVP consisting of the above four ODEs for some suitable choices of m. The missing condition m is updated by using Newtons's scheme (3.31). The iterative procedure is stopped when the following condition is met:

$$\mid m^{(k+1)} - m^{(k)} \mid < \epsilon,$$

for an arbitrarily small positive value of ϵ. Throughout this chapter ϵ has been taken as $(10)^{-10}$.

In a similar manner equation (3.20) can be treated numerically by the shooting techniques by considering f as a known function.

3.4 Results and Discussion

The main objective is about to observe the effects of various parameters against the velocity, the temperature and the concentration profiles. We compared the obtained results with those of [25] for verification. The influence of various parameters like squeezing parameter S, Eckert number E_c, Prandtl number Pr, Schmidt number S_c and the chemical reaction parameter γ is observed graphically. Numerical results of the Sherwood number, the coefficient of skin friction and the Nusselt number for the distinct values of S with some fixed parameters are shown in Tables 3.1-3.3.

Figure 3.2 displays the impact of S on the dimensionless velocity profile. It can be noted that the fluid velocity reduces by enhancing the values of squeezing parameter. Figure 3.3 present the impact of both positive and negative squeezing parameters on the temperature distribution. The larger values of S gives the significant reduction in the temperature profile. When plates move towards each other, it can be noted that the temperature profile is escalated. Figure 3.4 illustrated the impact of Pr on the θ. Due to the presence of viscous distraction impact, the temperature profile θ increases. Prandtl number $Pr < 1$ describe the liquid material with high thermal diffusivity but low viscosity, whereas the viscosity of liquid material is high for the Prandtl number $Pr > 1$. Figure 3.5 is delineated to show the impact of E_c on the temperature field θ. This graph exhibits that by enhancing the estimations of Ec, the temperature field θ is also increased. The thermal boundary layer thickness experiences a decrement by enhancing the values of Pr and E_c. Figure 3.6 is delineated to show the impact of E_{cx} on the θ of the fluid. It can be noted that, the field θ is also increased by enhancing the values of E_{cx}. Figure 3.7 illustrated the impact of S on the field ϕ of the fluid. The outcomes of S_c on the field ϕ are presented in the Figure 3.8. The molecular diffusivity is found to become weaker and the thickness of the layer at the boundary is thinner due to the gradational increment in the S_c. Figure 3.9 delineate the impact of γ on the concentration elds. For $\gamma > 0$, the concentration field ϕ decline significantly, whereas an increase in the concentration profile ϕ is very much visible for the $\gamma < 0$. Steeper curves are observed when larger estimations of γ are accompanied with severe condition of the reaction that is presented in Figure 3.9.

	$-f''(1)$	
S	Present	Mustafa et al. [25]
-1.0	2.170090	2.170090
-0.5	2.617403	2.614038
0.01	3.007133	3.007134
0.5	3.336449	3.336449
2.0	4.167389	4.167389

TABLE 3.1: Values of the skin friction coefficient for different parameters

				$-\theta'(1)$
S	P_r	E_c	E_{cx}	Present
-1.0	0.2	0.2	0.2	0.177287
-0.5				0.170885
0.01				0.168035
0.5				0.167292
2.0				0.170609
2.0	0.4	0.2	0.2	0.337108
	0.6			0.499663
	0.8			0.658433
2.0	0.2	0.4	0.2	0.215029
		0.6		0.259449
		0.8		0.303870
2.0	0.2	0.2	0.4	0.296798
			0.6	0.422987
			0.8	0.549176

TABLE 3.2: Values of the Nusselt number for the different parameters

.

			$-\phi'(1)$	
S	S_c	γ	Present	Mustafa et al. [25]
-1.0	1.0	1.0	0.804608	0.804558
-0.5			0.781427	0.781402
0.01			0.761224	0.761225
0.5			0.744199	0.744224
2.0			0.701719	0.701813

TABLE 3.3: Values of the reduced Sherwood number for $S_c = \gamma = 1.0$,

.

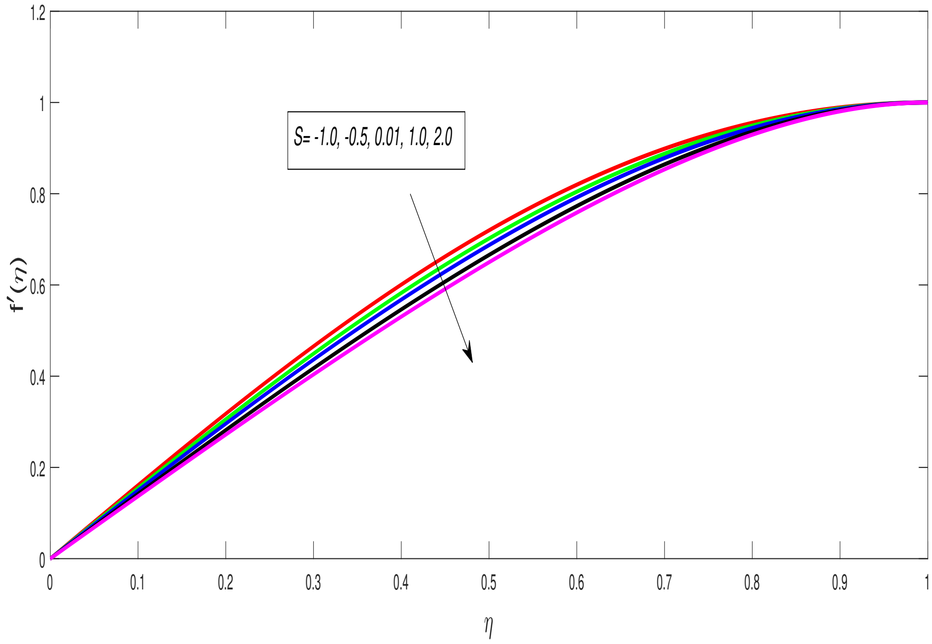

FIGURE 3.2: Influence of S on f'

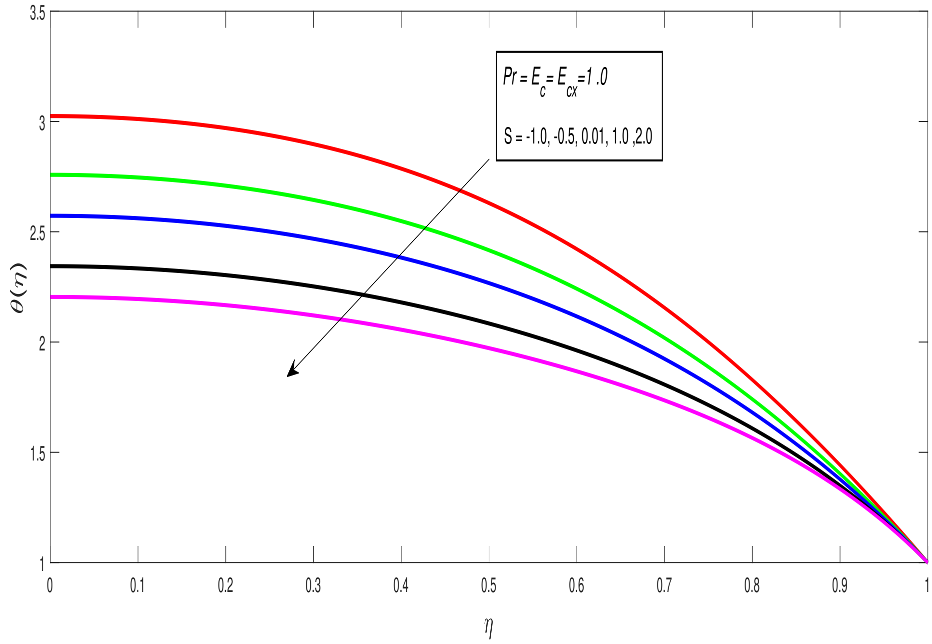

FIGURE 3.3: Influence of S on θ

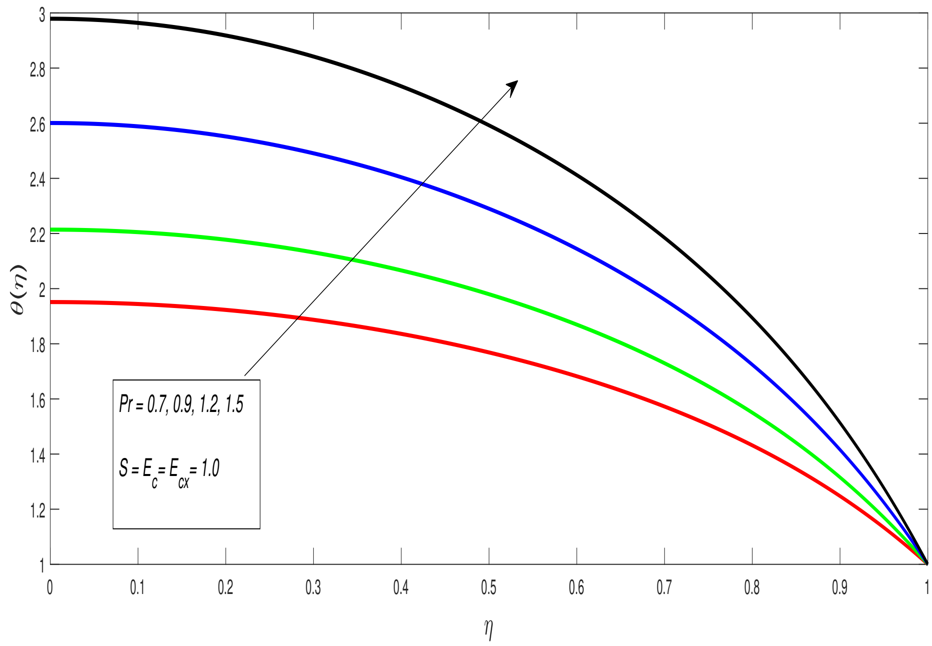

FIGURE 3.4: Influence of Pr on θ

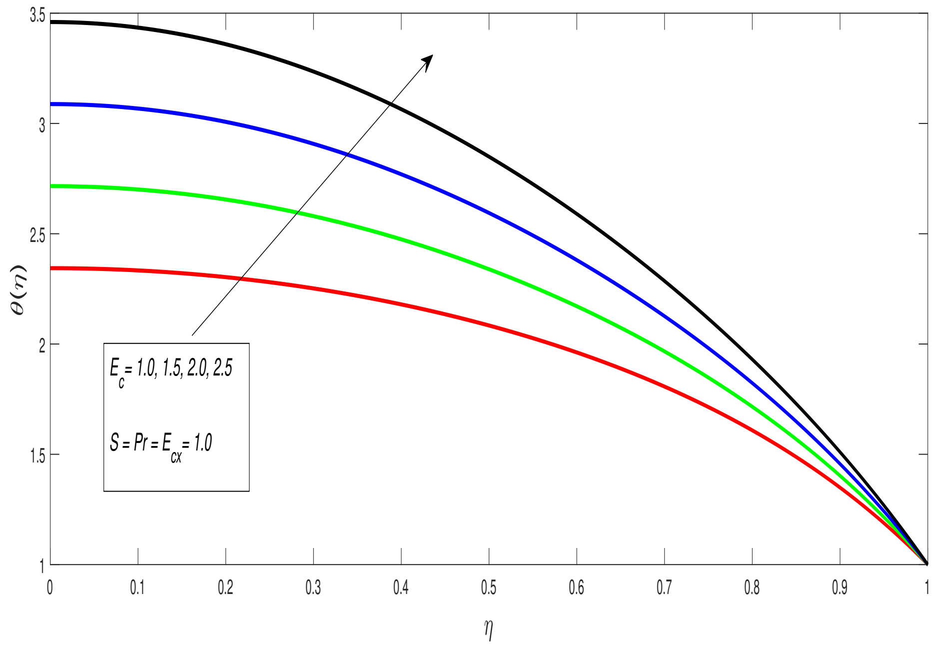

FIGURE 3.5: Influence of E_c on θ

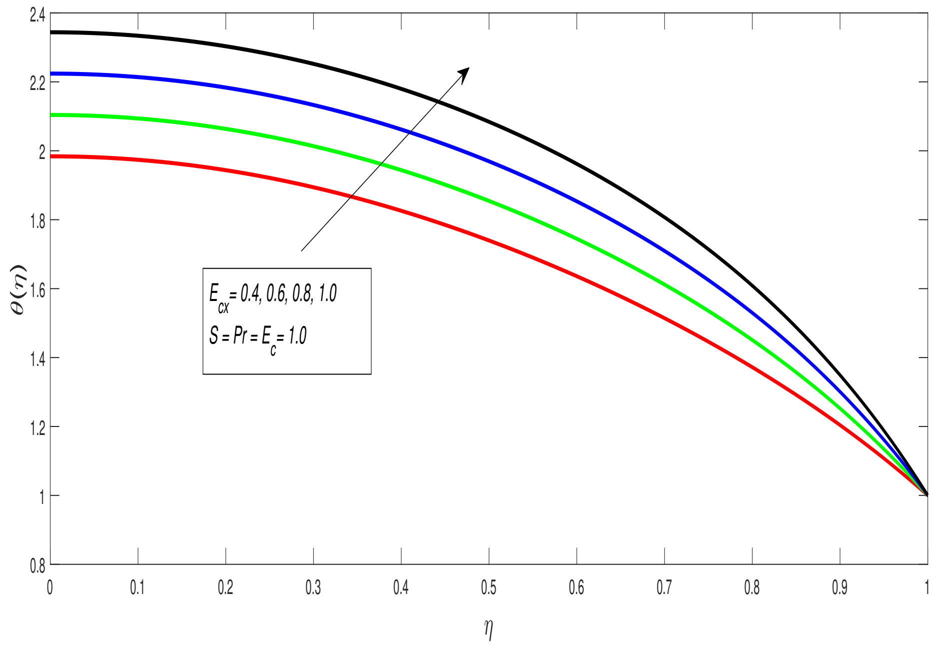

FIGURE 3.6: Influence of E_{cx} on θ

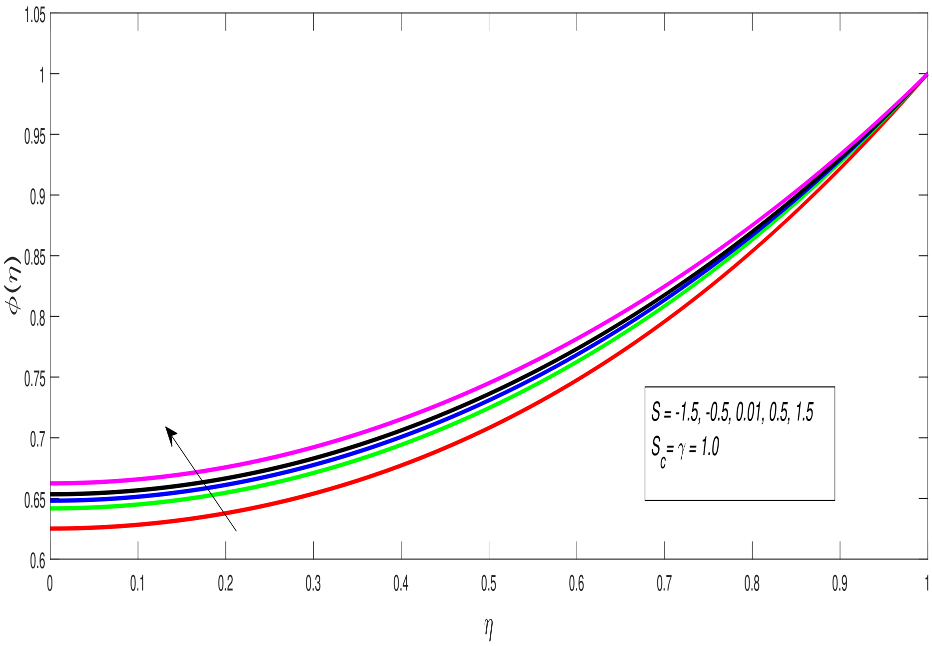

FIGURE 3.7: Influence of S on ϕ

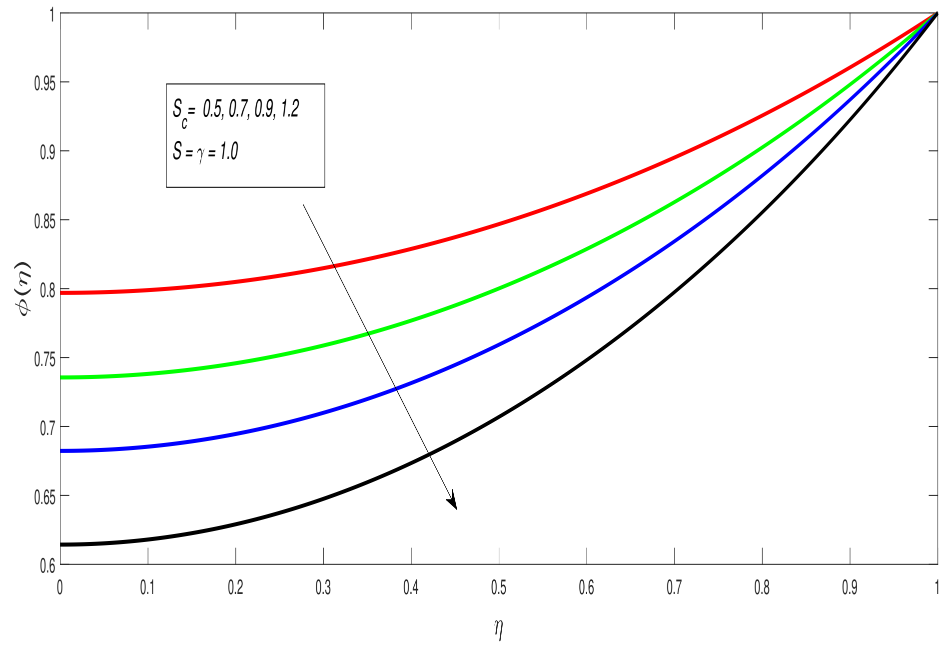

FIGURE 3.8: Influence of S_c on ϕ

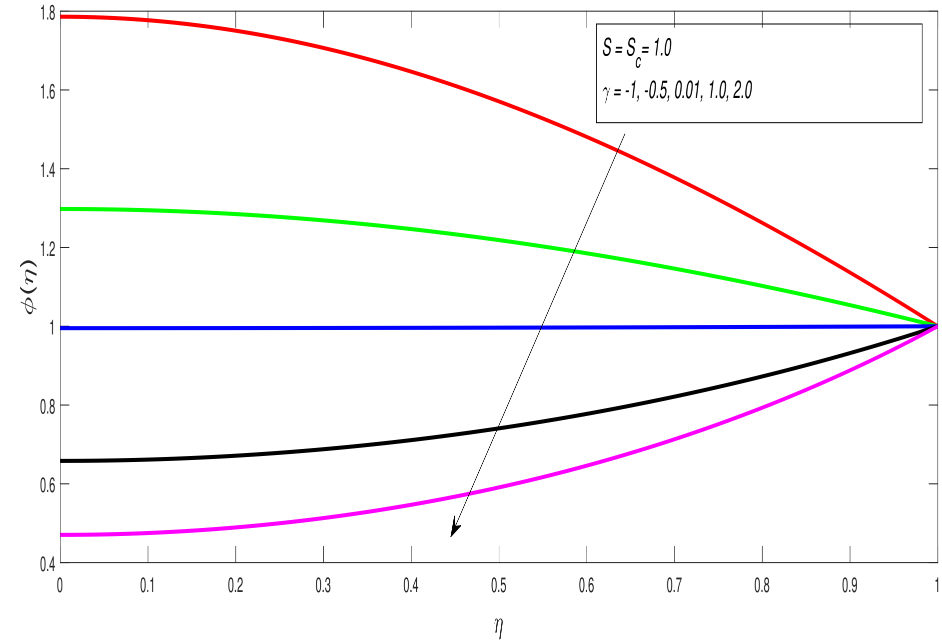

FIGURE 3.9: Influence of γ on ϕ

Chapter 4

Heat and Mass Transfer in the Unsteady Squeezing Flow of UCM Fluid Between Parallel Plates in the Presence of Inclined Magnetic Field

4.1 Introduction

In our work, the central objective is to extend the model of [25] with the effect of upper convected maxwell fluid (UCM) and inclined magnetic field between two parallel plates. We used the appropriate similarity transformations to convert the PDEs into the nonlinear ODEs. In order to solve ODEs, we used the shooting technique whose iterations are computed by MATLAB. By changing the values of different physical parameters, we observed the nature of velocity profile, temperature profile θ, and the concentration profile ϕ. At the end, the graphs and tables are shown which are obtained under this investigation.

4.2 Problem Formulation

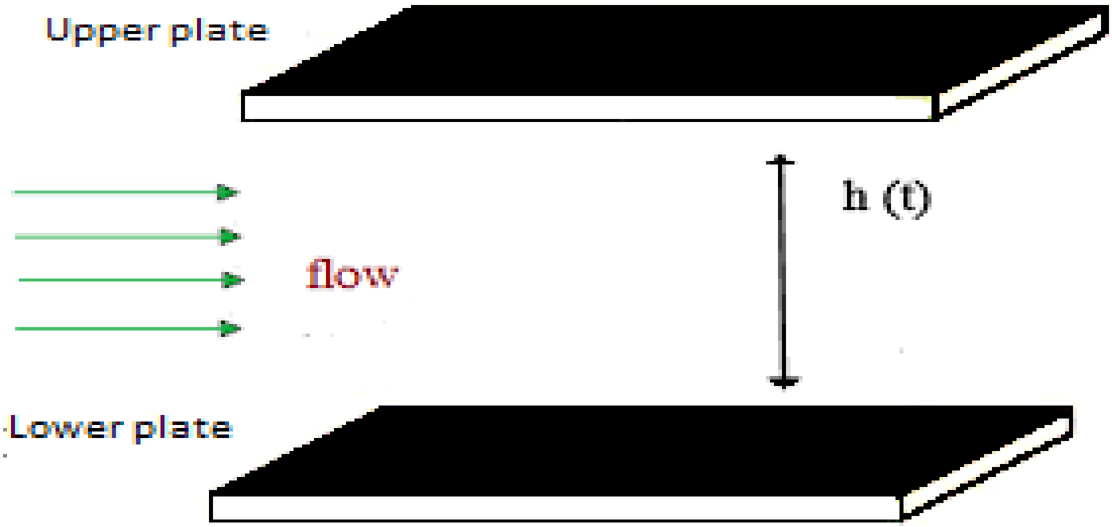

FIGURE 4.1: Physical model of the problem

An incompressible, 2D unsteady UCM fluid flow, which is squeezed between two plates held parallel to each other, along with the inclined magnetic field effect has been considered. The distance between plates is $y = l(1-\alpha t)^{\frac{1}{2}} = h(t)$. The viscous dissipation effect, the generation of heat due to friction caused by shear in the flow, is retained. Moreover the flow has been considered as symmetric.

The flow is explained by considering the two dimensional governing equations containing the equation of continuity, the equation of momentum, the equation of energy and the concentration equation are given as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\sigma B_m^2}{\rho}\sin(\omega)\Big(v\cos(\omega) - u\sin(\omega)\Big) + \beta\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right), \tag{4.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{\sigma B_m^2}{\rho}\cos(\omega)\Big(u\sin(\omega)$$
$$-v\cos(\omega)\Big)+\beta\left(u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+2uv\frac{\partial^2 v}{\partial x\partial y}\right), \quad (4.3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right], \quad (4.4)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-K_1(t)C. \quad (4.5)$$

The conditions associated with the boundary are as:

$$\left.\begin{aligned} &y=h(t): u=0, v=v_w=\frac{dh}{dt}, T=T_H, C=C_H,\\ &y=0: v=\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=\frac{\partial C}{\partial y}=0. \end{aligned}\right\} \quad (4.6)$$

Here, ρ is the fluid density, u an v are the velocity components in the the direction of x and y respectively, T denotes the temperature, C represents the concentration, p denotes the pressure, ν represents the kinematic viscosity, C_p is specific heat, D denotes the coefficient of diffusion and $K_1(t) = \dfrac{k_1}{(1-\alpha t)}$ defines the rate of reaction which depends on time.

For the conversion of the mathematical model (4.1)-(4.5) into the dimensionless form, the following similarity transformations has been introduced.

$$\left.\begin{aligned} &u=\frac{\alpha x}{2(1-\alpha t)}f'(\eta), \eta=\frac{y}{[l(1-\alpha t)^{\frac{1}{2}}]}, v=-\frac{\alpha l}{[2(1-\alpha t)^{\frac{1}{2}}]}f(\eta),\\ &\theta=\frac{T}{T_H}, \phi=\frac{C}{C_H} \end{aligned}\right\} \quad (4.7)$$

The detailed procedure for the verification of the continuity equation (4.1) has been discussed in Chapter 3. For the conversion of (4.2)-(4.5) into the dimensionless form has been described in the upcoming discussion.

The left side of (4.2) takes the following form:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\alpha^2 x}{2(1-\alpha t)^2}\bigg(f'(\eta)+\frac{\eta}{2}f''(\eta)+\frac{1}{2}(f'(\eta))^2 -\frac{1}{2}f(\eta)f''(\eta)\bigg). \tag{4.8}$$

$$\frac{\sigma B_m^2}{\rho}\sin(\omega)\bigg(v\cos(\omega)-u\sin(\omega)\bigg)=\frac{\sigma B_m^2}{\rho}\sin(\omega)\bigg(-\frac{\alpha l}{2\sqrt{1-\alpha t}}f(\eta)\cos(\omega) -\frac{\alpha x}{2(1-\alpha t)}f'(\eta)\sin(\omega)\bigg). \tag{4.9}$$

$$\beta\bigg[u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\bigg]=\frac{\alpha^3 x\beta}{8(1-\alpha t)^3}\bigg[\bigg(f(\eta)\bigg)^2 f'''(\eta) -2f(\eta)f'(\eta)f''(\eta)\bigg]. \tag{4.10}$$

Using (4.8),(4.9),(4.10), the dimensionless form of (4.2) takes the following form:

$$\frac{\alpha^2 x}{2(1-\alpha t)^2}\bigg(f'(\eta)+\frac{\eta}{2}f''(\eta)+\frac{1}{2}(f'(\eta))^2-\frac{1}{2}f(\eta)f''(\eta)\bigg)=-\frac{1}{\rho}\frac{\partial p}{\partial x} +\frac{\alpha\nu x}{2l^2(1-\alpha t)^2}f'''(\eta)+\frac{\sigma B_m^2}{\rho}\sin(\omega)\bigg(-\frac{\alpha l}{2\sqrt{1-\alpha t}}f(\eta)\cos(\omega) -\frac{\alpha x}{2(1-\alpha t)}f'(\eta)\sin(\omega)\bigg)+\frac{\alpha^3 x\beta}{8(1-\alpha t)^3}\bigg[\bigg(f(\eta)\bigg)^2 f'''(\eta)-2f(\eta)f'(\eta)f''(\eta)\bigg]. \tag{4.11}$$

Differentiating (4.11) w.r.t y, we get:

$$\frac{\alpha^2 x}{4(1-\alpha t)^2}\bigg(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\bigg)\frac{\partial\eta}{\partial y}=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y} +\bigg(\frac{\alpha\nu x}{2l^2(1-\alpha t)^2}f^{(iv)}(\eta)\bigg)\frac{\partial\eta}{\partial y}-\frac{\sigma B_m^2}{\rho}\sin(\omega)\bigg(\frac{\alpha l}{2\sqrt{1-\alpha t}}f'(\eta)\cos(\omega) +\frac{\alpha x}{2(1-\alpha t)}f''(\eta)sin(\omega)\bigg)\frac{\partial\eta}{\partial y}+\frac{\alpha^3 x\beta}{8(1-\alpha t)^3}\bigg[\bigg(f(\eta)\bigg)^2 f^{(iv)}(\eta)-2f(\eta)\bigg(f''(\eta)\bigg)^2 -2\bigg(f'(\eta)\bigg)^2 f''(\eta)\bigg]\frac{\partial\eta}{\partial y}. \tag{4.12}$$

The conversion of left hand side of momentum Eq. (4.3) has been shown below. The conversion derivatives is same as shown in Chapter 3.

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\Big(f(\eta)f'(\eta)-f(\eta)-\eta f'(\eta)\Big). \tag{4.13}$$

$$\frac{\sigma B_m^2}{\rho}\cos(\omega)\Big(u\sin(\omega)-v\cos(\omega)\Big)=\frac{\sigma B_m^2}{\rho}\cos(\omega)\Big(\frac{\alpha x}{2(1-\alpha t)} f'(\eta)\sin(\omega)+\frac{\alpha l}{2\sqrt{1-\alpha t}}f(\eta)\cos(\omega)\Big). \tag{4.14}$$

$$\beta\Big[u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+2uv\frac{\partial^2 v}{\partial x\partial y}\Big]=-\frac{\alpha^3 l\beta}{8(1-\alpha t)^{\frac{5}{2}}}\Big[\Big(f(\eta)\Big)^2 f''(\eta)\Big]. \tag{4.15}$$

Substituting (4.13),(4.14) and (4.15) in (4.3), it takes the following form:

$$\frac{\alpha^2 l}{4(1-\alpha t)^{\frac{3}{2}}}\Big(f(\eta)f'(\eta)-f(\eta)-\eta f'(\eta)\Big)=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\sigma B_m^2}{\rho}\cos(\omega)\Big(\frac{\alpha x}{2(1-\alpha t)} f'(\eta)\sin(\omega)+\frac{\alpha l}{2\sqrt{1-\alpha t}}f(\eta)\cos(\omega)\Big)-\frac{\alpha^3 l\beta}{8(1-\alpha t)^{\frac{5}{2}}}\Big[\Big(f(\eta)\Big)^2 f''(\eta)\Big]. \tag{4.16}$$

Differentiating (4.16) w.r.t x, we get:

$$\begin{aligned}0&=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y}+\frac{\sigma B_m^2}{\rho}\cos(\omega).\frac{\alpha}{2(1-\alpha t)}f'(\eta)\sin(\omega)\\ \Rightarrow\quad \frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y}&=\frac{\sigma\alpha B_m^2}{2\rho(1-\alpha t)}f'(\eta)\sin(\omega)\cos(\omega)\end{aligned} \tag{4.17}$$

Using (4.17) in (4.12), we get:

$$\begin{aligned}&\frac{\alpha^2 x}{4(1-\alpha t)^2}\Big(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\Big)\frac{\partial\eta}{\partial y}=-\frac{\sigma\alpha B_m^2}{2\rho(1-\alpha t)}\\ &f'(\eta)\sin(\omega)\cos(\omega)+\Big(\frac{\alpha\nu x}{2l^2(1-\alpha t)^2}f^{(iv)}(\eta)\Big)\frac{\partial\eta}{\partial y}-\frac{\sigma B_m^2}{\rho}\sin(\omega)\Big(\frac{\alpha l}{2\sqrt{1-\alpha t}}\\ &f'(\eta)\cos(\omega)+\frac{\alpha x}{2(1-\alpha t)}f''(\eta)\sin(\omega)\Big)\frac{\partial\eta}{\partial y}+\frac{\alpha^3 x\beta}{8(1-\alpha t)^3}\Big[\Big(f(\eta)\Big)^2 f^{(iv)}(\eta)\\ &-2f(\eta)\Big(f''(\eta)\Big)^2-2\Big(f'(\eta)\Big)^2 f''(\eta)\Big]\frac{\partial\eta}{\partial y}\end{aligned}$$

$$\Rightarrow \quad \frac{\alpha^2 x}{4(1-\alpha t)^2}\left(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\right)\left(\frac{1}{l\sqrt{1-\alpha t}}\right)$$
$$= -\frac{\sigma\alpha B_m^2}{2\rho(1-\alpha t)}f'(\eta)\sin(\omega)\cos(\omega)+\left(\frac{\alpha\nu x}{2l^2(1-\alpha t)^2}f^{(iv)}(\eta)\right)\left(\frac{1}{l\sqrt{1-\alpha t}}\right)$$
$$-\frac{\sigma B_m^2}{\rho}\sin(\omega)\left(\frac{\alpha l}{2\sqrt{1-\alpha t}}f'(\eta)\cos(\omega)+\frac{\alpha x}{2(1-\alpha t)}f''(\eta)\sin(\omega)\right)$$
$$\left(\frac{1}{l\sqrt{1-\alpha t}}\right)+\frac{\alpha^3 x\beta}{8(1-\alpha t)^3}\left[\left(f(\eta)\right)^2 f^{(iv)}(\eta)-2f(\eta)\left(f''(\eta)\right)^2\right.$$
$$\left.-2\left(f'(\eta)\right)^2 f''(\eta)\right]\left(\frac{1}{l\sqrt{1-\alpha t}}\right).$$

$$\Rightarrow \quad \frac{\alpha^2 x}{4l(1-\alpha t)^{\frac{5}{2}}}\left(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\right)=\frac{\alpha\nu x}{2l^3(1-\alpha t)^{\frac{5}{2}}}$$
$$f^{(iv)}(\eta)-\frac{\sigma\alpha x}{2\rho l(1-\alpha t)^{\frac{3}{2}}}\frac{B_0^2}{(1-\alpha t)}\sin(\omega)\left(\frac{2H(t)}{x}f'(\eta)\cos(\omega)+f''(\eta)\sin(\omega)\right)$$
$$+\frac{\alpha^3 x\beta}{8l(1-\alpha t)^{\frac{7}{2}}}\left[\left(f(\eta)\right)^2 f^{(iv)}(\eta)-2f(\eta)\left(f''(\eta)\right)^2-2\left(f'(\eta)\right)^2 f''(\eta)\right].$$

Multiplying each term by $\dfrac{2l^3(1-\alpha t)^{\frac{5}{2}}}{\alpha\nu x}$, we get:

$$\frac{\alpha l^2}{2\nu}\left(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\right)=f^{(iv)}(\eta)-\frac{\sigma B_0^2 l^2}{\rho\nu}\sin(\omega)$$
$$\left(2\delta f'(\eta)\cos(\omega)+f''(\eta)\sin(\omega)\right)+\frac{\alpha^2 l^2\beta}{4\nu(1-\alpha t)}\left[\left(f(\eta)\right)^2 f^{(iv)}(\eta)-2f(\eta)\right.$$
$$\left.\left(f''(\eta)\right)^2-2\left(f'(\eta)\right)^2 f''(\eta)\right]$$

$$\Rightarrow \quad S\left(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\right)=f^{(iv)}(\eta)-M^2\sin(\omega)$$
$$\left(2\delta f'(\eta)\cos(\omega)+f''(\eta)sin(\omega)\right)+S\lambda_r\left[\left(f(\eta)\right)^2 f^{(iv)}(\eta)-2f(\eta)\left(f''(\eta)\right)^2\right.$$
$$\left.-2\left(f'(\eta)\right)^2 f''(\eta)\right].$$

$$\Rightarrow \quad S\Big(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\Big)=\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]$$
$$f^{(iv)}(\eta)-M^2\sin(\omega)\Big(2\delta f'(\eta)\cos(\omega)+f''(\eta)\sin(\omega)\Big)-2S\lambda_r\Big[f(\eta)\Big(f''(\eta)\Big)^2$$
$$+\Big(f'(\eta)\Big)^2 f''(\eta)\Big].$$

Dividing each term by $\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]$, we get:

$$f^{(iv)}(\eta)-\frac{S}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\Big(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)-f(\eta)f'''(\eta)\Big)$$
$$-\frac{M^2}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\sin(\omega)\Big(f''(\eta)\sin(\omega)+2\delta f'(\eta)\cos(\omega)\Big)$$
$$-\frac{2S\lambda_r}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\Big[f(\eta)\Big(f''(\eta)\Big)^2+\Big(f'(\eta)\Big)^2 f''(\eta)\Big]=0. \tag{4.18}$$

Finally, the ODEs describing the proposed flow problem can be re-collected in the following system.

$$f^{(iv)}(\eta)-\frac{S}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\Big(3f''(\eta)+\eta f'''(\eta)+f'(\eta)f''(\eta)$$
$$-f(\eta)f'''(\eta)\Big)-\frac{M^2}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\sin(\omega)\Big(f''(\eta)\sin(\omega)+2\delta f'(\eta)\cos(\omega)\Big)$$
$$-\frac{2S\lambda_r}{\Big[1+S\lambda_r\Big(f(\eta)\Big)^2\Big]}\Big[f(\eta)\Big(f''(\eta)\Big)^2+\Big(f'(\eta)\Big)^2 f''(\eta)\Big]=0, \tag{4.19}$$

$$\theta''+PrS\Big(f(\eta)\theta'(\eta)-\eta\theta'(\eta)\Big)+PrE_c\Big(f'(\eta)\Big)^2+PrE_{cx}\Big(f''(\eta)\Big)^2=0, \tag{4.20}$$

$$\phi''(\eta)+S_cS\Big(f(\eta)\phi'(\eta)-\eta\phi'(\eta)\Big)-S_c\gamma\phi(\eta)=0. \tag{4.21}$$

The dimensionless boundary conditions are as follows:

$$\left.\begin{aligned} &f(0)=0, f''(0)=0, \theta'(0)=0, \phi'(0)=0,\\ &f(1)=1, f'(1)=0, \theta(1)=\phi(1)=1. \end{aligned}\right\} \tag{4.22}$$

Different parameters used in the above equations have the following formulation:

$$\left.\begin{aligned} &Pr=\frac{\mu C_p}{k},\ S=\frac{\alpha l^2}{2\nu}, S_c=\frac{\nu}{D},\ \gamma=\frac{k_1 l^2}{\nu},\ E_c=\frac{\alpha^2 l^2}{T_H C_p(1-\alpha t)},\\ &E_{cx}=\frac{\alpha^2 x^2}{4C_p T_H(1-\alpha t)^2},\ M^2=\frac{\sigma B_0^2 l^2}{\rho\nu},\ \lambda_r=\frac{\alpha\beta}{2(1-\alpha t)},\\ &B_m=\frac{B_0}{\sqrt{1-\alpha t}},\ H(t)=l\sqrt{1-\alpha t}. \end{aligned}\right\}$$

The skin friction coefficient, is given as follows:

- $$C_f=\frac{\mu\left(\frac{\partial u}{\partial y}\right)|_{y=h(t)}}{\rho v_w^2}$$

$$\Rightarrow\ C_f=\frac{\mu\alpha x}{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}}f''(\eta)|_{h(t)}. \tag{4.23}$$

$$\eta=\frac{y}{l(1-\alpha t)^{\frac{1}{2}}}$$

$$\Rightarrow\ \eta=\frac{l(1-\alpha t)^{\frac{1}{2}}}{l(1-\alpha t)^{\frac{1}{2}}}. \qquad \because\ \ y=h(t)=l(1-\alpha t)^{\frac{1}{2}}$$

$$\Rightarrow\ \eta=1 \qquad at\ \ y=h(t) \tag{4.24}$$

Using (4.24) in equation (4.23), we get the following form:

$$C_f=\frac{\mu\alpha x}{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}}f''(1)$$

$$\Rightarrow\ \frac{2l\rho v_w^2(1-\alpha t)^{\frac{3}{2}}}{\mu\alpha x}C_f=f''(1)$$

$$\Rightarrow\ \left(\frac{2\rho v_w^2 x\sqrt{1-\alpha t}}{\alpha l\mu}\right)\left(\frac{l^2(1-\alpha t)}{x^2}\right)C_f=f''(1)$$

$$\Rightarrow\ \frac{l^2}{x^2}(1-\alpha t)R_{ex}C_f=f''(1). \tag{4.25}$$

where,

$$R_{ex} = \frac{2\rho v_w^2 x\sqrt{1-\alpha t}}{\alpha l \mu}.$$

Local Nusselt number is defined as the follow:

$$\begin{aligned}
\bullet \qquad & Nu = -\frac{lk\left(\frac{\partial T}{\partial y}\right)|_{y=h(t)}}{kT_H}. \\
\Rightarrow\ & Nu = -\frac{lkT_H}{lkT_H\sqrt{1-\alpha t}}\theta'(\eta)|_{y=h(t)}. \\
\Rightarrow\ & Nu = -\frac{1}{\sqrt{1-\alpha t}}\theta'(1) \\
\Rightarrow\ & \sqrt{1-\alpha t}Nu = -\theta'(1). \qquad (4.26)
\end{aligned}$$

The local sherwood number is defined as:

$$\begin{aligned}
\bullet \qquad & Sh = -\frac{lD\left(\frac{\partial C}{\partial y}\right)|_{y=h(t)}}{DC_H} \\
\Rightarrow\ & Sh = -\frac{lDC_H}{lDC_H\sqrt{1-\alpha t}}\phi'(\eta)|_{y=h(t)} \\
\Rightarrow\ & Sh = -\frac{1}{\sqrt{1-\alpha t}}\phi'(1) \\
\Rightarrow\ & \sqrt{1-\alpha t}Sh = -\phi'(1). \qquad (4.27)
\end{aligned}$$

4.3 Solution Methodology

The shooting method has been used to solve the ordinary differential equation system (4.19)-(4.21). Equation (4.19) is numerically solved and then its solution is used in equations (4.20)-(4.21). To solve the equation (4.19) independently by using the shooting method, the following notations have been considered:

$$\begin{aligned}
f &= y_1, \\
f' &= y_1' = y_2,
\end{aligned}$$

$$f'' = y_1'' = y_2' = y_3,$$
$$f''' = y_1''' = y_2'' = y_3' = y_4.$$

By using above notations in equation (4.19), the following system of ODEs is obtained:

$$y_1' = y_2, \qquad y_1(0) = 0,$$
$$y_2' = y_3, \qquad y_2(0) = r,$$
$$y_3' = y_4, \qquad y_3(0) = 0,$$

$$y_4' = \frac{1}{1 + S\lambda_r f^2}\Bigg[S\Big(\eta y_4 + 3y_3 + y_2 y_3 - y_1 y_4\Big) + M^2 \sin(\omega)\Big(\sin(\omega) y_3 + 2\delta\cos(\omega) y_2\Big) + 2S\lambda_r\Big(y_2^2 y_3 + y_1 y_3^2\Big)\Bigg], \qquad y_4(0) = s.$$

The above initial value problem will be numerically solved by Runge-Kutta technique of order four. In the above system of equations, the missing conditions r and s are to be chosen such that:

$$y_1(\eta, r, s)_{\eta=1} - 1 = 0 \quad , \qquad y_2(\eta, r, s,)_{\eta=1} = 0. \tag{4.28}$$

To solve the above system of algebraic equation (4.28), we use the Newton's method which has the following iterative scheme:

$$\begin{pmatrix} r^{(k+1)} \\ s^{(k+1)} \end{pmatrix} = \begin{pmatrix} r^{(k)} \\ s^{(k)} \end{pmatrix} - \begin{pmatrix} \frac{\partial y_1}{\partial r} & \frac{\partial y_2}{\partial r} \\ \frac{\partial y_1}{\partial s} & \frac{\partial y_2}{\partial s} \end{pmatrix}^{-1}_{\eta=1} \begin{pmatrix} y_1(\eta, r^{(k)}, s^{(k)}) - 1 \\ y_2(\eta, r^{(k)}, s^{(k)}) \end{pmatrix}_{\eta=1}.$$

To incorporate Newton's method we further use the following notations:

$$\frac{\partial y_1}{\partial r} = y_5, \quad \frac{\partial y_2}{\partial r} = y_6, \quad \frac{\partial y_3}{\partial r} = y_7, \quad \frac{\partial y_4}{\partial r} = y_8,$$
$$\frac{\partial y_1}{\partial s} = y_9, \quad \frac{\partial y_2}{\partial s} = y_{10}, \quad \frac{\partial y_3}{\partial s} = y_{11}, \quad \frac{\partial y_4}{\partial s} = y_{12}.$$

As a result of these new notations,the Newton's iterative scheme gets the form:

$$\begin{pmatrix} r^{(k+1)} \\ s^{(k+1)} \end{pmatrix} = \begin{pmatrix} r^{(k)} \\ s^{(k)} \end{pmatrix} - \begin{pmatrix} y_5 & y_6 \\ y_9 & y_{10} \end{pmatrix}^{-1}_{\eta=1} \begin{pmatrix} y_1(\eta, r^{(k)}, s^{(k)}) - 1 \\ y_2(\eta, r^{(k)}, s^{(k)}) \end{pmatrix}_{\eta=1}. \tag{4.29}$$

Here k is the number of iterations ($k = 0, 1, 2, 3...$). Now differentiating the above system of four first order ODEs with respect to r and s, we get another system of eight ODEs. Writing all these twelve ODEs together,we have the following initial value problem (IVP):

$$\begin{aligned}
y_1' &= y_2, & y_1(0) &= 0, \\
y_2' &= y_3, & y_2(0) &= r, \\
y_3' &= y_4, & y_3(0) &= 0, \\
y_4' &= \frac{1}{1 + S\lambda_r f^2}\Bigg[S\Big(\eta y_4 + 3y_3 + y_2y_3 - y_1y_4\Big) \\
&\quad + M^2\Big(sin(\omega)y_3 + 2\delta cos(\omega)y_2\Big) \\
&\quad + 2S\lambda_r\Big(y_2^2y_3 + y_1y_3^2\Big)\Bigg], & y_4(0) &= s, \\
y_5' &= y_6, & y_5(0) &= 0, \\
y_6' &= y_7, & y_6(0) &= 1, \\
y_7' &= y_8, & y_7(0) &= 0, \\
y_8' &= -\frac{2S\lambda_r y_1y_5}{\Big(1 + S\lambda_r f^2\Big)^2}\Bigg[S\Big(\eta y_4 + 3y_3 + y_2y_3 - y_1y_4\Big) \\
&\quad + M^2\Big(sin(\omega)y_3 + 2\delta cos(\omega)y_2\Big) + 2S\lambda_r\Big(y_2^2y_3 + y_1y_3^2\Big)\Bigg] \\
&\quad + \frac{1}{\Big(1 + S\lambda_r f^2\Big)}\Bigg[S\Big(\eta y_8 + 3y_7 + y_2y_7 + y_3y_6 - y_1y_8 \\
&\quad - y_4y_5\Big) + M^2\Big(sin(\omega)y_7 + 2\delta cos(\omega)y_6\Big) + 2S\lambda_r\Big(y_2^2y_7 \\
&\quad + 2y_2y_3y_6 + y_3^2y_5 + 2y_1y_3y_7\Big)\Bigg], & y_8(0) &= 0,
\end{aligned}$$

$$y_9' = y_{10}, \qquad y_9(0) = 0,$$
$$y_{10}' = y_{11}, \qquad y_{10}(0) = 0,$$

$$\begin{aligned}
y_{11}' =& y_{12}, & y_{11}(0) =& 0, \\
y_{12}' =& -\frac{2S\lambda_r y_1 y_9}{\left(1+S\lambda_r f^2\right)^2}\Bigg[S\left(\eta y_4 + 3y_3 + y_2 y_3 - y_1 y_4\right) \\
&+ M^2\left(sin(\omega)y_3 + 2\delta cos(\omega)y_2\right) + 2S\lambda_r\Big(y_2^2 y_3 \\
&+ y_1 y_3^2\Big)\Bigg] + \frac{1}{\left(1+S\lambda_r f^2\right)}\Bigg[S\Big(\eta y_{12} + 3y_{11} + y_2 y_{11} \\
&+ y_3 y_{10} - y_1 y_{12} - y_4 y_9\Big) + M^2\Big(sin(\omega)y_{11} \\
&+ 2\delta cos(\omega)y_{10}\Big) + 2S\lambda_r\Big(y_2^2 y_{11} + 2y_2 y_3 y_{10} \\
&+ y_3^2 y_9 + 2y_1 y_3 y_{11}\Big)\Bigg], & y_{12}(0) =& 1.
\end{aligned}$$

The RK-4 method has been used to solve the IVP consisting of the above twelve ODEs for some suitable choices of r and s. The missing conditions r and s are updated by using Newtons's scheme (4.29). The iterative procedure is stopped when the following condition is met:

$$\max\{|r^{(k+1)} - r^{(k)}|,\ |s^{(k+1)} - s^{(k)}|\} < \epsilon,$$

for an arbitrarily small positive value of ϵ. Throughout this chapter ϵ has been taken as $(10)^{-10}$.

For the numerical results of equations (4.20) and (4.21) the similar procedure has been followed as discussed in Chapter 3.

4.4 Results and Discussion

The principle object is about to examine the impact of distinct parameters against the velocity, the temperature profile and concentration profile. The impact of different parameters, e.g squeezing parameter S, Pr, S_c, γ and E_c is observed graphically. Numerical results of the Sherwood number, the coefficient of skin friction and the Nusselt number for the distinct values of S with some fixed parameters are shown in Tables 4.1-4.3.

Figure 4.2 and 4.3 shows the impact of S on the dimensionless velocity profile. It can be noted that the fluid velocity reduces by enhancing the values of squeezing parameter. Figure 4.4 and 4.5 present the impact of both the positive and negative squeezing parameters on the temperature distribution. The greater values of S gives the noteworthy decrease in temperature profile. Figure 4.6 and 4.7 shows the impact of Pr on the field θ. The field θ is rising due to viscous dissipation effect. Prandtl number $Pr < 1$ describe the liquid material with high thermal diffusivity but low viscosity, whereas the viscosity of liquid material is high for the Prandtl number $Pr > 1$. Figure 4.8 and 4.9 are delineated to show the impact of E_c on the temperature field θ. These figures describes that on rising the estimations of E_c, the temperature profile θ is also increased. Thus the compactness of the thermal layer at the boundary is reduced by increasing the estimations of Pr and E_c. Figure 4.10 present the temperature distribution for different values of the magnetic parameter M. Figure 4.11 shows the behaviour of temperature profile by increasing the values of inclination angle ω of the applied magnetic field. So the influence of the magnetic inclination angle on temperature profile of fluid is similar to the magnetic parameter. Thus in practical application related to controlling the momentum and heat transfer of fluid in squeezing flow, the effects produced by changing the magnetic field strength can also be approximately achieved through adjusting the inclination angle of the magnetic field. Figure 4.12 and 4.13 are shows the impact of S on the field ϕ of the fluid. The outcomes of S_c on the field ϕ are presented in the Figure 4.14 and 4.15. It can be noted that the molecular diffusivity turns more fragile and the boundary layer thickness ends up more slender because of the gradual increase in S_c. Figure 4.16 and 4.17 are delineated the impact of γ on the concentration elds. For $\gamma > 0$, the concentration field ϕ decline significantly, whereas an increase in the concentration profile ϕ is very much visible for the $\gamma < 0$. Steeper curves are observed when larger estimations of γ are accompanied with severe conditions of the reaction that is presented in the Figure 4.16 and 4.17.

	$-f''(1)$				
S	M	ω	δ	λ_r	shooting
-1.0	3.0	$\frac{\pi}{4}$	0.2	0.2	3.036639
-0.5					3.357444
0.01					3.612109
0.5					3.808987
2.0					4.232647

TABLE 4.1: Values of the skin friction coefficient for different parameters

.

				$-\theta'(1)$
S	P_r	E_c	E_{cx}	shooting
-1.0	0.2	0.2	0.2	0.170218
-0.5				0.168694
0.01				0.168541
0.5				0.169009
2.0				0.171515
2.0	0.4	0.2	0.2	0.339028
	0.6			0.502699
	0.8			0.662681
2.0	0.2	0.4	0.2	0.215831
		0.6		0.260147
		0.8		0.304463
2.0	0.2	0.2	0.4	0.298714
			0.6	0.425913
			0.8	0.553112

TABLE 4.2: Values of the reduced Nusselt number for $\omega = \frac{\pi}{4} rad, M = 3.0, \lambda_r = \delta = 0.2$

.

			$-\phi'(1)$
S	S_c	γ	shooting
-1.0	1.0	1.0	0.800351
-0.5			0.779759
0.01			0.761250
0.5			0.745138
2.0			0.702388

TABLE 4.3: Values of the reduced Sherwood numbers for $\omega = \frac{\pi}{4} rad$, $M = 3.0$,$\lambda_r = \delta = 0.2$
.

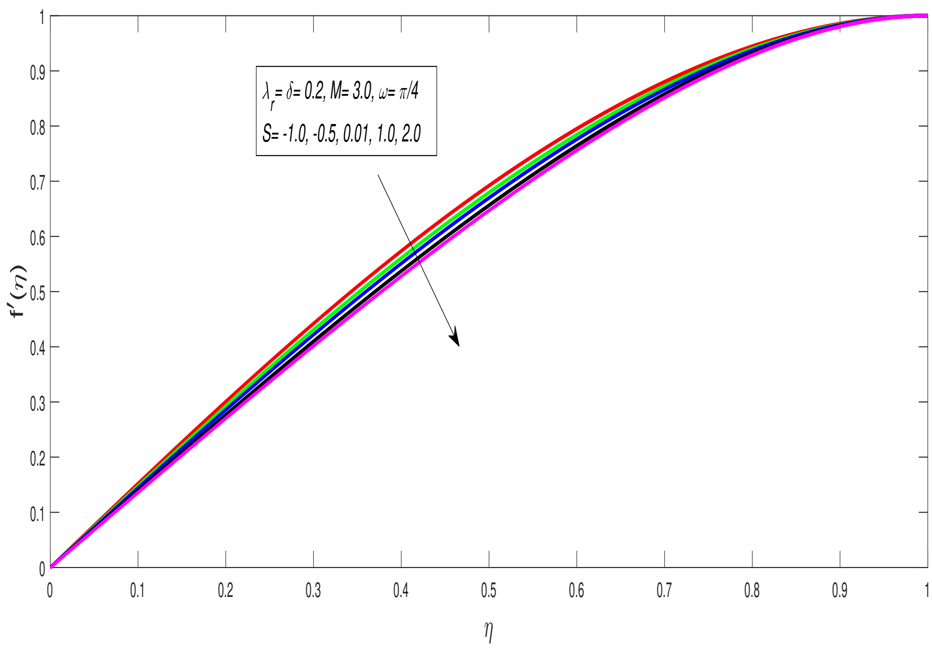

FIGURE 4.2: Effect of S on f'

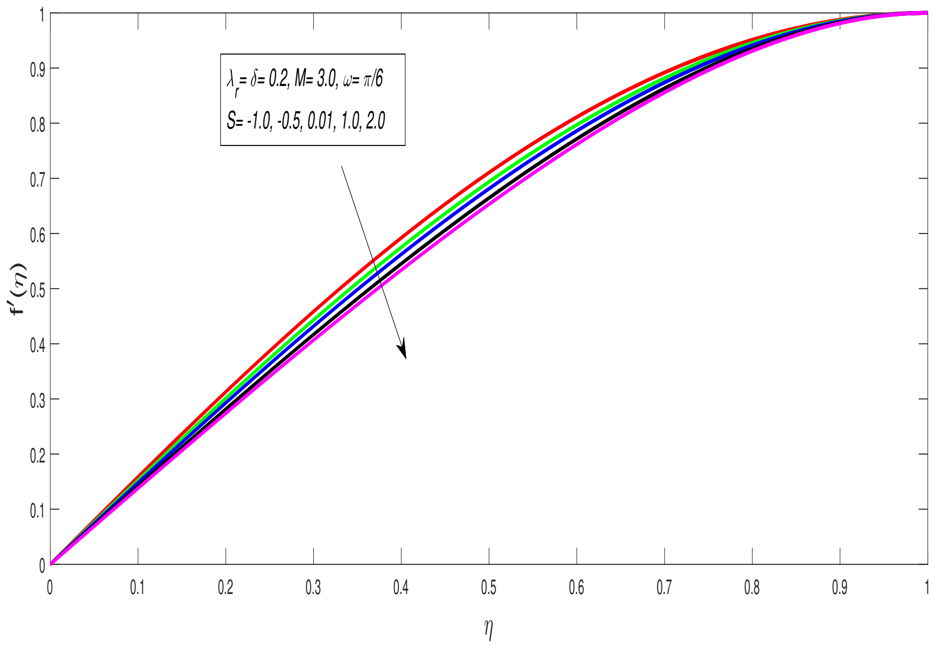

FIGURE 4.3: Effect of S on f'

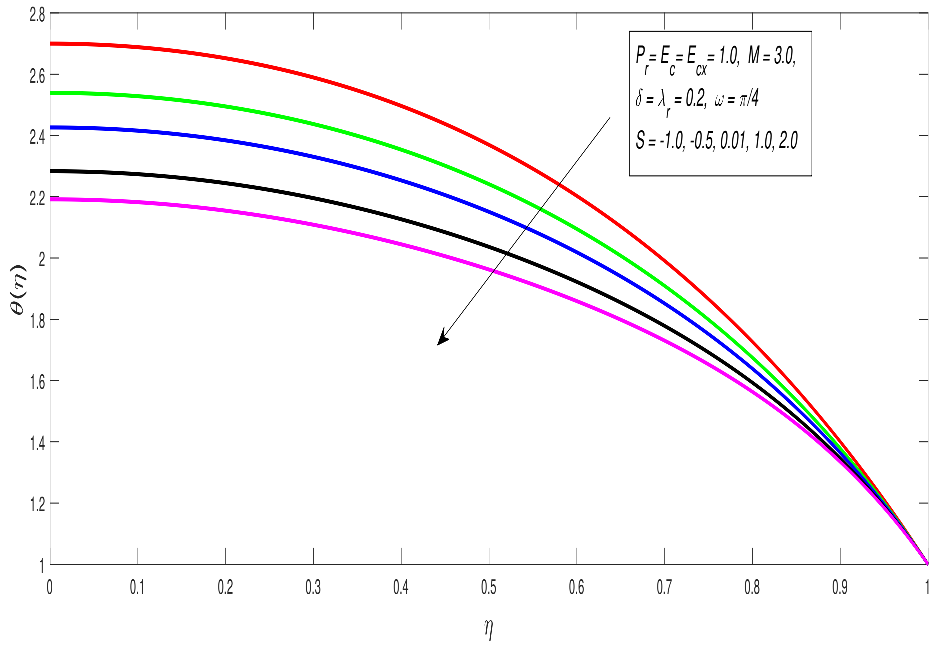

FIGURE 4.4: Influence of S on θ

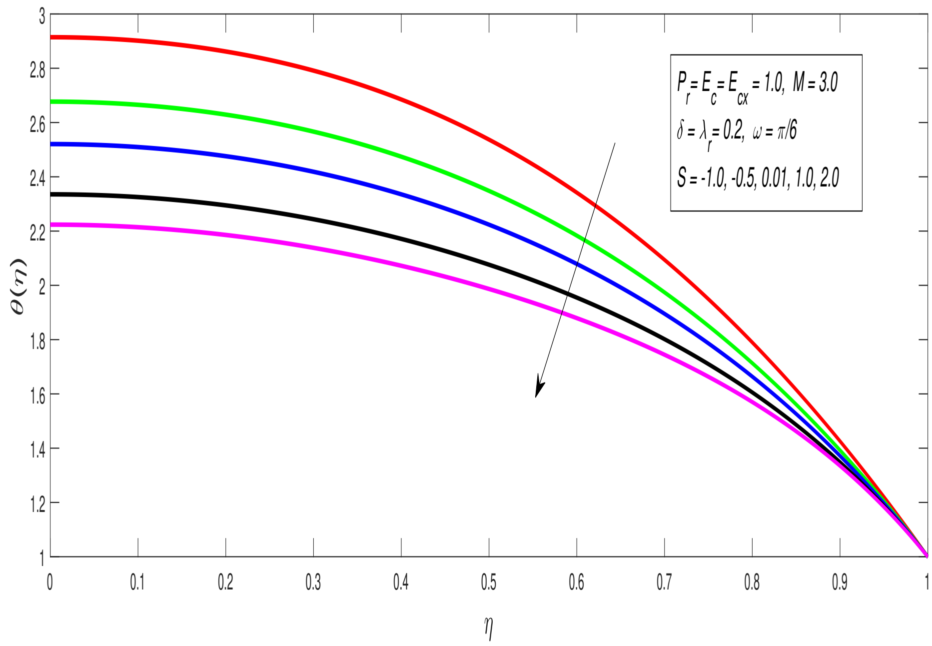

FIGURE 4.5: Influence of S on θ

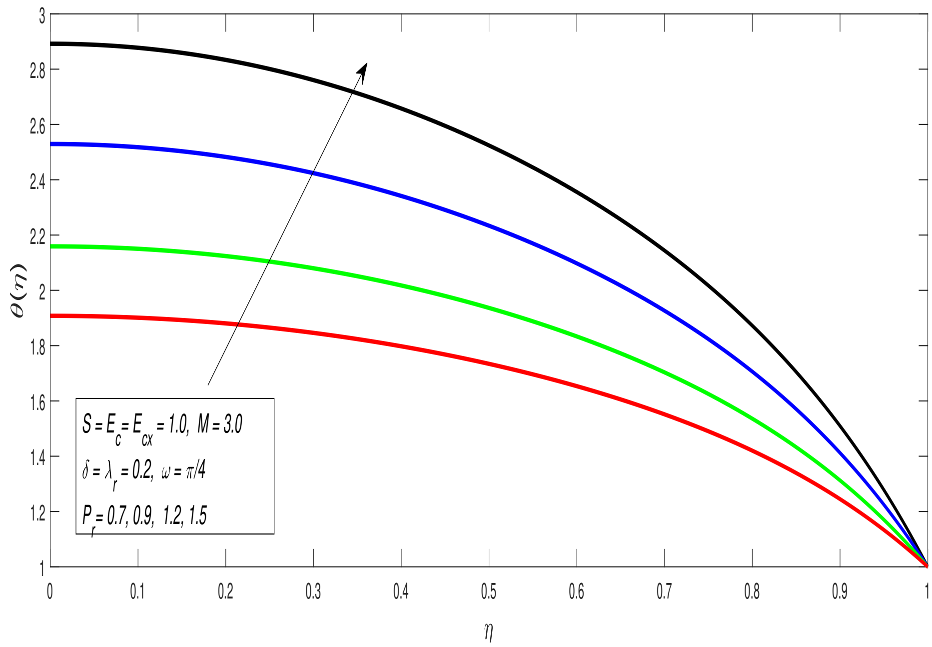

FIGURE 4.6: Influence of P_r on θ

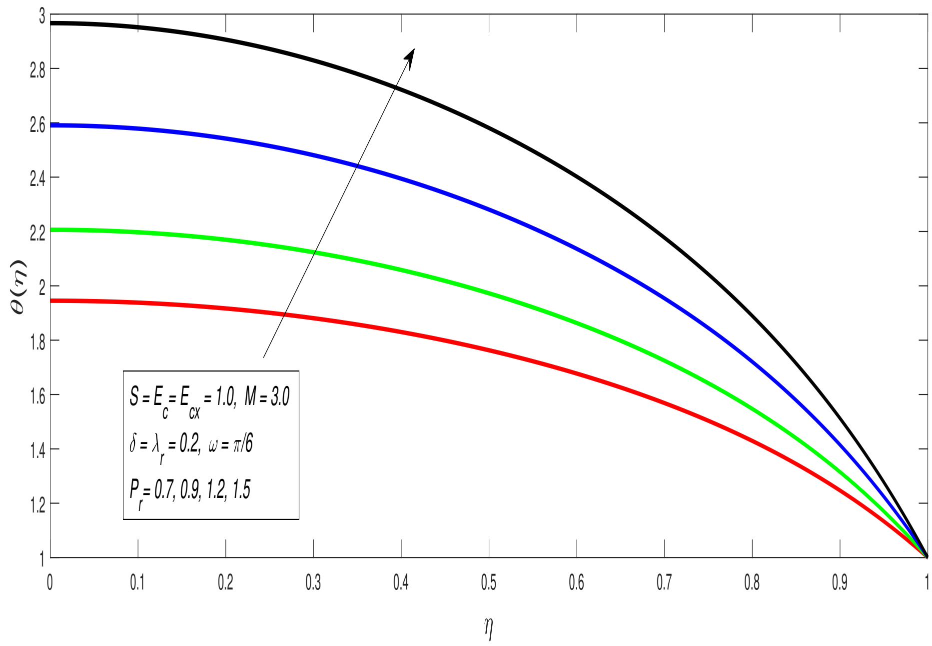

FIGURE 4.7: Influence of P_r on θ

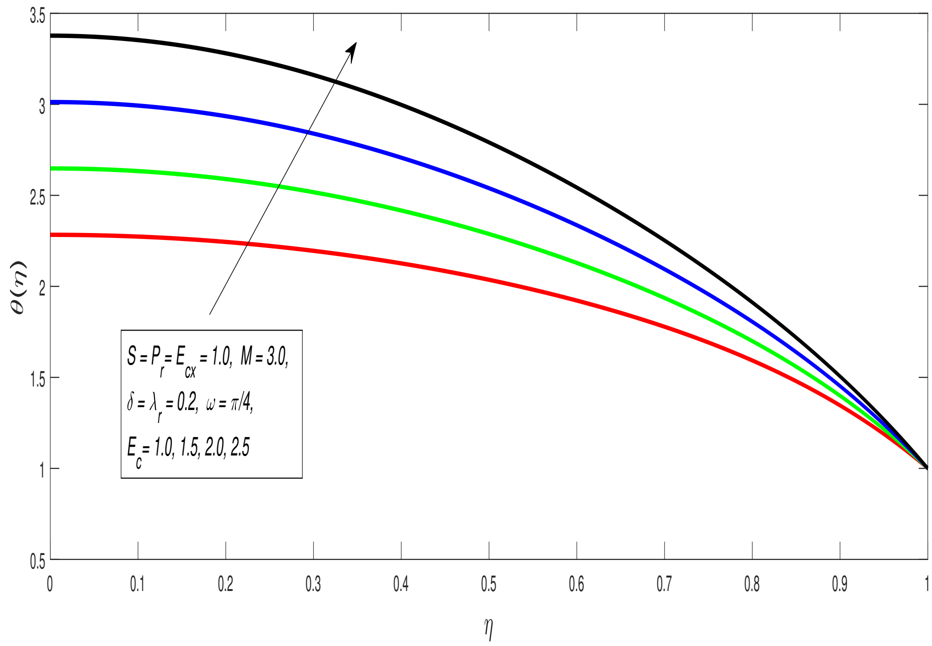

FIGURE 4.8: Influence of E_c on θ

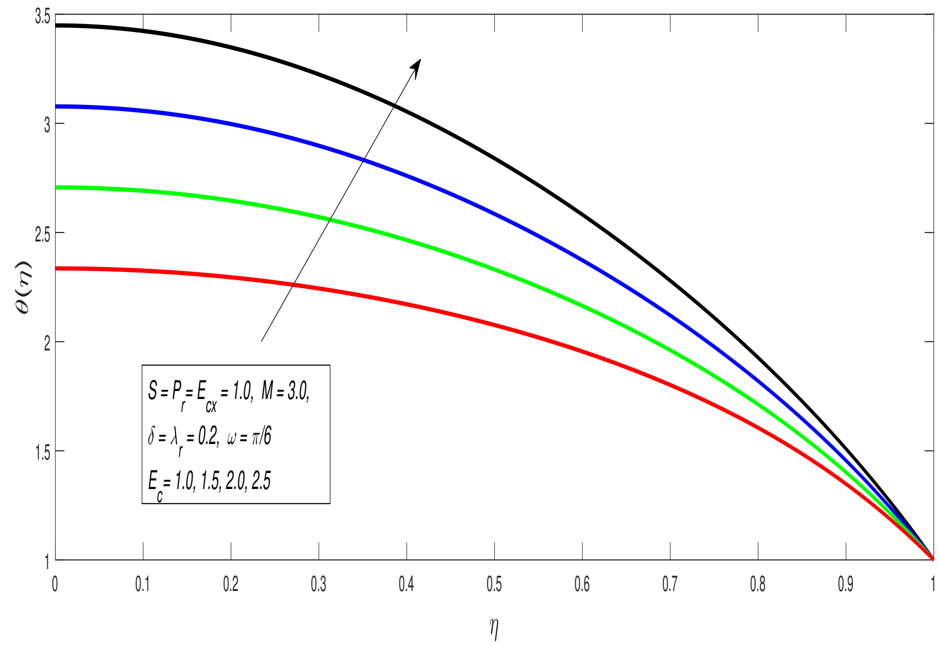

FIGURE 4.9: Influence of E_c on θ

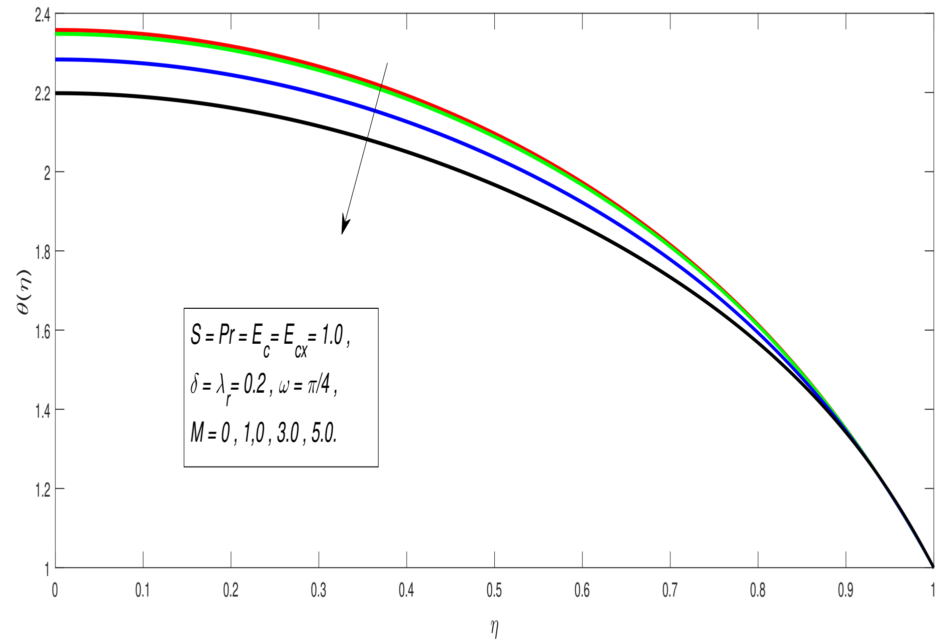

FIGURE 4.10: Influence of M on θ

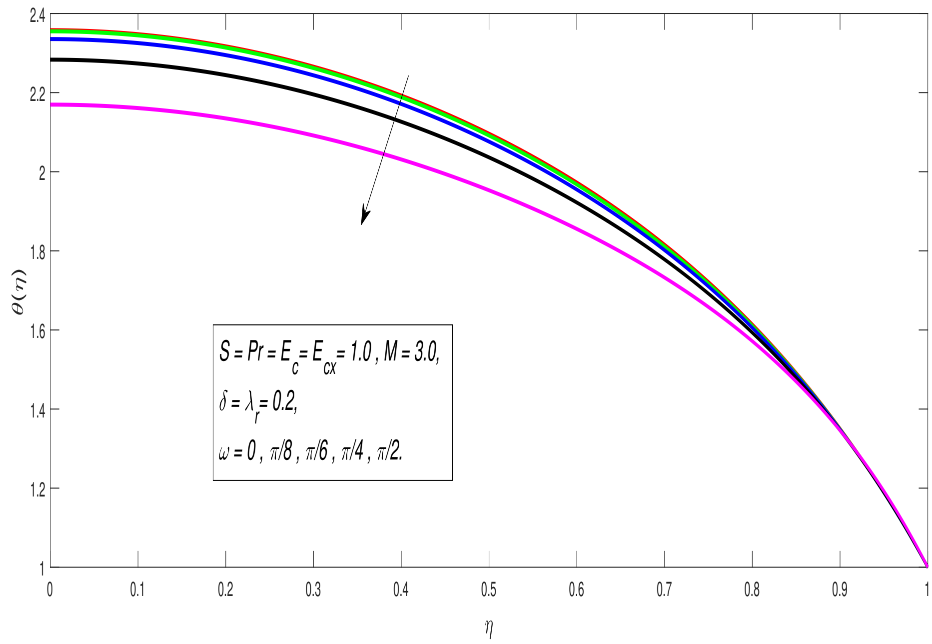

FIGURE 4.11: Influence of ω on θ

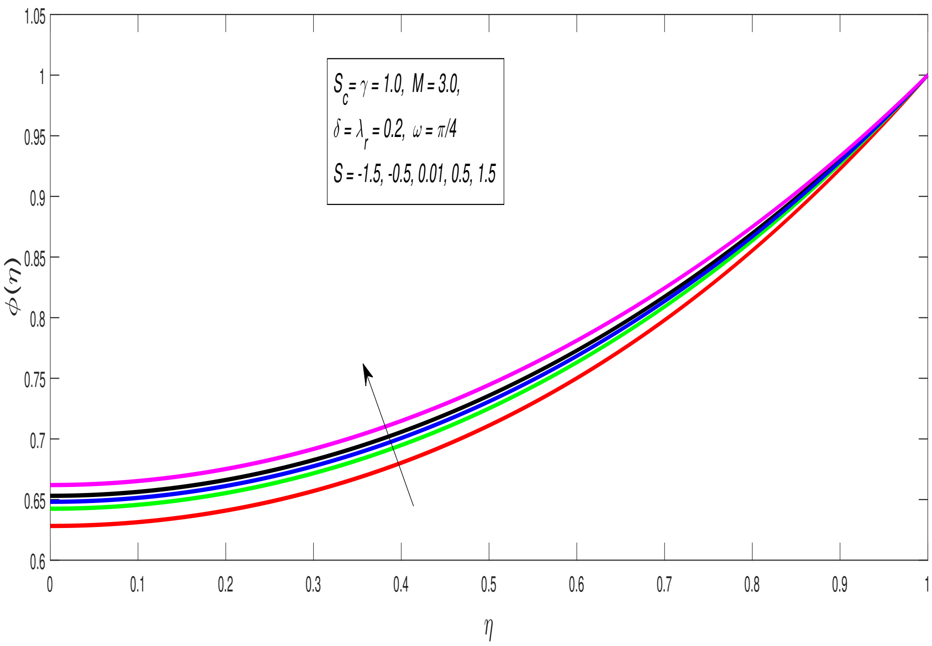

FIGURE 4.12: Effect of S on ϕ

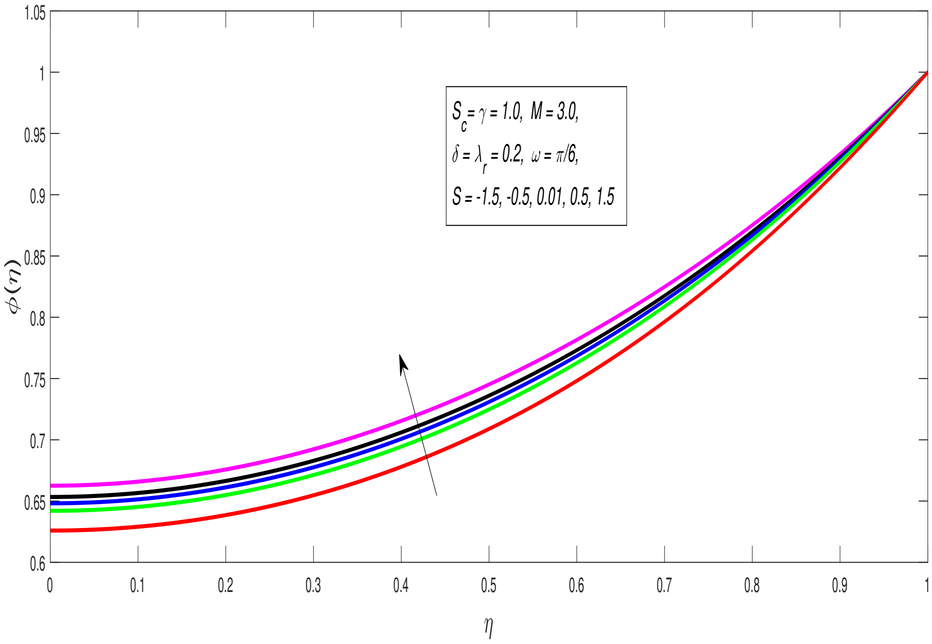

FIGURE 4.13: Effect of S on ϕ

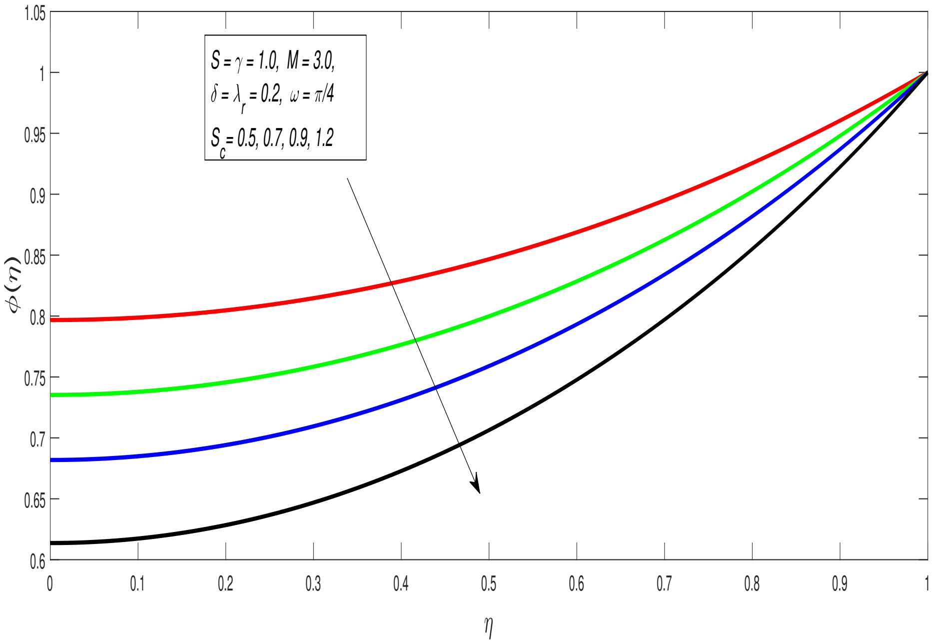

FIGURE 4.14: Influence of S_c on ϕ

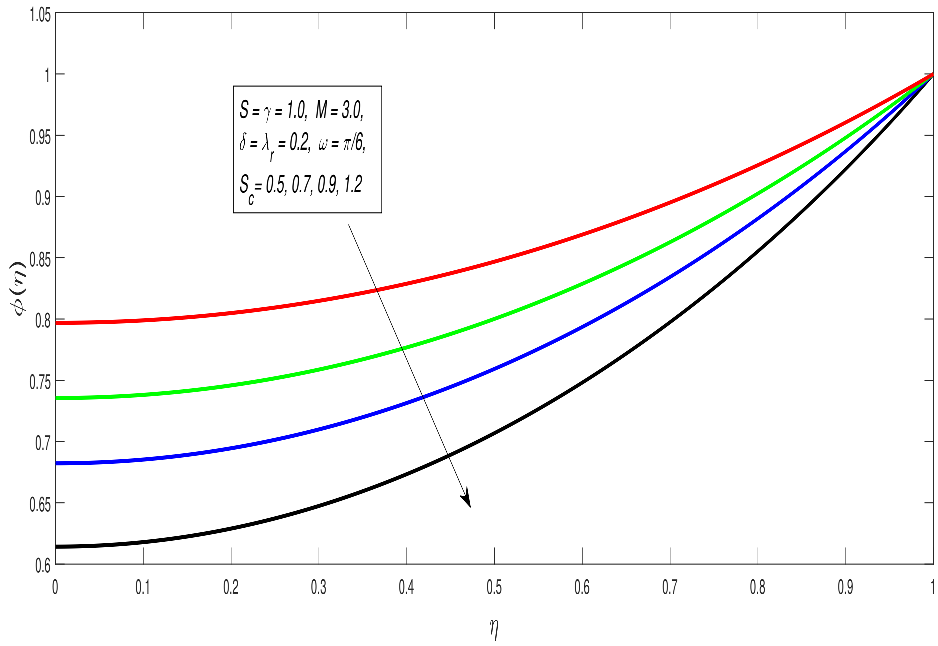

FIGURE 4.15: Influence of S_c on ϕ

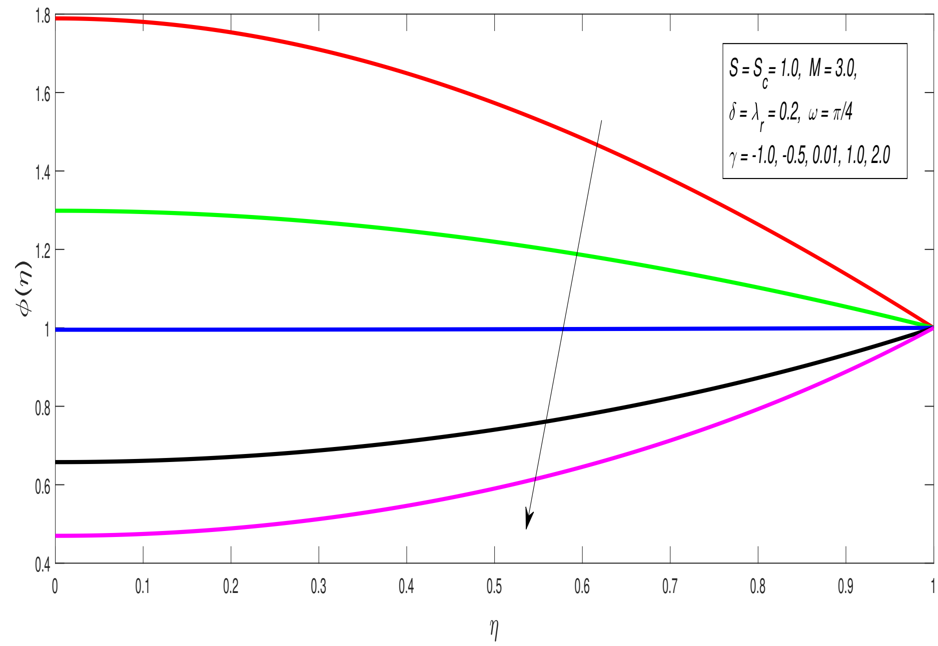

FIGURE 4.16: Influence of γ on ϕ

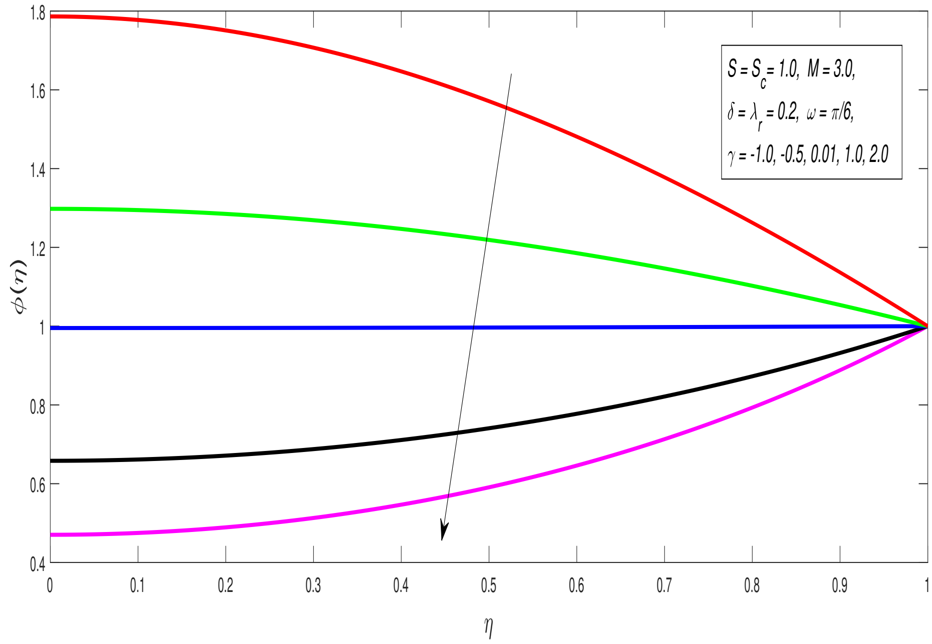

FIGURE 4.17: Influence of γ on ϕ

Chapter 5

Conclusion

In this research work, we first analyzed the two-dimensional, incompressible viscous fluid flow which is squeezed between two parallel plates. Secondly, the UCM flow is investigated by considering the inclined magnetic field effect in the velocity equations. The set of non-linear momentum, energy and concentration equations are transformed into the dimensionless ODEs by an appropriate transformation. Numerical solutions are obtained by using the shooting technique. The influence of distinct physical parameters such as, Eckert number E_c, Schmidt number S_c, squeezing parameter S, Prandtl number Pr, and the chemical reaction parameter γ on the velocity profile, temperature field and the concentration profile are elaborated in the graphical and tabular form. The above mentioned analysis of the UCM flow has led us to the following conclusion:

- A decrement in the temperature profile is noted for increasing the values of the squeezing parameter.
- The temperature rises for the larger estimation of the Prandtl number considering the UCM fluid.
- It is observed that an increment in the temperature p occurs for the increasing values of the Eckert number.

- An increment in the Schmidt number is observed to decelerate the concentration profile.
- The temperature distribution decelerate due to the boosting value of magnetic parameter.
- The concentration profile decreases for increasing the values of the chemical reaction parameter, the concentration profile is decreased while on the other hand, the concentration profile increases by a decrement in the chemical reaction parameter.

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