
بسم الله الرحمن الرحيم

IDENTIFICATION OF ELASTIC PROPERTIES OF ORTHOTROPIC COMPOSITES BASED ON A GENETIC ALGORITHM

By

Shummaila Rasheed
Reg. No. MME141003

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Certificate

This has to certify that Miss. Shummaila Rasheed (MME141003) has incorporated all observations, suggestions and comments made by external as well as the internal examiners and thesis supervisor. The title of her thesis is: "Identification of Elastic Properties of Orthotropic Composite based on Genetic Algorithm".

....

Dr. Nadeem Ahmed Sheikh

(Thesis Supervisor)

DEDICATION

To my parents
Mr. Muhammad Rasheed
Mrs. Ameer Akhter (Late)
My Husband
Shafqat Ali

And

It is with my deepest gratitude and warmest affection that I dedicate this thesis to my Supervisors "Dr. Nadeem Ahmed Sheikh "and "Prof. Carlos Antonio" who have been constant source of knowledge and inspiration for me in this whole period.

CAPITAL UNIVERSITY OF SCIENCE & TECHNOLOGY
ISLAMABAD

CERTIFICATE OF APPROVAL

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By

Shummaila Rasheed

MME141003

THESIS EXAMINING COMMITTEE

S No	Examiner	Name	Organization
(a)	External Examiner	Dr. Saeed Badshah	IIU, Islamabad
(b)	Internal Examiner	Dr. Muhammad Mahabat Khan	CUST, Islamabad
(c)	Supervisor	Dr. Nadeem Ahmed Sheikh	CUST, Islamabad

Dr. Nadeem Ahmed Sheikh

Thesis Supervisor

January, 2016

Dr. Saif ur Rahman

Head of Department

Department of Mechanical Engineering

Dated : 19th January, 2016

Dr. Imtiaz Ahmad Taj

Dean

Faculty of Engineering

Dated : 19th January, 2016

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DECLARATION

It is declared that this is an original piece of my own work, except where otherwise acknowledged in text and references. This work has not been submitted in any form for another degree or diploma at any university or other institution for tertiary education and shall not be submitted by me in future for obtaining any degree from this or any other University or Institution.

Shummaila Rasheed
Reg. No.MME141003
January, 2016

ABSTRACT

An inverse approach combining numerical and experimental results with full-field displacement measurements, will allow the identification of all the in-plane elastic properties from experimental tests. The off-axis tensile test was chosen, and in order to calibrate the numerical method an 8-harness satin weave glass fiber reinforced phenolic composite was selected. Instead the use of a non-destructive technique an alternative hybrid approach is proposed to obtain the field displacement. In this way a reference displacement field is generated by finite element method considering the loading and boundary conditions used in tensile tests and the mechanical properties obtained from experiments.

In this work effort is made to develop a technique for nondestructive characterization of laminated composites. Elastic properties of the composite are determined through this approach using an inverse technique based on finite element analysis and evolutionary algorithm supported by the experimental results. Mechanical properties of composite develop a link between the load applied in off axis tensile test and displacement field. The implementation of surrogate model ANN (Artificial Neural Network) eliminates the exhaustive calculation of elastic properties and displacement fields. Using UDM (Uniform Design Method) set of design points is generated. This will enables a uniform exploration of domain values that will be used in the development of ANN (Artificial Neural Network) approximation model.

Then, Reference displacement field is generated by FEM (Finite Element Method) taking into account the geometry of specimen, considering experimental loading and boundary condition. Experimental output data acquired for displacement field is providing as a reference in the optimization problem. The design variables of the optimization problem are the independent elastic engineering constants E_1 (Longitudinal), E_2 (Transversal), G (Shear Modulus) and ν (Poisson's ratio).

Keywords: Elastic properties, composite materials, inverse formulation, genetic algorithm, displacement field

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Chapter 1

INTRODUCTION OF RESEARCH WORK

1.1 Overview

The successful illustrations of innovative materials developments are, composite materials based lighter viable aircraft, bio-materials used in orthopedic surgical procedure and acoustic absorbing foams optimization. One of the key to this innovation is the evaluation of accurate mechanical properties. Trustworthy, identification approaches to find constitutive material parameters are essential in various applications like accurate detection of strength of composite using micromechanical model, damage recognition in structural health observing and prediction of spatial distribution of tissues belongings for abnormal tissues finding in medical analysis. Classical compression and tension tests (direct methods) can be applied to determine only a limited mechanical properties through a single test including the supposition that materials are isotropic and homogenous. Conversely, for the adequate characterization of heterogeneous/non-isotropic materials like composite, such identification techniques are more challenging [1]. Furthermore, this type of mechanical tests are incapable of identifying in-situ local mechanical properties in heterogeneous materials like reinforced composites/nano composites. Several literatures discovered that in-situ mechanical properties composite (constituent phases) are dissimilar from those determined in bulk material like result of manufacturing processes (consolidation/curing procedure) [2].

A very auspicious mode for concurrently finding mechanical properties of material is to implement an inverse approach together with full field measurement [3]. Many developments have been made to approximate mechanical properties using this technique. The most primitive and widely used inverse approach is based on FEMU (Finite Element Model Updating) strategy allows to rebuild unknown material properties from full field displacement / strain data. The Mixed numerical-experimental identification approach is an influential mean for cases where enough knowledge of geometry of specimen and boundary condition is obtainable.

1.2 Statement of Problem

Parameter identification methods integrating optimization techniques and the finite element method (FEM), offer an alternative tool for material characterization. Conventional test methods for measuring elastic constants are often destructive and require multiple complex tests to acquire all data. Non-destructive methods to determine the effective elastic constants of composite plates can be based on static measurements [4], vibration testing [4] or on the measurements of ultrasonic wave velocities [5]. Static or quasi-static methods are generally based on direct measurement of strain components and calculation (by measuring loads and geometry) of stress components during mechanical tests (tensile, compressive, flexural, torsional, etc.). The development of methodologies using a single specimen for measuring all the in-plane elastic constants could result very advantageous.

Recently, an inverse method was developed [6] with the aim of identifying all the four engineering constants from the measurement of displacement field of a single square plate transversally loaded. The inverse techniques are based on the minimisation of an error functional which describes the discrepancy between experimentally measured and numerically calculated response values.

1.3 Purpose of the Research

The main objective of this work is to suggest a reliable inverse identification strategy both in term of their accuracy and computational time to identify mechanical properties of composite. To this end, an attempt is made to find out the preferable approach for the non-destructive characterization of laminated composites depending on laboratory facilities. On that reason, a static approach using the off-axis tensile tests and an inverse procedure based on the planning of experiments methodology, are applied for determination of elastic properties of laminated composite plates.

The proposed numerical approach is based on FEA (Finite Element Analysis), EA (Evolutionary Algorithm) supported by experimental results. The idea is to determine the optimal estimation of the model parameters by minimizing a selected measure-of-fit between the responses of the tested system and the numerical model. The objective is to obtain the four independent elastic engineering constants for an orthotropic medium, based on the measurement of a heterogeneous displacement field.

1.4 Theoretical bases and Organization

The thesis is structured as follow. Chapter 2 explains inverse identification methods for material properties Chapter 3 tells about the non-destructive approach used in this research. Implementation of the inverse approach is discussed in the Chapter 4. Chapter 5 is about the result and discussion, and research work is concluded in Chapter 6.

Summary

The suggested inverse approach will permit the identification of all the in plane elastic properties, combining numerical and experimental results with full field displacement measurements, from experimental trials. As an alternative, the use of a non-destructive technique an alternative hybrid approach is proposed to obtain the field displacement.

Chapter 2

INVERSE IDENTIFICATION METHODS FOR MATERIAL PROPERTIES

2.1 Difference between Direct Method & Inverse Problem

With known input parameters and a reliable model the response of a complex system can be predicted. Such problems in which the response of a system is obtained with respect to the input parameters, is stated as a *direct problem*. Conversely, based on the system response and a given model the estimation of set of parameters is *inverse problem* method. A physical entity may be directly represented by many parameters, e.g. density, voltage, seismic velocity, mass or may be by coefficients and other constants of functional relationship which describes the physical system [7], [8]. A direct problem might be defined in a simplified form by assuming that a physical system is modeled through an adequately understood function $\boldsymbol{g}$, given an input parameters $\boldsymbol{p}$ and collected output vector $\boldsymbol{z}$:

$$\boldsymbol{g}\,(\boldsymbol{p}) + \boldsymbol{\epsilon} = \boldsymbol{z} \tag{2.1}$$

Where error vector $\boldsymbol{\epsilon}$ total is the sum of data and modeling error of the problem. Thus a process of calculating unknown parameters $\boldsymbol{p}$ given real measured output data for $\mathbf{z}$ is an inverse problem. An operator $\boldsymbol{g}$ takes the shape of linear or nonlinear system of algebraic equations, partial differential equation or ordinary differential equation.

In mechanics, direct methods count the execution of standardized (conventional) experimental skills so as to define constitutive parameters of a specified material. A graphical explanation between displacement and load (stress/strain) can be obtained in such cases. There are always some types of unavoidable systematic and random errors which the final data contains in it. Some common examples of direct measurement methods like compressive and tensile test, 3-point/4-point bending tests, pure shear tests, biaxial tensile tests, etc. used in experimental mechanics. On account of electrostatics, direct problem is mentioned as a set of boundary value PDEs (partial differential equations) or unknown displacement field (U(x)) is referred in direct problem, assuming that the geometry, mechanical properties and boundary conditions are known. Systematic representation of such problem is showed in figure 1.1(a).

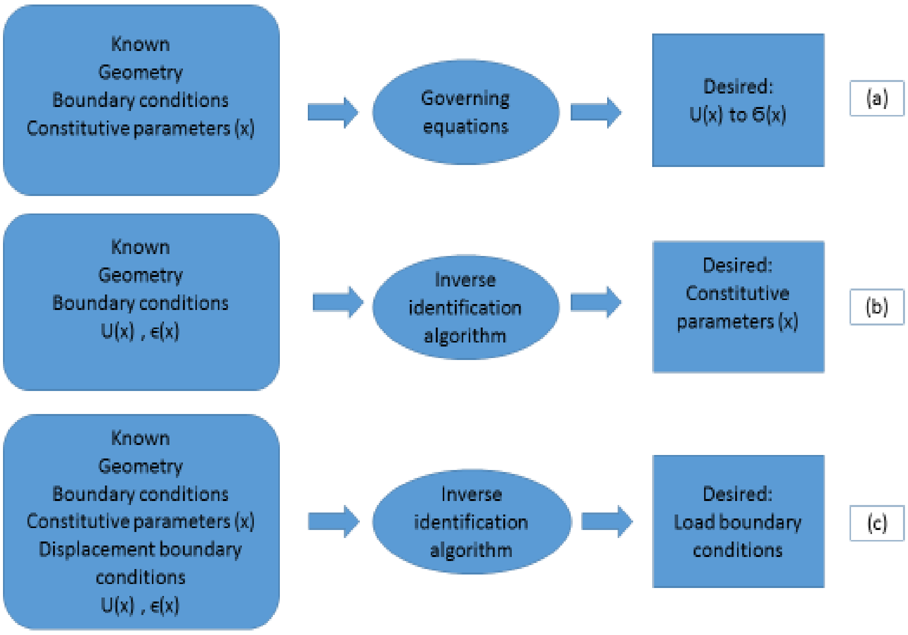

Figure 1 Direct & inverse identification in elasticity; (a) Direct Problem, (b) inverse problem of type 1, (c) inverse problem of type 2

Taking into account the benefits of modernized mathematical techniques analytical techniques have been developed to calculate distribution of properties. However, such kind of closed form solutions can be implement for known boundary condition, simple geometry and simplified assumptions on material model [9].

In experimental mechanics, inverse problem falls into two main categories: (type 1, Figure 1b), the problems associated with identifying unknown constitutive parameters, (type 2, Figure 1c), in which constitutive equations parameters are assumed to be known. Inverse problem of type 1 classify the identification of material properties distribution. In such problems, observed data is taken as a given values, basically the displacement/strain fields U(x) or ϵ(x), external forces and boundary conditions. This case deals with stiffness tensor elements which represent the unknown mechanical properties by using boundary conditions with a set of overdetermined displacements. Therefore, one search error minimization in governing equations. Many inverse identification methods have been developed for residual minimization [10]

2.2 Inverse Problem Defies

Lack of solution uniqueness is one of the main challenges in the inverse methods which means that many solutions might fulfil the optimality conditions given by the measured data [11]. Round-off error in calculations, noise in measured data and inexact direct model creates such situations in practice. It is vital to determine what solutions have been obtained, which of them are physically conceivable and maybe which of them are stable in different states of constraints. So, some important parameters that must be taken into account are solution *existence*, *uniqueness* and *instability* [12].

Solution existence: Inverse problems might contain no admissible set of parameters that produce the model exactly match the measured data. This may take place when the mathematical model is approximate, (i.e. does not contain constraints and exact boundary conditions) or consists of noisy measured data [12].

Solution uniqueness: There is no assurance that the solution is unique even if it is an exact solution. It shows that inappropriate solutions may fulfil Eq. 2.1. This is the case mostly in rank deficient discrete linear inverse problems and may direct to a biased estimated model [12].

Solution instability: Inconsistency of estimated model verses trivial changes of measurements is one of the major issue in inverse problems. They are mentioned as *ill-conditioning* and *ill-posedness* in the case of discrete and continuous inverse problems. In 1902 Hadamard's definition gave the concept of ill-posed and well-posed problems [12], so, if the solution of inverse problem is not unique or if the solution is discontinues function of data the inverse problem is considered as ill-posed. In practice, many factors in engineering might create ill-posedness of inverse problems based on following three categories: (i) imperfect forward problem (i.e. finite element or mathematical model) that might generate from inappropriate definition of boundary condition or incorrect mathematical models.

A part of modeling error which is produced due to the discontinuous measured data and availability at discrete points is called discretization error. Meshing/discretization of geometry in FEM and constitutive relationship as a linear combination lies within this category. The effect of discretization in inverse problem has been studied many time in literature [13]; (ii) approximation in numerical analysis (round-off errors) and

(iii) model inconsistency verses trivial perturbations of measured data (presence of noise).

Data errors exist inherently in measurement algorithm and experimental arrangements. These types of errors can be reduced by careful experimental setup, calibration or through statistical characteristic analysis, and become the source of uncertainty.

When inverse process is ill-posed, additional information like physical or mathematical constraints must be implemented to alleviate the inversion process. Such a method which can be imposed to avoid unwanted oscillating behaviors of the solution is called *regularization*. Further important aspects will be discussed respectively [13].

2.3 Linear Elasticity Governing Equations

Taking into consideration a deformable solid which is subjected to a couple of external traction $\mathbf{T}^{(1)}$, $\mathbf{T}^{(2)}$ and sustained by part of its external surface. Under static conditions basic governing equations to be solved in linear elasticity and changes into equilibrium equations [14], after neglecting the body forces (weight) and considering small material element, reads:

$$div.\sigma(x) = \sum_{j=1,2,3} \frac{\partial\sigma ij(x)}{\partial xj} = 0 \ (for \ i = 1, 2, 3) \qquad (2.2)$$

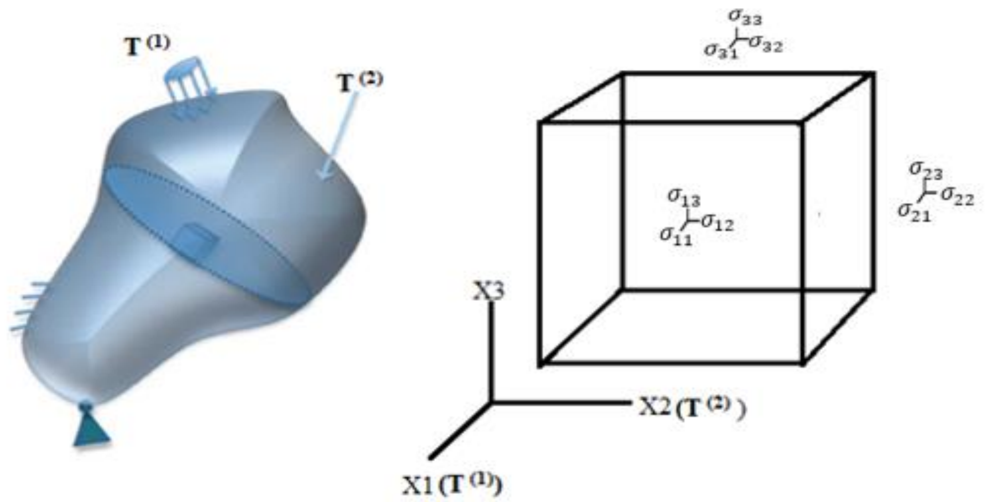

Figure 2 Stress component on a typical element of a deformable solid [12]

Where,

Second-order stress tensor $\sigma = \{\sigma_{i\,j}\}$
Position vector of a material point $\boldsymbol{X} = (x_1, x_2, x_3)^{\mathrm{T}}$
Deformable body stress tensor component is depicted in figure 2.2. Strains are explained as:

$$\epsilon\,(x) = ½\,\nabla\boldsymbol{u}\,(x) + \nabla\boldsymbol{u}^T\,(x) \tag{2.3}$$

Where, u is displacement vector. Finally, Hook's Law relates stress and deformation for a linearly elastic material like this:

$$\sigma_{\,i\,j} = C_{ijkl} \in_{kl} \tag{2.4}$$

C_{ijkl} = Symmetric fourth-order local stiffness (or elasticity) tensor having dependent coefficients which rely on material symmetry. The matrix shape of the stress strain relation for a specific case of isotropic material in a plane case of stress reduces to (with the usual definition of simplified indices for strain and stress): 11 →1, 22→2, 12→6

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_6\end{Bmatrix} \tag{2.5}$$

Or

$$\sigma = Q\epsilon \tag{2.6}$$

Where, **Q =** in-plane stiffness matrix and has following components;

$$\begin{cases} Q_{11} = Q_{22} = \frac{E}{(1-\mathrm{v}^2)} \\ Q_{12} = Q_{21} = \frac{\mathrm{v}\,E}{(1-\mathrm{v}^2)} \\ Q_{66} = \frac{(Q_{11} - Q_{12})}{2} \end{cases} \tag{2.7}$$

After loading, the linear elastic material stored strain energy in it that can be obtained by integrating the strain energy density over the *(V)* volume.

$$\begin{aligned} W &= \frac{1}{2}\int_v \sigma_{ij}\,\epsilon_{ij} dV \\ &= \frac{1}{2}\int_v C_{ijkl}\,\epsilon_{ij} dV \end{aligned} \tag{2.8}$$

2.4 Techniques for Full-Field Measurement

The analytical/numerical models and experimental results integration in inverse problem has given grow to the progress of various non-contact full-field measurement methods. Recently, the development in computer version and imaging systems (2D/3D), started the improvements of optical methods like moiré interferometry [15], ESPI (Electronic Speckle Pattern Interferometry) [16], speckle photography [17] and DIC (Digital Image Correlation in 2D and 3D) [18], [19]. These techniques are used to measure physical quantities like strains and displacements for a big number of discrete points inside a specimen and at the surface of specimen. The DIC is the most popular method due to its straightforwardness and simplicity among the other mentioned techniques.

These measuring techniques are more advantageous because they can provide the requisite heterogeneous kinematics fields without damaging the material with a single trial. Consequently, this saves the time and money for sample preparation. Full-field measurement methods are able to capture inclusion localization and local phenomena such as stress concentration and damage, unlike point-wise techniques performed by displacement transducer and strain gauges. For finding modified polymer properties in a nano clay nanocomposites and in-situ mechanical properties like composite fiber-matrix strength, such a large information is very important. Many unknown parameters can be consecutively recognized by using a single and undetermined setup and full-field measured data. This is due to the reason that heterogeneous strain fields are influenced by a larger number of constitutive constraints than identical one. Using full-field measuring technique, samples with any complicated geometry can be used in an in-situ mechanical testing without using a manufacturing standard coupons. The benefits of full-field measurement method and their uses to composite has been introduced by Grediac in [3].

Based on full-field measurement many inverse identification methods have been stated in literature for determining mechanical measurements of different models e.g. viscoelastic, elastic, hyperelastic, or even plastic models [20].

The next section introduce previous inverse identification techniques which are most commonly used (i) Virtual Fields Method (ii) Finite Element Model Updating

2.5 Full Field Measurements Based Identification Methods

2.5.1 VFM (Virtual Fields Method)

The Virtual Field Method based on the principle of virtual work to govern/find unknown constitutive parameters as represented by its name. Grediac proposed this method first time and has been successfully implemented to different mechanical characterization problems [21], [22], [23]. This method (VFM) has been theoretically proven similar to the Finite Element Method Updating (FEMU), as the first mentioned method gives maximum like-hood results and the second one is established rely on displacement gap minimization [20].

Virtual Fields Method has the limitation, it uses displacement field differentiation to acquire the full-field strain measurements which might introduce uncertainty in the measurements. Regardless of this shortcoming, this method is more advantageous than the other identification methods because of both less computational efforts and decreasing the ill-posedness intrinsic in the practice [20]. This VFM is established on principle of virtual work by writing the global equilibrium of a body exposed to a particular load. Based on the virtual works, relevant equation is written by the applied load on various part of body, mentioned below:

$$-\int_v \boldsymbol{\sigma} : \epsilon[\boldsymbol{u}^*]\, dV + \int_s \boldsymbol{T} .\ \boldsymbol{u}^* dS + \int_v \boldsymbol{f} .\boldsymbol{u}^* dV = \int_v \rho\, \boldsymbol{a} .\ \boldsymbol{u}^* dV \qquad (2.9)$$

Where, $\boldsymbol{\sigma}$ = stress field, $\epsilon[\boldsymbol{u}*]$ = virtual strain field attained by differentiating virtual displacement field$[\boldsymbol{u}^*]$, V= solid volume, $\boldsymbol{T}$= external loading, S= surface, $\boldsymbol{f}$= external body force/unit volume, ρ = material density, $\boldsymbol{a}$ = acceleration**.** In the Eq. (2.9) left hand side integrals give the virtual work done by external and internal forces, while the right hand side is related to the inertial effects that are created by acceleration. The above mentioned basic equation may be implemented to the type of constitutive relationship of material and mechanical problem, which produce two types of approaches called as linear or nonlinear Virtual Field Method [24].

If the aim is to find the constitutive law factors that has already been fixed with the supposition that volume (V) is the deformation field, and quasi static applied load then **a** is negligible. Neglecting the body force and including the plane stress condition, substituting equation (2.4) into (2.9), we will get

$$\int_v \boldsymbol{C} : \boldsymbol{\epsilon}[\widehat{\boldsymbol{u}}] : \boldsymbol{\epsilon}[\boldsymbol{u}^*] \, dV = \int_s \boldsymbol{T} . \boldsymbol{u}^* dS \qquad (2.10)$$

Where, $\boldsymbol{\epsilon}[\widehat{\boldsymbol{u}}]$ = strain tensor, $\widehat{\boldsymbol{u}}$ = measured displacements. Virtual field method has been used to inflate Eq. 2.10 using as many virtual displacement fields (independent) equal to the parameters (unknown). This leads to a ***(A.q = b)*** system of linear equation for a linearly elastic material which are used to directly calculate the unknown parameters. One of the key factors in this method is selection of a suitable data of test function, virtual displacement field functions has been selected from the infinite number of possibilities. Some specific conditions must be satisfied by test functions: (i) function must be differentiable, (ii) kinematically admissible (KA) and C^o continuity, (i.e. must fulfill actual displacement B.C (boundary conditions).

However, the main hindrance is related to the tactic for searching appropriately virtual fields which is an important aspects that explains the sensitivity of the virtual field method [25]. With a sensible choice of virtual fields, the constancy of techniques in contradiction of measurements noise increases. In point of fact, a higher independence of virtual fields, a lower sensitivity of the linear system to noisy data (and therefore to more reliable solutions) are proportional to each other. So-called special virtual fields [26], can be obtained automatically to ensure the independency of relative equations by rendering matrix ***(A)*** equal to the identity matrix which has been shown. Unlike intuitive selection of virtual fields, virtual field parameterization is necessary for special selection for which the unknown coefficients are obtained by imposing appropriate constraints well-suited the applied boundary conditions. The technique entails an optimization process combined with identification of optimum geometrical and testing arrangement suitable for a specified material.

The concept behind virtual field optimal choice was also given by Avril [25] to handle Virtual Fields Method, ill-posedness by reducing its sensitivity to noise. Furthermore, an approach established on piecewise virtual field was suggested for distinguishing heterogeneities in functionally classified material and defining local mechanical properties. In conclusion, it is stating that the Virtual Fields Method (VFM) has been lately exploited to determine solutions for inverse problems of identification of load/boundary conditions (type 2) [25].

2.5.2 FEMU (Finite Element Model Updating Method)

Finite element method (FEM) is one of the most dominant numerical tool to resolve solid mechanics problems. This type of analysis is denoted as *forward* or *direct analysis*. This can be implemented to calculate the inverse problem of type 1 (Figure 2 (b)). The FEMU method is also recognized as *Displacement Gap Method* [27]. In FEMU method, material parameters put into the FE model are iteratively restructured relied on the contrast of corresponding yield data after their investigational counterparts and forward analysis (basically strain fields or displacements). The aim is to figure out the set of values that give the superlative match between real performances and numerical yields, took through experiments.

To this end, optimization methods are exploited to reduce the inconsistency between experimentally calculated strain fields/displacement and numerically predicted (FE), relating to the mechanical properties in a "least square objective function" as:

$$r\,(\boldsymbol{p}) = \|\widehat{\boldsymbol{u}} - \boldsymbol{u}(\boldsymbol{p})\|^2 w = (\widehat{\boldsymbol{u}} - \boldsymbol{u}(\boldsymbol{p}))T.W\,.(\widehat{\boldsymbol{u}} - \boldsymbol{u}(\boldsymbol{p})) \qquad (2.11)$$

Where $\widehat{\boldsymbol{u}}$ **=** measured displacement field, $\boldsymbol{u}$ = displacement field (determined with the Finite Element Model (FE)), and W = symmetric positive definite weight matrix. Update the vector of model values $\boldsymbol{p}$ at every iteration until convergence is attained. Relevant algorithm rely on a gradient based optimization method is illustrated by figure 2.3. The precision of the given value in the experimentally calculated data used in the optimization method rests strongly on the level of uncertainty. The most important disadvantage of the process is the requirement for an iterative computational development for modifying the sought data. This can be excessive when dealing with identification problems which include 3D FE models. Another disadvantage is related with the algorithm convergence which is mostly affected by some familiar factors such as material model, boundary conditions, optimization technique and sensitivity of algorithm to noisy calculated parameter.

Unlike basic mechanical tests accomplished under the supposition of homogeneity of material, FEMU can be implemented to find the properties distribution in material with heterogeneous stress and strain fields. One of the benefits of the technique is that it does not need full field measurement on the entire field, therefore, using partial facts of displacement data the parameter identification can be conceded [28]. Certainly, inside smaller region of interests inverse solutions are usually preferred as these could be more stable and give rise to less discretization error.

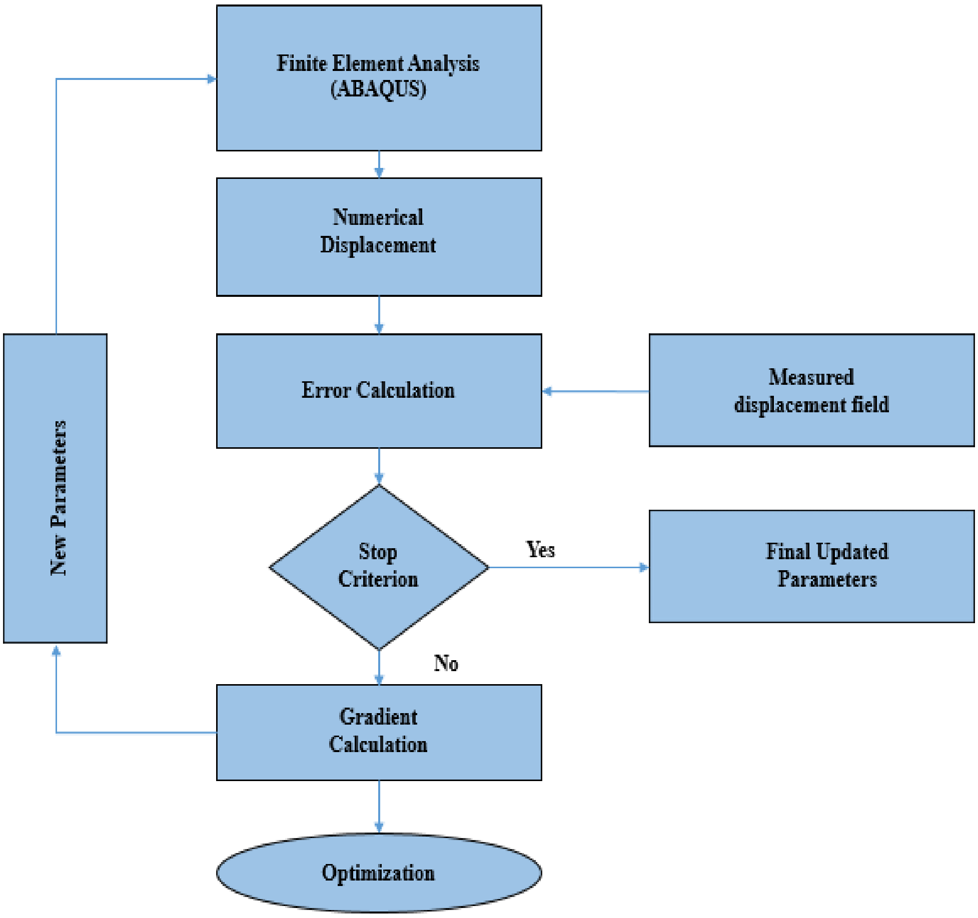

Figure 3 FEMU updating algorithm

Moreover, with intricate geometries and boundary conditions material models can be categorized taking the advantages of FE (finite element) simulation skills. Over the last decades, FEMU technique has been exploited in several systematic branches like medical application and structural dynamics [29], [30], [31] and is still being implemented to identify mechanical properties of material [32]. Meanwhile there is no limitation for constitutive equation, a wide range of behavior of material can be identified from elasticity to hyper visco-elasticity.

For example, in linear elasticity Genovese determined elastic constants based on Phase Shifting (ESPI) measurements [33]. Elastic properties of a synthetic polymer plate under plane stress conditions is determined by Cardenas Garcia by using displacement field obtained from moiré interferometry technique [34]. Similarly, four in plane elastic parameters of orthotropic composite are reconstructed by Lecompte which are subjected to biaxial loads. Conversely, except reducing the mismatch among the displacements, they took strains as output data. An optimization technique used for the identification of in plane elastic parameters of an eight ply woven composite was investigated by Genovese [35]. With the objective of finding visco-elastic parameters of wood based panels, an inverse method (mixed numerical experimental) was proposed by Magorou based on optical metrology and genetic optimization [36].

Mechanical behavior of composite like toughness and strength depends on mechanical characteristic of constitutive phase, specifically at the matrix and reinforcement interface [37]. This important issue has arisen the development of many identification method for in-situ local mechanical properties. Kang has suggested an inverse/ genetic approach for interfacial identification of parameters [38]. After using GA (Genetic Algorithm) optimization approach, at interface region in planar matrix composite, shear and tensile mechanical properties were determined consist of failure information at the same region.

An inverse method to evaluate mechanical properties of cohesive law of adhesive layer with the help of full field displacement [39]. It should be stated that there are some other model updating strategies in which forward problem could have closed form solution or can be performed through the alternative numerical approaches (boundary element method) [40].

2.5.3 CEGM (Constitutive Equation Gap Method)

Initially in FE (Finite Element), this approach was implemented for error estimation and then for elastic properties identification [41]. The objective of this method is to reduce the error in constitutive equation 2.4 through a criterion same as the following function:

$$\boldsymbol{\epsilon}\,(\boldsymbol{C},\boldsymbol{\sigma},\hat{\boldsymbol{u}}) \;=\; \frac{\mathbf{1}}{\mathbf{2}}\int_{\Omega}(\,\boldsymbol{\sigma}-\boldsymbol{C} :\; \boldsymbol{\epsilon}\,[\;\hat{\boldsymbol{u}}\,]\,) : \boldsymbol{C}^{-\mathbf{1}} : (\boldsymbol{\sigma}-\; \boldsymbol{C} : \boldsymbol{\epsilon}\,[\hat{\boldsymbol{u}}])\; dV \qquad (2.12)$$

Where $\boldsymbol{C}^{-\mathbf{1}}$= compliance tensor (which deliberates physical dimension of energy to $\epsilon(\boldsymbol{C},\boldsymbol{\sigma},\hat{\boldsymbol{u}})$. Weight tensor $\boldsymbol{C}^{-\mathbf{1}}$smoothened the above functional error. The Constitutive

Equation Gap Method (CEGM) could be inferred as reducing the inconsistency between a provided $\boldsymbol{\sigma}$ (stress field) and other stress field coming from product of strain data and unidentified constitutive matrix. In reality, the above function is another way of reducing the sum of stress/strain energy potentials [14]. The beauty of the method is that it does not require the full field measured data for solution, but it integrates available data (full-field) to increase the reliability of acquired results. For example, to classify the $\boldsymbol{C}$ which is established on the experimental accessibility of stresses and displacement, following minimization problem is achieved.

$$\boldsymbol{C} = min\, \boldsymbol{Y}\,(\boldsymbol{C}*) \qquad (2.13a)$$

$$\boldsymbol{Y}\,(\boldsymbol{C}*) = min\, \boldsymbol{\epsilon}\,(\boldsymbol{C}*, \boldsymbol{\sigma}, \widehat{\boldsymbol{u}}) \qquad (2.13b)$$

Where, $\boldsymbol{C}$ = summation of admissible elasticity tensor. Recently a new development in Constitutive Equation Gap Method has been suggested in which optimized stress fields are built with the intention of minimizing experimental observation variance and then regulate the problem [42].

2.5.4 EGM (Equilibrium Gap Method)

The EGM (Equilibrium Gap Method) comprises of finding the parameters which reduce the errors in interior equilibrium equation in a bulk substantial [43]. Main objective of this method is to reduce the error in nodal forces taken from theoretically measured displacements. A reasonable form of equilibrium equation can be estimated using FE discretization by stating the displacement field with respect to nodal displacement interposed by some shape functions. The relation will take following form:

$$\boldsymbol{K}\,(\boldsymbol{p})\,\boldsymbol{u} = \boldsymbol{f} \qquad (2.14)$$

Where, $\boldsymbol{K}\,(\boldsymbol{p})$ = stiffness matrix (function of known shape function), $\boldsymbol{p}$ = unknown elastic parameter, $\boldsymbol{u}$ = nodal displacement, $\boldsymbol{f}$ = external force vectors. So, the EGM (Equilibrium Gap Method) can be answered with following:

$$\boldsymbol{f_{res}} = \| K\,(\boldsymbol{p})\,\hat{\boldsymbol{u}} - \hat{\boldsymbol{f}} \|^2 \qquad (2.15)$$

Where, $\widehat{\boldsymbol{u}}$ = measured displacement field. This can be used to find the isotropic damage of materials and local reduction of properties. Mostly this method has been used to repossess spatial elastic properties distribution within domain [44]. This can be used to find the isotropic damage of materials and local reduction of properties.

2.5.5 RGM (Reciprocity Gap Method)

It is the combination of Maxwell- Betti theorem and virtual field method [45]. Maxwell-Betti theorem states, owing to second load, the work done by one load on the displacement is equivalent to the work done by second load on displacement owing to first load. Consider two elastic bodies which occupy the identical region and identified by two discrete elasticity distribution tensors $\boldsymbol{C}$ and $\boldsymbol{C}^{*}$, displacements fields $\widehat{\boldsymbol{u}}$ and $\boldsymbol{u}^{*}$, tempted by traction distributions $\boldsymbol{T}$ and $\boldsymbol{T}*.(\boldsymbol{u}*,\boldsymbol{T}*)$ is auxiliary state mentioned as virtual/adjoint state.

$$\int_{S}(\widehat{\boldsymbol{T}}\,.\boldsymbol{u}^{*}\;-\;\boldsymbol{T}^{*}.\widehat{\boldsymbol{u}})\;dS=\int_{S}\boldsymbol{\epsilon}\,[\,\boldsymbol{u}^{*}]:(\boldsymbol{C}^{*}-\boldsymbol{C}):\boldsymbol{\epsilon}\,[\widehat{\boldsymbol{u}}]\;dV=R\,(\boldsymbol{C}^{*}-\boldsymbol{C},\boldsymbol{u}^{*},\widehat{\boldsymbol{u}}) \quad (2.16)$$

$R\,(\boldsymbol{C}*\;-\boldsymbol{C},\boldsymbol{u}*,\widehat{\boldsymbol{u}})$ = reciprocity gap. The assumption for this theorem is that if the elasticity tensor of two conditions are undistinguishable the reciprocity gap can appear. So, if $\boldsymbol{C}$ and $\boldsymbol{C}^{*}$ are different by small disturbance. I.e.$\boldsymbol{C}*=\;\boldsymbol{C}\;+\;\delta\,\boldsymbol{C}$, eq. 2.16 will be accomplished in which by reducing the reciprocity gap in an inverse unidentified elastic tensor can be repossessed.

2.6 Comparison of Above Methods

All identification methods discussed above are beneficial using large experimental data which is obtained from full field measurement. The CEGM and FEMU techniques do not need full field. FEMU is widespread among all because it does not require full field measurement on the whole domain and as a result the computational result for identification process is shortened. Furthermore, this strategy incomes from robustness of the FE method when sample with intricate geometries and boundary condition are to be considered. The main disadvantage is that it is computationally expensive and time consuming process particularly when dealing 3D models. On the other side, VFM and EGM utilize full field data within domain and give direct solution of inverse problem.

2.7 Optimization Methods

For inverse problems many optimization methods have been introduced. They have three major groups: (i) derivative free optimization, (ii) gradient based optimization, (iii) hybrid optimization techniques.

2.7.1 Derivative Free Optimization Method

- **Evolutionary Algorithm**

EA (Evolutionary algorithms) like GA (Genetic Algorithm) are established built on genetic descriptions and use iterative development to reduce unbiased function and develop initial results. In GA (Genetic Algorithm) every optimization value is denoted as “chromosomes” (individuals). The process is prepared by arbitrarily creating a population of m individuals. The objective function is calculated for the all individuals. Individual are then organized according to their objective function (fitness) values, like individual with smaller fitness values are located on peak. Then GA uses evolutionary approach, for instance development (mutation and crossover) or growth in population to expand it and make a novel generation. Maximum scoring individuals permit to the following generation without damage.

Mostly, a greater population size suggests additional objective function assessments and an improved concluding solution. Conversely, they consist of time taking black-box calculations, greater population size directs to extensive computational time. On the other side, small population possibly will directs to early convergence and suboptimal result. Quicker optimal results can be accomplished by the higher value of crossover rate. But again with very high value, this may be a premature convergence. The performance of inverse identification/genetic measures does not rest on initial solution.

However, conventional GAs are computationally expensive, time consuming and have deficiency of indigenous search ability [38] [32]. To speed up the performance and procedures of GA, PGA (Parallel Genetic Algorithm) are introduced in 2003 [46]. These are of two types: (i) coarse grain method, (ii) goal method [47]. In coarse grain, population is distributed into many sub populations and every population grasps a serial

GA autonomously. Secondly, the global method contains single population in which individuals are divided into several cores to increase the assessment of objective function. Every individual accomplishes mutation/crossover actions and a global selection is performed over the complete population. It is also important to mention that EA (Evolutionary Algorithms) relates with experiential optimization approaches with global convergence classifications.

- **Direct search Algorithm**

It is based on the objective function during iterations and do not have need of guesstimate of these function gradient. Due to the reliability and flexibility, these functions are still very famous. These methods were established on sequential examination of experimental results created by a firm approach [48]. Nelder Mead Simplex Algorithm is a most commonly used algorithm in which objective function is done on a set of points that make a Simplex [49].

GPS (Generalized Pattern search) is an alternative method applied for unconstrained optimization [50]. The drawback of GPS was further resolved by MADS (Mesh Adaptive Direct Search) [51]. The ideas used in both process is the same while Polling is quite different. MADS is for nonlinear optimization algorithms. Another feature of MADS is that it neglected infeasible trial points according to the given constraints during optimization process. Revisions have been conceded out that reveals MADS overtakes GPS [52].

2.7.2 Gradient Based Optimization Method

The Conjugate Gradient, Gauss Newton, Levenberge Marquardt and the Trust Region are widely used Gradient Based Optimization Method in literature. Main objective of this method is its quick convergence in the locality of global minimum.

Nonetheless, its performance muscularly depends on initial solutions and it might easily fall in local minima if there is a lot of difference between initial guess and global minimum [53].

2.7.3 Hybrid Optimization Method

It is a blend of both gradient based and derivative free method to improve the precision of solutions. The concept is to bring close global searching property (Derivative Free Algorithm) and high convergence rate (Gradient Based Method). Literature show the dominances of this approach in contrast with gradient based and derivative free techniques [54]

2.8 Micromechanical Models

The objective of homogenization method is to estimate the whole performance of heterogeneous materials through the mechanical properties of elements [55]. For fruitful homogenization in composite materials, geometrical and mechanical properties in addition to the morphology of constituents must be wisely determined. The HSB (Bound of Hashin Strikman), SCS (Self Consistent Scheme), MT (Mori Tanaka), TOA (Third Order Optimization), Lielens's Model, primarily used for linear elasticity [56].

Numerically homogenization can be carried out through FFT (Fast Fourier Transform) and FE (Finite Element). Moreover, FEM (Finite Element Method) is user demanding due to problems come across during meshing. The main benefits of such type of material is that they give precise predictions, providing that they put on RVE (Representative Volume Element). It was established that the MT (Mori Tanaka) Model for fiber reinforced composite, TOA (Third Order Optimization) and Lielens's Model for particle reinforced composites were extra precise than the further for approximating effective properties [57].

Chapter 3

NON DESTRUCTIVE APPROACH USED IN THE RESEARCH

The suggested inverse approach will permit the identification of all the in plane elastic properties, combining numerical and experimental results with full field displacement measurements, from experimental trials. As an alternative, the use of a non-destructive technique an alternative hybrid approach is proposed to obtain the field displacement. Since the off axis tensile tests observed in this work, displacement field obtained from experiment was substituted by numerically achieved nodal displacement values using the finite element approach (FEM).

The mechanical properties of the composite create the connection between applied load to the displacement field and off-axis. The use of a surrogate model based on ANN (Artificial Neural Network) enables to establish the relationship between the elastic properties and the displacement field avoiding the exhaustive calculations based on FEM. In UDM (Uniform Design Method) design points are produced over a domain located on the mean reference data of random variables. UDM (Uniform Design Method) generated points are used as an input and output configurations, then ANN (Artificial Neural Networks) is recognized founded on evolutionary learning procedure [58].

3.1 Uniform Design Approach

The objective of this guesstimate method, at the minimum cost, reveal the association between input and response variables. The important thing is to get a better idea of the predictable output to define the set of points appropriately. In literature [59] [60] [61] [62], an inaccuracy bound for the credible yield value is explained. This is a rate of inconsistency of response time deviation of established data that resolved the entire domain. Error will be small taking into account the disparity when even points are scattered on the limit of input variables.

Therefore, uniformly distributed points are needed. The objective of the method proposed by Fang is to obtain the facts that are consistently spread in the s- dimensional unit cube C^s [60] that is centered on quasi Monte Carlo approach. In reality, UDM (Uniform design Method) can be dignified as an experimental design with main idea of decreasing divergence. Here, divergence is the extent of “universally acknowledged uniformity “.

Suppose P = set of points at $C^{s,}$ for $\mathbf{g} = (g_1, g_2, g_s)$ suppose $N(\mathbf{g}, P)$ = total hyper cube points $[0, g_1] * [0, g_2] * \ldots\ldots[0, g_s]$. The divergence of “*P*” is characterized as: $\Psi(n, P) = sup_{g \in C^s} \,|N(\mathbf{g}, P)/n - g_1, g_2, g_s$.I While $N(\mathbf{g}, P)/n$ = point’s percentage (lies in $[0, g_1] * [0, g_2] * \ldots\ldots [0, g_s]$), with volume g_1, g_2, g_s [69]. g_1, g_2, g_s = percentage of volume covered by P. $\Psi(n, P)$ is good when there is small difference between $N(\mathbf{g}, P)/n$ and g_1, g_2, g_s. $U_n(q^s)$, represented a UDM (Uniform design Method) table, where U is uniform design, n is no. of samples, q = level of each input variables, s = max. No. of columns.

There is an equivalent auxiliary table which comprehends a references of columns with fewer divergence for a given group of inputs [61]. Construction of UDM (Uniform design Method) table is given by:

- For n calculate $H_n = \{h_1, h_2, h_3\}$ with m.d.c $(n, h_i) = 1$ and $h_i <= n$, $i = 1\ldots m$, with $m = \Phi(n)$, where Φ is Euler function. $\Phi(n) = n(1 - 1/p_1)(1 - 1/p_2) \ldots (1 - 1/p_t)$.
- $n = p_1 p_2 \ldots p_t$ is the prime decomposition.

For s = distinct element of H_n, create $n * s$ (table) where $u_{ij} = ih_j \pmod n$ for ($i = 1\ldots n$ and $j = 1\ldots s$), $0 < u_{ij} <= n$,

Finally the UDM (Uniform design Method) will be in the form with discrepancy $\Psi(n,P) = 0.1189$ in table A. [59]

For each UDM design point, the longitudinal displacements, u, are obtained by FEM considering the specimen geometry, the experimental loading and boundary conditions. In the proposed approach the analysis is performed using the degenerated Amahd shell element [63]. The BC (Boundary Conditions) implemented to the numerical technique are in contract with the non-rotating testing machine grips and rigid support.

Table A: UDM design points for discrepancy [55]

UDM design points for discrepancy Ψ(n,P)=0.1189				
Design points	**1**	**4**	**6**	**9**
1	1	11	15	25
2	2	22	2	22
3	3	5	17	19
4	4	16	4	16
5	5	27	19	13
6	6	10	6	10
7	7	21	21	7
8	8	4	8	4
9	9	15	23	1
10	10	26	10	26
11	11	9	25	23
12	12	20	12	20
13	13	3	27	17
14	14	14	14	14
15	15	25	1	11
16	16	8	16	8
17	17	19	3	5
18	18	2	18	2
19	19	13	5	27
20	20	24	20	24
21	21	7	7	21
22	22	18	22	18
23	23	1	9	15
24	24	12	24	12
25	25	23	11	9
26	26	7	26	6
27	27	17	13	3

3.2 ANN (Artificial Neural Network)

The methodology used here is alike to the RMS (Response Surface Method). The objective of implementing ANN (Artificial Neural Network) is to overpower the difficulties interrelated with affluent assessment of operational dependability for the study of respond inconsistency. UDM (Uniform design Method) generated points are used as input and output arrangements to advance an ANN (Artificial Neural Network) placed on evolutionary learning technique. The input and output factors are random variable and bound state function, consistency and related sensitivities. [58]

3.3 ANN (Artificial Neural Network) Topology Definition

The recommended ANN (Artificial Neural Network) is arranged into three layers of neurons (nodes): (i) input layers, (ii) hidden layers, (iii) output layers. The term synapses is an association between input and hidden nodes, hidden and output nodes. These linkages develop connection between D_i^{inp} (input data) and D_j^{out} (input data). In the proposed ANN (Artificial Neural Network), $\mathbf{D}^{\mathbf{inp}}$ is expressed by a group of values "Л ", (a random variable), which comprehends strength and elastic properties of composites.

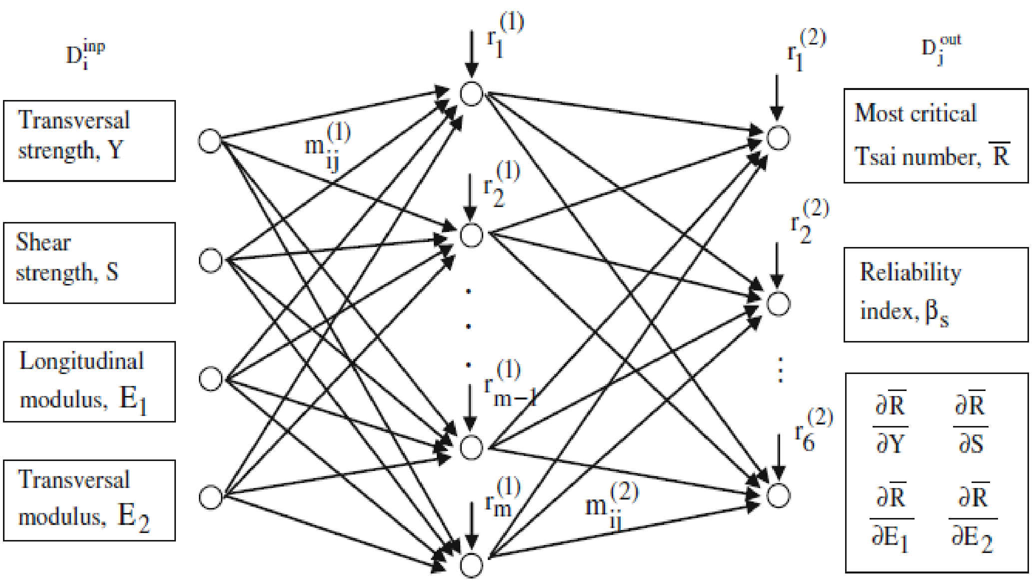

Figure 1 Artificial Neural Network Topology

ANN (Artificial Neural Networks) is supplied by the input variables $\boldsymbol{\pi} = [E_1, E_2, Y, S]$ that represent E_1(Longitudinal elastic modulus), E_2(transversal elastic modulus), Y (transversal tensile strength) and shear strength S, each group of values for the design variable vector $\boldsymbol{\pi}$ is labelled over and done with UDM (Uniform Design Method).

The subsequent output data vector $\mathbf{D}^{out}$ encompasses the displacements, **u,** measured along longitudinal axis of symmetry of the specimen used in experimental tests. Figure 4 displays the topology of the ANN (Artificial Neural Network), viewing input and output factors [58]. Each configuration includes of an input and output vectors, entails to be normalized to diminish numerical inaccuracy proliferation all over the ANN (Artificial Neural Network) learning procedure. Data standardization is

$$\bar{D}\,k = (Dk - Dmin)\,(DN\,max - DN\,min)\,/\,(D\,max - D\,min) + D\,N\,min \quad (3.1)$$

Where,

D_k = real value of variable before normalization, D_{min} = minimum values of normalization and D_{max} = maximum value of normalization, correspondingly in input and output data. With respect to equation 2.18, the condition $Dmin <= \bar{D}$ k $<= DN\,max$.. Relying on the input and output configuration, most commonly used various values of input output variable are 0.1 and 0.9.

The synopsis of adapted signals is proficient through function, considered as an $A(x)$, Activation Function. Accordingly, using sigmoid functions the activation of kth node of hidden layer (p = 1), output layer (p = 2) is achieved as stated below:

$$A_k^{(p)} = 1/\,(1 + e^{-\eta(p)} C_k(p)) \quad (3.2)$$

Where, p = activation layer, this could be the hidden layer or output layer and $C_k(p)$ = component of $\mathbf{C}^{(p)}$ which is provided by

$$\boldsymbol{C}\,(p) = \boldsymbol{M}\,(p)\,\boldsymbol{D}\,(p) + \boldsymbol{r}\,(p) \quad (3.3)$$

Where,

$\boldsymbol{M}^{(p)}$ = matrix of weights of synapses, $\boldsymbol{r}^{(p)}$ = biases vector, $\boldsymbol{D}^{(p)}$ = input data vector. $C_k(p)$ is scaling function effect the sensitivity of sigmoid functions. In the neurons the weights of synapses $m_{ij,}$ and biases at the output and hidden layers are distinguished with the help of the learning procedure [58].

3.4 GA (Genetic Algorithm)

These are search algorithms well-known as natural genetic and selection process. Based on "Darwinian Survival of fitness", consecutive generations develop more fit entities. A GA (Genetic Algorithm) for a definite problem requisite following stages.

- Genetic operators that are used for the offspring modification.
- Method to produce an initial population of potential solutions.
- Mention a genetic example for latent solution to problem.
- Parameters value for application in the genetic algorithm.
- An evolution function for rating evolution regarding to their fitness.

In GA (Genetic Algorithm) every individual is characterized by fixed sequence of signs, programming a possible solution in an assumed problematic planetary. This interplanetary, contains all potential solutions of problem mentioned as a search space. Customary GAs (Genetic Algorithm) play with fixed length strings of symbol called as "chromosomes" and contain identical amount of entities (in consecutive generations). Every single positions in string is connected to a gene. Genes grouping, inhabiting a detached site in string, put forward the methodology of a detailed proposed problem.

In GAs (Genetic Algorithm), a gene can be represented by any sign, even though both for theoretical and practical accessibilities the ordinary numbers are typically used; for example {0, 1} is a binary gene. In a genetic algorithm, code is recognized either by length of string/gene beside it or through mapping between the potential solutions to the problem and string [64]. GA (Genetic Algorithm) starts with randomly created initial populations. A combinatory progression is performed with the implementation of selection, crossover and mutation (genetic operators).

Every individual relating to initial population is evaluated and decoded with respect to some defined criteria represented as "fitness function", ranking is performed afterward. After this in initial solutions the pair of string are arbitrarily selected and every single of them showing to crossover/mutation which produce off springs that will be interpreted and then calculating their fitness. Afterward, a new ranking of solution is done and fitness individual will keep on creating the novel population.

The portents will retain up until the junction is attained. Global optimum is considered as a solution when stopping criteria is defined and verified, decoded from "chromosomes" of fitness solution [64] [65].

3.5 GA (Genetic Algorithm) Problem Formulation

In the framework of GA (Genetic Algorithm), every solution of a problem is enlightened with a group of c that are design variable, a_i, which describe the order of real numbers A [66].

$$A = a1 * a2 * \ldots * ac, ai \in IR \quad (3.4)$$

Available solutions for a_i is identified through restricting the size of their search interplanetary. Afterward, the programming for a_i is developed and length of binary string, l_{ci} is set aside for a_i is identified rendering to the succeeding equation:

$$li = 2lci \quad (3.5)$$

Where, l_i = entire number of locations on binary strings regarding variable a_i. For illustration of a_i related to positon p_i. This position is used to calculate the real value of result relating to the particular domain.

$$ai = ai + (ai - ai) / (li - 1) * pi \quad (3.6)$$

Where, a_i = lower limit, $\underline{a}_i$ = upper limit of values estimated by the design variable a_i. Chromosomes are obtained by encoding a_i to binary codes which changes phenotype to string of bits [66].

$$A(t) = \prod_{i=1}^{c} a I = \prod_{i=1}^{c} (e : a i \rightarrow \{0, 1\}) \quad (3.7)$$

An EA (Evolutionary Algorithm), (t = 0, 1, 2 ...) must have the ability to adapt the population of solution, A (t), to accomplish improved presentation in an iteration with the circumstances E.

GAs (Genetic Algorithm) require info, I (t) related to the adaptation of population A (t) and to its surroundings E.

$$GA: A(t) * I(t) \rightarrow A(t+1) \quad (3.8)$$

Initialization progression holds the establishment of introductory population of arbitrarily shaped chromosomes. Each chromosome is then designed regarding objective function, y (A), unveiling its elasticity to the settings in which it is rooted. GAs (Genetic Algorithm) increase fitness when it actuates and fitness function is described as:

$$f\ (A)\ =\ k - y\ (A) \tag{3.9}$$

Where, k = arbitrary huge positive assessment, that promises that the fitness f (A) certainly not becomes negative.

After creation of randomly generated potential solutions the algorithm continues evaluating fitness and generate a new population with same size. From preceding individuals new population will be elaborated, after directing them to combination of genetic operators.

Afterwards, offspring fitness is completed and solutions are equated with the reference values. The algorithm will end when solution values and reference values are close to each other [67].

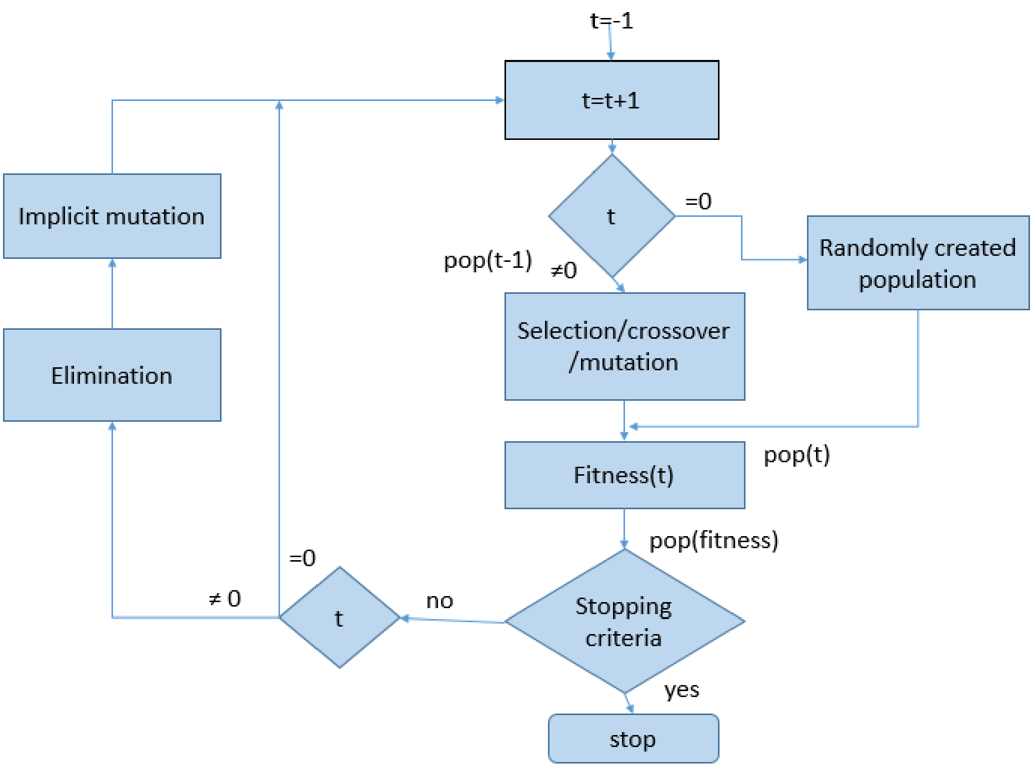

Figure 2 Genetic Algorithm Process Scheme

3.4 Elastic Properties Identification Using GA (Genetic Algorithm)

- **Problem Formation**

When a body is exposed to load, material point that vacates space point Q_i originally will move to alternative space point Q_i^*, so the associated displacement vector is articulated by $\bar{u}(Q_i)$, illustrated by Figure 6.

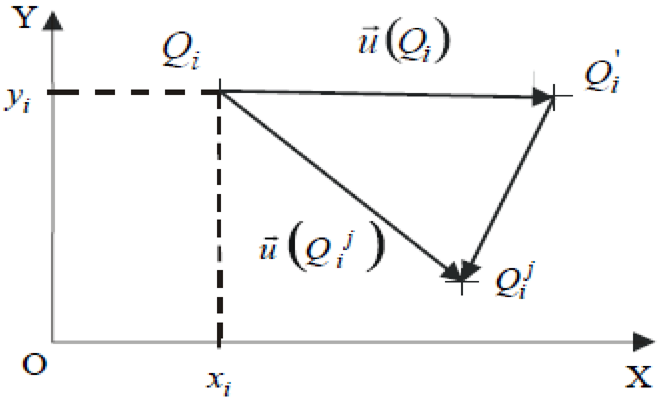

Figure 3 Reference displacement u̅(Qi), of point Qi and displacement

Mechanical properties of material shown in table B, create relationships concerning the load applied to sample and displacement field. In numerical identification methodologies they act as design variables. The in plane characteristics are represented in GA by following sequence.

$$A = \prod_{i=1}^{4} a\,\mathrm{i} = E1 * E2 * v12 * G12, \quad ai \in IR \qquad (3.10)$$

The minimized objective function was obtained by the quadratic mean variance between reference displacement field, $\bar{u}(x_i, y_i)$, and displacement field gained from GA (Genetic Algorithm) [68].

$$y(A) = \frac{1}{N}\left\{\sum_{i=1}^{N}\left\|u_i^j - u_i\right\|^2\right\} \qquad (3.11)$$

3.5 Genetic Operators

Researchers are making effort to develop GAs (Genetic Algorithm) to enhance their efficiencies and for better adaptation of different optimization problems [69]. Keeping in mind, numerous advances have been suggested in a group of elastic parameters that will be explained in order:

- **Selection**

Rendering to the objective functions values, after ranking of solutions, the total number n of possible solutions are further divided in three groups that contained an even number of results, $n_T, n_M, n_{B,}$ explained in Figure 7. According to the approach used in the selection operator, n_T solutions from S_T subgroups may combine together or with whom, who is belonging to S_M, succeeding a random logic and ensuing in S_T subgroups possible solutions. Bigamy can occur among the solution's group. S_M, then gives two set of $n_{MS}$and n_{Mi} solutions. These two are coupled together. This combination is randomly generated, but unlike the preceding set, operate except the existence of bigamy.

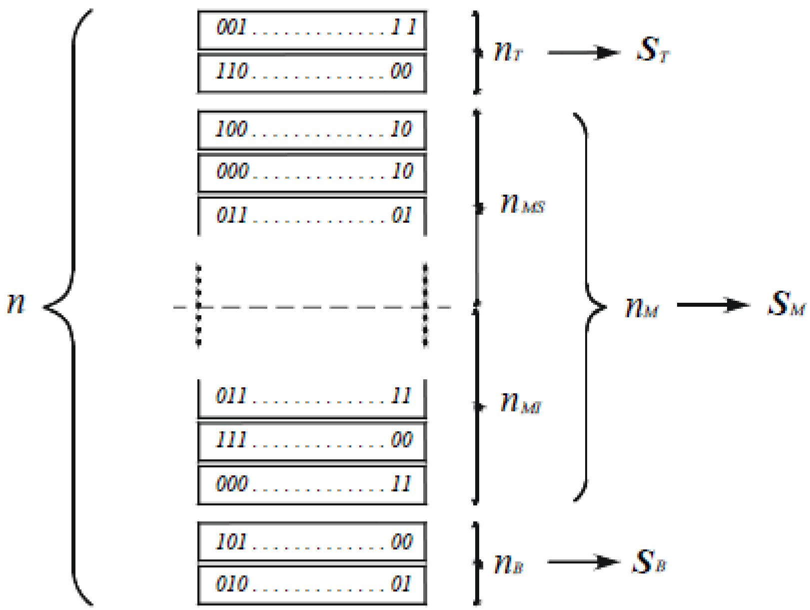

Figure 4 Selection Operator

- **Crossover**

This operator is responsible for the consolidation of genetic material of a formerly formed group of results, made by the group of chromosomes, which turns to a list joining pairs, chaotically shaped as open in Selection. Crossover Operator begins creating an arbitrary quantity, r_i amongst the varieties of span of design string as shown in Figure 8.

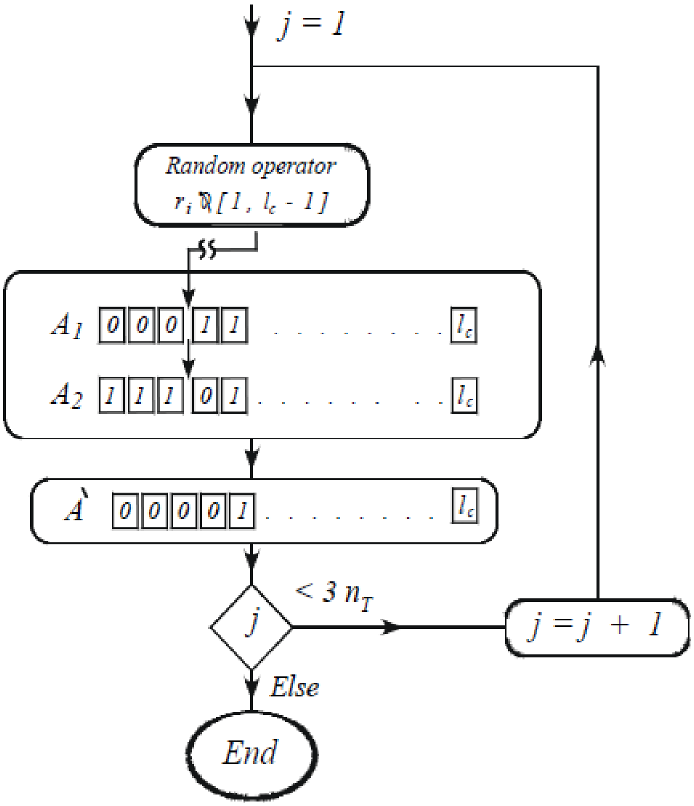

Figure 5 Crossover Operator

The number, r_i is applied to designate the place of cross site in coupling strings. Previously, entire gene ideals from one coupling element to the child section till, r_i the long-term sub string is derived from the other coupling element. Offspring generation increases the design space investigation, like, that resembles to an uninterrupted searching of global optimum.

- ## Mutation

Mutation operator is performed through arbitrarily flipping a gene in a chromosomes of a recently created results. It works with a predefined probability p_m.

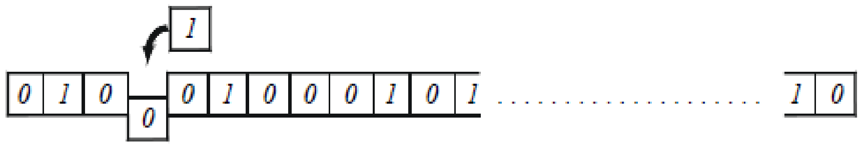

Figure 6 Mutation flipping gene

Therefore the Mutation consents the relocation of evidence ultimately vanished in Crossover, approves the penetrating of solution that further narrates to search space. This has a tributary status so it has attributed a small possibility to take place.

- ## Elimination

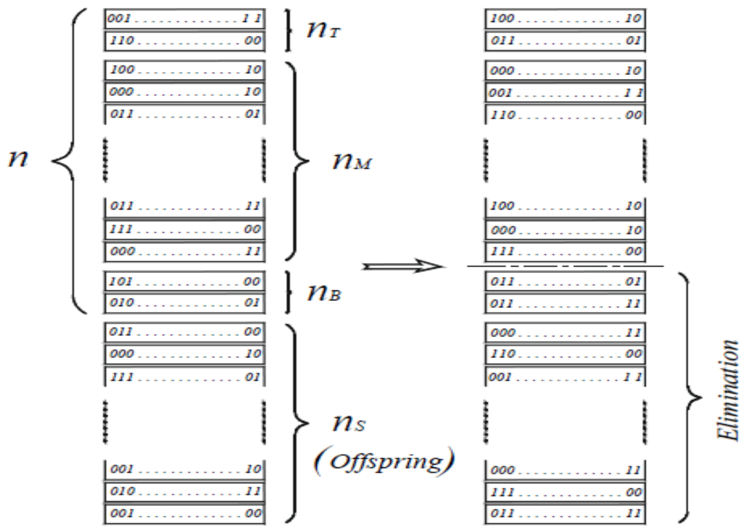

Figure 7 Elimination

After the conclusion of Selection, Crossover and Mutation operator, Fitness of novel descendant's solution is assessed. This gratitude lets the achievement of solution ranking, confirming the accessory of greater number of positions and an order as shown in Figure 10. The topping of a new result to a high position in ranking, forbid their elimination over “Natural Selection”.

Uninterruptedly, all results that consist of the half inferior place from list specified in Figure 10. This operation put on the of individual’s natural selection, substitute as a natural agent for the removal of individual that are less fit [70].

3.6 Stopping Criteria

Stopping criteria was developed in terms of relative error for each elastic parameter and large set of generations with uninterrupted fitness value. The algorithm convergence is attained when relative error in used values are very close or less than 22% [71].

Summary

In the proposed study an attempt is made to find out the preferable approach for the non-destructive characterization of laminated composites depending on laboratory facilities. On that reason, a static approach using the off-axis tensile tests and an inverse procedure based on the planning of experiments methodology, are applied for determination of elastic properties of laminated composite plates.

The recommended numerical method is built on FEA (Finite Element Analysis) and EA (Evolutionary Algorithms) supported by experimental results. The idea is to determine the optimal estimation of the model parameters by minimizing a selected measure-of-fit between the responses of the tested system and the numerical model. The objective is to obtain the four independent elastic engineering constants for an orthotropic medium, based on the measurement of a heterogeneous displacement field.

Chapter 4

IMPLIMENTATION OF EVILUTIONARY ALGORITHM

The proposed approach for identification of mechanical properties of composite laminates is addressed according to the following steps:

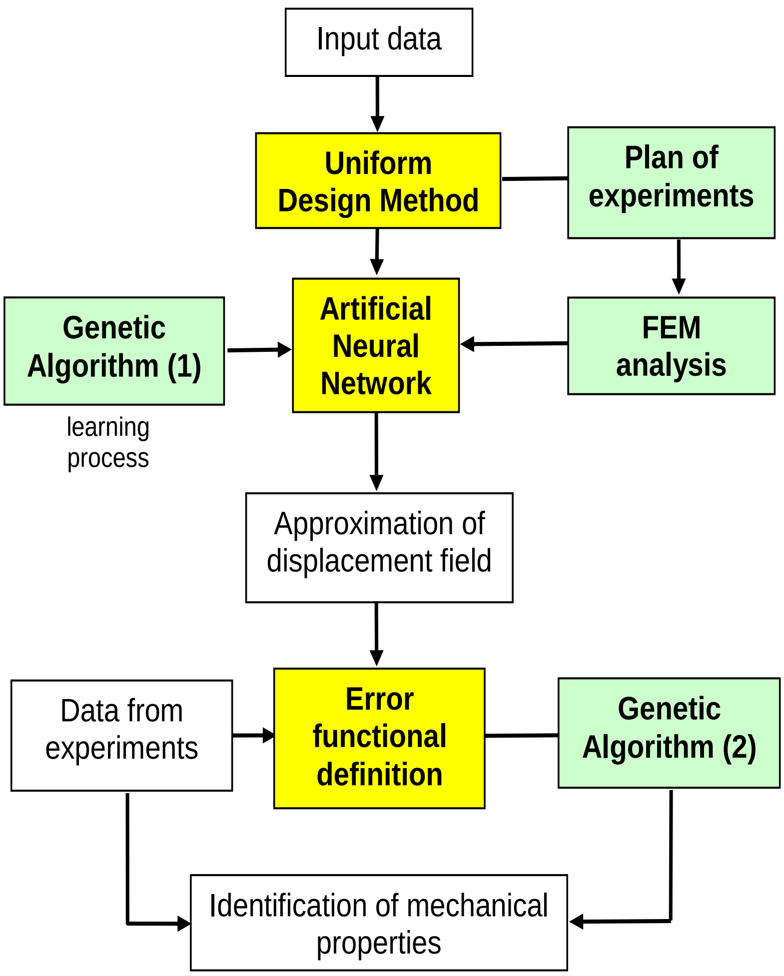

Figure 11 Flowchart of proposed approach for identification of mechanical properties

4.1 Implementation of UDM (Uniform Design Method)

Using the UDM (Uniform Design Method) a set of design points belonging to the interval $\left[\bar{\pi}_i - \alpha\,\bar{\pi}_i\,,\ \bar{\pi}_i + \alpha\,\bar{\pi}_i\right]$ is generated covering a domain centred at mean reference values of the design variables. This method enables a uniform exploration of the domain values necessary in the development of ANN (Artificial Neural Networks) approximation model. The mechanical properties obtained from experimental tests are considered the mean reference values, $\overline{\pi}_i$, for composite laminates.

Table B: Reference values obtained from experiment [68]

Longitudinal elastic modulus, E_1	20.33 GPa
Transversal elastic modulus, E_2	20.03 GPa
In-plane shear modulus, G	3.843 GPa
Poisson ratio, v	0.16

The UDM (Uniform Design Method) points are considered as experimental input values to be used in the ANN (Artificial Neural Networks) learning method. A number of 27 training data sets is selected inside the interval $\left[\overline{\pi}_i - 0.06\overline{\pi}_i, \overline{\pi}_i + 0.06\overline{\pi}_i\right]$, with mean reference value $\overline{\pi}_i$ set as a design variable for each mechanical property considered in the proposed approach.

The mean reference value, $\overline{\pi}_i$ is the elastic constant value obtained from experimental tests and corresponds to UDM design point number 14 in Table A. After considering Table $U_{27}(27^{10})$ of the UDM (Uniform Design Method), columns 1, 4, 6 and 9 requisite nomination rendering to the certain accessory. Table C give four variables and discrepancy $\Psi(n,P) = 0.1189$. Then the interval $\left[\overline{\pi}_i - 0.06\overline{\pi}_i, \overline{\pi}_i + 0.06\overline{\pi}_i\right]$ is equally discretized with 27 points and, using the integer code format, the definite configuration for $\boldsymbol{\pi} = [E_1, E_2, G, \nu]$ is obtained, as shown in Table D.

Table C: UDM points in sequence

27 UDM points in sequence				
Design points	**E_1 (E+10)**	**E_2 (E+10)**	**G_{12} (E+09)**	**v**
1	1.911020	1.882820	3.612420	0.150400
2	1.920403	1.892065	3.630157	0.151138
3	1.929786	1.901309	3.647894	0.151877
4	1.939169	1.910554	3.665631	0.152615
5	1.948552	1.919798	3.683368	0.153354
6	1.957935	1.929043	3.701105	0.154092
7	1.967318	1.938288	3.718842	0.154831
8	1.976702	1.947532	3.736578	0.155569
9	1.986085	1.956777	3.754315	0.156308
10	1.995468	1.966022	3.772052	0.157046
11	2.004851	1.975266	3.789789	0.157785
12	2.014234	1.984511	3.807526	0.158523
13	2.023617	1.993755	3.825263	0.159262
14	2.033000	2.003000	3.843000	0.160000
15	2.042383	2.012245	3.860737	0.160738
16	2.051766	2.021489	3.878474	0.161477
17	2.061149	2.030734	3.896211	0.162215
18	2.070532	2.039978	3.913948	0.162954
19	2.079915	2.049223	3.931685	0.163692
20	2.089298	2.058468	3.949422	0.164431
21	2.098682	2.067712	3.967158	0.165169
22	2.108065	2.076957	3.984895	0.165908
23	2.117448	2.086202	4.002632	0.166646
24	2.126831	2.095446	4.020369	0.167385
25	2.136214	2.104691	4.038106	0.168123
26	2.145597	2.113935	4.055843	0.168862
27	2.154980	2.123180	4.073580	0.169600

Table D: Input experimental Values used in ANN learning process

Input experimental values used in ANN learning process			
E1 (E+10 Pa)	E2 (E+10 Pa)	G (E+09 Pa)	v
1.911020	1.975266	3.860737	0.168123
1.920403	2.076957	3.630157	0.165908
1.929786	1.919798	3.896211	0.163692
1.939169	2.021489	3.665631	0.161477
1.948552	2.123180	3.931685	0.159262
1.957935	1.966022	3.701105	0.157046
1.967318	2.067712	3.967158	0.154831
1.976702	1.910554	3.736578	0.152615
1.986085	2.012245	4.002632	0.150400
1.995468	2.113935	3.772052	0.168862
2.004851	1.956777	4.038106	0.166646
2.014234	2.058468	3.807526	0.164431
2.023617	1.901309	4.073580	0.162215
2.033000	2.003000	3.843000	0.160000
2.042383	2.104691	3.612420	0.157785
2.051766	1.947532	3.878474	0.155569
2.061149	2.049223	3.647894	0.153354
2.070532	1.892065	3.913948	0.151138
2.079915	1.993755	3.683368	0.169600
2.089298	2.095446	3.949422	0.167385
2.098682	1.938288	3.718842	0.165169
2.108065	2.039978	3.984895	0.162954
2.117448	1.882820	3.754315	0.160738
2.126831	1.984511	4.020369	0.158523
2.136214	2.086202	3.789789	0.156308
2.145597	1.938288	4.055843	0.154092
2.154980	2.030734	3.825263	0.151877

4.2 Implementation OF FEM Using Ahmad Shell Element Method

For each UDM (Uniform Design Method) design point, the displacement field, u is obtained as a reference using the same experimental testing conditions. The displacement field used as a reference is created by FEM (Finite Element Method) considering the specimen geometry, the experimental loading and boundary conditions [72]. In the FEM input find the connectivity of each element (total eight nodes per element). There are 20 elements and a total of 79 nodes. The element uses three translations and two rotations (5 degrees of freedom per node). The Ahmad degenerated shell element has an implicit procedure to define the shell thickness: is defined upper coordinates and lower coordinates at each node. Using these coordinates are calculated the local shell thicknesses. Further the 2D integration this FEM use an integration procedure along the thickness of the shell. It is used the original Ahmad developments together further changes to avoid locking phenomena. There are some drawbacks (as shear locking) that were studied by several authors. The version used in our approach is immune to shear locking for example [73].

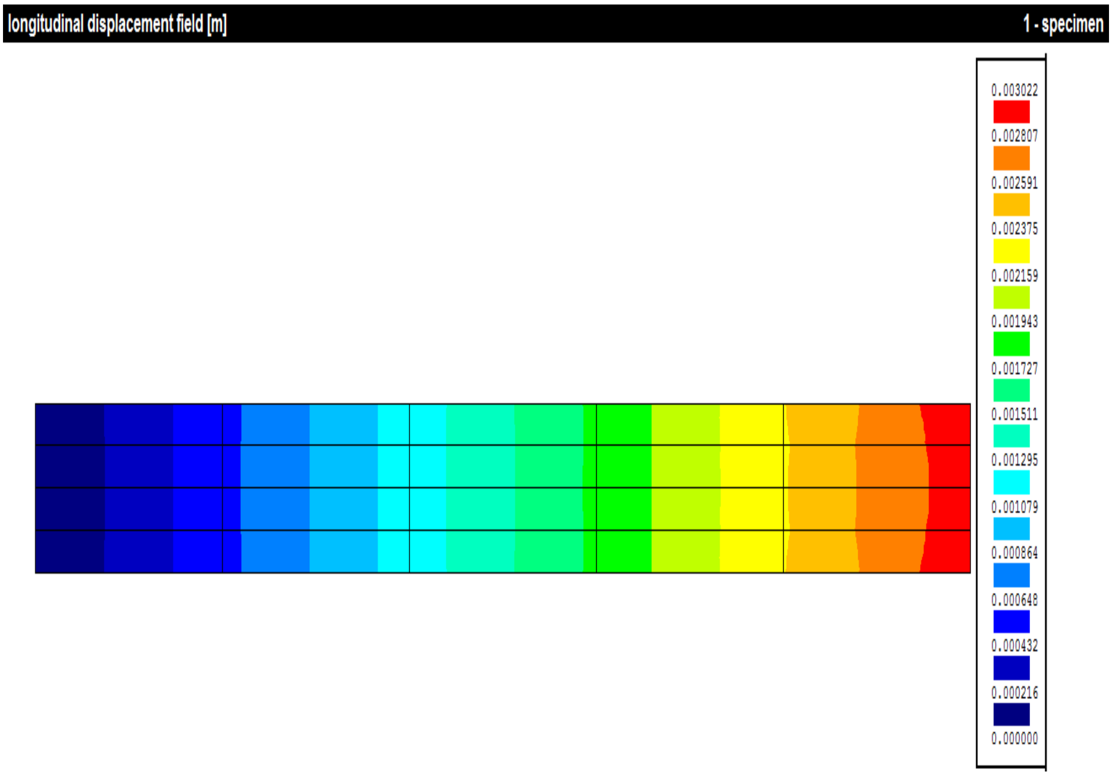

Figure 12 Reference displacement field generated by "Draw mesh" along x-axis

In the above figure the XX axis (longitudinal) displacement due to traction applied load is plotted. The displacement corresponds to UDM (Uniform Design Method) design point number 14 in figure 12.

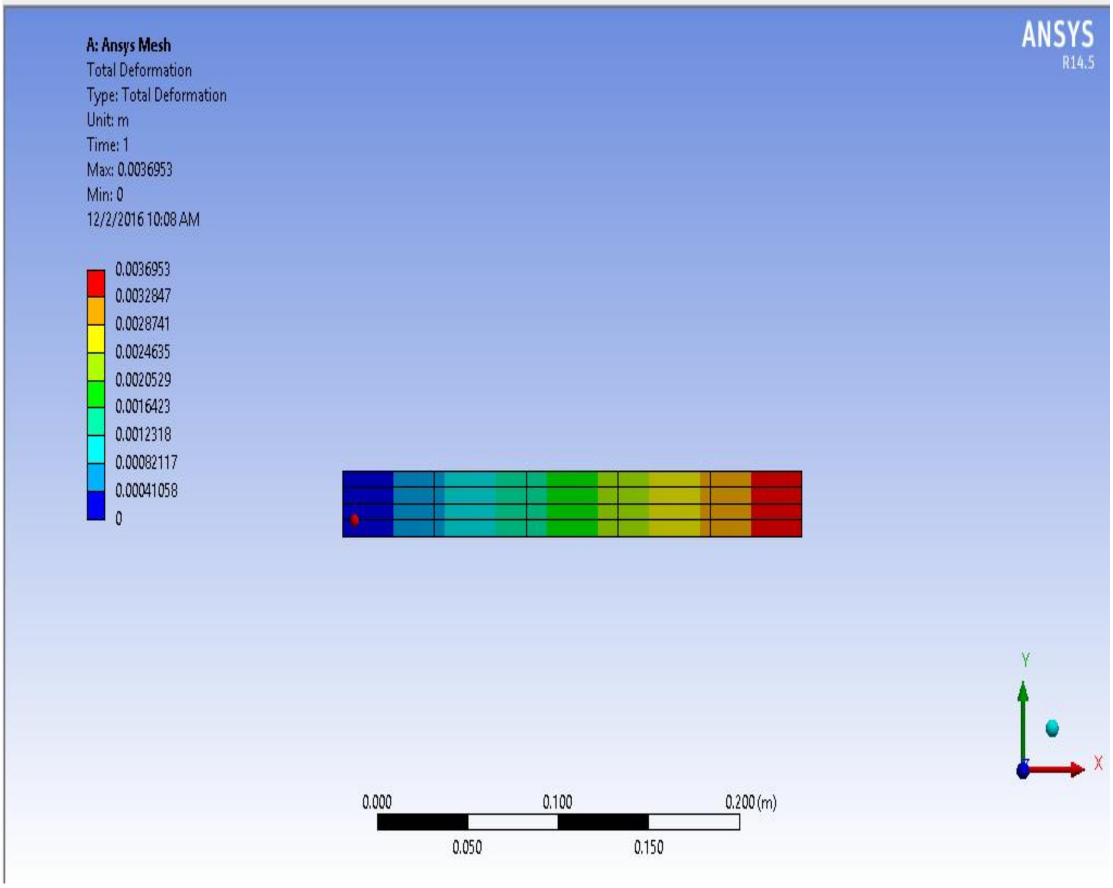

Figure 13 Displacement field generated by Ansys

In the proposed approach the analysis is performed using the degenerated Amahd shell element [74]. This displacement field treated as experimental data used to rectify the numerical method suggested in this work. The BC (Boundary Conditions) used in the numerical model are the rigid and non-rotating testing machine grips.

4.3 Implementation of ANN (Artificial Neural Network)

An artificial neural network ANN (Artificial Neural Network) is established grounded on supervised evolutionary learning. The formerly produced UDM (Uniform Design Method) design points and their calculated output response values are used as input and output configurations in the learning procedure of the optimal topology of ANN (Artificial Neural Network).

In the ANN development a weight value is associated with each synaptic connection between processing units (PEs) that is defined as the connection importance as shown in Figure 13. The weight value acts as a multiplicative filter together with the activation procedure performed by an appropriated function.

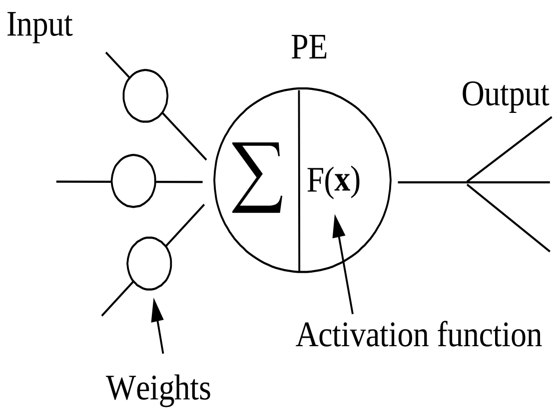

Figure 14 Processing units of ANN

The ANN architecture is formed by several layers of neurons and different matrices with synaptic weights can be identified as linkage elements between layers. Learning of ANN occurs while modification of connection weight matrix is undertaken at the learning process.

The surrogate model is based on ANN (Artificial Neural Network) developments using planned input/output patterns of results supported by UDM (Uniform Design Method). The proposed ANN (Artificial Neural Network) is organized into three layers of nodes (neurons): input, hidden and output layers. The linkages between input and hidden nodes and between hidden and output nodes are denoted by synapses. These are weighted connections that establish the relationship between input data D_i^{inp} and output data D_j^{out}. In the developed ANN (Artificial Neural Network), the input data vector $\mathbf{D}^{inp}$ is defined by a set of values for design variables $\boldsymbol{\pi}$, which are the mechanical properties of composite laminates, such as elastic or strength properties. In this methodology, respective set of values for the design variable vector $\boldsymbol{\pi}$ is nominated using the UDM (Uniform Design Method). The conforming output data vector $\mathbf{D}^{out}$ comprises of the displacements, u, measured along longitudinal axis of symmetry of the specimen used in experimental tests. Figure 14 shows the topology of the ANN (Artificial Neural Network), display the input and output parameters. Every configuration, comprising of an input and output vector, wants to be normalized to evade numerical error propagation throughout the ANN (Artificial Neural Network) learning procedure. Which is attained using data normalization (input data between 0.01 and 0.99).

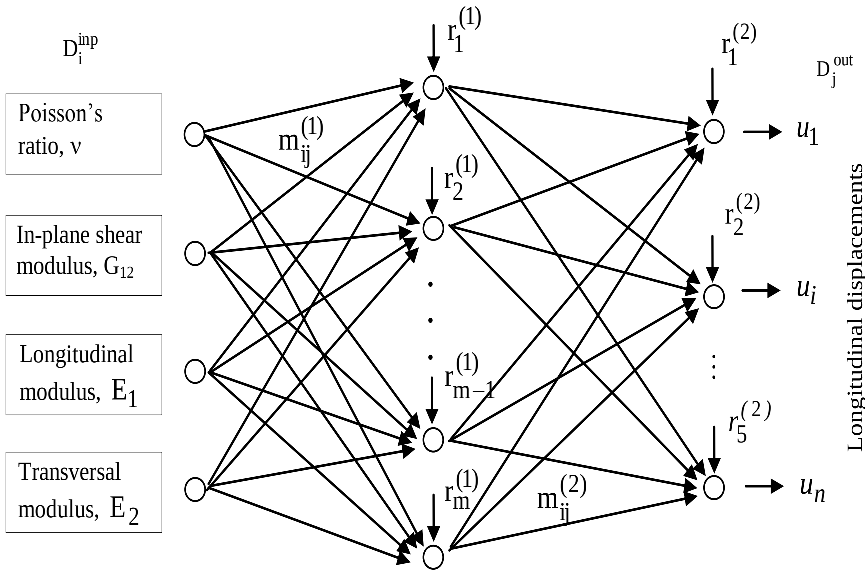

Figure 15 Artificial Neural Network topology

- **Performance evaluation of ANN (Artificial Neural Network)**

The difference between ANN (Artificial Neural Network) and already defined output data is used to regulate the learning procedure, which is proposed to attain a complete process model. ANN (Artificial Neural Network) is introduced by set of input data, it utilizes weights of synapses and biases value to yield reliable simulated results by a process known as learning.

Each group of input data weight matrix $\mathbf{M}^{(p)}$ and biases $\mathbf{r}^{(p)}$, output is acquired. Simulated results of output are related with previously explained values to approximate the error, which is further minimized through optimization process. The equations for error:

- **Mean relative error**

$$E_1(\boldsymbol{M},\boldsymbol{r}) = \frac{1}{2N_{exp}} \sum_{i=1}^{N_{exp}} \left(\left| \frac{\bar{R}^{exp} - \bar{R}^{sim}}{\bar{R}^{exp}} \right| + \left| \frac{ß_s^{exp} - ß_s^{sim}}{ß_s^{exp}} \right| \right) \qquad (4.1)$$

- **Absolute error**

$$E_{\mathbf{2}}(\boldsymbol{M},\boldsymbol{R}) = \sum_{i=1}^{N_{exp}}(\left|\bar{R}^{exp} - \bar{R}^{sim}\right| + \left|ß_{s}^{exp} - ß_{s}^{sim}\right|) \qquad (4.2)$$

Where, N_{exp}= number of experiment measured in UDM (Uniform Design Method) method, subscripts *exp* and *sim* represent the experimental and simulated data. The error obtained from the above equations and equation for biases [58] are reflected in ANN (Artificial Neural Network) learning process.

- **Evolutionary process based ANN (Artificial Neural Network)**

The adopted supervised learning process of the ANN (Artificial Neural Network) based on a Genetic Algorithm (GA) uses the weights of synapses, $\mathbf{M}^{(p)}$, and biases of neural nodes at the hidden and output layers, $\mathbf{r}^{(p)}$, as design variables. A binary code format is used for these variables.

The number of digits of each variable can be different depending on the connection between the input-hidden layers or hidden-output layers. The formulation of optimization problem related to the ANN (Artificial Neural Networks) learning procedure is grounded absolute and relative errors minimization obtained through the input and output configurations obtained for the UDM (Uniform Design Method) design points. Biases related to regularization in the hidden/ output neurons is intricate in the learning scheme and soothing and accelerating the numerical practice.

In the meantime, the objective of evolutionary search is to improve a FIT global fitness function accompanying with ANN (Artificial Neural Network) presentation, the optimization problem is assumed as:

$$Maximize\ FIT = K - \left[\sum_{k=1}^{3} c_k\ E_k(\boldsymbol{M},\boldsymbol{r}) + c_4\Gamma(\boldsymbol{r}^{(1)},\boldsymbol{r}^{(\mathbf{2})})\right] \qquad (4.3)$$

Where,

K = arbitrary constant to acquire positive fitness c_k= weight constants for regularization, and $\Gamma(\boldsymbol{r}^{(1)},\boldsymbol{r}^{(\mathbf{2})})$ = mean quadratic values for biases [58]. These ANN (Artificial Neural Networks) simulated output results are associated with the experimental (FEM) output values ($u_i^{exp(FEM)}$ i=1… n) attained for the same input data to estimate the variance (or error), which must be diminished during the learning procedure [25]. The supervised learning of the suggested ANN (Artificial Neural Network) is based on numerous measures of the error with the aim to accelerate and stabilize the learning process. The first measure is the root-mean-squared error defined as

$$RMSE = \sqrt{\frac{1}{N_{exp}} \sum_{i=1}^{N_{exp}} \left[\left(u_i^{sim} - u_i^{exp(FEM)} \right)^2 \right]_i} \tag{4.4}$$

Where $N_{\exp}$ is the number of experiments considered in the set of design points of UDM and the superscripts sim and exp denote the simulated and experimental data of displacement field? To reinforce the error minimization a second measure is introduced based on the following mean relative error component:

$$RE = \frac{1}{N_{exp}} \sum_{i=1}^{N_{exp}} \left[\left(\frac{u_i^{sim} - u_i^{exp(FEM)}}{u_i^{exp(FEM)}} \right)^2 \right]_i \tag{4.5}$$

The influence of the biases of the neurons of the hidden and output layers is also included to stabilize the learning process:

$$\Gamma = \sqrt{\frac{1}{N_{exp}} \sum_{i=1}^{N_{exp}} \left[\frac{1}{N_{hid}} \sum_{k=1}^{N_{hid}} \left(r_k^{(1)} \right)^2 \right]_i} + \sqrt{\frac{1}{N_{exp}} \sum_{i=1}^{N_{exp}} \frac{1}{N_{out}} \left[\sum_{k=1}^{N_{out}} \left(r_k^{(2)} \right)^2 \right]_i} \tag{4.6}$$

Where N_{hid} and N_{out} are the number of neurons of the hidden layer and of the output layer, respectively. The error measures presented from Equations (4.4) and (4.5) and biases component in Equation (4.6) are aggregated using the following formula:

$$\Gamma_1 \left(\mathbf{M}^{(1)}, \mathbf{r}^{(1)}, \mathbf{M}^{(2)}, \mathbf{r}^{(2)} \right) = c_1 \, RMSE + c_2 \, RE + c_3 \, \Gamma \tag{4.7}$$

Being the constants c_k used to regularize the numerical differences between the three error terms aiming to stabilize the numerical procedure. The weights of the synapses and biases can be changed until the value of Γ_1 falls within a prescribed value.

The ANN supervised learning procedure is based on the minimization of the function defined in equation (4.7) using the weights of synapses $\mathbf{M}^{(p)}$, and biases of neural nodes at the hidden and output layers $\mathbf{r}^{(p)}$, as design variables as shown in Figure 14. The search for ANN optimal configuration is performed by a Genetic Algorithm denoted $\mathbf{GA}^{(1)}$. At this stage a population of solutions for ANN configuration denoted by $\mathbf{P}^{(t)}$ is considered at each t-generation.

A binary code format is used for these variables. The number of digits of each variable can be different depending on the connection between the input-hidden layers or hidden-output layers. The domain of the learning variables $\mathbf{M}^{(p)}$ and $\mathbf{r}^{(p)}$ (p=1 and p=2) and scaling parameter η can be tuning together the code format of design variables of the ANN learning procedure. The optimization problem formulation associated with the ANN learning process is based on the minimization of the function defined in Equation (4.7) without constraints, as follows

$$\text{Maximize } FIT^{(1)} = K^{(1)} - \Gamma_1\left(\mathbf{M}^{(1)}, \mathbf{r}^{(1)}, \mathbf{M}^{(2)}, \mathbf{r}^{(2)}\right) \quad \text{over} \quad \mathbf{M}^{(p)} \text{ and } \mathbf{r}^{(p)} \tag{4.8}$$

$$\text{Subject to } \mathbf{M}^{(p)}, \mathbf{r}^{(p)} \in \Omega_1 \quad \text{(p=1 and p=2)},$$

where Ω_1 is the domain of design variables in learning procedure, $FIT^{(1)}$ is the fitness function in GA search to obtain the optimal ANN configuration, $\mathbf{P}_{\mathrm{ANN}}{}^{\mathrm{opt}}$ for the weight of synapses and biases in neurons. Since the selection operator of GA is fitness-based the function $FIT^{(1)}$ must take positive values. So, the constant $K^{(1)}$ must be large enough to obtain always positive fitness values.

4.4 Inverse Optimization through GA (Genetic Algorithm)

The experimental output data obtained for displacement field is used as reference in the inverse optimization problem. The design variables of the optimization problem are the independent elastic engineering constants *E1*, *E2*, *G* and Poisson's ratio *v*.

The objective is to obtain the four independent elastic engineering constants for an orthotropic medium, based on the measurement of a heterogeneous displacement field. The optimal estimation of the model parameters is performed by minimizing an error functional defined as the difference between the experimental measurements and the simulated output results from ANN approximation model.

4.6 Error Minimization

By minimising the error functional the identification parameters i.e., elastic properties, are obtained. The minimisation procedure is based on a genetic algorithm (GA). The GA run arbitrarily created an individual's population called the initial population. Using a genetic operators like selection and crossover and mutation, evolutionary search procedure is achieved.

4.5 Learning and Inverse Design Procedures Based on GA (Genetic Algorithm)

GA (Genetic algorithms) denoted by $\mathbf{GA}^{(1)}$ and $\mathbf{GA}^{(2)}$ used in both phases of the recommended procedure have the alike group. But, parameters labelled in independent mode are helpful in different genetics.

GA (Genetic algorithms) are evolutionary search approaches based on the survival of the fittest and natural selection theory presented by Charles Darwin. Both proposed $\mathbf{GA}^{(1)}$ and $\mathbf{GA}^{(2)}$ mostly perform on three parts: (1) coding/decoding design variables into strings; (2) weighing the fitness of individual solution string; and (3) implementing genetic operators to crop the next generation of solution strings in a new population. In this paper, four basic genetic operators, (1) Selection, (2) Crossover, (3) Elimination and (4) Replacement from control similarity and Mutation are utilized. An elitist tactic recognised on conservation of the best-fit group transfers the best-fitted solution into a new population for the next generation.

The operators are applied in the following sequence:

- **Step1**: Initialization. Using a uniform probability distribution function, the initial population is arbitrarily created
- **Step 2**: Selection. The population is ranked according to individual fitness. The elite group is found counting highly-fitted individuals. The selection of the couple of parents p_1 and p_2, is fitness-based: one from the best-fitted group (elite) and another from the least fitted one. The current population S^k is shifted to an intermediate stage gained by the Crossover operator where they will link the offspring group B.

- **Step 3**: Crossover. Consuming genes from both progenitors, crossover operator transforms the couple of chromosomes parents into a new chromosome (offspring). The offspring genetic material is acquired using the multi-point combination technique known as parameterized uniform crossover .This is functional to both binary string of the couple selected chromosomes p_1 and p_2. Then, this Crossover is applied with a predefined probability to choose the offspring genetic material from the highest fitted chromosome (offspring) .The offspring group B created by recombination will be joined to the original population S^k generating the enlarged population $S^k \cup \mathbf{B}$.
- **Step 4**: Deletion/Replacement by similarity control. Keeping into account the fitness the enlarged population of solutions $S^k \cup \mathbf{B}$ is ranked. Then, the similarity control is performed on the genes of separate design variables and after on all of them following an updating scheme throughout the evolutionary progression. The objective is to evade the existence of very similar individuals into the population decreasing the endogamy properties of recombination. This is followed by deletion of solutions with similar genetic properties and subsequent replacement by new haphazardly generated individuals. The new enlarged population $S^k \cup \mathbf{B}$ is graded and the individuals with low fitness are omitted. Now, the dimension of the current population is not as much of the original one. The original size population will be enhanced after taking into account a group of new solutions attained from the Mutation operator.
- **Step 5**: Mutation. The Mutation genetic operator is used to overwhelm the problem encouraged by Selection and Crossover operators where some generated solutions have a great percentage of equal genetic material in the offered approach. This is linked with an absence of population diversity inducing premature convergence of the evolutionary process by anchorage at local minima. So, pointing to recover the diversity level a chromosome set group which genes are created in an arbitrary way is presented into the population. This operation is called Implicit Mutation and is fairly different from classic practises where a condensed number of genes are transformed. Undeniably, this group of chromosomes will be recombined with the remaining individuals into the population during next generations. The new population

S^{k+1} is achieved and the evolutionary process will last until the stopping criteria are touched after mutation.

- **Step 6**: Stopping criterion. The stopping criterion used in the convergence analysis is centred on the relative variation of the mean fitness of a reference group through a fixed number of generations and the feasibility of the equivalent solutions. This reference group usually is the elite group. The search is stopped if the mean fitness of the reference group does not evolve after a finite number of generations. Otherwise, the new population S^{k+1} evolution continues and the process goes on to Step 2.

Figure 15 shows the flow diagram of both genetic algorithms $\mathbf{GA}^{(1)}$ and $\mathbf{GA}^{(2)}$.

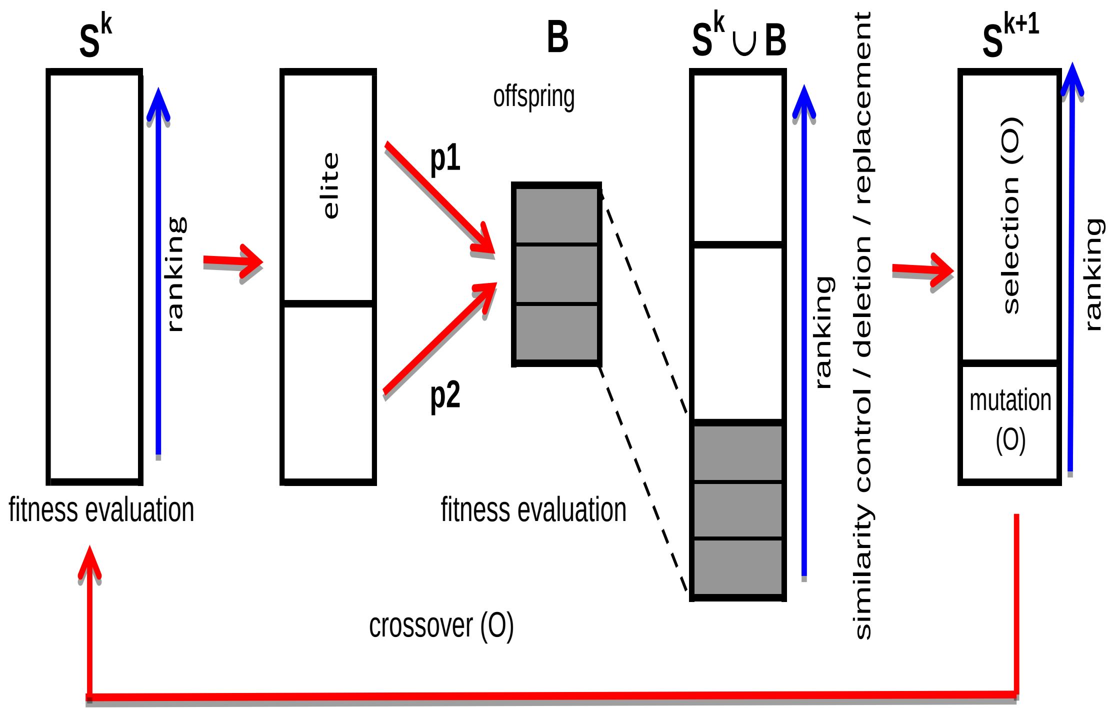

Figure 16 Evolution steps and genetic operators of $\mathrm{GA}^{(1)}$ and $\mathrm{GA}^{(2)}$ at each *k-th* generation.

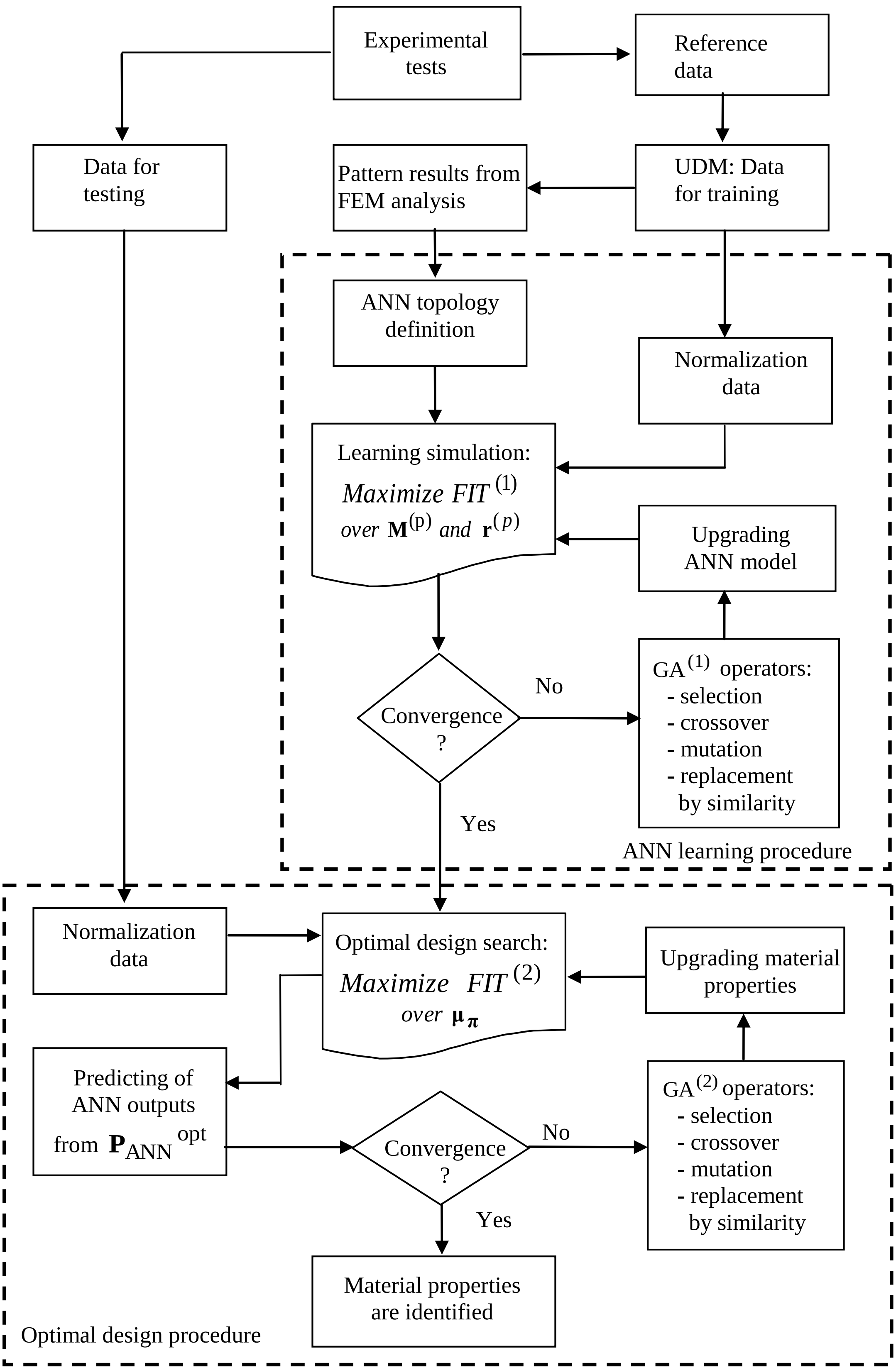

Figure 17 learning and inverse design procedures

Chapter 5

RESULTS AND DISCUSSION

5.1 Experimental Details

The previous literature show that there are authors who proposed a micromechanical model for analyzing the orthotropic elastic properties of 8 satin weave composite. These proposed models [72] are further restricted by three limitations i) yarn is modelled as a homogenous solid and material properties are independent of manufacturing process ii) exact post cure geometry of weave is not considered iii) they ignore the effect of void content on yarn and matrix properties. But these cannot be ignored as manufacturing process parameter have a strong effect on post cure geometry, volume fraction, within yarn and void content.

So, the model proposed in this paper study the effect of all the parameters which were discussed above. This FE Based micromechanical model no only consider the correct post cure geometry but also include the volume fraction within yarn and void contents.

Three sets of experiments were performed, that are

- Tensile and shear test for mechanical properties testing
- Non-destructive estimation of UC (unit cell) architecture of cured composite with the help of X-ray micro- tomography (XMT)
- Characterization of physical properties including void content and volume fraction

In tensile test the specimen is cut using the quickstep method. Poisson ratio is calculated using the strain gauges and displacement along the tensile force axis was measured using extensometer. Finally the graphical representation of stress and strain is given. Shear modulus is obtained by rail shear test method. X-ray micro tomography is used to clearly imaging the reinforcement microstructures of the whole sample in the final cured state not limited in depth. The density of composite is computed with mettler Toledo analytical balance. The average density of specimen is 1.595 gram per cubic centimeter with standard deviation of .0088. Volume fraction is measured using the

ASTM standard. The fraction of fibers present in yarn is defined as the fiber volume fraction of yarn. Volume mathematical relation is used to calculate the volume fraction of fibers in yarn assuming that the shape of yarn is lenticular. Measurement taken from the XMT are used to create a realistic geometric model. In this study the XMT images are utilized to identify the repetitive pattern in the given composite.

It is assumed during developing model that constituent materials are assumed to be free of cracks and bonded perfectly to each other before and after loading. TexGen a geometric modeling software is used to generate a realistic geometries of textile composites. FE analysis are performed using the FE software Abaqus. The author assumed that the UC is subjected to quasi static loading and undergoes small deformation. Presented governing equations which shows UC fulfil the conservation and mass momentum equation. Constitutive model, UC effective response for stress and strain, analytical model for yarn effective properties , periodic boundary conditions, FE homogenization, all these methods are completely discussed in this paper to describe the model validation and result various simulated cases. Before further analysis the micromechanical model is subjected to the sanity check. Which was done by setting all the input properties of both constituents. The input material properties of yarn are obtained by considering the fiber volume fraction effects due to resin infiltration and presence of voids. Comparison of different homogenization strategies depending upon the material input data showed that there is a good interrelation between the elastic properties obtained from the FE simulation and available experimental data for the case where the fiber volume fraction within yarn and void content is accounted analytically homogenization process. Finally the author reached to a result that a realistic model of cured composite is required to accurately find out the effective properties of 8 harness strain weave composites. Moreover it is also concluded that fiber volume fraction within the yarn and void content effects on elastic material properties can be accounted for analytically using the methodology described in this paper for the 8-harness satin weave composites [72].

An off axis tensile test was chosen to calibrate the numerical method using 8-harness satin weave glass fiber reinforced phenolic (GFRP) composite. The tensile tests were performed on composite (Primco SL246/40= according the specifications set out in ISO 527-4:1997(BS 2782-3) shown below.

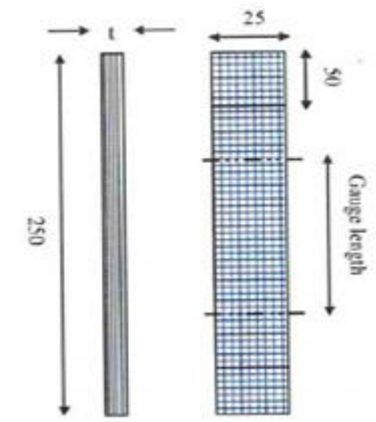

Figure 18 Specimen geometry and dimensions, tensile test, BS EN ISO 527-4 / BS 2782-3

Structural investigation is achieved for the input values by using Table 2, 27 input and output formations are developed and used in ANN (Artificial Neural Network) learning process. For every, UDM (Uniform design Method) design point, the longitudinal displacements, u, are obtained by FEM considering the specimen geometry, the experimental loading and boundary conditions.

A number of 5 (five) neurons are deliberated for the hidden layer of the ANN (Artificial Neural Network) topology. The ANN (Artificial Neural Network) learning procedure is expressed as an optimization problem with 55(fifty five) design variables corresponding to 45 (forty five) weights of synapses and 10 (ten) biases of neural nodes. The ANN (Artificial Neural Network) based GA (Genetic Algorithm) learning technique is accomplished using a population of 21 individuals/solutions. The elite and mutation groups consist of 7 and 4 solutions, respectively (Conceição António, 2001). The binary code format with 5 digits is accepted for both designing the values of the weights of synapses, $\mathbf{M}^{(p)}$, and biases of neural nodes at the hidden and output layers, $\mathbf{r}^{(p)}$. The learning development is resolved after 30000 generations of the GA. The ANN learning process is performed using a genetic algorithm as referred in flowchart of Figure 11. The evolutionary history is shown in Figure 12.

The optimal estimation of the model parameters is performed by minimizing an error functional defined as the difference between the experimental measurements of longitudinal displacements and the simulated output results calculated from optimal ANN approximation model. The search based on GA performs during 10000 generations. In this case the genetic parameters are equal to the previous ones used in

ANN learning process. The results show good convergence of the proposed methodology.

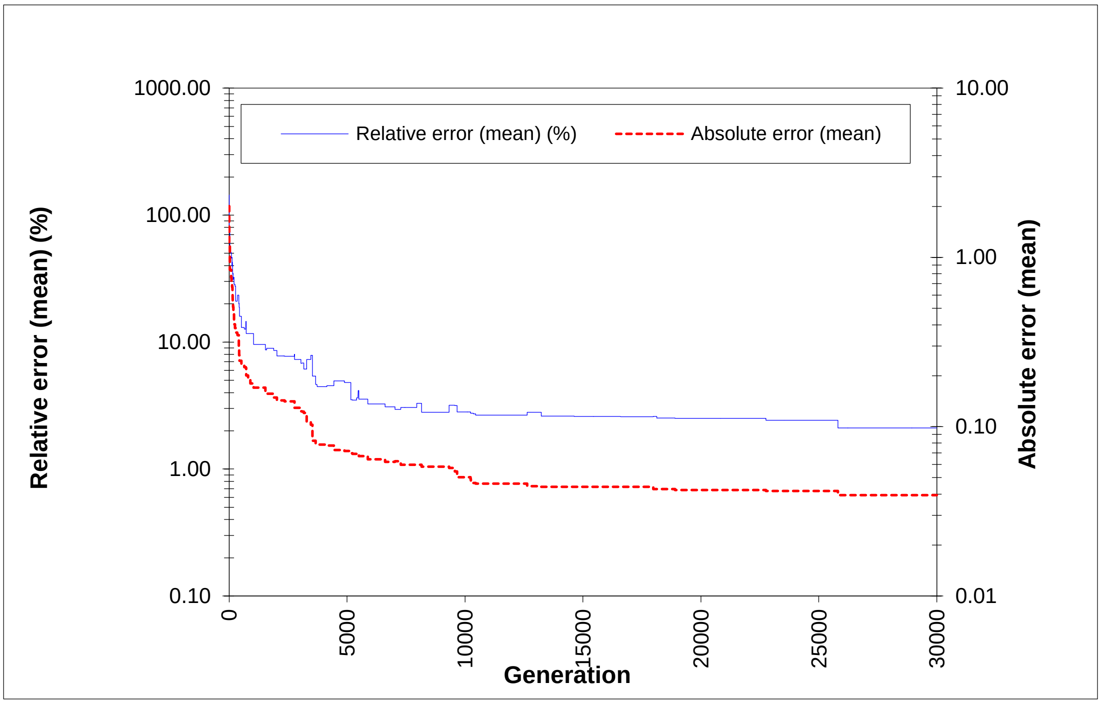

Figure 19 Absolute and relative errors evolutions in learning process of ANN-based on GA

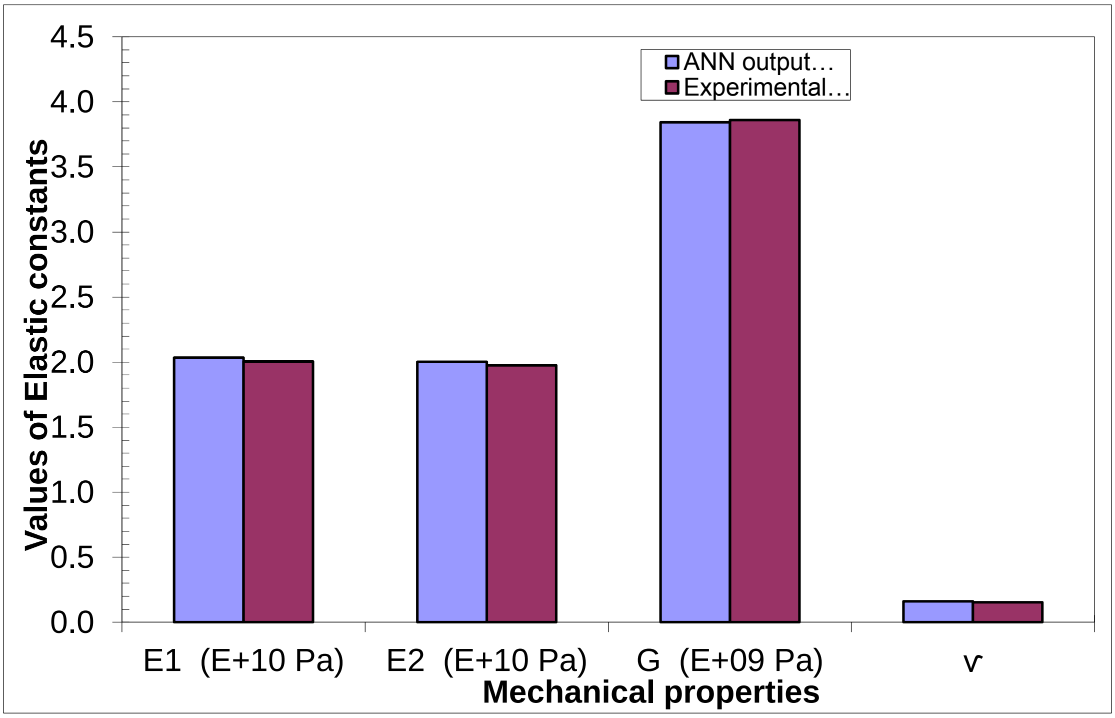

Figure 20 Comparison of simulated and experimental elastic properties

The results for the inverse optimization problem corresponding to 5th and 6th steps of the proposed approach are presented in Table E. The design variables of the optimization problem are the independent elastic engineering constants E1, E2, G and *v*. The objective is to obtain the four independent elastic engineering constants for an orthotropic medium, based on the measurement of the displacements field along longitudinal axis of symmetry of the experimental testing specimen.

Table E: Comparison between experimental and simulated values

Mechanical Properties	**E1 (E+10 Pa)**	**E2 (E+10 Pa)**	**G (E+09 Pa)**	**V**
Experimental values	2.05100	2.42800	3.855	0.25000
Numerical Results	2.03300	2.00300	3.84300	0.16000
Simulated values	2.00546	1.97527	3.86074	0.15345
Relative error (%) b/w simulated And Experimental values	1.35	1.38	0.46	0.66

Summary

The proposed inverse approach combining numerical and experimental results with measurement of full field displacement, will permit the identification of all the in plane elastic properties from experiments. The off axis tensile test was selected to calibrate the numerical method using an 8-harness satin weave glass fiber reinforced phenolic (GFRP) composite. Instead the use of a non-destructive technique an alternative hybrid approach is proposed to obtain the field displacement. In this way a displacement field used as a reference is created by FEM (Finite Element Method) considering the loading and boundary conditions, and the mechanical properties obtained from experimental tests.

The analysis is performed using the degenerated Amahd shell element [74]. This displacement field acts as an experimental data used to calibrate the numerical method proposed in this work. Rigid and non-rotating testing machine grips are used as a BC (Boundary Conditions) applied to the numerical model. Minimization of functional identification parameters i.e., elastic properties, are obtained. The minimisation technique is built on a GA (Genetic Algorithm).The GA (Genetic Algorithm) initialize with randomly generated individual’s population are called the initial population.

With the application of selection, crossover and mutation evolutionary search development is performed. Taking into account the off axis tensile tests inspected in this work, the experimental displacement field was substituted by the numerically determined nodal displacement values through the Ahmad finite element. The mechanical properties of the composite launch the association between the loads applied to the off axis and displacement field. Results show good convergence of the proposed methodology. An alternative objective function based on strain analysis is also analyzed.

Chapter 6

CONCLUSION OF RESEARCH

An inverse approach combining numerical and experimental results with full field displacement measurements used as a reference data, will permit the identification of all the in plane elastic properties from experimental trials. As an alternative the use of a non-destructive technique an alternative hybrid approach is proposed to obtain the field displacement. Taking into account the off axis tensile tests inspected in this work, the experimental displacement field was replaced by the numerically determined nodal displacement values obtained from the FEM (Finite Element Method) using degenerated Ahmed Shell Element Method. The use of a surrogate model consisted on an ANN (Artificial Neural Network) supports to establish the relationship between the elastic properties and the displacement field avoiding the exhaustive calculations based on FEM (Finite Element Method).

ANN (Artificial Neural Network) developments using planned input/output patterns of FEM (Finite Element Method) results supported by UDM (Uniform Design Method) structured data. The ANN (Artificial Neural Network) surrogate model is built through a learning procedure based on evolutionary search. The ANN (Artificial Neural Network) is a nonlinear dynamic modelling system inspired by our understanding and abstraction on the biological structure of the human brain. Its architecture and operating procedures are based on a large number of highly interconnected processing units denoted by neurons and the linkages are similar to the brain synapses as in biological sense.

The optimal estimation of the model parameters is performed by minimizing an error functional defined as the difference between the experimental measurements and the simulated output results from ANN (Artificial Neural Network) approximation model. By minimising the error functional the identification parameters i.e., elastic properties, are obtained. The minimisation procedure is based on a GA (Genetic Algorithm). The results show good convergence of the proposed methodology for identification of mechanical properties of composite materials.

Recommendations

In the present research authors considered the full field displacement as a reference field to proceed with the inverse approach, for future work the stress fields and strain fields can be used as a reference following the same methodology used in this work.

In order to develop the existing methodology in this work additional progresses must be prepared for improved performance. Different strategies of crossover can be developed to execute the convergence. Alternative objective function evaluation that are correspondingly workable to elastic properties. Furthermore, ANN (Artificial Neural Network) other than supervised learning can be used for more accurate results.

Moreover, revisions have been done to recognize interfacial properties by means of FEMU (Finite Element Model Updating Method). Measurement uncertainties happening precisely in interfacial area can badly effect the related identified properties.

Even though the displacement field used as a reference displacement field in this thesis was calculated numerically, in future for further advancements stress/strain fields could be obtained.

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