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Effect of Thermal Radiation on an Magnetohydrodynamics Nanofluid Flow

by

Maria Bibi

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I dedicate this sincere effort to my beloved Parents and my elegant Teachers whose devotions and contributions to my life are really worthless and whose deep consideration on part of my academic career, made me consolidated and inspired me as I am upto this grade now.

CERTIFICATE OF APPROVAL

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Abstract

The main objective of this thesis is to investigate the MHD stagnation point flow of water based nanofluids. Similarity transformations are applied to convert the PDEs into a system of nonliner ordinary differential equations. The system of ODEs has been solved with the help of the computational software MATLAB to compute the numerical results utilizing the shooting method. Effects of various physical parameters such as Eckert parameter Ec, Prandtl number Pr and radiational parameter R, magnetic parameter M, volume fraction ϕ, Schmidt S_c, convective heat transfer N_c, convective mass transfer N_d on the velocity, temperature and concentration profile are illustrated by graphs. The temperature profile are increased for the increasing values of the magnetic parameter M.

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Abbreviations

MHD	Magnetohydrodynamics
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations

Symbols

ρ	fluid density
μ	viscosity
τ	stress tensor
η	dimensionless similarity variable
ψ	stream function
τ_w	wall shear stress
α	thermal diffusivity
M	Magnetic parameter
R	Radiation parameter
h	Convective heat flux
Pr	Prandtl number
Cf_x	Skin friction coefficient
Nu_x	Nusselt number
Sh_x	Sherwood number
c_p	Specific heat at a constant pressure
T	Temperature
T_f	Temperature of the hot fluid
ϕ	Volume fraction
k^*	Absorption coefficient
B_0	Uniform transverse magnetic field
q_w	Rate of heat transfer
Ec	Eckert number
Re_x	Local Reynolds number

σ^*	Stefan-Boltzmann constant
σ	Electric conductivity
D_m	Mass diffusivity coefficient
T_∞	Free stream temperature
U_∞	Free stream velocity
k_f	Thermal conductivity
D	Rate of heat transfer diffusivity
$(\rho c_p)_f$	Heat capacity of the fluid
$(\rho c_p)_p$	Heat capacity of the nanoparticle
(u, v)	Velocity components
(x, y)	Cartesian coordinates

Chapter 1

Introduction

During the past few decades, the work on stagnation point flow of an incompressible liquid over a permeable surface has got significant importance due to extensive number of applications in manufacturing industry. Some of its main applications are refrigeration of electrical gadgets by fan, atomic receptacles cooling for the duration of emergency power cut, solar receiver, etc. The study of (2D) stagnation point flow was first investigated by Hiemenz [1], whereas for getting an accurate solution, Eckert [2] extended this problem by adding the energy equation. Heat transfer analyzed in the stagnation point stream induced by stretching sheet have been stuided by Mahapatra and Gupta [3], Ishak et al. [4] and Hayat et al. [5]. Ibrahim et al. [6] induced the impact of heat transfer on stagnation point fluid flow because of shirinking sheet. In this review several parameters Nt, Nb, Le, Pr were involved temperature field and mass transfer field.

In future, advancement in nano-technology is expected for making unbelievable changes in our lives. A very big number of researchers are working in this area due to its great use in the engineering space. In the process of air cleaning, development of microelectronics, safety of nuclear reactors biomedicine, transportation fuel cell hybrid-powered engines, domestic refigerators etc. Choi and Eastman [7] was the first who introduced the idea of nanofluid and presented the opinion of heat transfer properties of nanofluid. Tasi [8] was discussed the impact of wall suction and thermophoresis particles on a laminar flow towards a flate surface. Khan and Pop

[9] explained a laminar fluid flow of a nanofluid through a stretching sheet. In this article the impact of Brownian motion and theromphoresis on a naofluid. Ahmad and Pop [10] and Hamad et al. [11] were discussed the free convective bounday layer flow of a nanofluid over a flate vertical plate by using the Cu, AIO_2 and TiO_2 nanoparticles. Yazdi et al. [12] inspected the (2D) magnetohydrodynamic stagnation point flow within the nearness of thermal radiation. Casson fluid flow in the presence of nanoparticles is discussed by Nadeem et al. [13]. The impact of thermal radiation and magnetic field by using a micropolar fluid flow examind by Krishnamurthy et al. [14]. Mansur et al. [15] was examined that the stagnation point flow past a shirinking sheet. In this review, by increament of Browian motion parameter and thermophhoresis parameter diminishes the heat transfer rate at the surface. Hayat et al. [16] inspected the impact of Joule heating and thermal radiation over a Peristaltic Jaffrey nanofluid flow.

The study of MHD fluid flow was first introduced by Swedish Physicist, Alfven [17]. Numerous analysts are inquisitive about the think about of MHD liquid flow because of its significant applications within the forms of designing, vitality generators, planetary and sun powered plasma liquid flow frameworks, attractive field control of fabric handling framework, half breed attractive impetus framework for space travel, businesses [18], biomedical sciences [19]. Yih [20] explored that the impact of heat and mass transfer on MHD along a continously moving sheer surface. Kesvaiah et al. [21] was talked about the time dependent MHD flow against a semi-infinite flate plate. Zheng et al. [22] decipted the effect of heat transfer against a permeable extending sheet. It was presented a HAM technique, the impact of physical parameter pertinets parameter on axisymmetic and heat transfer rate. Mahmoud and Waheed [23] inspected the influence of thermal radiation on MHD stagnation point flow over a moving plate. He is numerically performed the shooting strategy. Mustafa et al. [24] analyzed the non-uniform fluid flow becacuse of continusly moving flate suraface along convective boundary conditions. He was using numerically shooting technique. In the abbsence of magnetic parameter$M$ Yasin et al. [25] discussed the (2D) MHD flow with the impact of Joule heatiing, viscous dissipation and velocity slip. In different devices, the effect of viscous dissipation plays an important role in regular convection. Sheikholeslami et al. [26] examined the affect of thermal radiation on MHD flow among the flat plates. He is numerically using the RK-4 approach. Reynolds number, magnetic parameter, schmidth number, thermophoretic have inspected. By the increment of ratiation parameter the boundary layer thickness becomes decrease. The symbolic impacts of thermophoresis and Brownian motion have been comprised in the geometry of Sheikholeslami et al. [26] nanofluid. Chutia and Deka [27] numerically discussed the heat transfer and steady MHD flow in an electrically protected rectangular pipe in the existence of the attractive field. The inclined magnetic field effects on fluid flow were explored by Singh et al. [28].

1.1 Thesis Contributions

In this thesis a review of Mabood et al. [29] has been conducted and extending by considering the thermal radiation and Joule heating. The suitable transformation is used to transform PDEs into a system of ODEs and after that treated numerically with the help of shooting approach.

1.2 Thesis Outline

This proposal is organized within the taking after manner:

Chapter 1 : This chapter describes the current work briefly.

Chapter 2 : It contains the basic definitions.

Chapter 3 : It is focused on the detailed review of [29] to present a nanofluid stagnation point flow over a moving plate.

Chapter 4 : This chapter expands the ides of [29] by counting the effect of Joule heating and heat radiation.

Chapter 5 : The results of the current thesis are concluded.

All the references utilized in this thesis are recorded in Bibliography

Chapter 2

Preliminaries

2.1 Basic Definition

The current chapter contains a few fundamental definitions of fluid flow, essential concepts and concepts of fluid dynamics, dimensionless numbers and physical laws. The terminologies relevant to the rest of the thesis have been specially focused.

Definition 2.1.1. (Fluid) [30]
"A matter which continuously changes its shape under the influence of shear stress is called fluid. It yields easily to shear stress and repeatedly deforms its shape as long as the shear stress acts. Fluid has no fixed shape and conforms to the shape of a container in which it is placed."

Definition 2.1.2. (Fluid Statics) [30]
"The study of fluid mechanics is concerned with various properties of fluid and the forces acting on them. Fluid mechanics is mainly divided into two categories: fluid static which deals with the study of fluid at rest and fluid dynamics which deals with the study of fluid in motion."

Definition 2.1.3. (Flow) [31]
"It is the deformation of the material under the influence of different forces. If the deformation increase is continuous without any limit then the process is known as flow."

Definition 2.1.4. (Uniform and Non-uniform Flows) [31]

"The flow is said to be uniform if the magnitude and direction of flow velocity are the same at every point and flow is said to be non-uniform if the velocity is not the same at each point of the flow, at a given instant."

Definition 2.1.5. (Steady and Unsteady Flows) [32]

"A flow whose flow state expressed by velocity, pressure, density, etc, at any position, does not change with time, is called a steady flow. A flow whose flow state does change with time is called an unsteady flow."

Definition 2.1.6. (Compressible and Incompressible Flows) [31]

"Flow in which variations in density are negligible is termed as incompressible otherwise it is called compressible. The most common example of compressible flow is the flow of gases, while the flow of liquids may frequently be treated as incompressible. Mathematically,

$$\frac{D\rho}{Dt} = 0,$$

where ρ denotes the fluid density and $\frac{D}{Dt}$ is the material derivative given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V.\nabla. \tag{2.1}$$

In Eq. (2.1), V denotes the velocity of the flow and ∇ is the differential operator. In Cartesian coordinate system, ∇ is given as

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}."$$

Definition 2.1.7. (Stress) [31]

"Stress is a force acted upon a material per unit of its area and is denoted by Mathematically, it can be written as;

$$\tau = \frac{F}{A}, \tag{2.2}$$

where F denotes the force and A represents the area."

Definition 2.1.8. (Shear Stress) [31]

"It is a type of stress in which the force vector acts parallel to the material surface or cross section of a material."

Definition 2.1.9. (Viscosity) [31]

"This is the internal property of a fluid by virtue of which it offers resistance to the flow. Mathematically, it is defined as the ratio of the shear stress to the rate of shear strain. i.e,

Viscosity $= \mu =$shear stress/rate of shear strain.

In the above definition, μ is the coefficient of viscosity or absolute viscosity or dynamics viscosity or simply viscosity having dimension $[\frac{M}{LT}]$. Water is thin having low viscosity and on other hand, honey is thick having higher viscosity. Usually liquids have non-zero viscosity. Its unit is $Pa.s = \frac{kg}{s.m}$."

Definition 2.1.10. (Nanofluid)

"A nanofluid is a fluid involving nanometer-sized particles, called the nanoparticles. These fluids are engineered colloidal suspension of nanoparticles in a base fluid. The nanoparticles used in nanouids are usually made of metals, ethylene glycol and oil. The term nanouid distinguishes as itself from base fluid on the basis of thermal conductivity, as the base retains less heat transfer abilities as compared to nanofluid."

Definition 2.1.11. (Newtonian and Non-Newtonian fluid) [31]

"A fluid is said to be a Newtonian fluid in which the stress arising from its flow at every point is linearly proportional to the local strain rate. Newtonian fluid behaviour is described by the relation

$$\tau = \mu \frac{du}{dy}.$$

In the above equation, τ is the stress tensor, μ the viscosity and $\frac{du}{dy}$ is the deformation rate. Fluids are said to be non-Newtonian fluids for which the shear stress is not directly proportional to the deformation rate. Mathematically,

$$\tau = \alpha\left(\frac{\partial u}{\partial y}\right)^n; n \neq 1,$$
$$\tau = \eta\left(\frac{\partial u}{\partial y}\right)^n; n \neq 1,$$
$$\eta = \mu\left(\frac{\partial u}{\partial y}\right)^{n-1},$$

where η is the apperent viscosity, μ the viscosity , n is the flow behaviour index and n can be grater or less then one."

Definition 2.1.12. (Generalized Continuity Equation) [31]
"Coherence condition is gotten from the law of preservation of mass which states that mass can not one or the other be made nor be devastated interior a control volume. The mass interior the settled control framework will not alter on the off chance that we look at a differential control volume framework encased by a surface settled in space, at that point the condition of coherence can be composed as,

$$\frac{\partial \rho}{\partial t} + \nabla.\,(\rho V) = 0. \tag{2.3}$$

On the off chance that the thickness is consistent and spatially uniform, in that case Eq. (2.3) ended up

$$\nabla.\,(\rho V) = 0.''$$

Definition 2.1.13. (Law of Conservation of Mass) [33]
"Law of conservation of mass states that mass can nethier be created nor destroyed. The mathematical equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla.\,(\rho V) = 0,$$

∇ and V are define as
$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and $V = (u, v, w)$

The above stated equation is also called continuity equation. In above equation ρ is the density of the fuids and V as the velocity field. For incompressible fuid the density of the fuid is assumed to remain constant. So, the continuity equation yields

$$\nabla.V = 0.''$$

Definition 2.1.14. (Law of Conservation of Momentum) [33]
"Every fluid particle obeys Newtons second law of motion which is at rest or in steady state or accelerated motion. This law states as the rate of change of momentum is equivalent to applied force. The mass of the framework is consistent, in this manner Newtons second law can be composed as

$$m\frac{DV}{Dt} = F.$$

The flow of the fluid is represented by the differential equation as

$$\rho\frac{DV}{Dt} = \bigtriangledown.\tau + \rho b$$

where ρb is the net body force, τ is the Cauchy stress tensor and $\bigtriangledown.\tau$ are the surface forces."

Definition 2.1.15. (Law of Conservation of Energy) [33]
"The work done on a material element by both body and surface forces, together with the heat transferred into or out of the element should lead to a change in its internal and kinetic energies. According to the energy conservation equation one has

$$\rho\frac{De}{Dt} = \tau.L - div\ q + \rho r.$$
$$\rho c_p\frac{DT}{Dt} = \bigtriangledown.\tau.\nabla V + \nabla^2 T,$$

where $k(kapa)$ is the thermal conductivity of material heat flux, T is the temperature which decreases with the increase of distance, c_p the specific heat of fuid and

$\frac{D}{Dt}$ is the material time derivative."

Definition 2.1.16. (Magnetohydrodynamics) [31]

"The study of the dynamics of electrically conducting aids for example plasmas or electrolytes, is known as magnetohydrodynamics (MHD)."

Definition 2.1.17. (Stagnation Point) [31]

"It is a point in a flow field where the fluid velocity is zero. It exists at the surface of objects in the field where fluid is at rest by the object."

Definition 2.1.18. (Joule Heating) [33]

"It is the process in which heat is generated by passing an electric current through a metal. Joule heating also referred to as resistive heating and ohmic heating."

Definition 2.1.19. (Mass Transfer) [31]

"Mass exchange is the total movement of mass from one place to another."

Definition 2.1.20. (Heat Transfer) [31]

"It is the energy transfer due to the temperature difference. At the point when there is a temperature contrast in a medium or between media, heat transfer must take place. Heat transfer is normally conducted from a high temperature region to that at a lower temperature."

Definition 2.1.21. (Conduction) [31]

"It is the transfer of heat with in the objector between two objects in direct contact. Regions with greater molecular kinetic energy will pass their thermal energy to regions with less molecular energy through direct molecular collisions which is principally conduction. Conduction requires medium to transfer heat. "

Definition 2.1.22. (Convection) [31]

"Convection heat transfer occurs when a liquid (fuid) comes in contact with a material of a different temperature. Convection also requires medium to transfer heat. There are three kinds of convection, forced convection, free convection and mixed convection. Forced convection is carried out when the fuid is moving due to some outside force, like a pump or a fan. Free convection happens when heat

is transferred to a still fuid, and the heating of part of a fuid causes motion in a fuid, like hot air rising, bringing cooler air to move in its place. In forced convection, the fuid movement causes the heat transfer, in free convection, heat transfer causes motion. Mixed (combined) convection is a combination of forced and free convections."

Definition 2.1.23. (Radiation) [31]
"It is the transfer of heat from one object to another by means of electromagnetic waves. Radiation is entirely diverse from both conduction or convection. Radiation does not require any medium to transfer heat. A surface will emitt energy and the amount of energy emitted by an object in this way does not depend on the material, only the temperature, like feeling the heat of a replace or radiator without actually touching it. In physics radiation is a process in which energetic particles or energetic waves travel through a vacuum, or through matter-containing media that are not required for their propagation. For example light, radio waves, heat and sound."

Definition 2.1.24. (Thermal Conductivity) [31]

"Thermal conductivity (k) is the property of a material related to its ability to transfer heat. Mathematically,

$$k = \frac{q \bigtriangledown l}{S \bigtriangledown T}.$$

where q is the heat passing through a surface area S and the effect of a temperature difference $\bigtriangledown T$ over a distance is $\bigtriangledown l$. Here l, S and $\bigtriangledown T$ all are assumed to be of unit measurement. In system unit of thermal conductivity is $\frac{W}{m}$ and its dimension is $[MLT^3\theta^{-1}]$."

Definition 2.1.25. (Reynolds Number *Re*) [31]

"It is a dimensionless number which is used to clarify the different behaviours like turbulent or laminar flow. It helps to measure the ratio between inertial force and the viscous force. Mathematically,

$$Re = \frac{\frac{\rho U^2}{L}}{\frac{\mu U}{L^2}} \Rightarrow Re = \frac{LU}{v},$$

where U denotes the free stream velocity, L the characteristics length. At low Reynolds number, laminar flow arises where the viscous forces are dominant. At high Reynolds number, turbulent flow arises where the inertial forces are dominant."

Definition 2.1.26. (Prandtl Number (Pr)) [31]

"It is the ratio between the momentum diffusivity (ν) and thermal diffusivity (α). Mathematically, it can be defined as:

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/C_P} = \frac{\mu C_p}{k},$$

where μ represents the dynamic viscosity, C_p denotes the specific heat and k stands for thermal conductivity. The relative thickness of thermal and momentum boundary layer is controlled by Prandtal number. For small Pr, heat distributed rapidly corresponds to the momentum."

Definition 2.1.27. (Eckert Number (Ec)) [31]

"It is the dimensionless number used in continuum mechanics. It describes the relation between flows and the boundary layer enthalpy difference and it is used for characterized heat dissipation. Mathematically,

$$Ec = \frac{u^2}{C_p \bigtriangledown T}."$$

Definition 2.1.28. (Skin Friction Coefficient (Cf_x)) [31]

"Skin friction coefficient occurs between the fluid and the solid surface which leads to slow down the motion of the fluid. The skin friction coefficient can be defined as

$$Cf_x = \frac{2\tau_w}{\rho U^2},$$

where τ_w denotes the wall shear stress, ρ the density and U the free-stream velocity."

Definition 2.1.29. (Nusselt Number (Nu_x)) [31]

"It is the ratio of the convective to the conductive heat transfer to the boundary. Mathematically,

$$Nu_x = \frac{hl}{k},$$

where h stands for convective heat transfer, L for the characteristics length and k stands for the thermal conductivity."

Definition 2.1.30. (Sherwood Number(Sh_x)) [31]

"It is the nondimensional quantity which shows the ratio of the mass transport by convection to the transfer of mass by diffusion. Mathematically:

$$Sh_x = \frac{kl}{D},$$

where L is characteristics length, D is the mass diffusivity and k is the mass transfer coefficient."

Definition 2.1.31. (Boundary Layer Flow) [31]

"The concept of boundary layer was first introduced by Ludwig Prandtl [37], a German aerodynamicist, in 1904. Prandtl introduced the basic idea of the boundary layer in the motion of a fluid over a surface. Boundary layer is a flow layer of fluid close to the solid region of the wall in contact where the viscosity effects are significants. The flow in this layer is usually laminar. The boundary layer thickness is the measure of the distance apart from the surface. There are two types of boundary layers:

- Hydrodynamic (velocity) boundary layer
- Thermal boundary layer."

Definition 2.1.32. (Hydrodynamic Boundary Layer) [31]

"A region of a fluid flow where the transition from zero velocity at the solid surface to the free stream velocity at some extent far from the surface in the direction normal to the flow takes place in a very thin layer, is known as the hydrodynamic boundary layer."

Definition 2.1.33. (Thermal Boundary Layer) [33]

"It is an area of the liquid nearest to the solid surface, where the fluid temperature is directly influenced by the heating or cooling from the surface."

Chapter 3

A Nanofluid Stagnation Point Flow over a Moving Plate

3.1 Introduction

In this section, an examination of the stagnation point boundary layer stream of Cu and Ag nanouids with water as the base liquid Mabood et al. [29] has been displayed. The heat capacities of conventional base fluid are improved by considering the Tiwari-Das [34]. Energy investigation is consolidated in the nearness of MHD and suction/injection marvels. Further, warm and mass transfer investigation is performed with convective boundary conditions. Using the similarity transformation, the governing PDEs are transformed into the ODEs. The numerical solution for the system of the dierential equations is achieved by using the shooting technique. The numerical comes about of Mabood et al. [29] have been replicated with a convincing agreement. The graphical and numerical comes about are moreover examined to appear the influence of different flow parameters included within the condition on speed, temperature, nanoparticles volume division, skin contact, nearby Nusselt and Sherwood numbers.

3.2 Mathematical Modeling

A (2D), laminar and boundary layer stream of Cu and Ag-nanofluids with water as a base-fluid has been considered past a level moving surface. Further more, a attractive field of uniform quality B_0 has been expected within the direction parameter to the surface. The geometry of the stream appear is showed up in Figure 3.1.

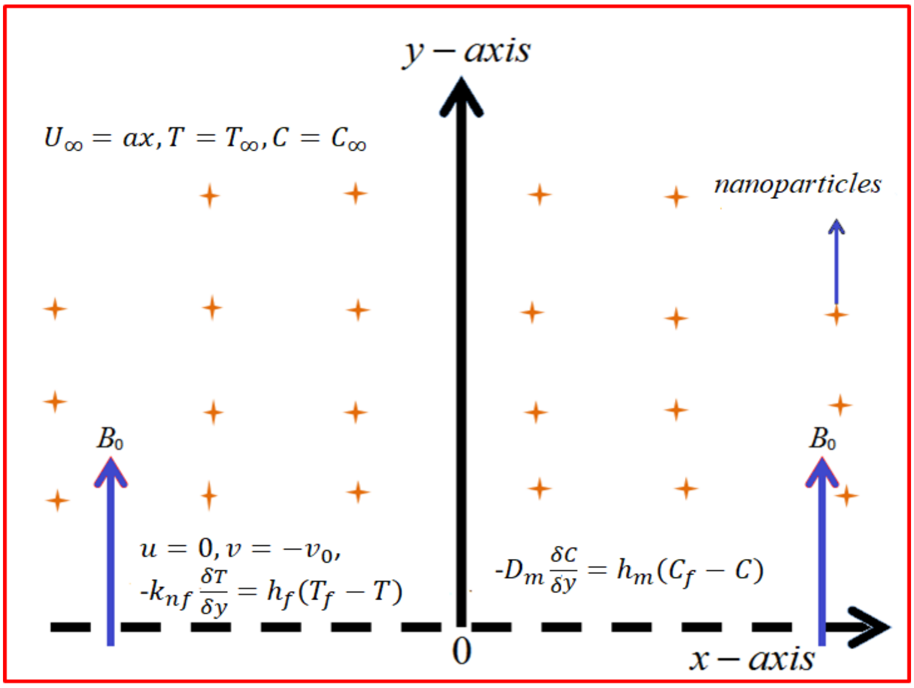


FIGURE 3.1: Flow geometry

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B_0}{\rho_{nf}}(U_\infty - u), \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{3.4}$$

The related boundary conditions as considered by Mabood et al. [29], are as follows.

$$\left.\begin{aligned} &y = 0 : u = 0,\ v = -v_0,\ -k_{nf}\frac{\partial T}{\partial y} = h_f(T_f - T),\\ &-D_m\frac{\partial C}{\partial y} = -h_m(C_f - C),\\ &y \to \infty :\ u = U_\infty = ax,\ T = T_\infty,\ C = C_\infty, \end{aligned}\right\} \tag{3.5}$$

where U_∞ is the free stream speed and D is species diffusity. Compelling thickness, warm diffusivity, electrical conductivity, kinematic thickness, thickness, particular warm and the coefficient of thermal extension of the nanofluid [29, 34] are given by

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{3.6}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{3.7}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{3.8}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s, \tag{3.9}$$

$$k_{nf} = k_f\left\{\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}\right\}. \tag{3.10}$$

Present the stream work ψ fulfilling the continuity condition within the taking after way

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x}. \tag{3.11}$$

The following process was implemented to convert the PDEs to ODEs:

$$\psi = \sqrt{U_\infty x\nu_f}f(\eta) = \sqrt{ax^2\nu_f}f(\eta) = x\sqrt{a\nu_f}f(\eta), \tag{3.12}$$

$$\eta = y\sqrt{\frac{U_\infty}{x\nu_f}} = y\sqrt{\frac{ax}{x\nu_f}} = y\sqrt{\frac{a}{\nu_f}}, \tag{3.13}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{3.14}$$

$$h(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{3.15}$$

The detailed procedure for the conversion of (3.1)-(3.4) has been described in the upcoming discussion.

- $$\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ &= x\sqrt{a\nu_f} f'(\eta)\sqrt{\frac{a}{\nu_f}} = axf'(\eta). \end{aligned}$$
- $$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial}{\partial x}(axf'(\eta)) \\ &= af'(\eta). \end{aligned} \tag{3.16}$$
- $$\begin{aligned} v &= -\frac{\partial \psi}{\partial x} \\ &= -\frac{\partial}{\partial x}(x\sqrt{a\nu_f} f(\eta)) \\ &= -\sqrt{a\nu_f} f(\eta). \end{aligned}$$
- $$\begin{aligned} \frac{\partial v}{\partial x} &= \frac{\partial}{\partial x}(-\sqrt{a\nu_f} f(\eta)) \\ &= -\sqrt{a\nu_f} f'(\eta)\sqrt{\frac{a}{\nu_f}} \\ &= -af'(\eta). \end{aligned} \tag{3.17}$$

Though the continuity equation (3.1) can now be seen to be satisfied very easily by using (3.15)-(3.16) as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = af'(\eta) - af'(\eta) = 0.$$

Now we include below the procedure for the conversion of equation (3.2) into the dimensionless form.

- $$\begin{aligned} u\frac{\partial u}{\partial x} &= axf'(\eta)af'(\eta) \\ &= a^2xf'^2(\eta) \end{aligned} \tag{3.18}$$
- $$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}(xaf'(\eta)) \\ &= axf''(\eta)\frac{\partial \eta}{\partial y} \end{aligned}$$

- $$\begin{aligned}\frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}(xaf'(\eta)) \\ &= axf''(\eta)\frac{\partial \eta}{\partial y} \\ &= axf''(\eta)\sqrt{\frac{a}{\nu_f}}.\end{aligned}$$
- $$\begin{aligned}v\frac{\partial u}{\partial y} &= -\sqrt{a\nu_f}f(\eta)axf''(\eta)\sqrt{\frac{a}{\nu_f}} \\ &= -a^2xf(\eta)f''(\eta). \qquad (3.19)\end{aligned}$$

using equation (3.17) and (3.18), the left side of (3.2) becomes.

$$\begin{aligned}u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= a^2xf'^2(\eta) - a^2xff''(\eta) \\ &= a^2x(f'^2(\eta) - a^2xff''(\eta)) \qquad (3.20)\end{aligned}$$

To convert the right side of (3.2) into dimensionless form, the upcoming procedure has been carried out.

- $$U_\infty\frac{dU_\infty}{dx} = ax\frac{d}{dx}(ax) = a^2x. \qquad (3.21)$$
- $$\begin{aligned}\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}(\frac{\partial u}{\partial y}) &= \frac{\partial}{\partial y}(axf''(\eta)\sqrt{\frac{a}{\nu_f}}) \\ &= axf''(\eta)\sqrt{\frac{a}{\nu_f}}\sqrt{\frac{a}{\nu_f}} = \frac{a^2}{\nu_f}xf'''(\eta).\end{aligned}$$
- $$\begin{aligned}\nu_{nf}\frac{\partial^2 u}{\partial y^2} &= \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right) \\ &= \frac{a^2\mu_f x}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\rho_s)}f'''(\eta) \qquad \left(\because \nu_f = \frac{\mu_f}{\rho_f}\right) \\ &= \frac{a^2x}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\frac{\rho_s}{\rho_f})}f'''(\eta). \qquad (3.22)\end{aligned}$$
- $$\begin{aligned}\frac{\sigma_{nf}B_0^2}{\rho_{nf}}(U_\infty - u) &= \frac{\sigma_{nf}B_0^2}{\rho_{nf}}(ax - axf'(\eta)) \\ &= \frac{\sigma_{nf}B_0^2}{\rho_{nf}}ax(1-f'(\eta)) \\ &= \frac{\sigma_{nf}B_0^2ax(1-f'(\eta))}{(1-\phi)\rho_f + \phi\rho_s} \\ &= \frac{\sigma_{nf}B_0^2a^2x(1-f'(\eta))}{((1-\phi) + \phi\frac{\rho_s}{\rho_f})}. \qquad (3.23)\end{aligned}$$

Using (3.20)-(3.22), the right side of (3.2) becomes:

$$U_\infty \frac{dU_\infty}{dx} + \nu_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B_0}{\rho_{nf}}(U_\infty - u)$$
$$= a^2 x + \frac{a^2 x f'''(\eta)}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\frac{\rho_s}{\rho_f})} + \frac{\sigma_{nf} B_0^2 a^2 x}{((1-\phi) + \phi\frac{\rho_s}{\rho_f})}(1 - f'(\eta)).$$

Hence the dimensionless form of (3.2) becomes:

$$a^2 x(f'^2(\eta) - f(\eta)f''(\eta)) = a^2 x + \frac{a^2 x f'''(\eta)}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\frac{\rho_s}{\rho_f})}$$
$$+ \frac{\sigma_{nf} B_0^2 a^2 x}{((1-\phi) + \phi\frac{\rho_s}{\rho_f})}(1 - f'(\eta)).$$

$$\Rightarrow f'^2(\eta) - f(\eta)f''(\eta) = 1 + \frac{f'''(\eta)}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\frac{\rho_s}{\rho_f})}$$
$$+ \frac{\sigma_{nf} B_0^2 a^2 x}{((1-\phi) + \phi\frac{\rho_s}{\rho_f})a\rho_f}(1 - f'(\eta)).$$
$$\Rightarrow \frac{f'''(\eta)}{\nu_f(1-\phi)^{2.5}((1-\phi)\rho_f + \phi\frac{\rho_s}{\rho_f})}$$
$$+ \frac{\sigma_{nf} B_0^2 a^2 x}{((1-\phi) + \phi\frac{\rho_s}{\rho_f})a\rho_f}(1 - f'(\eta)) - f'^2(\eta) + ff''(\eta) + 1 = 0. \quad (3.24)$$

Now, we include below the procedure for the conversion of (3.4) into the dimensionless form.

$$\bullet \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}$$
$$\Rightarrow \quad T = (T_f - T_\infty)\theta(\eta) + T_\infty$$
$$\bullet \quad \frac{\partial T}{\partial x} = \theta(\eta)(T_f - T_\infty)\frac{\partial \eta}{\partial x} = 0$$
$$\bullet \quad u\frac{\partial T}{\partial x} = 0 \quad (3.25)$$
$$\bullet \quad \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}((T_f - T_\infty)\theta(\eta) + T_\infty)$$
$$= (T_f - T_\infty)\theta'(\eta)\frac{\partial \eta}{\partial y} = (T_f - T_\infty)\sqrt{\frac{a}{\nu_f}}\theta'(\eta).$$

$$= (T_f - T_\infty)\sqrt{\frac{a}{\nu_f}}\theta'(\eta).$$

- $$v\frac{\partial T}{\partial y} = -\sqrt{a\nu_f}f(\eta)(T_f - T_\infty\sqrt{\frac{a}{\nu_f}}\theta'(\eta)$$
$$= -a(T_f - T_\infty)f\theta'(\eta). \tag{3.26}$$

Using (3.24) and (3.25), the left side of (3.3) gets the following form.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -a(T_w - T_\infty)f\theta'(\eta). \tag{3.27}$$

The proper side of condition (3.3) into dimensionless form, we continue as follows.

- $$\frac{\partial^2 T}{\partial y^2} = (T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta).$$
- $$\alpha_{nf}\frac{\partial^2 T}{\partial y^2} = \frac{k_{nf}}{(\rho c_p)_{nf}}(T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta)$$

$$= \frac{k_{nf}\rho_f(T_f - T_\infty)a}{(\mu_f)((1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s)}\theta''(\eta). \qquad \left(\because \nu_f = \frac{\mu_f}{\rho_f}\right) \tag{3.28}$$

Hence the dimensionless form of equation (3.3) becomes:

$$-a(T_w - T_\infty)f(\eta)\theta'(\eta) = \frac{k_{nf}\rho_f(T_f - T_\infty)a}{(\mu_f)((1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s)}\theta''(\eta).$$

$$\Rightarrow -f(\eta)\theta'(\eta) = \frac{\frac{k_{nf}}{k_f}\rho_f}{\mu_f(\rho c_p)_f((1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\theta''(\eta).$$

$$\Rightarrow \frac{k_{nf}/k_f}{Pr((1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\theta''(\eta) + \theta'(\eta)f(\eta) = 0. \qquad \left(\because Pr = \frac{(c_p)_f\mu_f}{k_f}\right)$$

Now we include below the procedure for the conversion of equation 3.4 into the dimensionless form.

- $$h(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

$$\Rightarrow \quad C = (C_w - C_\infty)h(\eta) + C_\infty$$

- $$\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}(C_w - C_\infty)h(\eta) + C_\infty) = 0.$$
- $$u\frac{\partial C}{\partial x} = 0. \tag{3.29}$$

- $$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}(C_w - C_\infty)h(\eta) + C_\infty) = (C_w - C_\infty)h'(\eta)\frac{\partial \eta}{\partial y}$$

- $$v\frac{\partial C}{\partial y} = (-\sqrt{a\nu_f}f(\eta))(C_w - C_\infty)\sqrt{\frac{a}{\nu_f}}h'(\eta) = -a(C_w - C_\infty)f(\eta)h'(\eta). \quad (3.30)$$

- $$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}\left((C_w - C_\infty)h'(\eta)\sqrt{\frac{a}{\nu_f}}\right) = (C_w - C_\infty)\frac{a}{\nu_f}h''(\eta). \quad (3.31)$$

Hence the dimensionless form of equation 3.4 becomes:

$$-a(C_w - C_\infty)f(\eta)h'(\eta) = D\left((C_w - C_\infty)\frac{a}{\nu_f}h''(\eta)\right)$$

$$\Rightarrow \frac{\nu_f}{D}f(\eta)h'(\eta) + h''(\eta) = 0.$$

$$\Rightarrow h''(\eta) + Scf(\eta)h'(\eta) = 0. \qquad \left(\because Sc = \frac{\nu_f}{D}\right) \quad (3.32)$$

The final dimensionless form of the governing model, is :

$$\frac{f'''}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)} + ff'' - f'^2 + \frac{M(1-f')}{\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)} + 1 = 0. \quad (3.33)$$

$$\frac{k_{nf}/k_f}{Pr((1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\theta'' + f\theta' = 0. \quad (3.34)$$

$$h'' + Scfh' = 0. \quad (3.35)$$

The boundary condition 3.5 get the taking after dimensionless frame:

$$\left.\begin{aligned} &f(\eta) = f_w,\ f'(\eta) = 0,\ \theta'(\eta) = -\frac{k_f}{k_{nf}}N_c(1-\theta(\eta)), \\ &h'(\eta) = -N_d(1-h(\eta)) \qquad at\ \eta = 0. \\ &f(\eta) \to 1,\ \theta(\eta) \to 0,\ h(\eta) \to 0 \qquad as\ \eta \to \infty. \end{aligned}\right\} \quad (3.36)$$

Different parameters in the above model have the following formulations:

$$Pr = \frac{(cp)_f \mu_f}{k_f},\ M = \frac{\sigma B_0^2}{\rho_f a},\ Sc = \frac{\nu_f}{D},\ N_d = \frac{h_m}{D_m}\sqrt{\frac{\nu_f}{a}}. \tag{3.37}$$

The surface drag coefficient, is obtained as:

$$\begin{aligned}
C_{fx} &= \frac{\tau_w}{\rho_f U_\infty^2} \\
&= \frac{\frac{-\mu_f}{(1-\phi)^{2.5}}\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho_f U_\infty^2} \\
&= \frac{\frac{-\mu_f}{(1-\phi)^{2.5}} ax\sqrt{\frac{a}{\nu_f}} f''(0)}{\rho_f (ax)^2} && (\because U_\infty = ax) \\
&= \frac{\sqrt{\nu_f}}{(1-\phi)^{2.5}\sqrt{ax}} f''(0). \\
\Rightarrow \sqrt{Re_x} C_{fx} &= -\frac{-1}{(1-\phi)^{2.5}} f''(0). && \left(\because Re_x = \frac{U_\infty x}{\nu_f} = \frac{ax^2}{\nu_f}\right)
\end{aligned}$$

The local heat transfer number is given as:

$$\begin{aligned}
Nu_x &= \frac{q_m x}{k(T_f - T_\infty)} \\
&= \frac{(-k_{nf}(\frac{\partial T}{\partial y})_{y=0})x}{k(T_f - T_\infty)} \\
&= \frac{\left(-k_{nf}(T_f - T_\infty)\sqrt{\frac{a}{\nu_f}}\theta'(0)\right)x}{k(T_f - T_\infty)} \\
&= -\frac{k_{nf}\sqrt{\frac{a}{\nu_f}}x}{k}\theta'(0) \\
&= -\frac{k_{nf}}{k}\sqrt{Re_x}\theta'(0). \\
\Rightarrow \frac{Nu_x}{\sqrt{Re_x}} &= \frac{k_{nf}}{k_f}\theta'(0).
\end{aligned}$$

The nearby mass exchange number is given as:

$$\begin{aligned}
Sh_x &= \frac{q_m x}{D(C_w - C_\infty)} \\
&= \frac{\left(-D(\frac{\partial C}{\partial y})_{y=0}\right)x}{D(C_w - C_\infty)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{-D(C_w - C_\infty)\sqrt{\frac{a}{\nu_f}}h'(0)x}{D(C_w - C_\infty)} \\
&= -\sqrt{\frac{a}{\nu_f}}xh'(0). \\
\Rightarrow \frac{Sh_x}{\sqrt{Re_x}} &= -h'(0).
\end{aligned}
$$

3.3 Solution Methodology

To solve the differential equation (3.32)-(3.34), the shooting method has been used. Since, equation (3.32) involves only f and its derivatives, it can be solved separately by the shooting. The solution of equation (3.32) will be used in equation (3.33) and equation (3.34) as a known input.

It can be observed that for the third order ODE equation (3.32), two initial conditions are given at $\eta = 0$. Let us denote the missing initial condition $f''(0)$ by r. For further proceeding, the following notations have been introduced:

$$
f = y_1,\ f' = y_2,\ f'' = y_3,\ \frac{\partial f}{\partial r} = y_4,\ \frac{\partial f'}{\partial r} = y_5,\ ,\frac{\partial f''}{\partial r} = y_6.
$$

For simplification, the following notations have also been opted.

$$
\left.
\begin{aligned}
\frac{1}{1 - \phi + \phi\frac{\rho_s}{\rho_f}} &= E, \\
\frac{E}{(1-\phi)^{2.5}} &= F.
\end{aligned}
\right\}
$$

Utilizing the above notations, one can effectively have the taking after system of first order ODEs:

$$
\left.
\begin{aligned}
y_1' &= y_2, & y_1(0) &= f_w, \\
y_2' &= y_3, & y_2(0) &= 0, \\
y_3' &= \frac{1}{F}(y_2^2 - y_1y_3 - EM(1 - y_2) - 1), & y_3(0) &= r,
\end{aligned}
\right\}
$$

$$\left.\begin{aligned} y_3' &= \frac{1}{F}(y_2^2 - y_1 y_3 - EM(1-y_2) - 1), && y_3(0) = r,\\ y_4' &= y_5, && y_4(0) = 0,\\ y_5' &= y_6, && y_5(0) = 0,\\ y_6' &= \frac{1}{F}(2y_2 y_5 - y_1 y_6 - y_3 y_4 + EM y_5), && y_6(0) = 1. \end{aligned}\right\} \tag{3.38}$$

The above initial value problem (IVP) will be solved numerically by using RK-4 approach. To implement the RK-4 method, the missing initial condition will be chosen as $r = r^0$. For the refinement of the missing condition, Newton's method for root-finding has been used which is governed by the following iterative formula:

$$r^{(k+1)} = r^{(k)} - \left(\frac{y_2(\eta_\infty,\ r^k) - 1}{y_5(\eta_\infty,\ r^{(k)})} \right). \tag{3.39}$$

For numerical solution of equation (3.37), the unbounded space $[0, \infty)$ has been supplanted by a bounded space $[0,\ M_\eta]$ where M_η is a positive number such that the variation in solution for $\eta > M_\eta$ is ignorable. the execution of the Newton's method can be presented in the following algorithmic form:

Step-1: Choose an initial guess $r = r^{(0)}$ in the equation (3.37) and solve it by the RK-4 method.

Step-2: If for a very small positive number ε,

$$\mid (y_1(r^{(k)})_{\eta = M_\eta} - 1) > \varepsilon \mid \text{ for } k = 0, 1, 2...,$$

then go to Step-3, otherwise the solution is there.

Step-3: Compute the next value of the missing initial condition $r^{(k+1)}, k = 0, 1, 2, ...$ by using the Newton's scheme given by (3.38).

Step-4: Repeat Step-1 with $r = r^{(k+1)}$.

In a similar manner the ODEs (3.33)-(3.34) along with the associated BCs can be solved by considering f as a known function.

Physical Properties	Water	Copper(Cu)	Silver(Ag)
$\rho\,(Kgm^3)$	997.1	8933	10500
$c_p\,(J/KgK)$	4179	385	235
$k\,(WmK)$	0.613	401	429

TABLE 3.1: Thermo-physical properties of water and nanopaarticles.

3.4 Code Validation

In this area the numerical comes about have been appeared within the frame of graphs and table. The effect of a few parameters ϕ, Pr, M, S_c, N_c, N_d on speed f' temperature θ and concentration h have been studied.
TABLE 3.2 is prepared to analyze the skin contact, heat transfer rate and mass transfer rate influenced by the variety in ϕ, M, N_c, N_d, f_w, and Sc. For the extending esteem of ϕ, M, f_w the skin grinding is found to amplify. Nusselt number is watched to increase for the extending esteem of ϕ. where its shows up an inverse conduct for N_c. Sherwood number is watched to amplify for the extending values of both N_d and S_c.

Figure 3.2 (a-c) show up the impact of the volume fraction suction parameter and M on the dimensionless velocity. It is seen in Figure 3.2 (a) that with an increase within the volume fraction parameter ϕ the speed of the fluid increases. It is effect to noted that Ag-water has higher speed profile but less momentum boundary layer thickness that of Cu-water. Figure 3.2 (b) reflects an increasing slant within the speed profile for the suction parameter f_w. In this case, Cu-water is ruling within the speed but has comparatively thinner boundary layer thickness. From Figure 3.2 (c), we watch that the speed profile increments with an increment within the attractive parameter M of nanoparticles. The velocity distribution of Ag-water is higher that of Cu-water but an inverse drift in noted for the boundary layer thickness.

ϕ	N_c	N_d	f_w	M	$f''(0)$	$-\theta'(0)$	$-h'(0)$	
		$S_c \rightarrow$					5	10
0	0.1	10^{-10}	0	0	1.2328	0.0918	1.0436	1.3391
0.1					1.4479	0.0626	1.0898	1.4019
0.2					1.5013	0.0409	1.1001	1.4162
0				1	1.5853	0.0922	1.1052	1.4251
0.1					1.6910	0.0627	1.1273	1.4548
0.2				5	2.4339	0.2575	1.2186	1.5854
0.1	0.5			5	2.4339	0.2575	1.2186	1.5854
	1				2.4339	0.4191	1.2186	1.5854
	10				2.4339	0.9629	1.2186	1.5854
	100				2.4339	1.1064	1.2186	1.5854
	1000				2.4339	2.4339	1.2186	1.5854
	2	1	0.3	2	2.1410	0.7868	0.6879	0.7867
		2			2.1401	0.7868	1.0486	1.2970
		3			2.1401	0.7868	1.2707	1.6547
0.2	3	5	-1.0	10	2.1429	0.6789	0.0125	0.0002
			-0.5		2,4579	0.2736	0.2273	0.0868
			0.5		3.2329	0.7652	1.1913	2.6265
				1.0	3.6898	0.8994	2.5804	3.3596

TABLE 3.2: Skin coefficient, diminished Nusselt and Sherwood numbers for Pr=6.2 and different values of the physical parameters

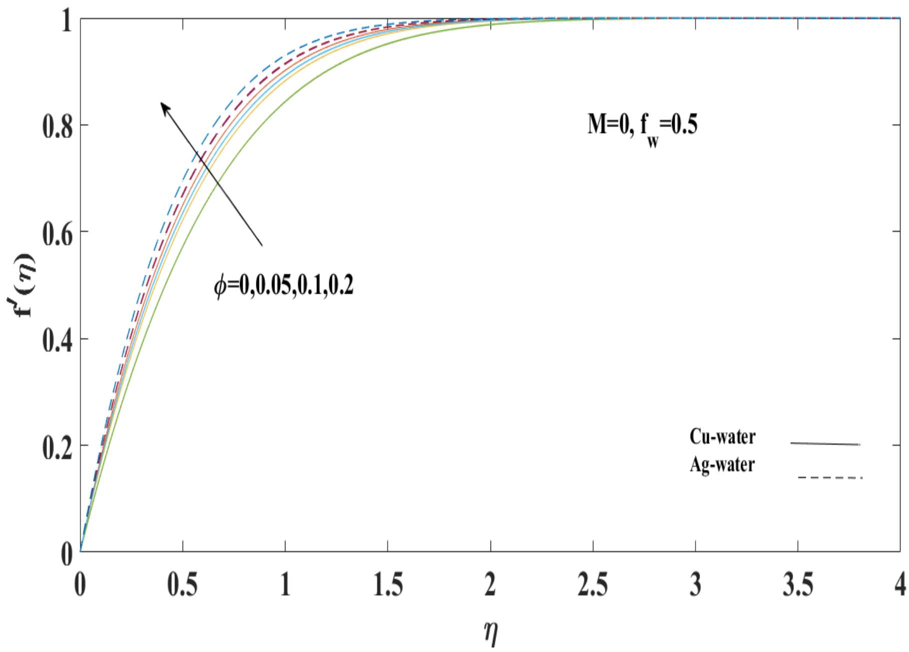
Figure 3.3 (a) and (b) are arranged to inspect to examine the impact of the nanoparticles volume fraction ϕ and the magnetic parameter M on the dimensionless temperature. It is seen that a rise within the volume division parameter and the magnetic parameter upgrades the fluid temperature. It is to be taken note that the warm boundary layer thickness gets to be thicker. The difference between the temperature profile of the Ag-water and Cu-water is negligible.

Figure 3.4 (a) and (b) are shown to seem the impact of Schmidt number S_c and the convective mass trade parameter N_d on the dimensionless concentration. It is observed that with the expanding values of Schmidt number S_c and the convective mass trade parameter N_d, the concentration profile appears an expanding drift. It is watched that the concentration profile with the same set of parameters for both Cu and Ag liquids are about overlaping, so their charts are not included.

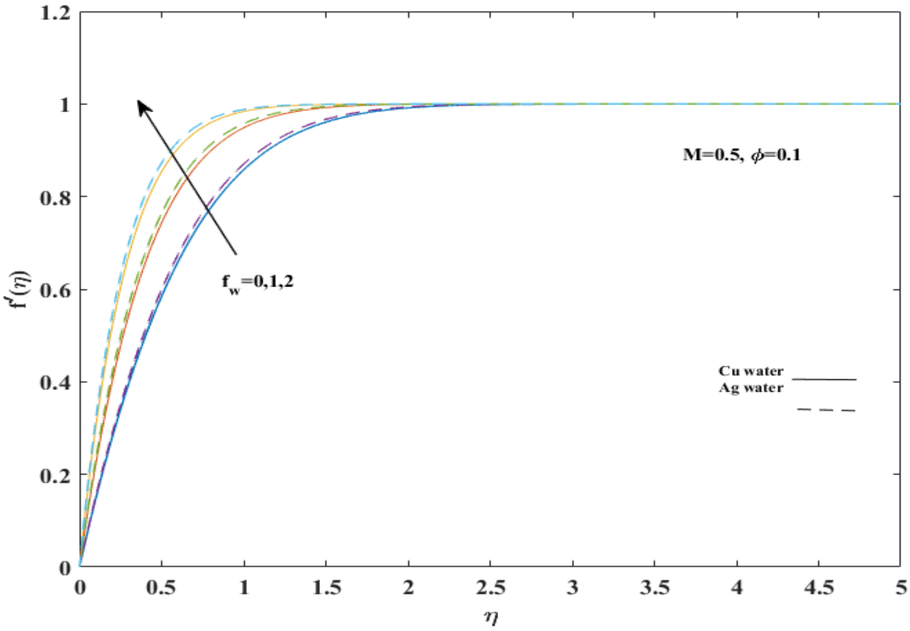
Figure 3.5 (a-b) are appeared to watch the affect of the suction parameter f_w and the volume division of nanoparticles for both Cu and Ag nanofluids. The skin division increases with an increment in both the volume division ϕ and the suction parameter f_w.

Figure 3.6 (a-b) portray the dissemination of wall Nusselt number$Nu_x/\sqrt{Re_x}$ for nanoparticles volume division ϕ, magnetic parameter M and convective heat trans fer parameter N_c. It can be observe that the nanoparticles volume division ϕ, the magnetic parameter M and the convective warm exchange parameter N_c increase the wall Nusselt number. Furthermore, it is additionally clear that the convective parameter enhances the warm exchange coefficient from the surface.

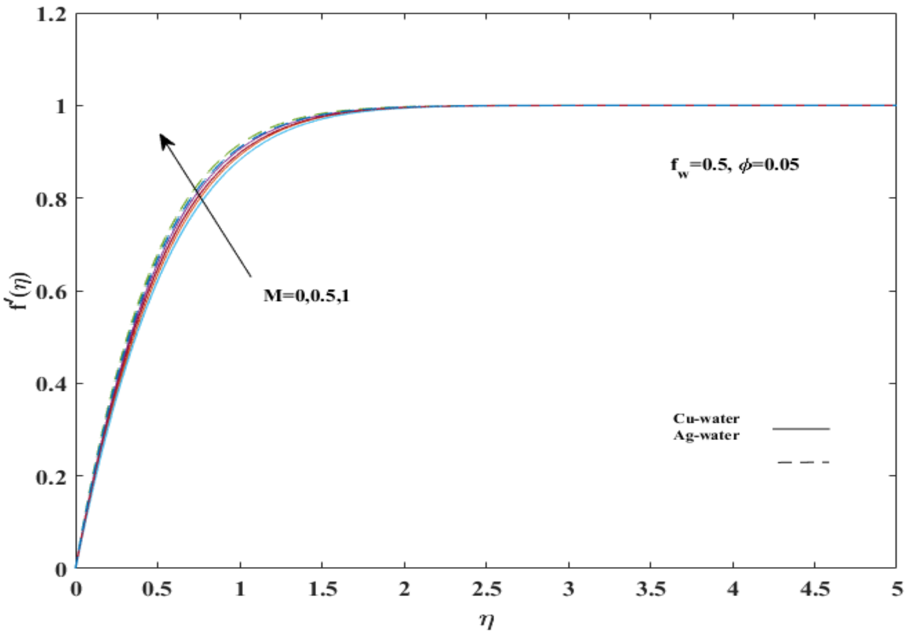
Figure 3.7 (a-b) are displayed to analyze the effect N_d with two different values of the magnetic parameter on Sherwood number against the volume division. We noticed that the Sherwood number increments with an increment within the magnetic parameter M and the mass exchange parameter N_d.



a



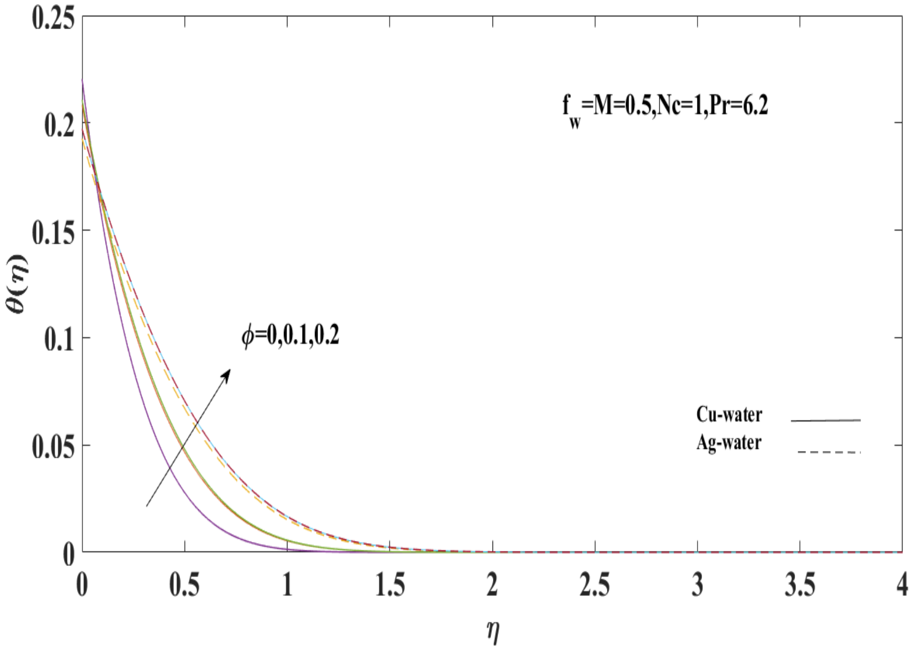
b



c

FIGURE 3.2: Impact of ϕ, f_w and M on f'

.



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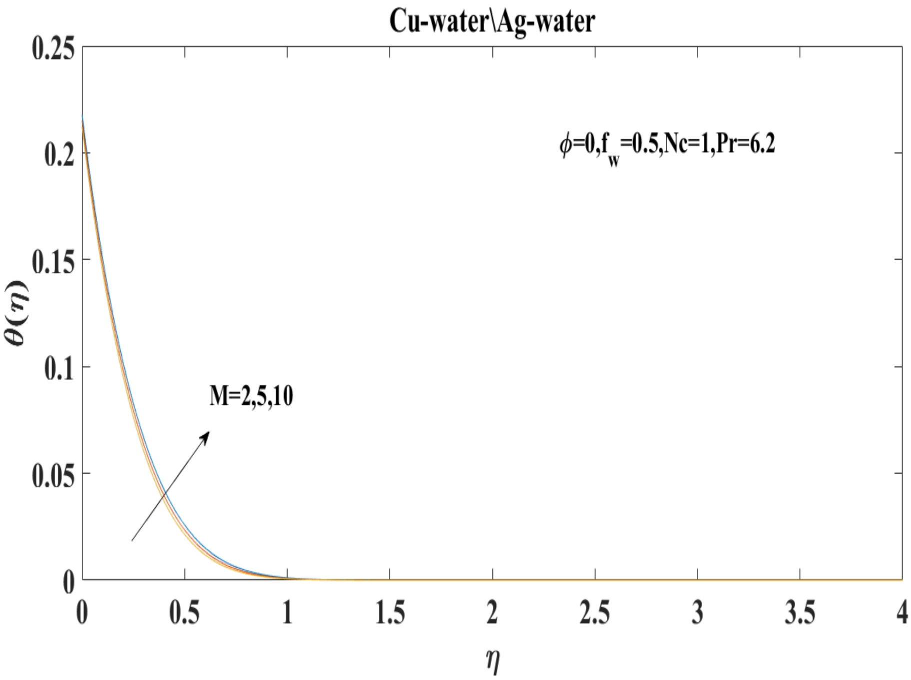
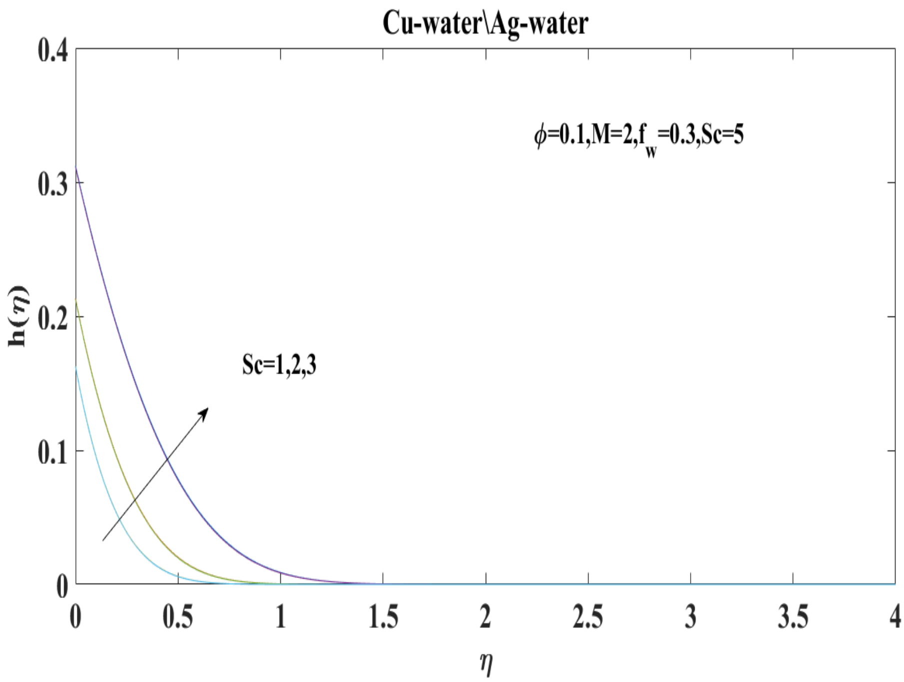


b

FIGURE 3.3: Impact of ϕ and M on θ.



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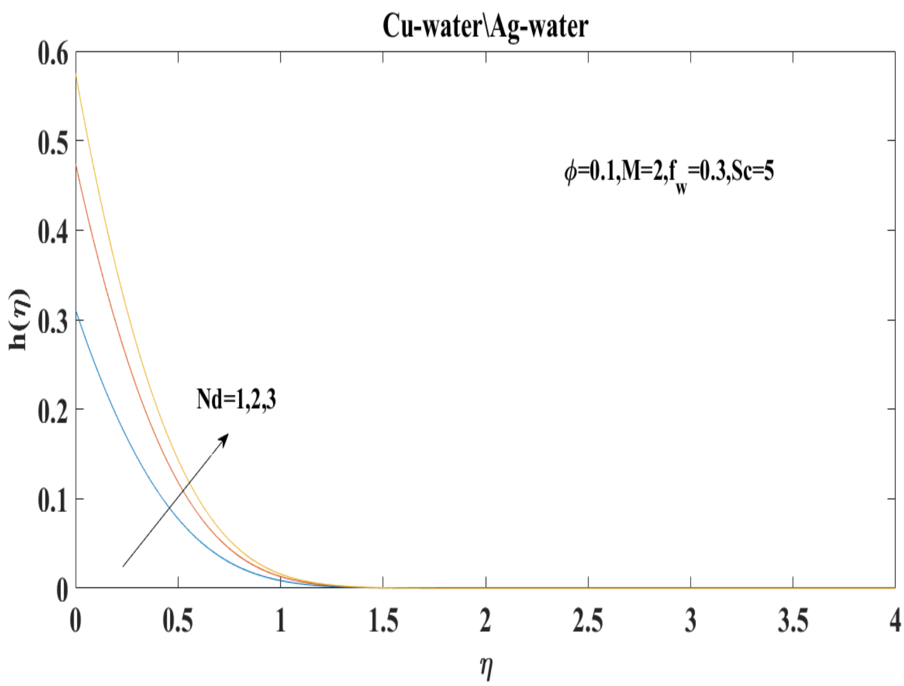
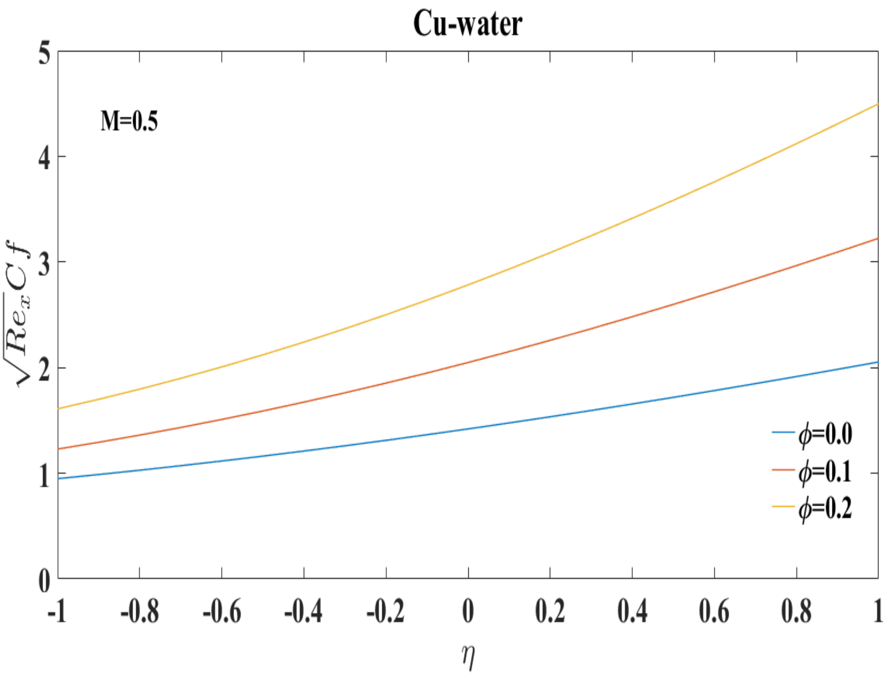
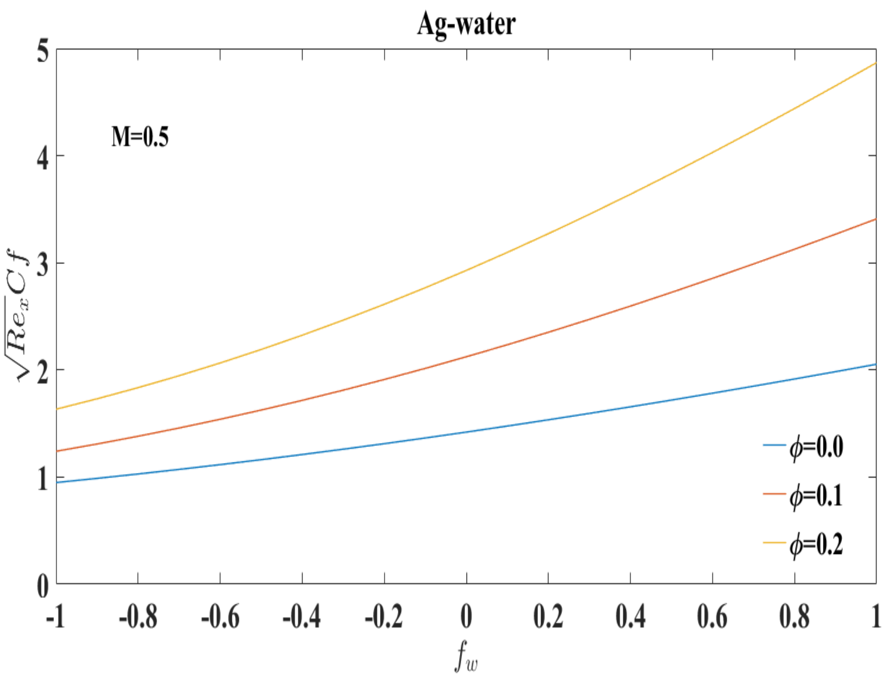


b

FIGURE 3.4: Impact of S_c and N_d on h.

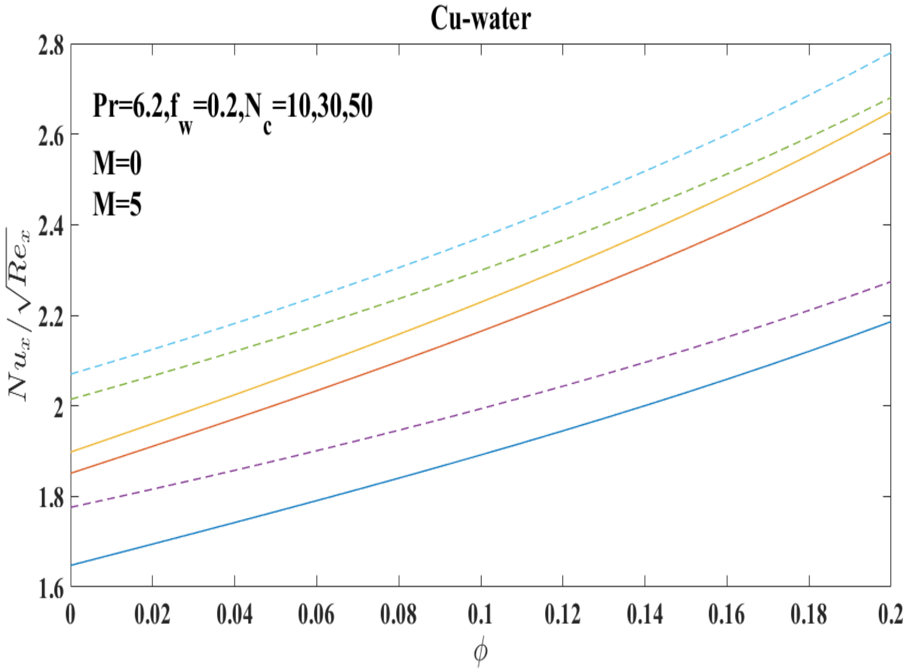


a



b

FIGURE 3.5: The variation of $\sqrt{Re_x}Cf$ with f_w for distinctive values of involved parameters.



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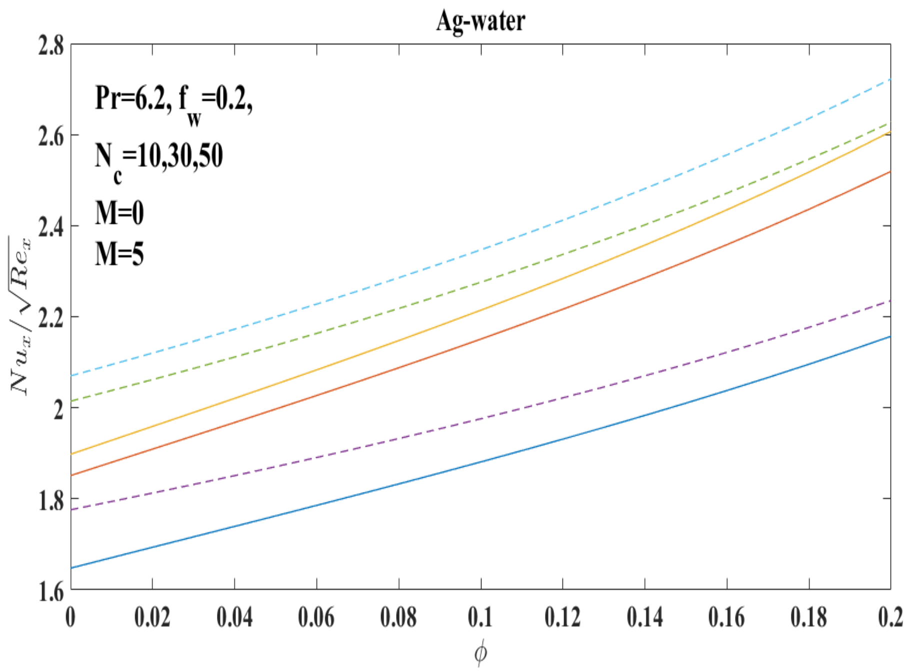
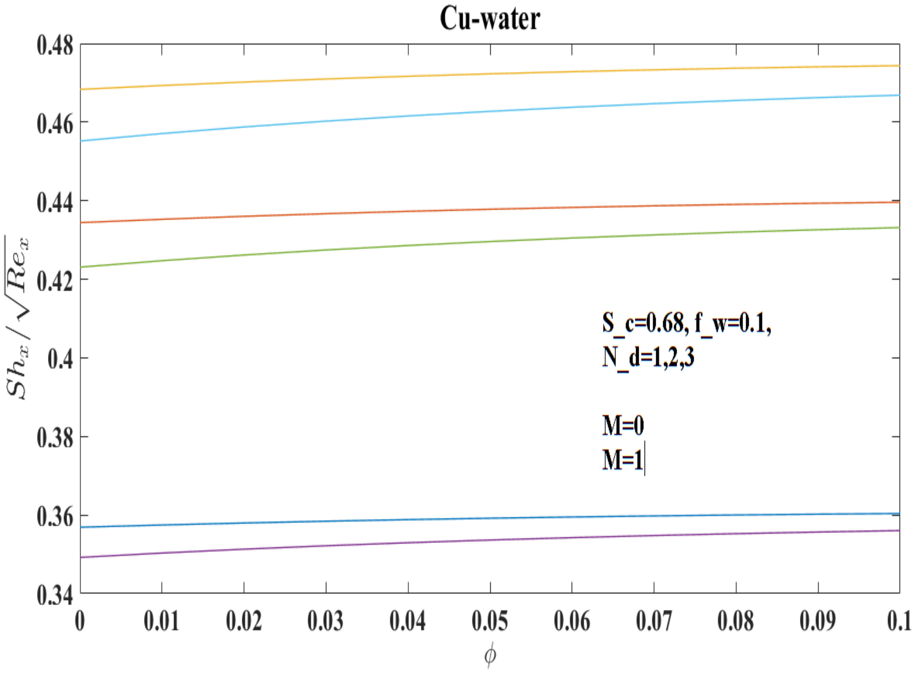


b

FIGURE 3.6: The assortment of $Nu_x/\sqrt{Re_x}$ with ϕ for distinctive values of included physical parameters.



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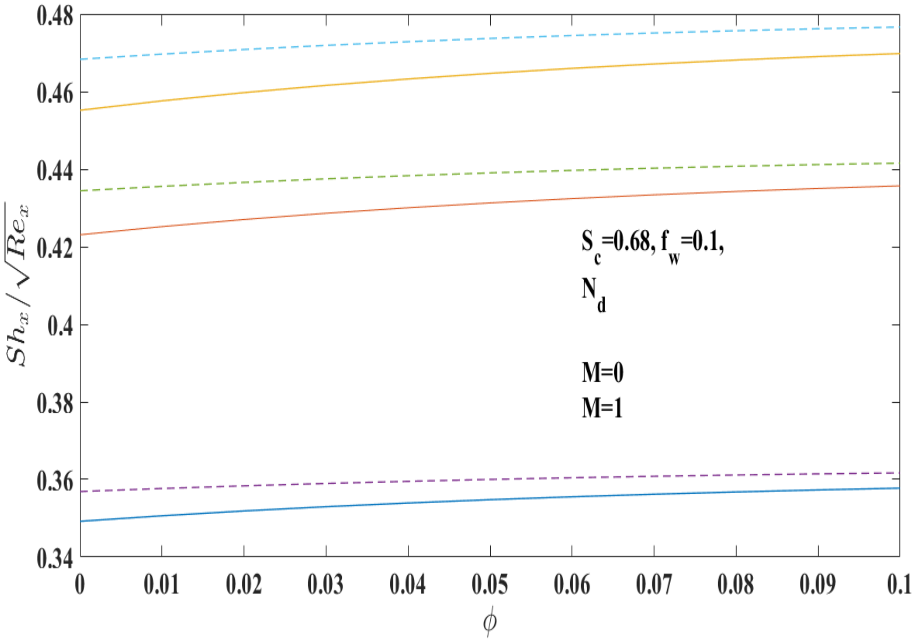


b

FIGURE 3.7: The variation of $Sh_x/\sqrt{Re_x}$ with ϕ for particular values of involved physical parameters.

Chapter 4

MHD Flow of Nanofluid Along with Joule Heating and Thermal Radiation

This chapter inspected the work of Mabood et al. [29] by counting the contributing effect the physical quantities Joule and heat radiation. Heat transfer analysis is performed together with the effect of thermal radiation and Joule heating. The thermal behaviour of base fluid is improved by considering the well-known Tiwari-Das model [34]. The nonlinear (PDEs) are converted into a system of (ODEs) by using an appropriate similarity transformation. The governing conditions of this show are illuminated numerically with the assistance of shooting procedure. The impact of distinctive parameters are established through graphs and table.

4.1 Problem Formulation

A laminar, two dimensional and relentless boundary layer flow of Cu and Ag-nanouids with water as a base-fluid has been considered over a flat moving surface. Warm exchange investigation is conducted with the impact of Joule warming and thermal radiation. The geometry of the flow demonstrate is considered the same as in Figure 3.1. The flow is depicted by the overseeing conditions of coherence, force, energy and the concentration condition portraying the two dimensional flow as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{4.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\nu_{nf}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_{nf}B_0}{\rho_{nf}}(U_\infty-u), \tag{4.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y}+\frac{\sigma_f B_0^2}{(\rho c_p)_{nf}}u^2, \tag{4.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \tag{4.4}$$

The associated boundary conditions as considered by Mabood et al. [29], in such a way:.

$$\left.\begin{aligned} &y=0: u=0,\ v=-v_0,\ -k_{nf}\frac{\partial T}{\partial y}=h_f(T_f-T),\\ &-D_m\frac{\partial C}{\partial y}=-h_m(C_f-C),\\ &y\to\infty:\ u=U_\infty=ax,\ T=T_\infty,\ C=C_\infty. \end{aligned}\right\} \tag{4.5}$$

where U_∞ is the free stream speed and D is species diffusity. Compelling thickness, warm diffusivity, electrical conductivity, kinematic thickness, thickness, particular heat and the coefficient of thermal extension of the nanofluid [29, 34] are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s, \tag{4.6}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \tag{4.7}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{4.8}$$

$$(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s. \tag{4.9}$$

The radiative heat flux q_r is given by,

$$q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4.10}$$

where σ^* is the stefan-Boltzman reliable and k^* is the retention coefficient. On the off chance that the temperature qualification is exceptionally small, at that point the temperature assortment can be expanded around T_∞ in a Taylor arrangement, as below:

$$T^4 = T_\infty^4 + \frac{4T_\infty^3}{1!}(T - T_\infty) + \frac{12T_\infty^2}{2!}(T - T_\infty)^2 + \frac{24T_\infty^2}{3!}(T - T_\infty)^3 + \frac{24T_\infty^2}{4!}(T - T_\infty)^4.$$

Disregarding the higher order terms,

$$\begin{aligned} & T^4 = T_\infty^4 = 4T_\infty^3(T - T_\infty), \\ \Rightarrow \quad & T^4 = T_\infty^4 + 4T_\infty^3 T - 4T_\infty^4, \\ \Rightarrow \quad & T^4 = 4T_\infty^3 T - 3T_\infty^4, \\ \Rightarrow \quad & \frac{\partial T^4}{\partial y} = 4T_\infty^3 \frac{\partial T}{\partial y}. \end{aligned} \tag{4.11}$$

Using equation (4.11) in equation (4.10) and then differentiating w.r.t. y, we obtain:

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \tag{4.12}$$

Then the equation (4.3) gets the following form.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho c_p)_f}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma_f B_0^2}{(\rho c_p)_{nf} u^2}. \tag{4.13}$$

Introduce the stream function ψ satisfying the continuity equation in the following way:

$$u = \frac{\partial \psi}{\partial y}, v = \frac{\partial \psi}{\partial x}. \tag{4.14}$$

In order to switch the PDEs to ODEs the following transformation has been introduced:

$$\psi = \sqrt{U_\infty x \nu_f} f(\eta) = \sqrt{a x^2 \nu_f} f(\eta) = x\sqrt{a\nu_f} f(\eta), \tag{4.15}$$

$$\eta = y\sqrt{\frac{U_\infty}{x\nu_f}} = y\sqrt{\frac{ax}{x\nu_f}} = y\sqrt{\frac{a}{\nu_f}}, \tag{4.16}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{4.17}$$

$$h(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{4.18}$$

The procedure for the conversion of equations (4.1), (4.2) and (4.4) is exactly the same as in chapter 3. However, the detailed procedure for the non-dimensionlization of (4.3) has been included below. The left side of (4.3) gets the following form.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -a(T_w - T_\infty)f(\eta)\theta'(\eta). \tag{4.19}$$

To transform the right side of equation (4.3) into dimensionless form, we continue as follows.

- $$\frac{\partial^2 T}{\partial y^2} = (T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta).$$
- $$\alpha_{nf}\frac{\partial^2 T}{\partial y^2} = \frac{k_{nf}}{(\rho c_p)_{nf}}(T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta)$$
  $$= \frac{k_{nf}\rho_f(T_f - T_\infty)a}{(\mu_f)((1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s)}\theta''(\eta). \tag{4.20}$$
- $$\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} = \frac{1}{(\rho c_p)_{nf}}\frac{-16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}$$
  $$= \frac{1}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s}\frac{-16\sigma^* T_\infty^3}{3k^*}(T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta) \tag{4.21}$$
- $$\frac{\sigma_f B_0^2}{(\rho c_p)_{nf}}u^2 = \frac{\sigma_f B_0^2 (axf'(\eta))^2}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s}. \tag{4.22}$$

Using (4.20)-(4.22) the dimensionless form of right side equation (4.3) is as follows.

$$\alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y} + \frac{\sigma_f B_0^2}{(\rho c_p)_{nf}}u^2$$
$$= \frac{k_{nf}\rho_f(T_f - T_\infty)a}{(\mu_f)((1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s)}\theta''(\eta)$$
$$+ \frac{1}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s}\frac{16\sigma^* T_\infty^3}{3k^*}(T_f - T_\infty)\frac{a}{\nu_f}\theta''(\eta) + \frac{\sigma_f B_0^2 (axf'(\eta))^2}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s}$$

$$= \frac{\frac{k_{nf}}{k_f}(T_f - T_\infty)a}{Pr\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)}\theta''(\eta) + \frac{16\sigma^* T_\infty^3 (T_f - T_\infty)a}{Pr\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)3k^*}\theta''(\eta)$$
$$+ \frac{\sigma_f B_0^2 a^2 x^2}{\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)} f'^2(\eta). \qquad \left(\because Pr = \frac{(cp)_f \mu_f}{k_f}\right)$$

Hence the dimensionless form of equation (4.3) becomes:

$$\begin{aligned}
-a(T_w - T_\infty)f(\eta)\theta'(\eta) &= \frac{k_{nf}/k_f(T_f - T_\infty)a}{Pr\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)}\theta''(\eta) \\
&+ \frac{16\sigma^* T_\infty^3 (T_f - T_\infty)a}{Pr\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)3k^*}\theta''(\eta) + \frac{\sigma_f B_0^2 a^2 x^2}{\left(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)} f'^2(\eta) \\
\Rightarrow -Prf(\eta)\theta'(\eta) &= \left(\frac{k_{nf}/k_f}{(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})} + \frac{4R}{3(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\right) \\
&+ \frac{MEcPr}{(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})} f'^2 \\
&\left(\because R = \frac{4\sigma^* T_\infty^3}{k_f k^*},\ M = \frac{\sigma B_0^2}{a},\ Ec = \frac{U_\infty^2}{(cp)_f (T_f - T_\infty)}\right)
\end{aligned}$$

The final dimensionless form of the governing model, is :

$$\frac{f'''}{(1-\phi)^{2.5}\left(1 - \phi + \phi\frac{\rho_s}{\rho_f}\right)} + ff'' - f'^2 + \frac{M(1 - f')}{\left(1 - \phi + \phi\frac{\rho_s}{\rho_f}\right)} + 1 = 0. \tag{4.23}$$

$$\left(\frac{k_{nf}/k_f}{(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})} + \frac{4R}{3(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})}\right)\theta'' + \frac{MEcPr}{(1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f})} f'^2 + Pr\theta f' = 0. \tag{4.24}$$

$$h'' + Scfh' = 0. \tag{4.25}$$

The boundary condition (4.5) obtain the following form:

$$\left.\begin{aligned}
&f(\eta) = f_w,\ f'(\eta) = 0,\ \theta'(\eta) = -\frac{k_f}{k_{nf}}N_c(1 - \theta(\eta)), \\
&h'(\eta) = -N_d(1 - h(\eta)) \qquad at\ \eta = 0. \\
&f(\eta) \to 1,\ \theta(\eta) \to 0,\ h(\eta) \to 0 \qquad as\ \eta \to \infty.
\end{aligned}\right\} \tag{4.26}$$

Different parameters in the above model have the following formulations:

$$\left.\begin{aligned} &Pr = \frac{(cp)_f \mu_f}{k_f},\ M = \frac{\sigma B_0^2}{\rho_f a},\ Sc = \frac{\nu_f}{D},\ N_d = \frac{h_m}{D_m}\sqrt{\frac{\nu_f}{a}},\ R = \frac{4\sigma^* T_\infty^3}{k_f k^*},\\ &Ec = \frac{U_\infty^2}{(cp)_f (T_f - T_\infty)}. \end{aligned}\right\} \tag{4.27}$$

The surface drag coefficient, is obtained as:

$$\begin{aligned} C_{fx} &= \frac{\tau_w}{\rho_f U_\infty^2}\\ &= \frac{\frac{-\mu_f}{(1-\phi)^{2.5}}\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho_f U_\infty^2}\\ &= \frac{\frac{-\mu_f}{(1-\phi)^{2.5}} ax\sqrt{\frac{a}{\nu_f}} f''(0)}{\rho_f (ax)^2} \qquad\qquad (\because U_\infty = ax)\\ &= \frac{\sqrt{\nu_f}}{(1-\phi)^{2.5}\sqrt{ax}} f''(0).\\ \Rightarrow \sqrt{Re_x} C_{fx} &= -\frac{-1}{(1-\phi)^{2.5}} f''(0). \qquad\qquad \left(\because Re_x = \frac{U_\infty x}{\nu_f} = \frac{ax^2}{\nu_f}\right) \end{aligned}$$

The local heat transfer number is given as:

$$\begin{aligned} Nu_x &= \frac{q_m x}{k(T_f - T_\infty)}\\ &= \frac{(-k_{nf}(\frac{\partial T}{\partial y})_{y=0})x}{k(T_f - T_\infty)}\\ &= \frac{\left(-k_{nf}(T_f - T_\infty)\sqrt{\frac{a}{\nu_f}}\theta'(0)\right)x}{k(T_f - T_\infty)}\\ &= -\frac{k_{nf}\sqrt{\frac{a}{\nu_f}}x}{k}\theta'(0)\\ &= -\frac{k_{nf}}{k}\sqrt{Re_x}\theta'(0).\\ \Rightarrow \frac{Nu_x}{\sqrt{Rex}} &= \frac{k_{nf}}{k_f}\theta'(0). \end{aligned}$$

The nearby mass exchange number is gotten as:

$$Sh_x = \frac{q_m x}{D(C_w - C_\infty)}$$

$$
\begin{aligned}
&= \frac{\big(-D(\frac{\partial C}{\partial y})_{y=0}\big)x}{D(C_w - C_\infty)} \\
&= \frac{-D(C_w - C_\infty)\sqrt{\frac{a}{\nu_f}}h'(0)x}{D(C_w - C_\infty)} \\
&= -\sqrt{\frac{a}{\nu_f}}xh'(0). \\
\Rightarrow \frac{Sh_x}{\sqrt{Re_x}} &= -h'(0).
\end{aligned}
$$

4.2 Numerical Treatment

For the solution of differential equation (4.23)-(4.25), the shooting method has been used. Since equation (4.24) is a different one compared with those in Chapter 3, so in the present section, a numerical treatment of only this equation is targeted. It can be solved by shooting, method as follows:.

The missing initial condition $\theta'(0)$ is denoted by s. For further proceeding, the following notations have been introduced,

$$\theta = y_1,\ \theta' = y_2,\ \frac{\partial \theta}{\partial s} = y_3,\ \frac{\partial \theta'}{\partial s} = y_4.$$

For simplification, the following notations have also been prefered.

$$
\left.
\begin{aligned}
\frac{k_f}{k_{nf}} &= C, \\
\rho_s cp_s &= A, \\
\rho_f cp_f &= B, \\
(1-\phi+\phi(A/B)) &= X, \\
\frac{1}{CX} &= I, \\
\frac{4R}{3X} &= Q, \\
MEcPr &= W.
\end{aligned}
\right\}
$$

Utilizing the above notations, one can effectively have the taking after framework of first order ODEs:

$$\left.\begin{aligned} y_1' &= y_2, & y_1(0) &= s,\\ y_2' &= \frac{-Prfy_2 - (Wf'^2)/X}{I+Q}, & y_2(0) &= -CN_c(1-s),\\ y_3' &= y_4, & y_3(0) &= 1,\\ y_4' &= \frac{-Prfy_4}{I+Q}, & y_4(0) &= 0. \end{aligned}\right\} \tag{4.28}$$

The over accomplished ODEs to illuminate the over framework numerically, we are going alter the space $[0,\infty)$ by the bounded space $[0, M_\infty]$. where M_∞ is a few reasonable limited actual number. To implement the RK-4 method, the missing initial condition will be chosen as $s = s^0$. For the refinement of the missing condition, Newtons method for root-finding has been used which is governed by following iterative formula:

$$s^{(k+1)} = s^{(k)} - \left(\frac{y_1(\eta_\infty,\ s^k) - 0}{y_3(\eta_\infty,\ s^{(k)})} \right).$$

Physical Properties	Water	Copper(Cu)	Silver(Ag)
$\rho\,(Kgm^3)$	997.1	8933	10500
$c_p\,(J/KgK)$	4179	385	235
$k\,(WmK)$	0.613	401	429

TABLE 4.1: Thermo-physical properties of water and nanopaarticles.

4.3 Code Validation

TABLE 4.2 is prepared to analyze the skin contact, heat transfer rate and mass transfer rate influenced by the varieties in ϕ, M, N_c, N_d, f_w, S_c, E_c and R. For the extending regard of ϕ, M, f_w the skin fricton is found to expand. Nusselt number is observed to decrease for the extending regard of ϕ, E_c, R. Whereas it

ϕ	N_c	N_d	f_w	M	E_c	R	$f''(0)$	$-\theta'(0)$	$-h'(0)$	
				$S_c \rightarrow$					5	10
0	0.1	10^{-10}	0	0	0.1	10	1.2328	0.0806	1.0436	1.3391
0.1							1.4479	0.0577	1.0898	1.4019
0.2							1.5013	0.0390	1.1001	1.4162
0				1			1.5853	0.0716	1.1052	1.4251
0.1							1.6910	0.0508	1.1273	1.4548
0.2				5			2.4339	0.0343	1.2186	1.5854
0.1	0.5						2.4339	0.0694	1.2186	1.5854
	1						2.4339	0.0974	1.2186	1.5854
	10						2.4339	0.1528	1.2186	1.5854
	100						2.4339	0.1620	1.2186	1.5854
	1000						2.4339	0.1630	1.2186	1.5854
	2	1			0.2	15	2.1410	0.2044	0.6879	0.7867
		2					2.1401	0.2044	1.0486	1.2970
		3					2.1401	0.2044	1.2707	1.6547
0.2	3	5			0.3	10	2.1429	-0.4673	0.0125	0.0002
			-0.5				2.579	-0.4924	0.2273	0.0868
			0.5				3.2329	-0.5119	1.1913	2.6265
				1.0			3.6898	-0.5066	2.5804	3.3596

TABLE 4.2: Skin coefficient, diminished Nusselt and Sherwood numbers for Pr=6.2 and diverse values of the physical parameters

shows up an inverse conduct for N_c. Mass transfer rate is observed to amplify for the growing values of both N_d and S_c.

Figure 4.1 (a-c) appear the affect of the volume fraction suction parameter and magnetic parameter on the dimensionless velocity. It is seen in Figure 4.2 (a) that with an increment within the volume fraction parameter ϕ the speed of the fluid increases. It is affect to note that Ag-water has higher speed profile but less momentum boundary layer thickness that of Cu-water. Figure 4.1 (b) reflects an increasing slant within the speed profile for the suction parameter f_w. In this case, Cu-water is ruling within the speed but has comparatively thinner boundary layer thickness. From Figure 4.1 (c), we watch that the speed profile increases with an increase within the attractive parameter M of nanoparticles. The velocity distribution of Ag-water is higher that of Cu-water but an inverse drift in noted for the boundary layer thickness.

Figure 4.2 are prepared to analyze the influence eckret number E_c on the dimensionless temperature for both Cu-water and Ag-water. From this graph it can be observed that by increasing the value of volume fraction E_c the temperature profile is going to increase.

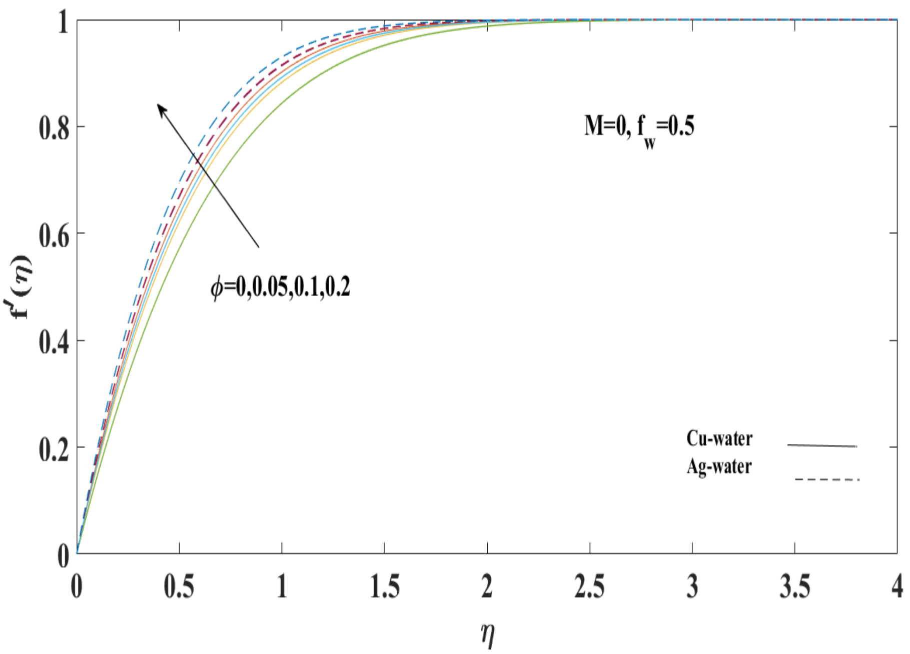
Figure 4.3 are organized to analyze the impact of Radiation parameter R on the dimensionless temperature for both Cu-water and Ag-water. From this charts it can be watched that by amplifying the Radiation parameter R the temperature profile is getting to decreased.

Figure 4.5 (a) and (b) are displayed to appear the impact of Schmidt number Sc and the convective mass exchange parameter N_d on the dimensionless concentration. It is observed that with the expanding values of Schmidt number Sc and the convective mass exchange parameter N_d, the concentration profile shows an increasing trend. It is observed that the concentration profile with the same set of parameters for both Cu and Ag fluids are nearly overlaping, so their charts are not included.

Figure 4.6 (a-b) are shown to observe the impact of the suction parameter f_w and the volume division of nanoparticles for both Cu and Ag nanofluids. The skin division increases with an increment in both the volume division ϕ and the suction parameter f_w.

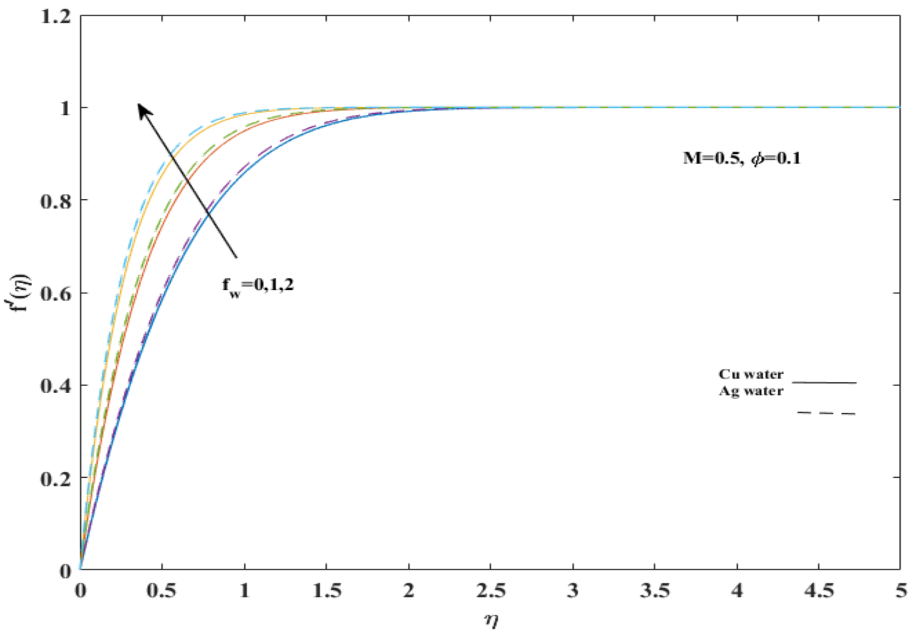
Figure 4.7 (a-b) portray the dissemination of wall Nusselt number$Nu_x/\sqrt{Re_x}$ for nanoparticles volume division ϕ, magnetic parameter M and convective heat trans fer parameter N_c. It can be observe that the nanoparticles volume division ϕ, the magnetic parameter M and the convective warm exchange parameter N_c increase the wall Nusselt number. Furthermore, it is additionally clear that the convective parameter enhances the warm exchange coefficient from the surface.

Figure 4.8 (a-b) are displayed to analyze the effect N_d with two different values of the magnetic parameter on Sherwood number against the volume division. We observe that the Sherwood number increments with an increase within the magnetic parameter M and the mass exchange parameter N_d.



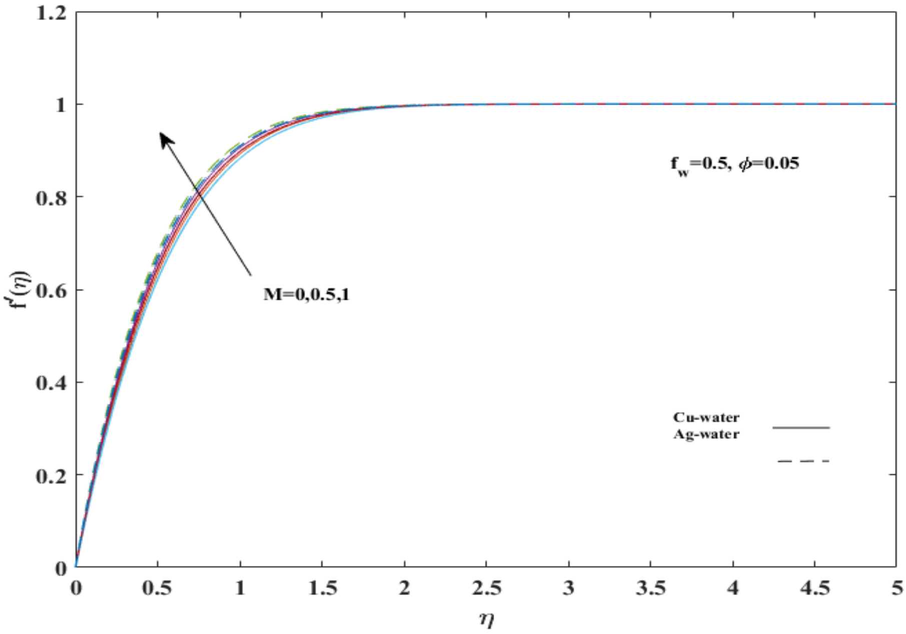
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
f′(η)
η
M=0, f_w=0.5
φ=0,0.05,0.1,0.2
Cu-water
Ag-water

a



1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
f′(η)
η
M=0.5, φ=0.1
f_w=0,1,2
Cu water
Ag water

b



c

FIGURE 4.1: Impact of ϕ, f_w and M on dimensionless f'.

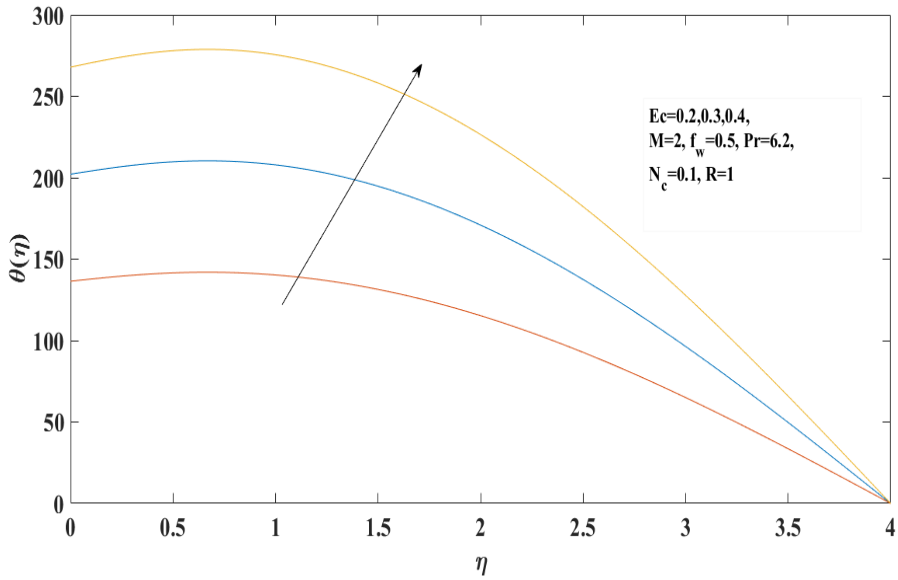


FIGURE 4.2: Impact of E_c on θ.

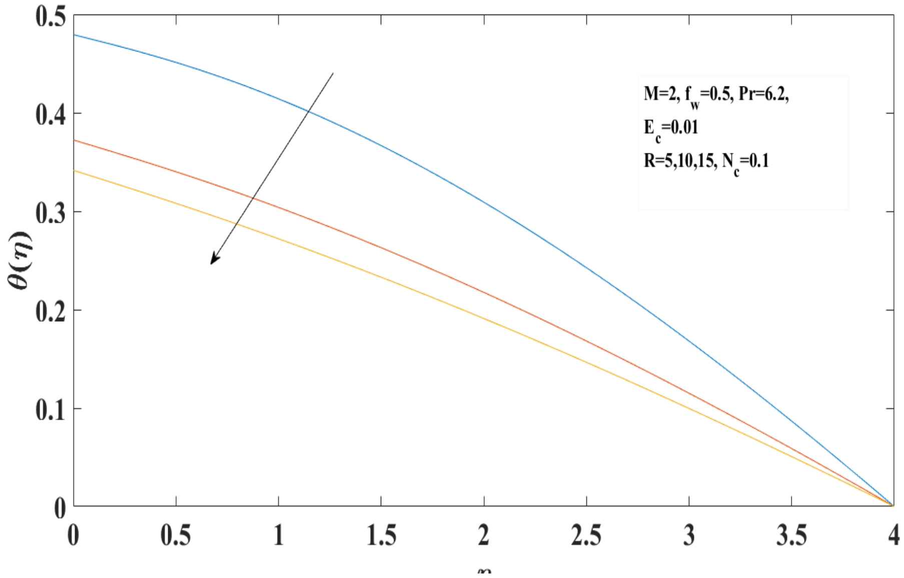
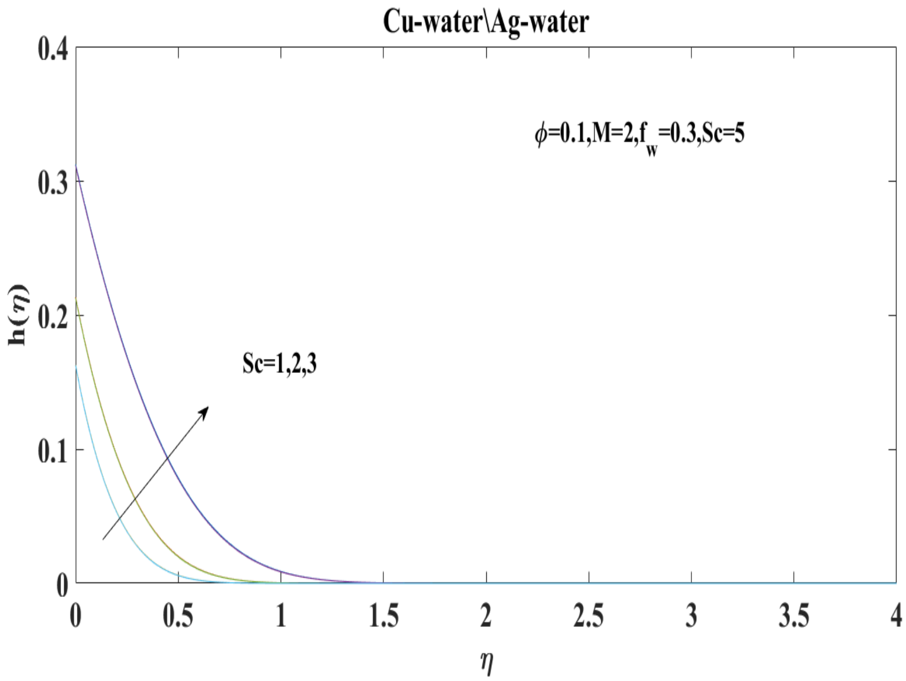
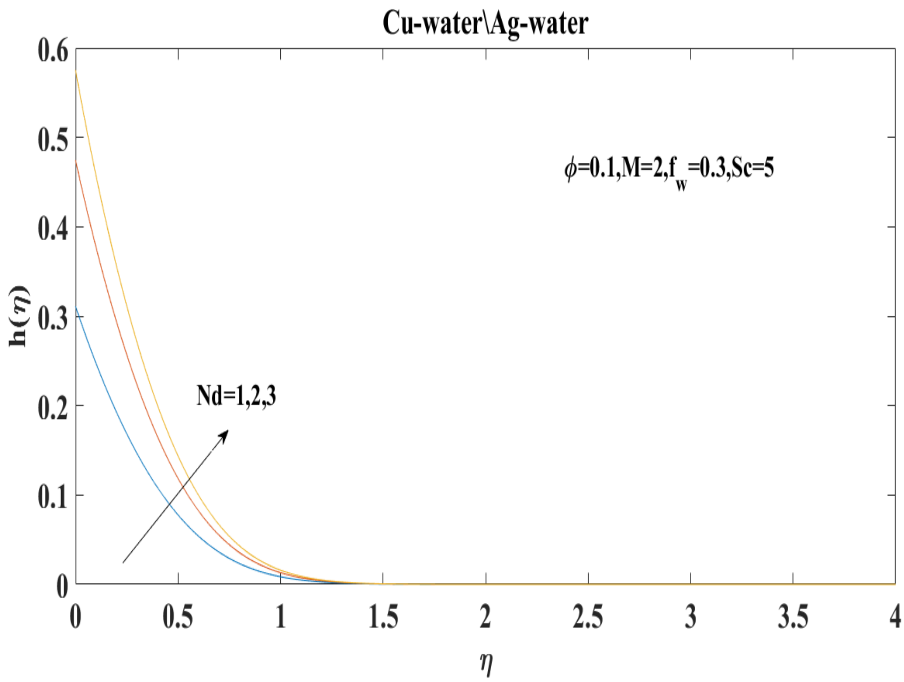


FIGURE 4.3: Behavior of R on θ.

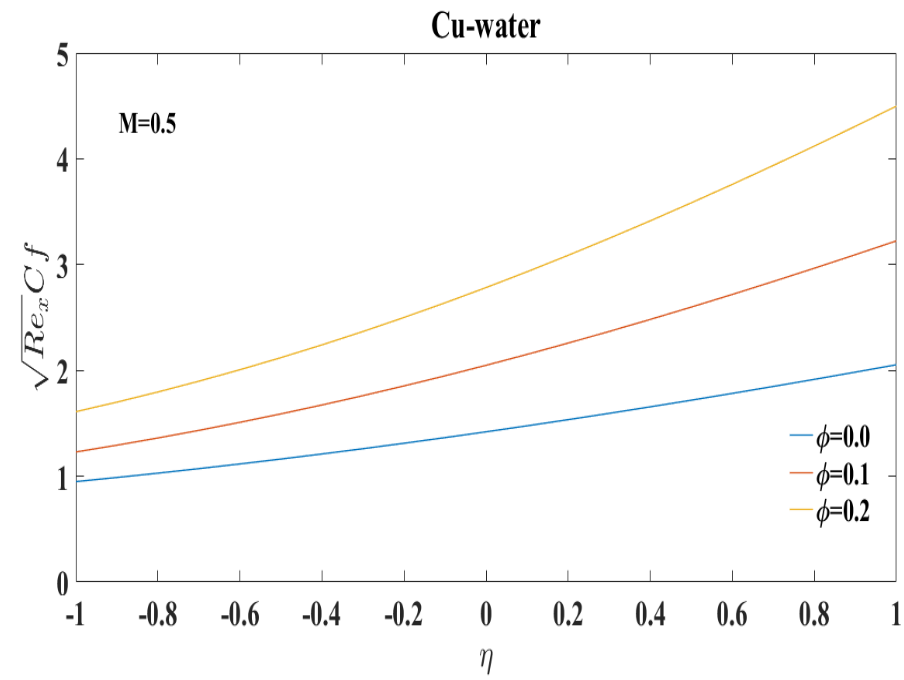


a

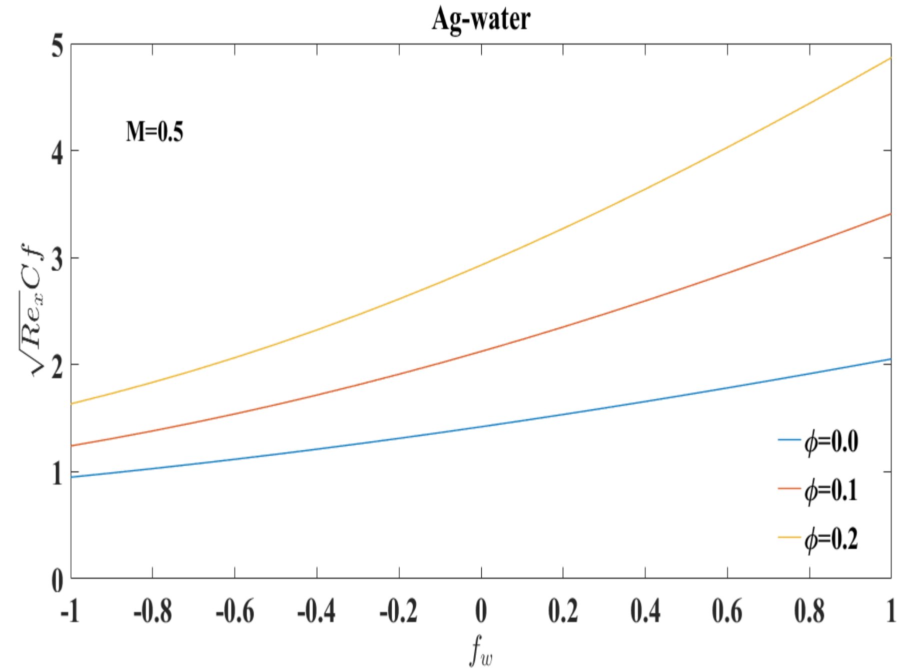


b

FIGURE 4.4: Behavior of S_c and N_d on h.

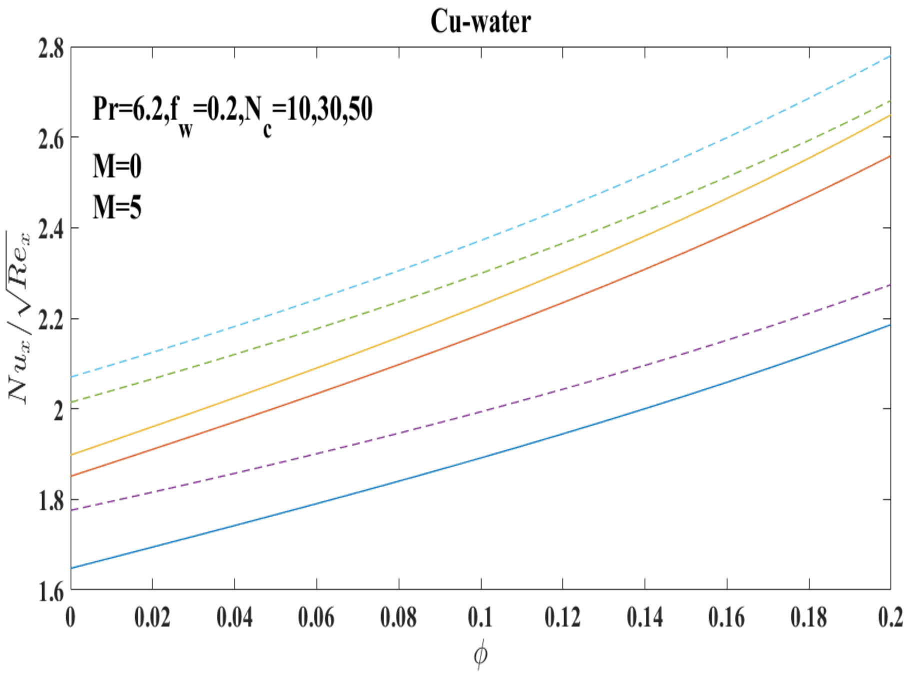


a

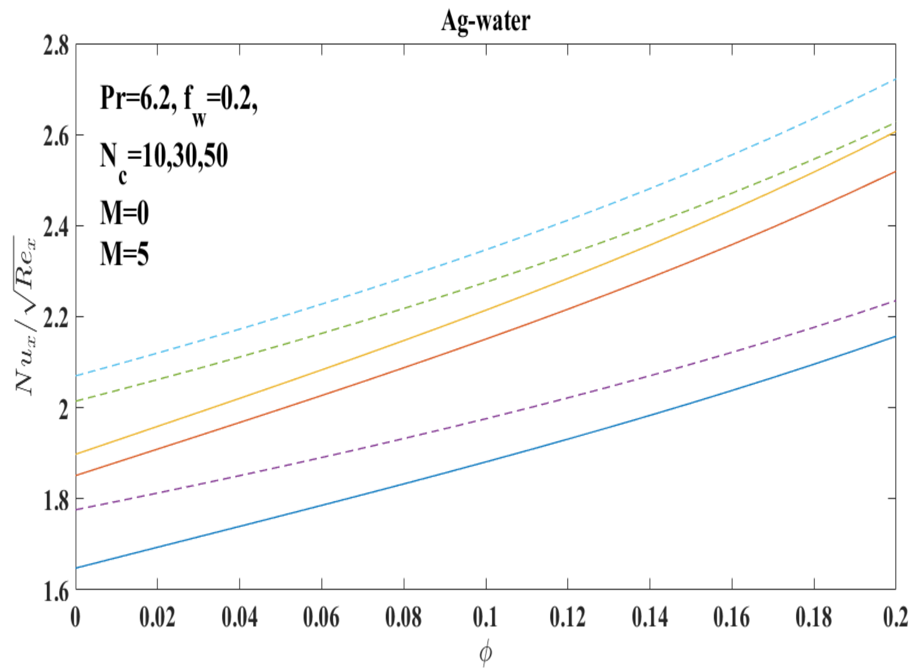


b

FIGURE 4.5: The variation of $\sqrt{Re_x}Cf$ with f_w for various values of involved parameters.

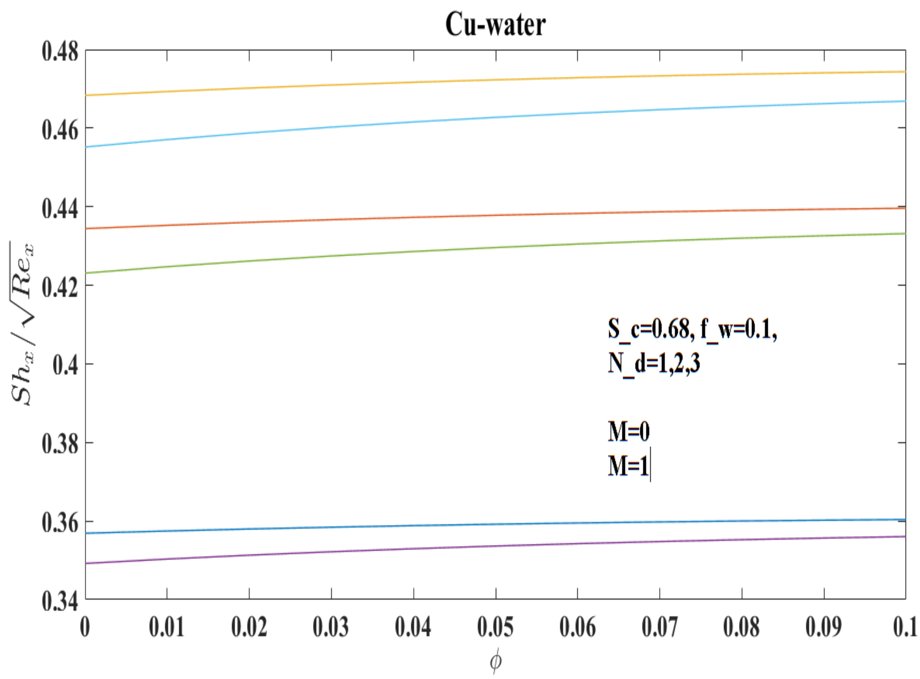


a

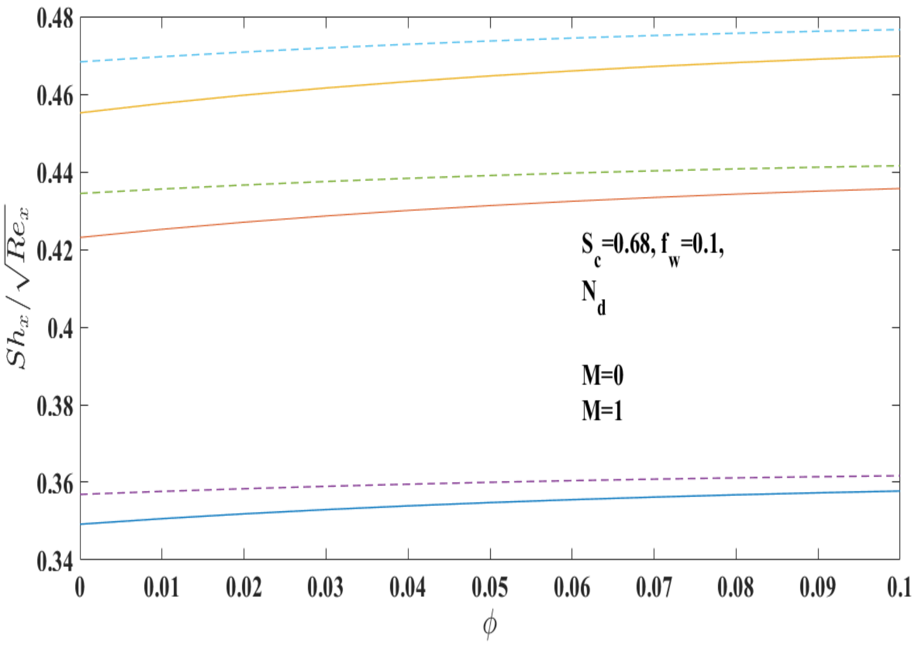


b

FIGURE 4.6: The assortment of $Nu_x/\sqrt{Re_x}$ with ϕ for distinctive distinctive values of included physical parameters.



a



b

FIGURE 4.7: The variation of $Sh_x/\sqrt{Re_x}$ with ϕ for distinctive values of involved physical parameters.

Chapter 5

Conclusion

We first looked at the work of Mabood et al. [29] in this preposition and extended it by counting the Joule heating and thermal radiation effects. The practices of velocity, temperature and concentration distribution are inspected both graphically by considering different values of different parameters. The significant findings have been listed underneath.

- An increase within the velocity profile is noted for the expanding values of the volume fraction ϕ.
- The temperature distribution increased for the expanding values of the magnetic parameter.
- Increment in thermal radiation decreases temperature profile.
- By expanding the values of Prandtl number comes about increment in temperature profile θ.
- The concentration distribution increases for an increment in Sc and N_d.
- The velocity profile is found to rise by enlarging suction parameter f_w.
- Due to the inclusion of Joule heating and thermal radiation , the temperature is seen to decline.

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