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Dynamic Measurement Noise Covariance Matrix R for Joint Probabilistic Data Association Filter

by

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Dedicated to my teachers who enlightened my soul and directed me towards the right path. Dedicated to my parents who always prayed for me and my elder sisters who have encouraged and supported me in every situation.

CERTIFICATE OF APPROVAL

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Abstract

In Multitarget Tracking (MTT), several targets of interest are being tracked simultaneously with the help of any optimal estimator. MTT tracking finds its applications in diverse fields like Pattern Recognition, Computer Vision, Radar Tracking, Robotics and many other research fields. In the literature, several algorithms have been implemented for MTT including Probabilistic Data Association Filter (PDAF), Joint Probabilistic Data Association Filter (JPDAF), Nearest Neighbor Standard Filter (NNSF), etc. JPDAF is the multitarget version of PDAF in which joint association probabilities are computed and tracks are then updated based upon these probabilities. Measurement noise covariance matrix R in JPDAF needs to be transformed from polar to Cartesian coordinate system. The optimal value of R should be calculated for the good performance of filter. In this thesis, measurement noise covariance matrix for JPDAF algorithm has been derived using standard radar parameters. 2D tracking is performed using scan radar and JPDAF algorithm. 3D tracking is also performed in a closed loop fashion using monopulse radar and JPDAF algorithm. For both 2D and 3D tracking, simulations are performed in MATLAB. Desired results are achieved and the error is reduced to such an extent that it lies inside the range bin for both cases.

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Abbreviations

AC	Amplitude Comparison
AGC	Automatic Gain Control
CA	Constant Acceleration
CNN	Convolutional Neural Networks
CT	Constant Turn
CTR	Continuous Tracking Radars
CV	Constant Velocity
DA	Data Association
DOA	Direction of Arrival
EKF	Extended Kalman Filter
GNN	Global Nearest Neighbor
GPS	Global Positioning System
JPDAF	Joint Probabilistic Data Association Filter
KF	Kalman Filter
LIDAR	Light Detection And Ranging
MHT	Multi-Hypothesis Testing
MTI	Moving Target Indicator
MTT	Multitarget Tracking
NNSF	Nearest Neighbor Standard Filter
PC	Phase Comparison
PD	Poisson Distribution
PDAF	Probabilistic Data Association Filter
PRF	Pulse Repetition Frequency
RCS	Radar Cross-Section

RNN	Recurrent Neural Networks
SNR	Signal to Noise Ratio
TWS	Track While Scan
UKF	Unscented Kalman Filter

Symbols

N_{TAR}	Total Number of Targets
Z_q	Observation Vector at the time q
θ_{it_i}	Event denoting that Measurement i has been originated from Target t_i
$\psi(\theta)$	Joint Association Event
ψ	The Validation matrix
$\gamma_t(\theta)$	Target Detection Indicator
$\beta_i(\theta)$	Measurement Association Indicator
$\alpha(\theta)$	Number of False Alarms in any event θ
λ	False Measurements Spatial Density
V_s	Surveillance Region Volume
P_d^t	Detection Probability for Target t
ϕ_{it}	Marginal Association Probabilities

Chapter 1

Introduction

1.1 Basic Idea of RADAR System

The term RADAR stands for **Radio Detection and Ranging** system. Radar is a device used to detect the specific target in the space with the help of electromagnetic waves. A radio signal is propagated towards the target and the processing techniques are then applied on the echo signal reflected back from the target. As a result, various parameters like range, speed and angles (azimuth and elevation) of target can be determined. RADAR system operates in the microwave and UHF range.

In the RADAR system, an electromagnetic signal is being generated by the transmitter which is then radiated with the help of antenna in the space. After striking with the target, transmitted signal is reflected back or may be re-radiated in different directions. The Radar receiver receives the echo signal and processing is done to find out the statistics of the target.

The range of target is determined with the help of round-trip time taken by the signal to travel from radar to target and then back from target towards the radar. The direction of target is determined by estimating the direction of arrival of reflected waves. If there is a relative motion between the target and the radar, the shift in the carrier frequency of reflected wave is observed. Doppler effect is then used to measure the velocity of the target.

Radars are mainly used to control the terrestrial and air traffic, for the guidance of missiles, for detecting the aircrafts and weather formations. Modern radars work in collaboration with the machine learning algorithms and are able to find out useful information in the presence of high noise levels. Initially, radar was developed as a device to warn about the approaching hostile aircraft and to direct the weapons towards the enemy target. Well-designed modern radar systems are capable of extracting more information about the target other than the range.

1.2 History of RADAR

In1900, a scientist Nikola Tesla observed that when the sound waves are made then echo is observed. In the same way, electromagnetic waves also bounce back and with the help of echo velocity and distance of object can be determined. In the late 19th century, Heinrich Hertz performed some experiments and the results depicted that radio waves were reflected back by metallic objects.

The systems based upon the principles of electromagnetism then started to develop. A German inventor, Hulsmever was the first person who used those principle of electromagnetism to build a detection device used in the ship in order to avoid the collision in case of foggy weather.

In 1935, the first radar system was developed by Sir Robert Watson who was a British physicist. By the end of 1939, England developed a chain of radar stations in order to prevent its coasts from the attackers that might be present in the air. During the WW2, radars were also used in the aircraft and ships. Germany was using the radar technology since 1940 but Japan was not able to use that successfully [1].

By 1940, the resonant cavity magnetron was invented by the physicist which was capable to generate radio pulses having high frequency and high power. Such cavity was operated with the help of very short wavelengths using LASERs. Microwave radars are also used these days in order to find out the atmospheric pollution.

After the WW2, the use of radar systems was extended and were used in numerous applications including navigation, guidance systems, meteorology and others. The major development after the post war period was TWT that eventually led to phased array radars. Signal processing capabilities were also enhanced with the passage of time.

FIGURE 1.1: Ship Radar of the United States Navy [2]

Monopulse, Pulse Doppler, Synthetic Aperture Radar and Phased Array radar are the four techniques that were used in post war radars. Phased array radar technology was matured after the WW2. An old radar system has been shown in the figure 1.1, which was the ship radar developed by US Navy.

Pulse Doppler radar are used to detect the moving target in the presence of clutter. Monopulse radar is the well-known tracking radar that can track the range and angles of target using the simultaneous lobing technique. Phased array radar consists of electronically scanned array that can help to steer the beam in any direction without mechanically moving the antennas. SAR is the type of radar that is used to create the 2D or 3D image of the terrain. In SAR successive pulses are obtained and high resolution image is then created from them.

1.3 Classification of RADAR

Depending upon the type of antenna used, transmitted pulse and function, radars are classified into the following different categories described below:

1.3.1 Functionality Based Classification

Radars can be classified on the basis of the functions they perform. Some of the categories defined on the basis of functionality are listed below:

1. **2D Radar**

   Such radars are used to detect the bearing angle and range of target of interest. Parabolic antennas are mostly used by such radar. In order to find out the third coordinate (elevation angle), 3D radars are required. After the WW2,2D search radars were used in collaboration with height-finding radar to detect the height of target. They are the oldest type of radar mostly used for detecting civil and military aircraft.

2. **3D Radar**

   In 3D radar system, the measurements of all three space coordinates are taken. These radars perform all the functions of a 2D radar, but they provide additional height information. They are also used for aerial surveillance and use Phased-array antenna. A special type of 3D radar is weather radar that is used to detect the weather conditions and uses pencil beam.

3. **Weather Radar**

   Such radar is used to locate the precipitation and also estimate its type like snow, rainfall or hail and is shown in figure 1.2. Pulse Doppler radars are mostly used as weather radars and have the ability to detect the motion of rain droplets and also their intensity.With the help of this type of radar, structure of storm and its strength can be analyzed.

Figure 1.2: Weather Radar [3]

4. **Synthetic Aperture Radar**

   SAR are used to get the high-resolution image of specific ground terrain as shown in figure 1.3. They are able to capture image irrespective of day or night and weather conditions.

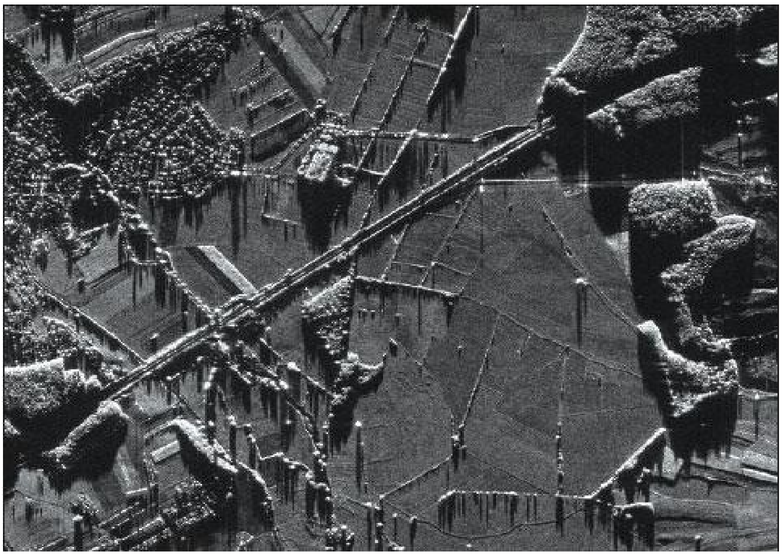

Figure 1.3: Synthetic Aperture Radar [4]

   SAR is mounted on the top of moving aircraft. If the antenna aperture is large high-resolution image will be obtained irrespective of the fact that aperture is physical or synthetic. In this way, SAR has the ability to produce high resolution image by using small physical antennas.

   In SAR, successive pulses are transmitted towards the ground terrain and echoes are received using the beam-forming antenna. As the aircraft moves, the location of antenna also changes with respect to target. By processing the multiple recorded beams obtained from different antenna positions good quality images are obtained.

5. **Early Warning Radar**

   Such radars can be either airborne or ground based. Their main function is to examine a large volume of space in which hostile attackers might be expected. These radars are mainly designed by using transmitters with high power and large antennas to detect the target at long ranges or the targets having small RCS. These radars allow the air defense system to be alerted earlier before the hostile attacker reaches the radar. In this way, defense system gets enough time to operate.

6. **Acquisition Radar**

   The function of acquisition radar is to search the region in space surrounded by the location of threat determined by the early warning radar. If the search volume contains more than one hostile aircrafts then the function of such radar is to determine that which aircraft may be a greatest threat to the defense system. These radars have proved to be less efficient if they are not coupled with other sensors.

7. **Terminal Guidance Radar**

   When the intended target is successfully located in the region of space identified by the early warning radar then the next step is to guide the interceptor. Terminal guidance can be done by using the autonomous active radar present on the interceptor.

1.3.2 Antenna Type Based Classification

Radars can also be classified based on the type of antenna used. Two main categories based upon the antenna type are described below:

1. **Mono-static Radar**

   In mono-static radar, transmitting and receiving antennas are placed closer to one another for detecting any object. The basic geometry of mono-static radar is shown in figure 1.4. In this type of radar gain of transmitting and receiving antenna is same $G = G_R = G_T$. The maximum range of a radar is the distance beyond which the target cannot be detected. Moreover, there are no synchronization issues with these radars.

2. **Bi-static Radar**

   This type of radar uses different antennas for transmission and reception as shown in the figure 1.4. Tight synchronization is required and their deployment is a difficult task.In this type of radar gain of transmitting and receiving antenna is different. This geometry is mainly used in case of weather radars and this technology is in use since several years. The system which consist of single transmitter and multiple receiver is knows as multi-static radar.

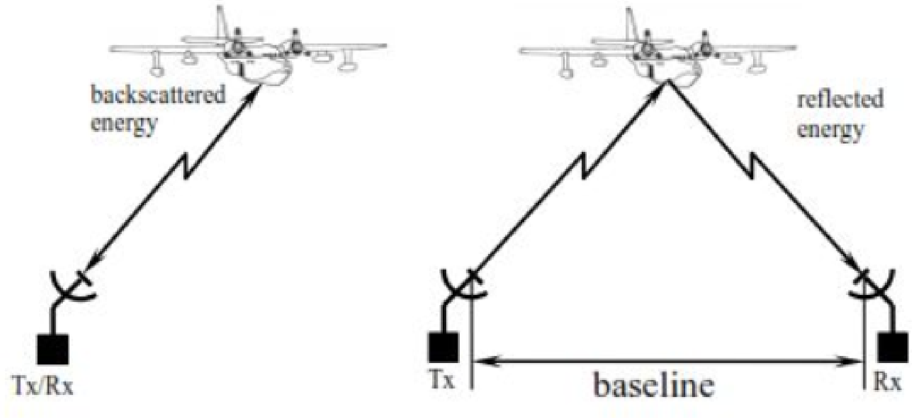

FIGURE 1.4: Monostatic and Bistatic Radar [5]

1.4 Basic Components of Radar System

Radars are mainly described in the terms of their modulation scheme. The most common and simplest type of radar used is known as Pulse radar. In such radar, pulses of short duration are generated and transmitted towards the target. The same antenna is used for transmission as well as reception by using a duplexer that helps in switching between the transmitting and the receiver circuits. With the help of pulse duration, the ability of radar is determined that how it will differentiate between two targets. If there is overlapping of skin returns obtained from two targets then radar will not able to differentiate between them.

The block diagram of traditional pulse radar is shown in the figure 1.5. The basic components of radar and their functions are described below:

1. **Transmitter**

   The transmitter may consist of a power amplifier like a Klystron, TWT or magnetron which is a power oscillator. Waveform generator is used to generate the signal which is then amplified by the power amplifier. High power RF pulses of short duration are being generated and radiated with the help of antenna in the space. A fan beam is transmitted by conventional 2D radar having narrow beamwidth in one dimension and wider in the other dimension.For the tracking radars, pencil beam is used instead of fan shaped beam.

2. **Duplexer**

   The function of duplexer is to switch the same antenna alternatively between the transmitter and the receiver which is required in the case of monostatic radar configuration. This switching is very important otherwise high-power pulses generated by the transmitter would destroy the receiver.

3. **Radar Antenna**

   The function of antenna is to radiate the energy in the space. Antennas mainly used in radar system include planar arrays, phased array that are steered electronically or parabolic reflectors. At the reception side, antenna receives the echo reflected from target and send it to receiver for further processing.

4. **Receiver**

   The RF signals are received by the receiver which are amplified and then demodulated. At the receiver side, signal processing is done to detect the signal and to recover useful information from it about the target [6].

5. **Signal Processor**

   Different signal processing algorithms are implemented on the echo pulses received at the receiver side. These algorithms include MTI, pulse compression, etc. In the radar system, the most difficult task is to implement the signal processing techniques. Some new algorithms are also used in the modern radars for detection of target [6].

6. **Display**

   Different type of displays are used in radar system that can present a continuous, comprehensible and graphical picture displaying the position of target.PPI plot is mostly used to display the target's position in polar form.

7. **Synchronizer**

   The main task of synchronizer is to create the timing pulses that may be used to start the transmitter for displaying sweep and ranging circuits.The receive pulses are plotted on the display screen based upon the time delay and in this way range of target can be determined [6].

8. **Support Systems in Radar**

   For the efficient working of radar system, some other systems are also in function that are discussed below:

   (a) Control system for transmit and receive antenna.
   (b) Power supply system.
   (c) Environment Control System (ECS)
   (d) For mobile radars transport system is also available.

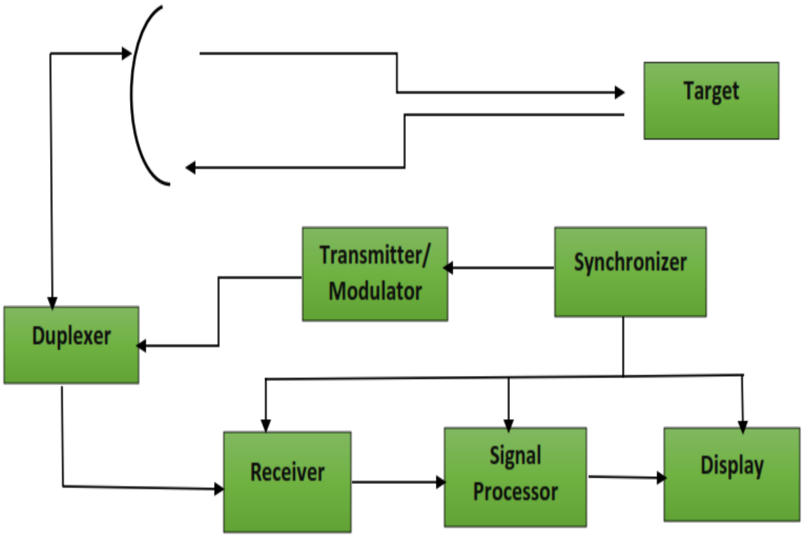

FIGURE 1.5: Basic Components of Pulse Radar

1.5 Tracking Radars

Tracking radar is used to measure the target coordinates and it provides information about the target which can help in predicting its future position.Such radar might track the target in range, doppler, angle or in any other combination. In the tracking radar, antenna beam is aligned with respect to the target to perform the tracking. Such radar is different from the search and TWS radar due to the fact that it ignores the volume of space which is not occupied by the target and its beam only follows the motion of target. In order to achieve the good resolution,tracking radar uses pencil beam having very narrow beam width in the order of 1°. The beam width varies depending upon the application but it should be same in both angular coordinates [7].

The tracking radar also tracks the range of desired target along with the angles (elevation and azimuth). It tries to keep the center of the range gate aligned with the target. During tracking, only the echoes or signals falling inside that gate are accepted and others are discarded.

For the angle tracking, if the target is deviated from the beam axis then an error signal is generated. The magnitude of error tells about the amount of deviation from the beam axis and polarity indicates the direction of deviation.Target is located and tracked with high efficiency as compared to search or TWS radar.With the help of tracking radars, the target is monitored continuously as compared to search radar where the the information about the target is not known between the successive scans.

The tracking is mostly done in a closed loop fashion using the signal processing techniques at receiver side. Single target tracker is used to track the single target in range and angles. Multitarget tracker has the ability of tracking multiple targets at the same time. TWS radar can perform scanning as well as tracking at the same time.

1.5.1 Types of Tracking Techniques

Tracking techniques that are mostly used in the radar system are described below:

1.5.1.1 Sequential Lobing

In this type of tracking technique, radar beam was used to switch rapidly between the two alternating positions instead of just pointing at the target of interest. The amplitude of echoes obtained from both positions were compared. If the target appears below or above the axis of two lobes the amplitudes of both echoes were not equal in magnitude [7].

1.5.1.2 Conical Scan

In the conical scan,beam of antenna is squinted at some angle with respect to the antenna axis.The beam is then rotated in a circular path around the axis.If the amplitude of signal returning from the target changes by changing the beam position it means that target is not aligned with the antenna axis.On the other hand,if target is aligned with the antenna axis then amplitude of returns by changing the beam position remain the same [7].

1.5.1.3 Monopulse

Monopulse radar can track the target in range and angles by comparing the beams received at four different receivers.Monopulse Radar has been divided into two major categories:Amplitude and Phase comparison Monopulse.In Monopulse radar,simultaneous lobing technique is used. Error signal is generated if target is not aligned with respect to the antenna axis.That error is then sent to servo to align the antenna axis with respect to the target.

1.5.2 Data Association

In data association, uncertainty is observed when different devices like SONAR and RADAR obtain the detections with origin uncertainty. When the skin return obtained from target is very weak then it becomes a difficult job to detect the target. So, the threshold value is lowered and as a result of this, false alarms and clutter might be detected.If these fake detections are used to update the track of target, it will lead to the divergence of the track.

For this purpose, the first step is to perform gating in which the validated measurements falling inside the gates of several targets are observed. After obtaining validated matrix, data association algorithms are implemented to find out the association probabilities of all the validated detections.With the help of marginal association probabilities obtained tracks are updated.

Gating is done to limit the number of measurements that are being processed by the data association algorithm. Measurement noise and process noise both are assumed to be Gaussian distributed. A chi-square test is performed to check whether the specific measurement falls inside the volume of probability distribution of the predicted target position. The volume of the hyper-ellipsoid is considered as a gate. Gating is actually a method to prune the measurements in order to reduce the computations and to remove the spurious measurements. Mahalanobis distance is used to compute the distance between the target predicted position (gate center) and the detections obtained.The gating threshold is defined by using Chi square distribution. If the distance is less than threshold, then it is assumed that the detection falls inside the validation gate.

PDAF is a Bayesian data association technique used for single target tracking in which track is updated based upon the weighted probabilities of the obtained detections. JPDAF is a Multitarget version of PDAF which can even work well when multiple detection may fall inside the overlapping region of multiple targets.

1.5.3 Filtering Techniques

Radar itself is not capable to estimate the targets true states so it works in the collaboration with KF to accurately estimate the states of target. Kalman Filter(KF) is used due to its simple implementation and efficient performance. Constant Velocity (CV) model of KF has been used.KF is a recursive technique so that with the help of obtained measurements it tries to minimize the estimation error to improve the values of estimated parameters. It is considered to be an eminent tool for the state estimation in dynamic systems.

A KF is known as the best estimator that can estimate the desired parameters by using the inaccurate and noisy measurements [8]. KF involves two basic steps: Prediction and Correction as shown in figure 1.6.

The performance of KF depends upon the prior information about the measurement noise covariance matrix R and process noise covariance matrix Q.

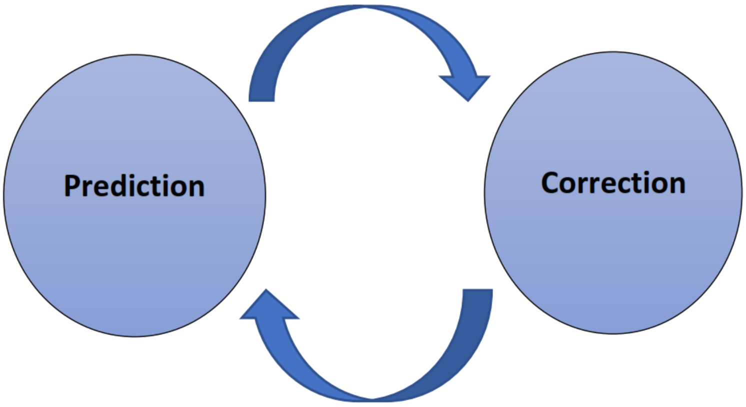

FIGURE 1.6: Kalman Filter Recursive Process

If the information about these covariance matrices in not adequate, this will result in the divergence of filter. If very small values are chosen for Q and R, unexpected results are observed. Contrary to this, if very large values are taken this will lead to filter divergence. So, the optimal values should be chosen for the Q and R. The basic function of filter is to estimate the states of any dynamical system by reducing the errors due to noise.

KF equations have been divided as: Time update and measurement update equations. System states and error covariance is projected in the forward direction with the help of time update equations [8]. Measurement update equations are used to provide feedback and try to minimize the error in the current iteration.

1.6 Thesis Organization

The thesis comprises of the following chapters:

Chapter 1: This chapter consist of introduction and working of radar system along with the functions of basic components of the radar. Classification of radar system on the basis of different parameters is also explained. Importance of tracking radars, data association and filtering is also a part of this chapter.

Chapter 2: This chapter includes the literature survey of different data association techniques along with their pros and cons. The concept of single and multiple targets in the presence of clutter is explained. Different types of filtering techniques like KF, EKF and UKF are explained along with their pros and cons. By the end of the chapter, motivation, problem statement, gap analysis and research methodology are also given.

Chapter 3: Chapter three involves the modeling of target dynamics by using the Constant Velocity (CV) model which is used in the JPDAF-KF. In this chapter, other dynamic models used for target like CA and CT have also been discussed. By the end of the chapter, the derivation for dynamic measurement noise covariance matrix R is given which is then used further in the simulations of MTT using JPDAF-KF.

Chapter 4: In this chapter,function of TWS radar using JPDAF-KF is explained.This chapter also covers the knowledge about the different types of scan patterns used in the radar system. The components used in the TWS radar are explained in detail along with their functions. By the end of the chapter, Joint Probabilistic Data Association Filter (JPDAF) is discussed in detail.

Chapter 5: This chapter explains in detail the basic concept and working of Monopulse radar. Different angle tracking techniques like simultaneous lobing and conical scan are explained in detail along with their drawbacks. This chapter also covers the knowledge about the two types of Monopulse radar:AC and PC. The concept of range tracking is also explained.This chapter is summarized with the concept of closed loop monopluse tracking radar using JPDAF-KF.

Chapter 6: In this chapter the simulation results for 2D tracking using Scan radar and JPDAF-KF are discussed in detail. The simulation results for 3D tracking using monopulse and JPDAF-KF are also discussed in detail. This chapter also involves the information about the important parameters of radar system used in the simulations. Simulations are performed in MATLAB ®.

Chapter 7: This chapter finally concludes the discussion done throughout the whole thesis and future direction is recommended for research work.

1.7 Chapter Summary

This chapter involves the basic concept of radar system and the history of radar. Classification of radar system based upon their function and antenna type used is also discussed. Importance of tracking radars is explained alongwith the concept of data association and filtering. By the end of the chapter, overview of the whole thesis is given.

Chapter 2

Literature Survey

2.1 Introduction

In the case of MTT, trajectories of multiple targets are being estimated with the help of noisy measurements acquired from the sensors. Algorithms used for MTT are capable of estimating the velocities and positions of moving targets. With the advancement in sensor technology, extensive number of sensors are accessible that can be utilized to observe a specific region.These sensors might include SONAR, RADAR, GPS etc. Many challenges are witnessed when the detections obtained from these sensors are affected by noise which might create problems in accurate estimation of trajectory of specific target [9] . To improve the estimation accuracy, the prior knowledge about the dynamics of target can be used .

Applications of MTT can be observed in numerous fields including surveillance, robotics, computer vision, pattern recognition, radar tracking, defense and many other research fields [10, 11]. Tracks of different targets are being created using MTT techniques and confirmed later by using track scores. Once the tracks are confirmed, velocities and positions of targets are determined from these tracks. MTT has been resolved into three major steps: Track initiation, deletion of track and maintaining the specific tracks if certain observations are associated to that track repeatedly as shown in Fig. 2.1.

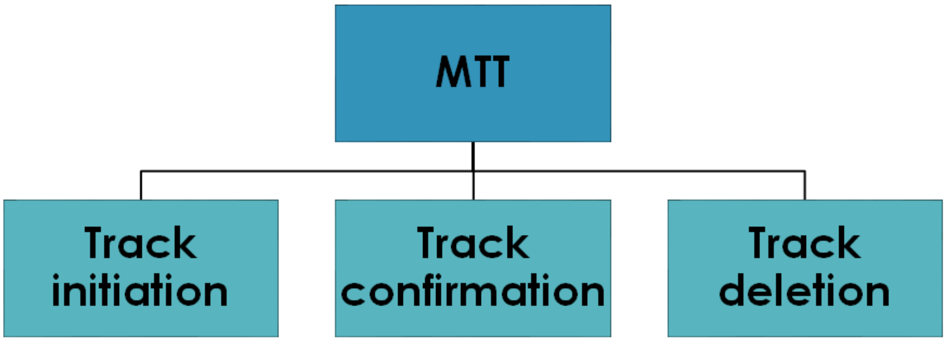

FIGURE 2.1: Steps Involved in MTT

Data association is another significant technique associated with MTT. Multiple detections are obtained from the scan radar, data association algorithm is then used to associate detections to their specific tracks. Uncertainty in data associations is observed when remote sensing devices obtain the detection having the origin uncertainty. The uncertainty occurs when echo reflected back from the target of interest is very weak and its detection becomes difficult.As a result, the threshold value is lowered in order to detect the true target resulting in increased False Alarms. If these false detections are used, they may cause the filter to diverge thus leaving the actual track of target.

The important step is to isolate the true detection among the spurious measurements, which is then used to update the states of filter that might be KF, EKF, UKF etc.The problem of origin uncertainty can be solved using different data association algorithms that have been discussed in literature. MTT accompanied by sensing and data fusion techniques has emerged as a renowned field since last decade. In MTT applications, the echo reflected back from target is obtained during the time interval being determined from the target range. The range gate is created and only the detections falling inside that specific gate are associated to that target of interest.With the help of those measurements range, elevation and azimuth information of the target of interest can be achieved.

While tracking a single target in the presence of clutter,there may be multiple detections appearing inside the validation region. The validated measurements include true detection as well as false alarms and clutter.

2.1.1 Tracking of Single Target in Clutter and FA

When only single target of interest is present then the validation gate is setup and validated measurements may include clutter and false alarms.

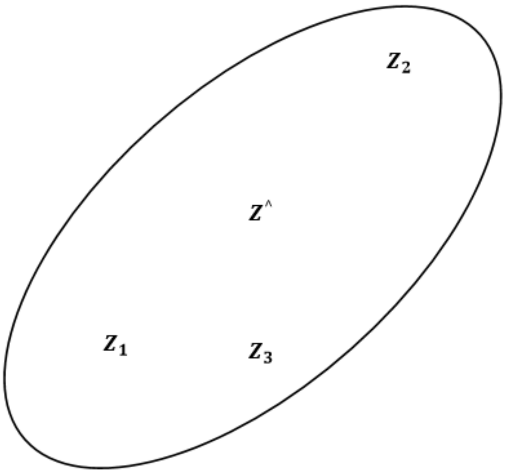

FIGURE 2.2: Single Target Tracking in Presence of Clutter and FA

In the figure 2.2, the validation gate appears to be an ellipse having center at the position $\hat{Z}$ predicted by KF.Z_1 ,Z_2 and Z_3 are the detections falling inside validation gate of specific target. With the help of residual error covariance matrix S the parameters of gate can be determined.Different association events can be generated from these validated measurements like Z_1 is originated from true target while other two measurements are assumed to be spurious or Z_3 is assumed to be originated from true target and others are considered as fake detection.

The association events are mutually exclusive so by using total probability theorem the association probability of detection for specific track is computed.The purpose of setting multidimensional gate is to reduce the search region and only the measurement falling inside the gate are considered while the remaining are assumed to be spurious measurements.Such a gate is known as **Validation gate**. The model for false detection is assumed to have uniform distributed across the entire validation region.

The validation region guarantees that target measurement falls inside that gate with the greater probability.In case of single target tracking if multiple detections appear inside the validation gate then the problem of origin unceratinity dominates.The detections observed outside the validation gate are ignored and assumed that they are not originated from target of interest and are far from the predicted target position.

2.1.2 Multitarget Tracking in the Presence of Clutter and FA

When more than one targets appear in the validation region along with the clutter and false alarm then data association probability becomes more difficult.

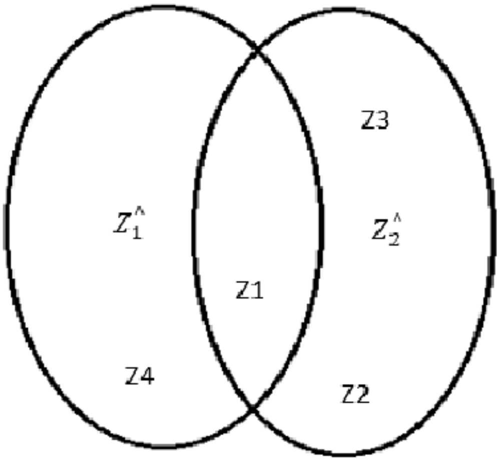

Figure 2.3: MTT in the Presence of Clutter and FA

In figure 2.3,two validation gates have been shown both centered at the predicted postions for target1 $\hat{Z}_1$ and target2 $\hat{Z}_2$ respectively.Z_4 lies inside the gate of T1, Z_1 lies in the overlapping region of both targets, Z_2 and Z_3 may belong to T2 or they may be false alarms.In this case different association events can be generated like Z_4 belongs to target1, Z_1 belong to target1 or target2 and others are assumed to be false alarms or belong to target2.In the case of overlapping gates, independent events not work well so joint associations events should be considered. In the case of Multitarget,there is also the interference from neighboring targets in addition to clutter and false alarms [12, 13] .

In the literature, numerous algorithms have been discussed related to data association and MTT. Bayesian and non-Bayesian approaches are being used for data association. Bayesian approach assumes that the observations belong to some specific probability distribution function. Non-Bayesian approach attempts to compute the likelihood of the specific distribution with the help of obtained data set.

2.2 Literature Survey of Filtering Techniques

2.2.1 Kalman Filter (KF)

Kalman published his paper in 1960 based upon the recursive method for the filtering process using the discrete data. Due to the advancement in digital communication from last few decades, KF has captured the attention of people and is widely used in many applications including the navigation and tracking. It involves mathematical recursive equations that are used for the estimation of states of any dynamical system. In the KF, mean squared error is tried to minimize. The filter is very attractive due to its certain properties like it can support the estimations of future, present as well as past.

KF works in a feedback fashion to estimate the states of a specific process. The estimation of states is done at some specific time and the feedback is attained in the form of measurements that are noisy. The KF equations are divided into two main categories: time and measurement update equations. The equations expressed for time update are used to project forward the states and error covariance of states in time [8].The equations used for updating the measurements help in providing the feedback in order to improve the estimations. The equations used for updating in time can also be called as predictor and those used for updating measurements can be expressed as corrector equations.

KF has been widely used in literature for the estimation purposes in different applications. In case of maneuvering targets,the KF is used for the estimation of position, velocity and also the acceleration in in terms of cartesian coordinate system with the help of noisy detections obtained from the radar.Detections obtained from the radar are in polar coordinate system (azimuth, elevation and range).In case of MTT, multiple detections are obtained from the radar containing false alarms as well. These detections need to be associated to the tracks already existing for targets.

KF aided by neural network enhances the adaptive abilities of KF discussed in [14].Tracking of maneuvering targets is being done using the acceleration model of KF. Neural network helps in removing the inaccuracies observed in tracking.KF is also used in the navigation applications using GPS discusses in [15].

2.2.1.1 Drawbacks of KF

The KF is used as optimal estimator to estimate the states of dynamical system but its drawback is that it is only used for the linear systems. Mostly the systems around us in the real world are nonlinear. For non-linear systems KF cannot be used directly instead non-linear filters are used.

2.2.2 Extended Kalman Filter (EKF)

KF is generally used for the estimation of states of a discrete-time system that is being expressed in terms of linear equations. In most of the applications the system encountered are non-linear in nature so the KF cannot be used directly. The EKF is then used which is linearized about the covariance and the mean. Such KF is referred as EKF. Some of the systems are close to linear so the linear approximations can be made for them but in other cases, linear approximations cannot be made as those system don′t behave linear even at the small ranges. Taylor series is used to deal with such non-linearities.

In Taylor series the approximation is done with the help of 1st order Taylor expansion that is being evaluated at the mean estimate. The EKF is widely used for the systems having state space model showing non-linearities due to its robustness property and is suitable for real time applications [16]. It is also used in the navigation applications using GPS discussed in [15]. Measurements obtained from the GPS receivers are sent to the software that are used for determining the states of the aircraft.

2.2.2.1 Drawbacks of EKF

In case of non-linear systems, non-linear filters are used and it is assumed to be a complex process. Estimation methods used for non-linear systems include Unscented Kalman Filter (UKF), EKF, particle filter etc. During the linearization process, some errors might occur that results in the degraded performance of EKF and the accuracy upto first order is achieved only .Due to this issue, robust EKF may be used discussed in [17].

2.2.3 Unscented Kalman Filter (UKF)

UKF is also a type of non-linear filter used for non-linear models. In EKF, the non-linear model is linearized by using the Taylor series. On the other hand, UKF tries to approximates the Gaussian distribution function with the help of the actual and available non-linear model without using Taylor series. In this type of filter sample points are carefully taken and the computation of Jacobian is avoided. It is assumed that the prior value of mean and covariance is accurate upto the second order is considered to be a good substitute for the estimation of states and parameters instead of EKF.Performance analysis of different types of UKF has been done and expressed in [18].More accuracy is achieved by using UKF instead of EKF at the same level of complexity.

2.2.3.1 Drawbacks of UKF

The UKF is a non-linear filter so it requires more computations and involves complexity.

Other types of KF, like Kalman-Bucy Filter is there which is used for continuous time system.Hybrid Kalman filter are also used in many application.

2.3 Data Association

Data association problem has also been divided into the following main steps [13]:

1. **Prediction**

   Filter predicts the states of target in time for next iteration in the presence of process noise that is assumed to be Gaussian. Such predicted position appears at the center of validation gate that is assumed to be elliptical.

2. **Gating**

   Gating is done to find out the validated measurements falling inside the gates of targets of interest.Spurious measurements appearing outside the validation region are ignored.

3. **Data association**

   Data association algorithm is used to find out joint association probabilities in case of Multitarget. Such probabilities are then fed to filter to update the estimates of tracks.

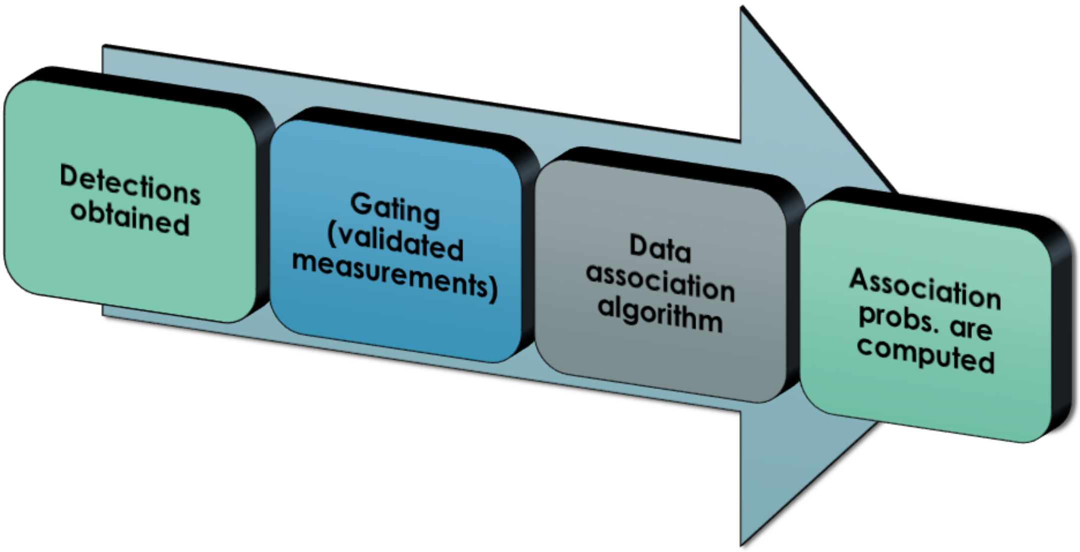

FIGURE 2.4: Steps Involved in Data Association

If these fake detections are used to update the track of target, it will lead to the divergence of the track. For this purpose, the first step is to perform gating in which the validated measurements falling inside the gates of several targets are observed. After obtaining validated matrix,data association algorithms are implemented to find out the association probabilities of all the detections for all the existing tracks.

2.4 Literature Survey of DA Algorithms

In the literature, numerous algorithms have been discussed related to data association and MTT [19–21] . Bayesian and non-Bayesian approaches are being used for data association. Bayesian approach assumes that the observations belong to some specific probability distribution function. Non-Bayesian approach attempts to compute the likelihood of the specific distribution with the help of obtained data set. Bayesian approaches involve PDAF, JPDAF , MHT ,MCMC-DA and particle filters.

Data association algorithms can also be categorized by the way in which they process measurements:

1. Single-scan algorithms estimate the current states of targets based on their previously computed tracks and the current scan of measurements.
2. Multi-scan algorithms may revisit past scans when processing each new scan, and can thereby revise previous association decisions in the light of new evidence.

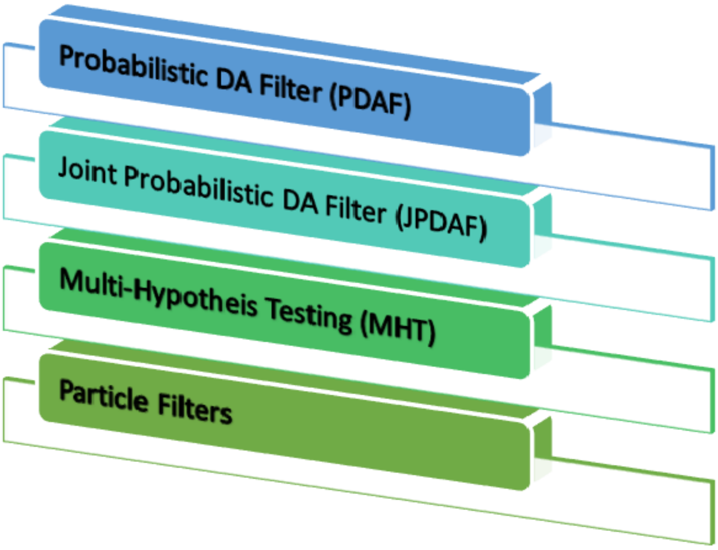

FIGURE 2.5: Bayesian Data Association Approaches

Non-Bayesian approaches include NNSF,GNN and Interception Tree. GNN and NNSF techniques do not work well in the presence of clutter or FAs so, Bayesian approaches are widely used. Neural networks including RNN and CNN have also proved very well for MTT.

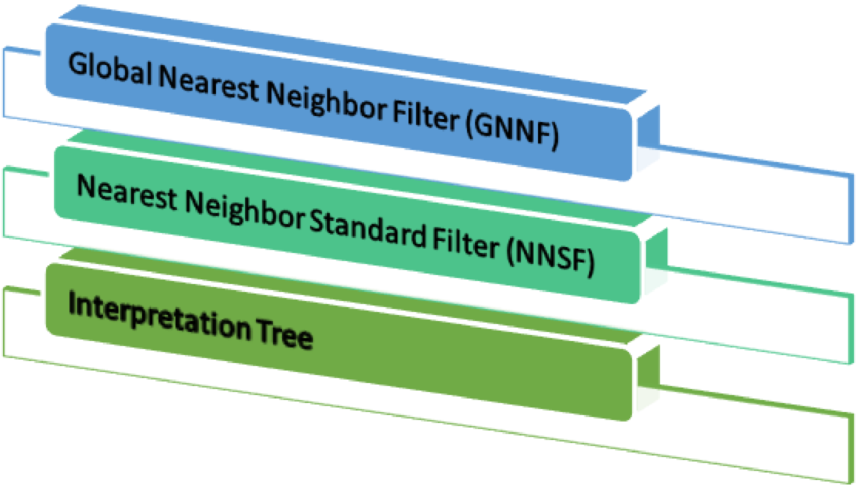

Figure 2.6: Non-Bayesian Data Association Approaches

2.4.1 Nearest Neighbor Standard Filter (NNSF)

In the NNSF, the measurement having the lowest cost is being assigned to each track. Mahalanobis distance is computed from the predicted position and the obtained measurements. Closest measurement is accepted and assumed that it belongs to actual track and other measurements are discarded [22]. In this way, NNSF is assumed that it takes a hard decision [23]. This measurement is then utilized to update the existing track of desired target.In some cases,one measurement can be used to update more than one tracks. In the case of accurate prediction and measurement model this technique performs well.

This data association algorithm is not much complicated and easy to implement as it just involves computing mahalanobis distance as compared to other techniques. Such technique does not work well in the presence of clutter and high false alarm density. If the chosen measurement is a false alarm or clutter it will then ultimately lead to filter divergence.

2.4.2 Global Nearest Neighbor Filter (GNNF)

GNN finds the best (most likely) assignment of input observations to the existing tracks. In this technique, measurement are assigned to the track such that the total cost should be minimized. The term global is used to refer to the fact that the assignment is made considering all possible (within gates) associations under the constraint that an observation can be associated with at most one track. This distinguishes GNN from the nearest neighbor (NN) approach in which a track is updated with the closest observation even if that observation may also be used by another track.

In GNN, measurements are obtained from the scan radar. In the beginning clusters are being created for each track. If the same measurement falls within the gate of two tracks, then those clusters are merged to form a super cluster. Assignment problem is done by using the Munkres algorithm. With the help of Munkres solution the associated measurements computed for each track are then fed to filter to update the states of specific track [20]. If any specific measurement is not associated to any track then a new track is being initiated. GNN works well when the targets are well seperated.

The GNN approach, which only considers the single most likely hypothesis for track update and new track initiation, only works well in the case of widely spaced targets, accurate measurements, and few false alarms in the track gates.

2.4.3 Probabilistic Data Association Filter (PDAF)

PDAF is also known as all neighbor data association filter. It uses Bayesian approach to find out the association probabilities of measurements related to tracks and is single target tracking algorithm. In PDAF, weights are being allocated to all measurements based upon their locations from the predicted position of the track contrary to GNN and NNSF where only single closest measurement is chosen. Track for specific target is updated by using the combined innovations of all the measurements [12]. PDAF does not work well in the presence of high clutter and false alarms .PDAF takes a soft decision where all the measurements falling inside gate of a specific target are considered [23].

When the multiple targets exist closer to each other then PDAF cannot estimate well which is the major drawback of PDAF.

2.4.4 Joint Probabilistic Data Association Filter (JPDAF)

JPDAF is used as a Multitarget extension of PDAF.JPDAF and MHT are widely used algorithms for MTT. JPDAF gives satisfactory results for high clutter and for crossing targets as well.In JPDAF joint association events are created with the help of validated measurements obtained from gating. Joint association probabilities are calculated for all validated measurements falling inside gates of all tracks [24]. These joint association probabilities are then utilized in updating the existing tracks of targets. In JPDAF,actual number of targets in the presence of false detections are already known and it is assumed that a single measurement is generated by each target.

The states in JPDAF are updated in the same way as PDAF.Combined innovation is being computed by using the sum of individual innovation that are being weighted by their marginal association probabilities. This combined innovation is then used to compute the updated state error covariance matrix.

PDAF and JPDAF both make the assumption that the tracks are uninitialized. By using two-point track initialization technique tracks can be initialized.

2.4.5 Multi-Hypothesis Tracking (MHT)

MHT is referred as multi-scan data association algorithm that maintains multiple hypotheses associating past measurements with targets. When a new set of measurements arrives, a new set of hypotheses is formed from each previous hypothesis. As a solution, this algorithm returns a hypothesis with the highest posterior. In MHT, the decision about removing an existing track or to create a new track is delayed until enough measurements are obtained. Due to this reason,MHT is categorized as a “deferred logic” method. MHT is capable of initiating and terminating a varying number of tracks and is suitable for autonomous surveillance applications [25].

As new measurements are received, they are gated with existing tracks. If the gates of tracks within more than one cluster enclose same measurements, the clusters are to be merged. Also new clusters are formed for the measurements that do not fall within the gates of tracks in existing clusters. When all the measurements have been processed, clusters may be split if it is possible. Then the formation of tracks and hypotheses begins for each cluster. Combining and pruning methods provide to limit the number of hypotheses. Finally, after all tracks are filtered for the prediction of the next scan, empty clusters are deleted and this cycle repeats itself.

When the number of detections increases the number of assignment hypotheses also increases proportionally. As a result, the implementation of such algorithm becomes quite complex and expensive. In order to solve this problem, hypothesis may be pruned or other gating techniques should be performed .

2.4.6 Neural Networks Data Association (NNDA)

The neural networks can be used to solve the data association problem in an efficient manner by defining energy function and minimizing it. The scoring function that needs optimization appears as sum of probabilities of measurement to track associations. The track files are computed and maintained by the KF [26]. Computer simulations are being performed and it is observed that neural networks have the ability to quickly converge to the hypothesis having the maximum score.

Different constraints are being considered for measurement to track associations like any measurement should not be associated to more than one track files. Other constraints may be that if no measurement is assigned to any track file such track may be deleted after some iterations and if any measurement is not associated to any track it may be assumed as a false alarm or clutter. When the number of targets increases such algorithm becomes complex.For the neural networks, we require large amount of data during training process.

2.5 Motivation

One of the core applications for radar in missile defense is target tracking.The ability of a radar to track many objects through the sky while associating their trajectories correctly is of grave importance to many radar missions.Tracking is done using any optimal estimator as the radar alone is not able to track the actual trajectories of the targets. Proper data association algorithm should be chosen for MTT.In the case of crossing targets JPDAF has ability to perform well and satisfactory results are achieved.

2.6 Gap Analysis

In the literature, mostly measurement noise covariance matrix R is not chosen appropriately as a result useful detections might fall outside the gates of existing tracks that may lead to inaccurate data association in JPDAF.

2.7 Problem Statement

To determine the dynamic measurement noise covariance matrix R using standard radar parameters for JPDAF.

2.8 Research Methodology

Research methodology involved the following steps:

1. Measurement noise covariance matrix R has been derived by using standard radar parameters.
2. MTT simulations have been performed in MATLAB® using data association algorithm JPDAF-KF.
3. In the JPDAF-KF the computed measurement noise covariance matrix R has been used and desired simulation results have been obtained from them.
4. 2D and 3D MTT tracking has been performed using JPDAF algorithm.

2.9 Chapter Summary

In this chapter, the concept of single and multiple targets in the presence of clutter has been discussed.Literature Survey of different filtering techniques as well as data association algorithms has been carried out and discussed in this chapter alongwith their pros and cons. It also involves the motivation, gap analysis, problem statement and research methodology.

Chapter 3

Modeling of Dynamic Measurement Noise Covariance Matrix R for JPDAF

3.1 Background

The measurements acquired by the tracking filters are not very accurate, filtration is required to remove the error and to improve the estimates. The obtained measurements are not true due to the presence of noise in the estimated values. Noise is of two types;the first one is the **measurement noise** which occurs due to inaccurate tracking device.Other one is **process noise** which is observed due to human error or other environmental issues. Noise may occur due to different factors like sudden change in target motion may occur due to atmospheric turbulence. Sometimes the measurements are not taken at the regular time intervals that may also lead to noisy measurements.

Radar itself is not capable of detecting the true position of target and cannot determine the velocity of target directly. To overcome these problems,linear estimators are being used that can predict the position and velocity of target in a more accurate manner. KF is preferred as it is simple to implement and shows the good performance.

KF follows a recursive method to filter out the variations in the noisy measurements to predict the true velocity and position of moving or maneuvering target. This recursive process of filtering is used in many applications of computer science. The recursive process is used to minimize the error in the measured values and to decrease the state error covariance as well [27].

KF is widely used in modern tracking systems due to its efficiency and simple implementation.This filter does not require the past data so less storage requirement is there and multiple targets can be tracked simultaneously. The estimation error goes on decreasing in the successive iterations which is the beauty of KF. The recursive process involves two main steps; prediction and updating the estimates based upon the error to improve the noisy estimates.

3.2 Modeling of Target Dynamics

The effective tracking of target can be done by extracting valuable information about the states of target from the observations. This information can be extracted in a better way if a good model related to target dynamics is available. From the past few decades,different mathematical models related to target dynamics have been developed. Very few people have the insight about these dynamical models.

The uncertainty in the target tracking occurs due to the lack of proper dynamical model of target that is being required by the tracker for appropriate tracking.The availability of proper model for target is enough but sometimes the tracker does not have the knowledge about the control input acting upon the system or the properties of noise are also unknown. So, for the maneuvering targets the most important step is modeling the proper motion of target.

Tracking techniques for the maneuvering targets are mostly based upon the knowledge of models.The assumption is made that motion of target can be expressed in terms of mathematical models.

Different dynamical models have been discussed in literature [28, 29]. These models are mostly in the form of state-space and are distinguised as:

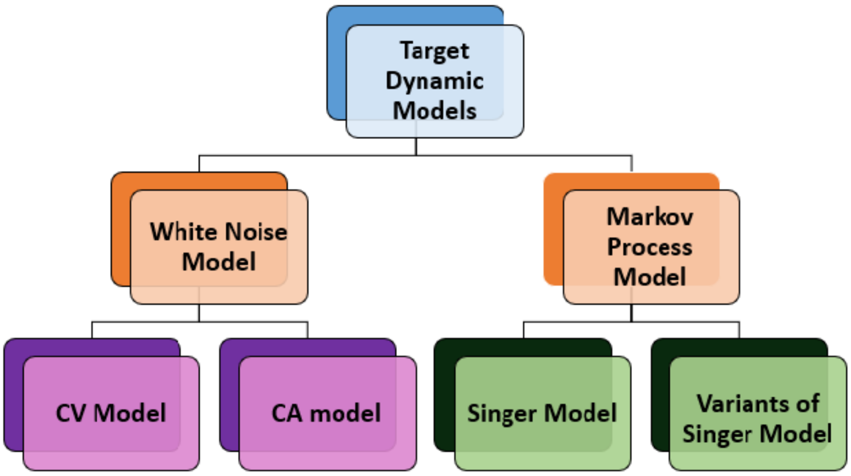

FIGURE 3.1: Target Dynamic Models

3.2.1 White Noise Model

The CA and CV models are included in this group and here white noise is being used to model the acceleration.

3.2.1.1 Constant Velocity (CV) Model

In the simulations of MTT, the CV model is taken for the two targets that are non-maneuvering and moving with constant velocities.The state-space model can be represented with the help of following equations:

$$X_{m+1} = AX_m + q_m \tag{3.1}$$

$$Y_m = HX_m + r_m \tag{3.2}$$

In the eq 3.1, X_{m+1} is the predicted state vector for iteration m+1 and X_m is a vector showing the current states of target which include positions and velocities along the x and y dimensions. In 2D, state vector of target can be represented by X having the dimensions of 4X1 and is represented as:

$$X = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{3.3}$$

The matrix A in eq 3.4 is the transformation matrix that is used to update the states of target. In 2D,dimensions of matrix A are 4X4. The update time is represented by ΔT which shows that after this time the target is being updated.

$$A = \begin{pmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.4}$$

The matrix Y_m represents the current measurements having the dimensions 2X1 and is represented as:

$$Y_m = \begin{pmatrix} x \\ y \end{pmatrix} \tag{3.5}$$

H is a matrix used for transformation to convert 4X1 state vector into the 2X1 vector to measure the position x and y in case of 2D.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{3.6}$$

In eq 3.1 and 3.2, q_m and r_m are process and measurement noises having the dimensions 4X1 and 2X1 respectively. Both noises q and r are assumed to be Gaussian having zero means and covariances that are represented by Q and R respectively.

$$p(q) \sim N(0, Q)$$

$$p(r) \sim N(0, R)$$

The performance of KF depends upon the prior information about the measurement noise covariance matrix R and process noise covariance matrix Q. If the information about these covariance matrices in not adequate, this will result in the divergence of filter [4-5].

If very small values are chosen for Q, unexpected results are observed. Contrary to this, if very large values are taken this will lead to filter divergence. So, the optimal values should be chosen for the Q and R. The basic function of filter is to estimate the states of any dynamical system by reducing the errors due to noise.

Time Update equations:

Time Update equations in KF are given as:

$$X_{m+1} = AX_m \tag{3.7}$$

$$P_{m+1} = AP_mA^T + Q \tag{3.8}$$

X_{m+1} is the predicted state vector and P_{m+1} is the predicted state error covariance matrix at the start of iteration $m+1$.The matrix P_{m+1} represents the error covariance in position (x,y) and in the velocities ($\dot{x}$,$\dot{y}$).The values of these error covariances go on decreasing with the course of time.

The Kalman gain can be computed using the following equation:

$$K_{m+1} = P_{m+1}H^T(HP_{m+1}H^T + R)^{-1} \tag{3.9}$$

The Kalman gain in eq 3.9 appears to be the gain for the residual term.This gain is then used to update the estimates in order to reduce the estimation error.

Measurement Update equations:

Measurement Update equations are given as follows:

$$\hat{x}_{m+1} = X_{m+1} + K_{m+1}(z_{m+1} - HX_{m+1}) \tag{3.10}$$

$$\hat{P}_{m+1} = (1 - K_{m+1}H)P_{m+1} \tag{3.11}$$

In eq 3.10, estimate of states are updated using the Kalman gain and it appears as a weighting parameter for the measurement residual $(z_{m+1} - Hx_{m+1})$. Residual is defined as the difference between the actual radar measurement and the measurement predicted by KF.States are updated to reduce the estimation error for appropriate tracking.

In the eq 3.11, state error covariance is updated based upon the computed Kalman gain and the predicted state error covariance. The updated state error covariance is always less than the predicted error covariance which depicts that with the passage of time the estimates get more accurate.

Process noise covariance matrix Q depends upon the update time and is given as:

$$Q = \begin{pmatrix} \frac{\Delta T^4}{4} & \frac{\Delta T^3}{2} & 0 & 0 \\ \frac{\Delta T^3}{2} & \Delta T^2 & 0 & 0 \\ 0 & 0 & \frac{\Delta T^4}{4} & \frac{\Delta T^3}{2} \\ 0 & 0 & \frac{\Delta T^3}{2} & \Delta T^2 \end{pmatrix} \tag{3.12}$$

The measurement noise covariance matrix R is given as:

$$R = \begin{pmatrix} \sigma_x^2 & \sigma_x\sigma_y \\ \sigma_y\sigma_x & \sigma_y^2 \end{pmatrix} \tag{3.13}$$

Note that the control input u=0 in the non-maneuvering models, although the actual thrust of the target has to be present to maintain the motion. Also, inclusion of any unnecessary component (e.g., acceleration) in the state vector would lead to a tracking performance deterioration

Optimal values of R and Q should be computed for the good and optimal performance of KF.Transformation has been done in order to find out the covariance matrix R in Cartesian coordinate system as the KF operates in this system. The equations involved in KF are shown in figure 3.2.

3.2.1.2 Constant Acceleration (CA) Model

For 2D CA model,the states taken are position,velocity and acceleration along the x and y axis.The state vector X in such case have the dimensions 6X1 and is given

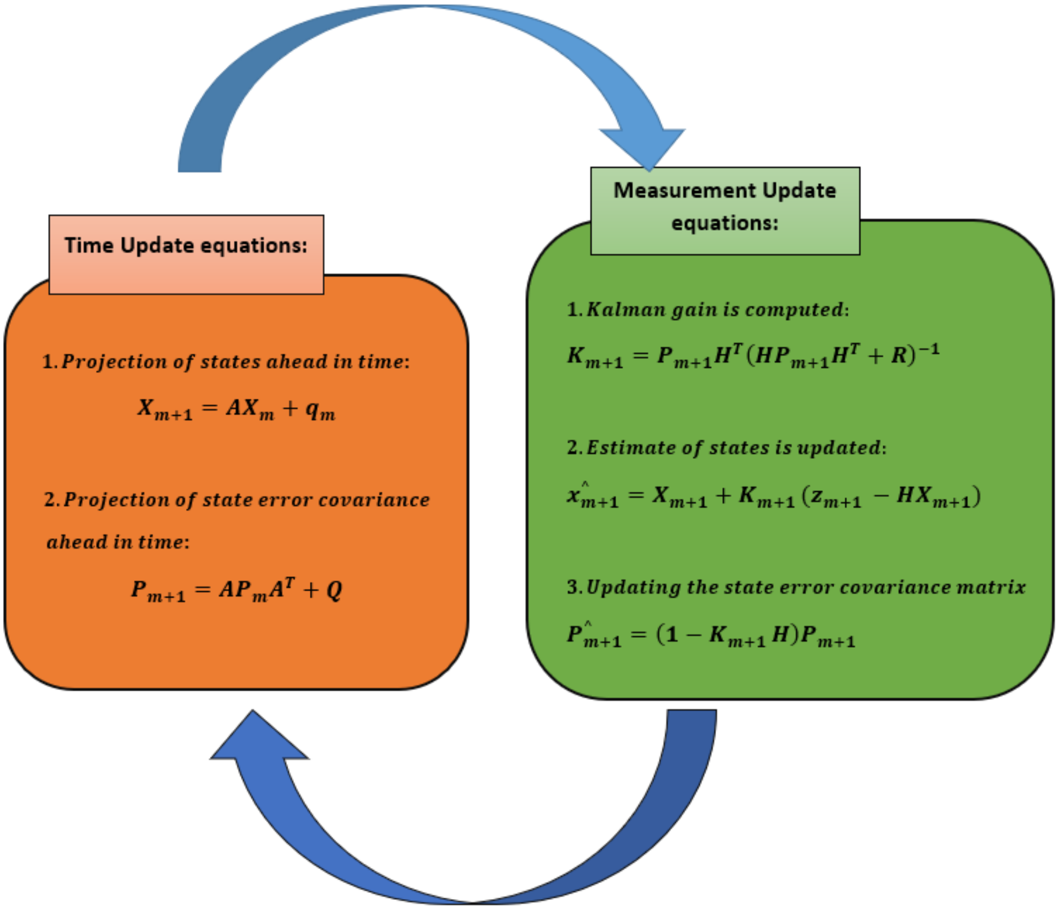

FIGURE 3.2: Kalman Filtering

as:

$$X = \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \end{pmatrix} \tag{3.14}$$

The state transition matrix used to update the states in time has the dimensions of 6X6 and is represented by the following matrix:

$$A = \begin{pmatrix} 1 & \Delta T & \frac{\Delta T^2}{2} & 0 & 0 & 0 \\ 0 & 1 & \Delta T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta T & \frac{\Delta T^2}{2} \\ 0 & 0 & 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.15}$$

The state space model remains the same given in eq 3.1 and eq 3.2. The output equation remains the same as expressed in eq 3.2. H is a matrix used for transformation to convert 6X1 state vector into the 2X1 vector to measure the position x and y in case of 2D. The output matrix Y_m represents the current measurements having the dimensions 2X1 and is represented as:

$$Y_m = \begin{pmatrix} x \\ y \end{pmatrix}$$

H is a matrix used for transformation to convert 6X1 state vector into the 2X1 vector to measure the position x and y in case of 2D.

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{3.16}$$

The process noise covariance matrix Q have now dimensions of 6X6 and is represented as follows:

$$Q = \begin{pmatrix} \frac{\Delta T^5}{20} & \frac{\Delta T^4}{8} & \frac{\Delta T^3}{6} & 0 & 0 & 0 \\ \frac{\Delta T^4}{8} & \frac{\Delta T^3}{3} & \frac{\Delta T^2}{2} & 0 & 0 & 0 \\ \frac{\Delta T^3}{6} & \frac{\Delta T^2}{2} & \Delta T & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta T^5}{20} & \frac{\Delta T^4}{8} & \frac{\Delta T^3}{6} \\ 0 & 0 & 0 & \frac{\Delta T^4}{8} & \frac{\Delta T^3}{3} & \frac{\Delta T^2}{2} \\ 0 & 0 & 0 & \frac{\Delta T^3}{6} & \frac{\Delta T^2}{2} & \Delta T \end{pmatrix} \tag{3.17}$$

The measurement noise covarinace matrix R remains the same as discussed in CV model.

3.2.1.3 Constant Turn (CT) model

It is very important to track a specific vehicle that might be taking turn in any direction.Turn is assumed to be in any direction ranging from 0° to 360°. In this model, it is assumed that the target is moving with a constant velocity V and a constant angular turn rate ω. CT model depends upon the choice of appropriate state components.The turn rate ω can also be integrated with the CV model to develop the CT model.

The state vector X in such case have the dimensions 4X1 and is given as:

$$X = \begin{pmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{pmatrix} \tag{3.18}$$

The state transition matrix used to update the states in time has the dimensions of 4X4 and is represented by the following matrix:

$$A = \begin{pmatrix} 1 & \frac{sin(\omega\Delta T)}{\omega} & 0 & -\frac{1-cos(\omega\Delta)T}{\omega} \\ 0 & cos(\omega\Delta T) & 0 & -sin(\omega\Delta T) \\ 0 & \frac{1-cos(\omega\Delta T)}{\omega} & 1 & \frac{sin\omega\Delta T}{\omega} \\ 0 & sin\omega\Delta T & 0 & cos\omega\Delta T \end{pmatrix} \tag{3.19}$$

The state space model remains the same given in eq 3.1 and eq 3.2. The output equation remains the same as expressed in eq 3.2. H is a matrix used for transformation to convert 4X1 state vector into the 2X1 vector to measure the position x and y in case of 2D. The matrix H is given as follows:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{3.20}$$

In this model,an assumption is made that the turn rate is known or it can be estimated.If the value of range rate is known, the turn rate can be estimated with the help of range rate value.If the turn rate that is assumed is very far from the true one then as a result the performance of tracker will deteriorate.In the framework containing multiple models for target tracking, CT model is also added.

3.2.2 Markov Process Model

It includes the very famous Singer model and also its variants.In this case,the control input is being modeled as a Markov random process. Some other types of model also appear in the literature [29].

The selection of model depends upon the behavior of target. Usually turn models like co-ordinated turn and constant turn are used along with CA or CV to estimate the movement of commercial planes. Whereas, CV and CA models with different bias terms are used for fighter jets or bomber planes.

3.3 Derivation of Dynamic Measurement Noise Covariance Matrix R

In the case of 2D KF,the measurements need to be transformed from polar to Cartesian coordinate system. Measurements obtained from radar are in polar coordinates.The matrix A, X, P and Q do not need transformation.The output measurement vector y can be represented as:

$$y = \begin{pmatrix} r \\ \alpha \end{pmatrix} \tag{3.21}$$

The relationship between Cartesian and polar coordinate system can be shown with the help of following figure:

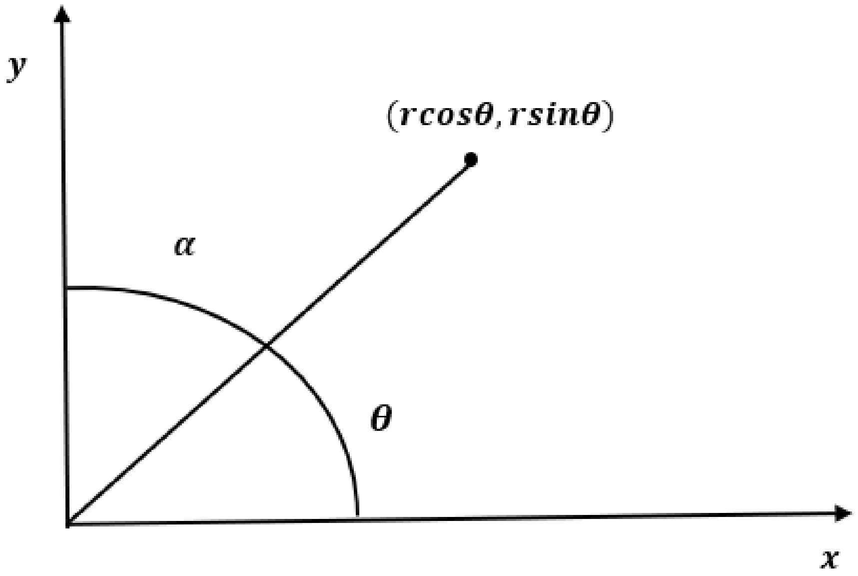

Figure 3.3: Relationship between Cartesian and Polar Coordinate System

In the figure 3.3, r is the range and α is the bearing angle measured by the radar in polar coordinate system. It is very important to transform the variances because in cartesian coordinate system x and y are not independent of each other. So the error ellipse is taken and it is rotated in the clockwise direction by the angle θ in the cartesian coordinate system. The rotation is done by using the matrix. In the other way, it can be said that variances in polar coordinates affect the variances in cartesian coordinate system. The error ellipse is shown in the figure 3.4. In the error ellipse, the distribution of data is such that it is negatively correlated. Error ellipse has one major and the other minor axis. The major axis shows the error in bearing angle and minor axis represents the error in range. After rotation, the correlation between error in x and y coordinate is assumed to be reduced.

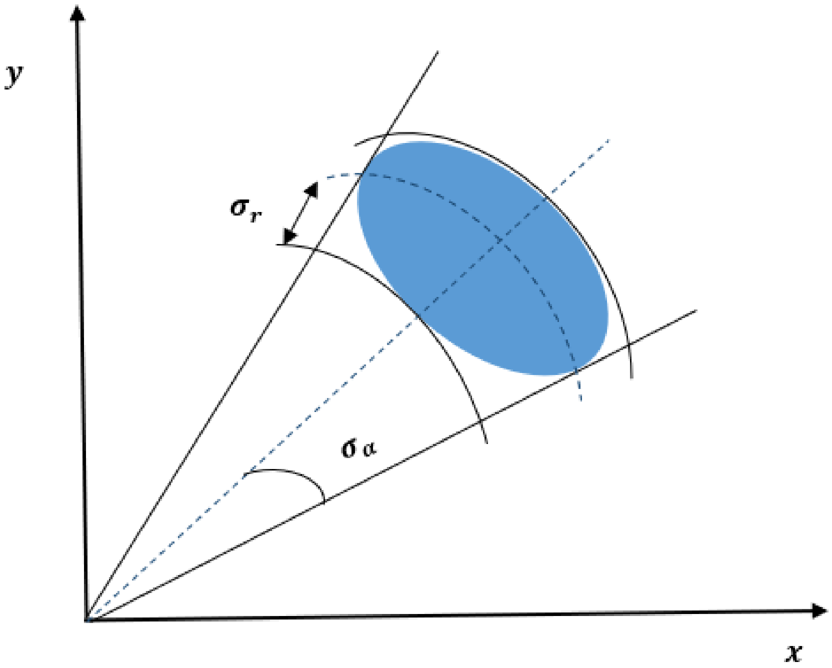

FIGURE 3.4: Error Ellipse Representing the Error in Radar Measurements

Consider the following:

$$\begin{pmatrix} e_x \\ e_y \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} e_r \\ re_\alpha \end{pmatrix} \tag{3.22}$$

$$\begin{pmatrix} e_x \\ e_y \end{pmatrix} \begin{pmatrix} e_x & e_y \end{pmatrix} = \begin{pmatrix} e_x^2 & e_x e_y \\ e_y e_x & e_y^2 \end{pmatrix} \tag{3.23}$$

By taking the expected value of matrix obtained above we get the following covariance matrix:

$$R = E \begin{pmatrix} e_x^2 & e_x e_y \\ e_y e_x & e_y^2 \end{pmatrix} \tag{3.24}$$

Consider the following relations:

$$\theta = \frac{\pi}{2} - \alpha \tag{3.25}$$

From the relation 3.22 we get:

$$e_x = e_r cos\theta + r e_\alpha sin\theta \tag{3.26}$$

$$e_y = -e_r sin\theta + r e_\alpha cos\theta \tag{3.27}$$

Squaring both eq. 3.26 and eq. 3.27 to get squared errors.

$$e_x^2 = e_r^2 cos^2\theta + r^2 e_\alpha^2 sin^2\theta + 2 e_r r e_\alpha cos\theta sin\theta \tag{3.28}$$

$$e_y^2 = e_r^2 sin^2\theta + r^2 e_\alpha^2 cos^2\theta - 2 e_r r e_\alpha cos\theta sin\theta \tag{3.29}$$

e_r and e_α are the standard deviation in range and bearing angle and their values are known.The measurements in cartesian coordinate system are assumed to be Gaussian distributed with the zero mean so the expected values are computed as follows:

$$E(e_x^2) = \sigma_x^2 = e_r^2 cos^2\theta + r^2 e_\alpha^2 sin^2\theta \tag{3.30}$$

$$E(e_y^2) = \sigma_y^2 = e_r^2 sin^2\theta + r^2 e_\alpha^2 cos^2\theta \tag{3.31}$$

Ignore the cross terms e_r and e_α as there is no correlation between them and they are assumed to be independent of each other.

$$E(e_x e_y) = \sigma_{xy} = E(-e_r^2 sin\theta cos\theta + r^2 e_\alpha^2 cos\theta sin\theta) \tag{3.32}$$

$$E(e_x e_y) = \sigma_{xy} = \frac{1}{2} sin2\theta(-e_r^2 + r^2 e_\alpha^2) \tag{3.33}$$

Finally the variances in x and y and cov(x,y) are given as follows:

$$\sigma_x^2 = e_r^2 cos^2\theta + r^2 e_\alpha^2 sin^2\theta \tag{3.34}$$

$$\sigma_y^2 = e_r^2 sin^2\theta + r^2 e_\alpha^2 cos^2\theta \tag{3.35}$$

$$\sigma_{xy} = \frac{1}{2} sin2\theta(-e_r^2 + r^2 e_\alpha^2) \tag{3.36}$$

The measurement noise covariance matrix R is given as:

$$R = \begin{pmatrix} \sigma_x^2 & \sigma_x\sigma_y \\ \sigma_y\sigma_x & \sigma_y^2 \end{pmatrix} \tag{3.37}$$

Finally,the transformed covariance matrix R becomes:

$$R = \begin{pmatrix} e_r^2 cos^2\theta + r^2 e_\alpha^2 sin^2\theta & \frac{1}{2} sin2\theta(-e_r^2 + r^2 e_\alpha^2) \\ \frac{1}{2} sin2\theta(-e_r^2 + r^2 e_\alpha^2) & e_r^2 sin^2\theta + r^2 e_\alpha^2 cos^2\theta \end{pmatrix} \tag{3.38}$$

Here, $e_r^2 = \sigma_r^2$ and $e_\alpha^2 = \sigma_\alpha^2$.

This transformed covariance matrix R will be used in the simulations of JPDAF-KF and CV model for target is assumed.

The value of σ_r^2 is taken as:

$$\sigma_r^2 = 625 \tag{3.39}$$

The value of σ_α^2 can be computed as follows:

$$\sigma_\alpha^2 = E(e_\alpha^2) = E(\frac{\pi}{2} - \theta)^2 \tag{3.40}$$

$$\sigma_\alpha^2 = E(\frac{\pi}{4}^2 + \theta^2 - \pi\theta) \tag{3.41}$$

$$\sigma_\alpha^2 = E(\frac{\pi}{4}^2) + E(\theta^2) - E(\pi\theta) \tag{3.42}$$

$$\sigma_\alpha^2 = \frac{\pi}{4}^2 + \sigma_\theta^2 \tag{3.43}$$

The value of σ_θ^2 is taken as:

$$\sigma_\theta^2 = (2.53°)^2 \tag{3.44}$$

The final covariance matrix derived is similar to that given in the document [30] but the complete derivation process is not given.

3.4 Chapter Summary

In this chapter, different dynamical models used for the target are discussed along with their equations. Mostly these dynamical models are used in the combination to track the maneuvering target. This chapter concludes with the derivation of dynamic measurement noise covariance matrix R using standard radar parameters.

Chapter 4

Data Association using TWS Radar

4.1 Introduction

Tracking radars are used to track the position of target which include information about range, velocity, azimuth and elevation angles. With the help of these obtained measurements tracks of targets are created and their values in future are predicted by using filters. Such radars are used in civilians as well as military applications.

Tracking in military radars is used to properly guide the missile towards its target otherwise this process could not be performed well and missile may lose its actual path. In civilian applications, such radars are used at airports to monitor the incoming and outgoing air traffic for security purposes.

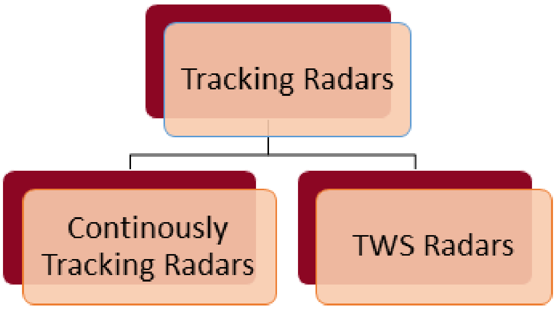

FIGURE 4.1: Types of Tracking Radars

4.2 Types of Tracking Radars

Tracking radars are used to track the range, angles and velocity of single or multiple targets depending upon the application. Such radars can be divided into two main types: **CTR** that can track single target at a time or **TWS** radar that can track multiple targets at a time. Tracking radars have the beams with the very narrow beam-width known as pencil beams.TWS radar uses scan radar to acquire the location of a target in a specific sector and that location is then fed to tracking radar to predict the future trajectory of target. Different search patterns may be used by TWS radar like **sector scan**,**raster scan**, **bidirectional scan** etc. The applications of tracking radars are shown in figure 4.2.

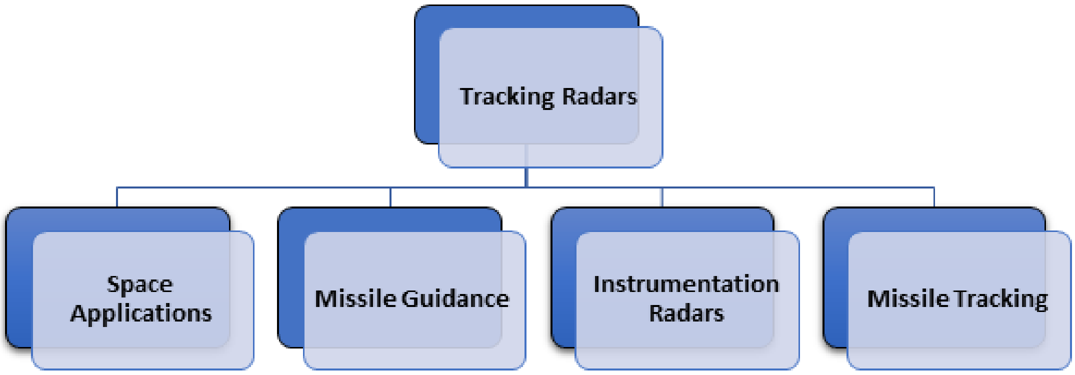

FIGURE 4.2: Applications of Tracking Radars

When any specific target is heading towards the radar then the antenna of radar should be rotated to track the targets location. In order to direct the antenna beams towards the target, the appropriate direction is determined by considering the two beam positions. The beam axis should be aligned with the position of target for proper tracking and in this way the null appears along the boresight. Two squinted beams are considered having specific squint angle with respect to boresight. The direction of boresight is determined by the point where both beams cross each other.

If the power received from one beam is more with respect to other, it means that target is oriented more towards that beam and we have to move our antenna so that target is aligned along with the boresight. Another angle of target can be tracked by considering two more addition beam positions in the orthogonal plane.

In angle tracking, the targets azimuth and elevation angle is measured continuously and the tracking accuracy depends upon the beamwidth of pencil beam used. Monopulse radars show very good tracking performance by using narrow beams.

4.3 Types of Scan Patterns

Scanning can be described as the search pattern of radar to detect targets. It may involve physically moving the antenna to change the orientation of the transmitted beam or alternatively alter the direction of the radar beam by electronic means.Scanning is different from tracking. Tracking involves the precise and continuous measurement of a targets range, angle or velocity to determine its flight path and predict its future position.

Different types of Scan patterns are used by the radars to search for target of interest in particular area. Scan patterns are classified as:

1. **Circular Scan**

   In circular scan pattern, the antenna scans continuously covering 360° in azimuth angle. The time taken to complete a scan pattern and return to the starting point, expressed in seconds is known as Scan Period.The number of complete scans in one minute or in one second is known as Scan Rate. Scan rate is often used to describe non-circular scanning radars.The total number of pulses reflected back from the target during the of one complete scan period is known as No of Hits Per Scan [31]. From the multiple hits obtained, the radar is able to collect enough information about the target.

   Scan radars with circular scan pattern use fan beam to search in a particular region. Fan beam has very small azimuth beam width and large elevation beam width thus a fan type shape is obtained. Long-range early warning, air defense and surveillance radars usually uses circular scan pattern.

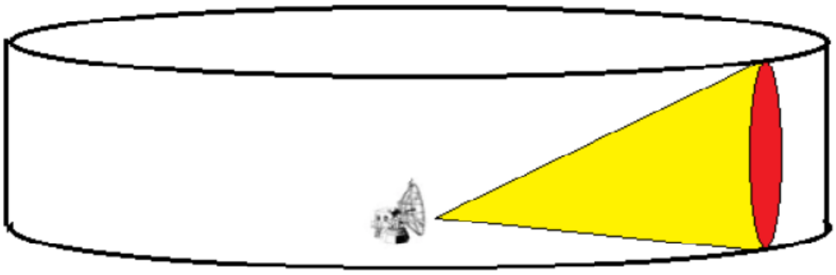

FIGURE 4.3: Circular Scan [31]

2. **Sector Scan**

   Target Acquisition Radar uses this type of scan pattern to search in a particular sector or particular area.Such radars may scan sectors that contain potential targets, allocated by early warning or long-range air defense radars. Scan rates of sector scanning radars vary according to angle scanned and function of the radar.

3. **Unidirectional Scan**

   In the helical and unidirectional scan pattern, the pencil beam is used to search the volume of 360°. The whole sector is being searched in the clockwise direction. The elevation of antenna is increased after the completion of one revolution. So,the three sweeps covering the 360° azimuth are completed and the elevation is reset to its initial value [31].

4. **Raster Scan**

   Air intercept radars uses raster scan to search a large volume of airspace in azimuth and elevation with good angle resolution. In the sector of interest, the beam is scanned in the azimuth. The elevation of beam is changed by the factor less than the overall beam width at the end of each scan and the beam is swept in opposite direction. Raster scanning can accurately fix a target position in 3 dimensions: range, azimuth and elevation. Scan rates vary according to the angle of sector, but may generally be in the region of 4 to 5 scans per second.

5. **Conical scan**

   In the conical scan type radars,the pencil beam is used that rotates around the target in a continuous manner. When the pencil beam is rotated in such a way then cone type scan pattern is obtained.In the center, there is an overlapping of the circular patterns when the beam is rotated continuously. As a result, very precise tracking results are obtained [32].

   These type of radars are operated at high frequencies having narrow beam width and PRF is also kept high. There may be more than 1800 revolutions/min which means that the information about elevation and azimuth of target are being updated about 30 times/sec.

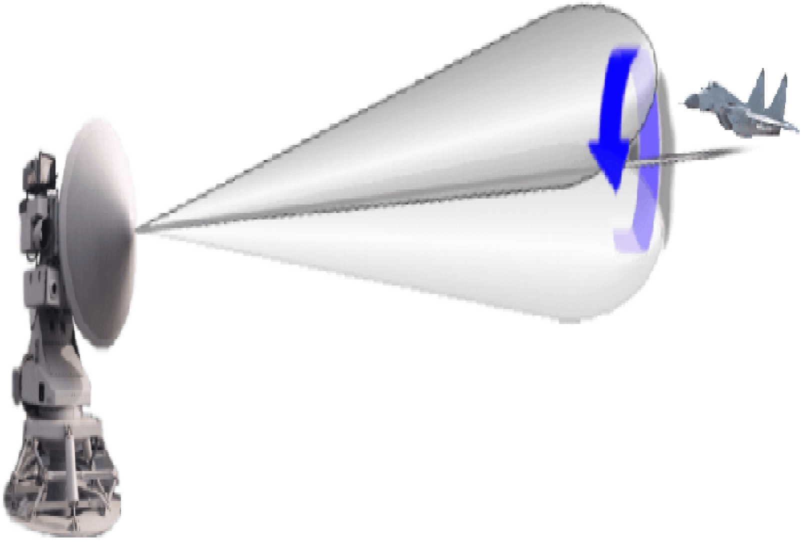

FIGURE 4.4: Conical Scan [31]

6. **Track While Scan (TWS)**

   Track while scan radar can perform searching as well as tracking operation at the same time. Raster scan is being done again and again to search the target of interest in the specific region. In this way, both the azimuth and elevation angles of target are acquired along with the range. Multiple scans are done and the detections obtained are then fed to filter to track the future trajectories of targets. These scans are assumed to be independent of each other. Whenever the range and angle estimates of certain target is obtained then the radar send this information to the corresponding TWS function.

   Phased array radar consists of array of multiple antennas with the help of which a beam is created that can be steered in any desired direction without the movement of antenna.

4.4 Function of TWS Radar

The information about the specific target is also referred as an observation or a detection. In TWS radar, judgement is made that whether the current detections obtained from the scan are similar to the previous ones or not. In these multiple scans, TWS radar has to maintain the tracks of true targets and if any false alarm occurs in some successive scans and then disappears, that track is deleted then.

This whole process is accomplished in five steps that are as follows:

1. Pre-processing of the obtained detections.
2. Correlation of obtained detections with the existing tracks.
3. Track initiation/deletion.
4. Filtering process to update the tracks.
5. Prediction of the gate center for next iteration.

1. **Pre-processing**

   The detections are obtained from the scan radar performing raster scan. If any detection obtained has same range and angle measurements as obtained in the previous scan these detections are then combined. The second step is that these observations are then transformed into the Cartesian coordinate systems as the filter operates on such coordinate system.

2. **Correlation**

   In this step it is decided that the whether the detection obtained belongs to the existing tracks or not. Different data association algorithms are used to solve this problem of assigning the true detection to the corresponding appropriate track. Measurement-to-track association probabilities are

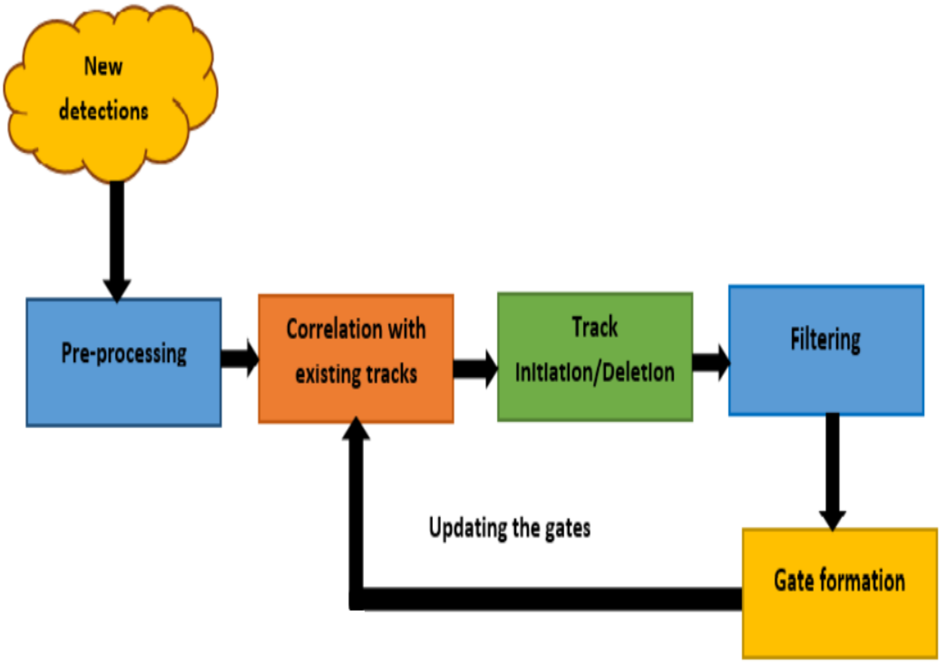

FIGURE 4.5: Steps involved in TWS Radar Processing

computed and then with the help of filter next positions of tracks are being predicted in time.

3. **Data Association**

   Bayesian or non-Bayesian data association techniques have been used and discussed in the literature [26]. These approaches compute the association probabilities of detections for the corresponding tracks. It is assumed that the detections belong to some specific pdf like Gaussian etc. In non-Bayesian approaches, the likelihood of detections is computed with the help of obtained data and association probabilities are not computed. Bayesian approach for data association using single target is known as PDAF. JPDAF is the multi-target version of PDAF that can work well even in the presence of clutter and high false alarm.

   Non-Bayesian approaches like NNSF, GNN and SNN do not give satisfactory results in the presence of high false alarm density and clutter.

The maximum error that can be observed in measurements is being represented by an ellipsoidal gate. With the help of filter, next prediction is being done and this predicted value then appears as a center of gate in the next iteration. The measurements that will be falling inside this gate are referred as validated measurements and the spurious ones are discarded.

4. **Track Initiation/Deletion**

   When any detection does not fall inside the both gates, like in figure 4.6, detection 4 is falling outside so this is associated to a new track. If in the successive scans any detection is correlated with this track,this tentative track will be confirmed. If the opposite case occurs,such detection is assumed to be false alarm and is ignored.If in successive scans no detection is assigned to any specific track,such track is then deleted. The concept of scores is also used for the track confirmation and deletion process.

5. **Filtering Process**

   In this process, the measurements obtained from the scan radar are fed to the filter.The error between the measured position and predicted position is being computed.This error is then used to update the states and covariance estimated of the target. Filtering is a recursive process which involves prediction and updating the estimates based upon the error. In this way, the track is then updated and the filter make the predictions for the future position of target.This prediction then appears as a center of gate for the next iteration.

6. **Gate formation**

   The predictions made by the filter are used to form new gates centered at these predicted positions. In the preceding scan again detections are obtained and validated measurements falling inside the gates of targets are determined. If the target is observed for the long time, estimates done by filter become accurate and the gates are then placed accurately. Hence the estimated track of target becomes closer to the actual track of target.

4.5 JPDAF

During multitarget tracking when the detections are very close to each other, a single detection may fall within the gates of multiple tracks. It becomes difficult to assign the true detection corresponding to its true track. In order to solve this problem, JPDAF algorithm is used that computes the joint association probabilities for all the validated measurements. Different hypothesis matrices are generated and their probabilities are then used to update the states of existing tracks. JPDAF also performs well in the case when two or more targets cross each other.

Consider the situation in which the detection 2 is lying in the overlapping region of both gates of target1 and target2 respectively shown in the figure 4.6.

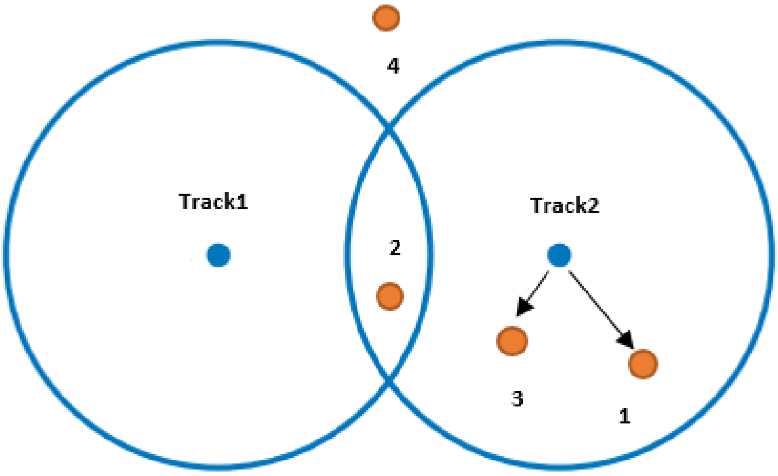

FIGURE 4.6: Conflict in Case of Closely Spaced Targets

The detections 3 and 1 are both falling within the vicinity of gate of target2.The detection 4 is not falling inside both gates so this may be due to false alarm or clutter.

Here the assignment problem can be solved as follows:

1. The detection 2 may be assigned to track1 as it is the only detection falling in its gate.
2. Detection 3 and 1 both fall inside the gate of track2. 3 is closer to the center of gate as compared to 1. So, it is assumed that detection 3 belongs to track2.
3. Detection 4 is falling outside the validation gates of both targets so it is already ignored.

4.5.1 Assumptions Taken in JPDAF

JPDAF is operated based upon the several assumptions which are as follows:

1. The total existing targets N are known even in the presence of clutter.
2. Detection originating from one target may fall inside the validation gate of another target.
3. It is assumed that the states have Gaussian distribution with zero mean.
4. Detections originating from multiple targets are not merged.
5. Models for multiple targets may be same or different.

4.5.2 Approach Adopted for the Algorithm

1. The marginal association probabilities are calculated jointly for all the validated detections.

2. Only the measurements obtained from the lastest scan are considered for the computation of association probabilities [24].
3. States of multiple targets are being estimated seperately like in PDAF.

For the computation of joint association probabilities, individual gates are not considered rather all the measurements falling inside all the gates are considered. This approach is adopted under the assumption that false measurements are uniformly distributed across the entire surveillance region.

4.5.3 Computing the Validation Matrix

Validation matrix can be defined as:

$$\psi = [w_{it}] \tag{4.1}$$

where =1,...,m; t=0,1,...,N;

This matrix comprises of binary elements indicating whether any measurement i lies inside the validation region of target t. t=0 means that none of the measurement is assigned to target t [24].

4.5.4 Finding the Event Matrix

In JPDAF, joint association events are created which are represented as:

$$\psi(\theta) = [w_{it}(\theta)] \tag{4.2}$$

This is also the binary matrix. An association event is assumed to be feasible if only single measurement is associated to any target and each target generates only one measurement at a time.

The binary variable $\gamma_t(\theta)$ is called as target detection indicator and its value is one if in any event θ,any measurement m is assigned to the target t.Another binary variable exists known as measurement association indicator $\beta_i(\theta)$.This indicates that in any event θ, measurement i belongs to target t.

For the feasible association events,following conditions should be fulfilled:

1. A measurement should belong to a single source not more than that.
2. At most,only single measurement could be originated from a target.

The number of false alarms in any event θ can be computed as:

$$\alpha(\theta) = \sum_{i=1}^{m} [1 - \beta_i(\theta)] \tag{4.3}$$

4.5.5 Evaluating the Joint Posterior Probabilities

Bayesian approach is used to compute the posterior probabilities of multiple hypothesis matrices being generated :

$$P(\theta_q|Z^q) = \frac{1}{d} p(Z_q|\theta_q, m_q, Z^{q-1}) P(\theta_q|m_q) \tag{4.4}$$

In 4.4, d is the constant used for normalization. Z_q is the vector of observations obtained at the time $'q'$. Z^{q-1} is the vector of past observations. The states of targets conditioned on the past observations are assumed to be mutually independent [24].The likelihood function appearing in the Bayesian formula can be written as:

$$p(Z_q|\theta_q, m_q, Z^{q-1}) = \prod_{i=1}^{mq} p(z_{i,q}|\theta_{q,it_i}, Z^{q-1}) \tag{4.5}$$

In 4.5, m_q represent the validated measurements falling inside the gates of both targets at the time instant $'q'$.

$$p(z_{i,q}|\theta_{q,it_i}, Z^{q-1}) = \begin{cases} G_{t_i}(z_{i,q}) & \text{if } \beta_i(\theta_q) = 1 \\ V_s^{-1} & \text{if } \beta_i(\theta_q) = 0 \end{cases} \tag{4.6}$$

Here,

$$G_{t_i}(z_{i,q}) = N(z_{i,q}; z_{q|q-1}, S_q) \tag{4.7}$$

In 4.7, $z_{q|q-1}$ is the measurement being predicted for target t_i and S_q is the residual error covariance matrix for the corresponding target.

Finally, after some more calculations the posterior probability can be computed by assuming the parametric JPDAF using the following formula:

$$P(\theta_q|Z^q) = \frac{\lambda^\alpha}{d_1} \prod_t (P_d^t)^{\gamma_t} (1 - P_d^t)^{1-\gamma_t} \prod_i [G_{ti}(z_{i,q})]^{\beta_i} \tag{4.8}$$

The parametric JPDAF uses Poisson pmf for the false alarms and d_1 is the normalization constant. λ is the false alarm spatial density.

In JPDAF, marginal association probabilities are computed for each validated measurement related to all the tracks. Such probabilities are obtained from joint probabilities by summing over all the joint events in which event of interest occurs. The expression for this probability can be expressed as:

$$\phi_{it} = \sum_{\theta:\theta_{it}\epsilon\theta} P(\theta|Z^q) \tag{4.9}$$

The states in JPDAF are updated in the same way as PDAF [24]. Combined innovation is being computed by using the sum of individual innovation that are being weighted by their marginal association probabilities.

$$E_q = \sum_{i=1}^{m_q} \phi_{i,q} \left[z_{i,q} - x^{'}_{q|q-1} \right] \tag{4.10}$$

In 4.10, $x^{'}_{q|q-1}$ is the predicted state vector in the start of the iteration $'q'$. The updated state error covariance is given by the relation:

$$P_{q|q} = P_{q|q-1} + \left[\phi_{o,q} - 1 \right] W_q S_q W^{'}_q + P^{'}_q \tag{4.11}$$

$P^{'}_q$ is the term representing spread of innovation and it is positive semi-definite.W_q is the Kalman gain and S_q is the residual error covariance. $\phi_{o,q}$ are the marginal association probabilities where no measurement is associated to target.

$$L = \left[z_{i,q} - x^{'}_{q|q-1} \right] \left[z_{i,q} - x^{'}_{q|q-1} \right]^{'} \tag{4.12}$$

$$P^{'}_q = W_q \left[\sum_{i=1}^{m_q} \phi_{i,q} L - E_q E^{'}_q \right] W^{'}_q \tag{4.13}$$

The $P^{'}_q$ term calculated in 4.13 is then added in equation 4.11 to update the state error covariance matrix P_q for the iteration $'q'$.

4.6 Chapter Summary

In the start of the chapter, different types of tracking radars are discussed followed by their applications. The types of scan pattern used by different radars are also explained. The working of TWS radar along with the data association filter (JPDAF) is also the part of this chapter.

Chapter 5

Closed Loop 3D Tracking using Monopulse JPDAF-KF

5.1 Introduction

The fundamental function of any radar system is to receive the echo pulses and to detect the information about the target from those pulses. In a typical radar system, the pulses generated at some specific frequency known as PRF and then propagated in the space. The antenna beam width may be same for both angles (elevation and azimuth) known as Pencil beam shown in figure 5.1.

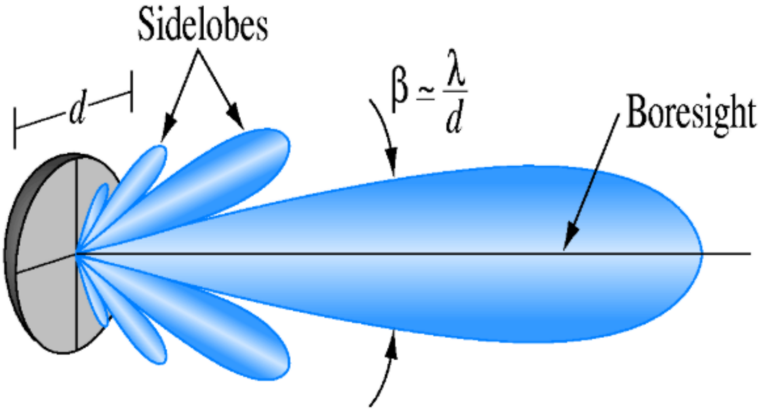

FIGURE 5.1: Pencil Beam used in Tracking Radar [33]

The beam width may be wide in one angular coordinate and narrow in other known as Fan shaped beam. Pencil beam is mostly used in tracking radars as it provides the good resolution whereas Fan shaped beam is mostly used in the search radars shown in figure 5.2.

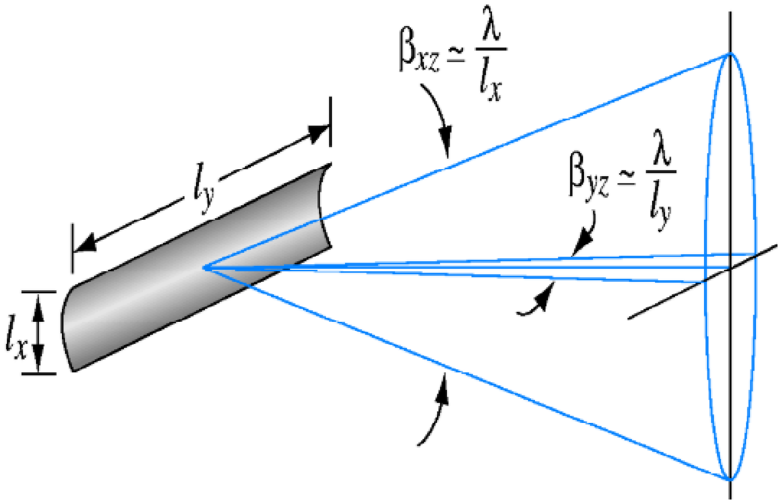

FIGURE 5.2: Fan Shaped Beam used in Search Radar [33]

Monopulse radar is included in the category of complex radars. The function of tracking radar is to automatically track the target of interest by aligning the axis of antenna beam with respect to that target. In order to update the position of the target, monopulse employs single pulse and derive the required information about target. From the same transmitting antenna two or more than two beams are transmitted and are then received at the separate receiving antennas.Comparison is done on the basis of amplitude as well as phase of received beams to derive the information about range and angles (azimuth and elevation) of target. Monopulse radar updates the position of specific target at a good speed thats why it is very eminent among the modern tracking radars. Monopulse is also named as simultaneous lobing technique and in this technique DOA is determined from the received pulses.

Tracking radar make use of very high directional pattern or very narrow beam width to achieve good resolution. The beam width is mostly very narrow and is of the order of 1° in both angular coordinates (azimuth and elevation). The beam width may vary depending upon the application but it should be same in both angular coordinates to get pencil beam.

In monopulse radar, if the target is deviated with respect to axis of beam then an error signal is generated. The magnitude of error signal depicts the amount of deviation from the beam axis and the polarity tells about the direction of deviation (right, left, down or upward). This error signal is tried to minimize in order to align the beam axis along with the target.

The tracking radar also tracks the range of desired target along with the angles. It tries to keep the center of the range gate aligned with the target. During tracking, only the echoes or signals falling inside that gate are accepted and others are discarded. The time is measured between the transmission of pulse and the range gate and in this way, range can be measured. If the echo pulse received from target of interest deviates from center of range gate then an error signal is also generated. This error is tried to minimize in order to keep range gate centered at the target.

5.2 Range Tracking

The range gate actually comprises of a switch. When the pulse is transmitted the switch is kept on during that time.The time delay between the pulse transmission and the reception of the echo inside the gate is termed as range.The echoes falling inside the gate are only considered and the spurious echoes outside the gate are ignored. Split -range gate tracker is widely used for tracking the target in range. The range gate is divided into two parts known as Early gate and Late gate.

The tracking loop designed for tracking the range tries that the energy of the echo should be equally divided among both gates. The error is then computed by subtracting the energy of echo in the late gate from the energy inside the early gate. The magnitude of error tells the difference between the echo and center of range gate.The polarity of error signal gives the information that in which direction the gate should be moved so that the echo may be aligned at the center of the range gate.

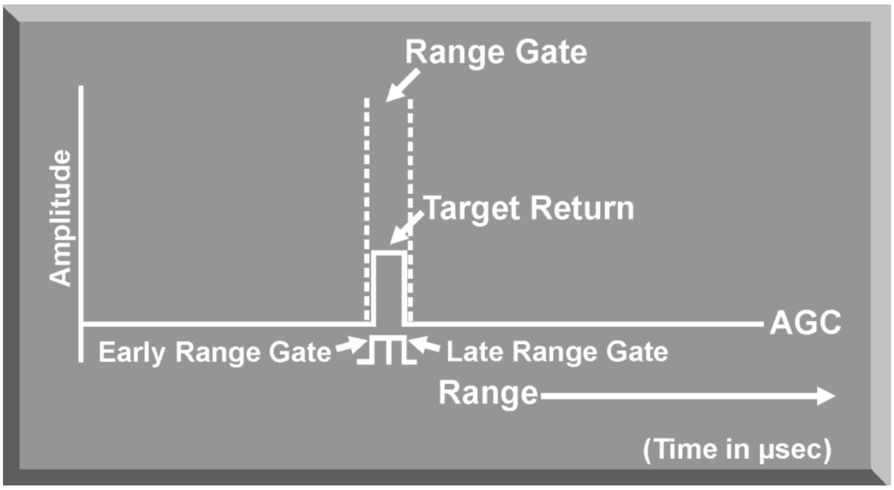

FIGURE 5.3: Range Gate in Range Tracker [34]

In some radar applications, the change in doppler frequency of target is also determined and similar to the range and angle a correction signal is being generated. This correction signal helps to keep the target frequency aligned with the center of the filter.

5.3 Angle Tracking Techniques

5.3.1 Sequential Lobing

Tracking radars are used in many applications nowadays. Before the concept of monopulse radar another technique was used for tracking the angles of target known as Lobe Switching shown in figure 5.4. In this type of tracking technique, radar beam was used to switch rapidly between the two alternating positions instead of just pointing at the target of interest. The amplitude of echoes obtained from both positions were compared. If the target appears below or above the axis of two lobes the amplitudes of both echoes were not equal in magnitude.Consider the figure 5.4 to understand the concept of Lobe switching.

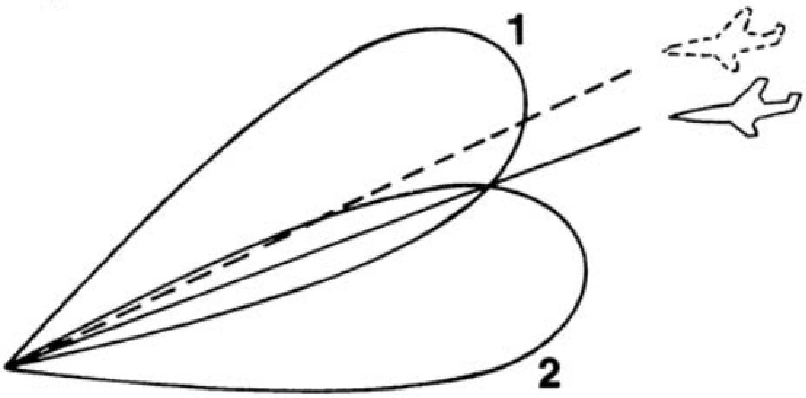

Figure 5.4: Sequential Lobing (Lobe Switching) [35]

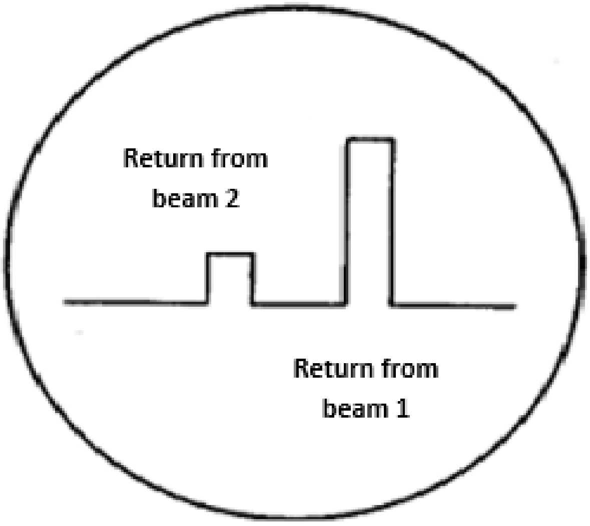

Figure 5.5: Amplitude Return from Two Beams in Sequential Lobing [35]

If the target of interest appears above the common axis of two beams then the amplitude of echo obtained from lobe1 appears to be greater as compared to that obtained from the other lobe. In order to align the antenna axis along with the target the the antenna should be pointed in a higher direction. The difference between the amplitudes of both lobes can be sent to the servo to reduce the error and to perfectly align the antenna axis along with the axis of target.

V1 is the voltage of echo received from Lobe 1 and V2 is the voltage of echo received from Lobe 2.

Consider the following conditions that may happen:

1. If V1-V2>0 (The antenna should point higher to reduce the error).
2. If V2-V1>0 (The antenna should point lower to reduce the error).
3. If V1=V2 (The antenna axis is aligned with the target).

The rotation of beam can be done by electronic or mechanical means. If the mechanical method is used for beam rotation then such techniques is known as Conical Scan. Both the lobe switching and conical scan technique are the part of Sequential Lobing.

5.3.2 Conical Scan

Conical scan is also another type of the Sequential Lobing. In conical scan antenna beam continuously rotates around the antenna axis for tracking a target shown in figure 5.6. In Conical scan method, first the target is being tracked in range. To determine the the angular coordinates (azimuth and elevation), the return obtained from the target is modulated using the frequency similar to the frequency at which beam is rotated. The angle between the direction of the target and the rotation axis determines the amplitude of the modulated signal. So, the conical scan modulation has to be extracted from the echo signal and then it is to be applied to servo control system, which moves the antenna beam axis towards the direction of the target [36].

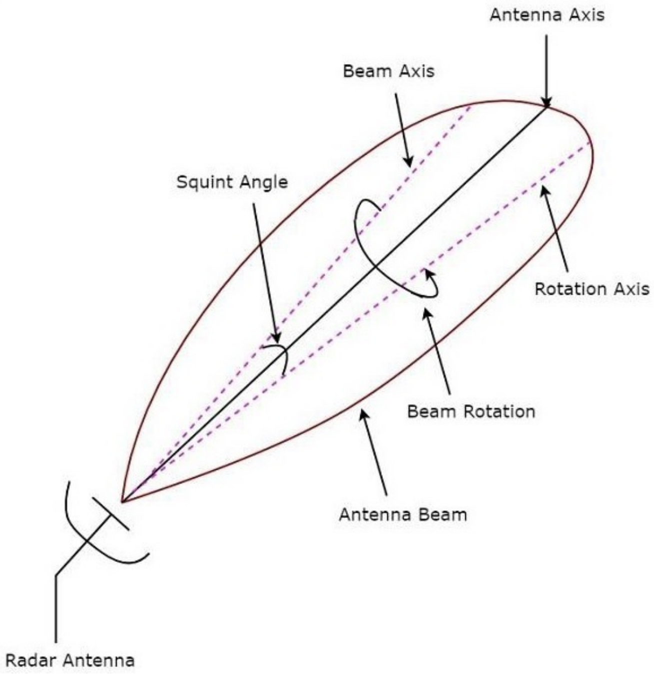

Figure 5.6: Conical Scan [36]

Drawbacks of Sequential Lobing

The main disadvantage confronted in the Sequential Lobing is in the case when there are variations in the echo strength of the target. These fluctuations may result in the wrong angle suggestions for target tracking. The radar is not able to differentiate between the pulse to pulse variations that might occurs due to variations in target RCS or due to the dislocation of target with respect to beam axis. Another main disadvantage faced is that when mechanical methods are used for the antenna rotation then due to vibrations it is not possible to perfectly align the antenna axis in the direction of desired target [34].

5.4 Monopulse Radar

Simultaneous Lobing also known as Monopulse is the solution to overcome the problems faced by Sequential Lobing. Monopulse radar can track the target in range and angles by comparing the beams received at four different receivers. Monopulse Radar has been divided into two major categories:Amplitude and

Phase comparison Monopulse.The first category appears to be same as Lobe switching. In lobe switching, the amplitudes of echoes obtained from four sequential beams are compared. Contrary to this, in monopulse radar four beams are obtained simultaneously at different receivers and then they are compared to derive the results about the range of target and angular error in tracking.

If the tracking is being done in just a single angular coordinate then two simultaneous beams are required. In the phase comparison monopulse technique, the phases of simultaneous received beams are compared. From every pulse that is received the angle information can be derived. Monopulse offers benefits over the other traditional angle tracking techniques but the cost and complexity also increases because several receivers are required. Succinct summaries about Monopulse are also proposed by Barton in [37, 38].

5.4.1 Amplitude Comparison Monopulse

The radar used for the amplitude comparison monopulse model consist of reflector antenna which is then fed by using four horns that appears to be symmetric about the axis. In the figure 5.7 ,two feed horns are visible and the remaining are on the other side.

Four squinted beams are produced by these four feed horns.Lower beams are produced with the help of upper horns. If the output of the beams is connected with four receivers identical in nature and a plane wave is incident upon them then their responses would have same phase but they may have different amplitudes.

In the amplitude comparison monopulse the squinted beams are shown in the figure 5.8.If the target lies on the common axis of all beams then amplitudes from

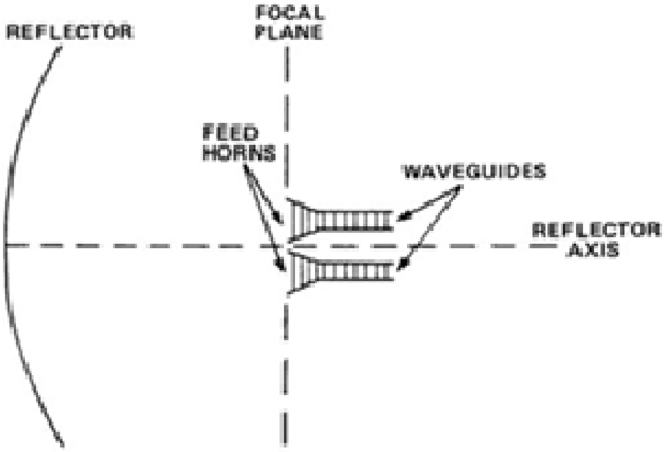

Figure 5.7: Amplitude Comparison Monopulse [37]

all the beams will be equal. By comparing these amplitudes, the error in angular coordinates of the target can be determined.

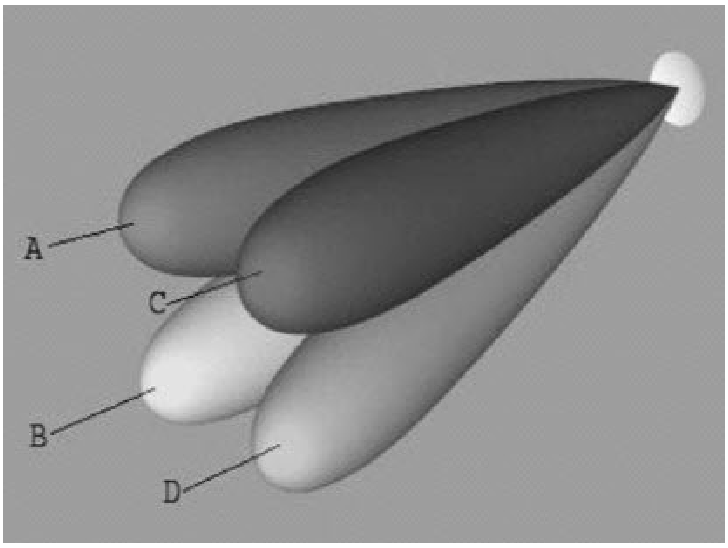

Figure 5.8: Four Squinted Beams in AC Monopulse [39]

The cross section of four squinted beams in amplitude comparison monopulse is shown figure 5.9.

Let A, B, C and D are the voltages obtained from such beams. The output obtained from feed horns is then fed to four receivers and then amplitude

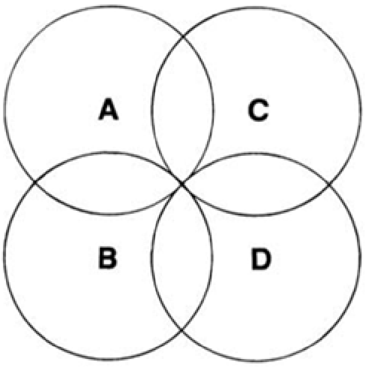

FIGURE 5.9: Cross Section of Squinted Beams in AC Monopulse [37]

comparison is done to derive the range and angular coordinates.

With the help of voltages obtained from the beams sum and difference equations can be expressed as:

$$S = \frac{1}{2}(A + B + C + D) \tag{5.1}$$

$$\Delta_{Az} = \frac{1}{2}[(A + B) - (C + D)] \tag{5.2}$$

$$\Delta_{El} = \frac{1}{2}[(A + C) - (B + D)] \tag{5.3}$$

The factor $\frac{1}{2}$ appearing in these equations is due to the fact that total input and output powers are equal if we assume that the combiners used in hardware are lossless. From the hardware point of view, comparators are used to create the sum and difference signals by adding and subtracting the received beams. These signals obtained are then sent to servo to align the beam with respect to the target.

5.4.2 Phase Comparison Monopulse

In this type of Monopulse radar, the beams are overlapping that are pointed towards the target shown in the figure 5.10. Within these beams interpolation of target angles is being done.In this type,it is assumed the amplitude of beams is same and the manipulation is done based upon their phases.

If the target appears to be along the boresight of antenna the outputs obtained from both the apertures are assumed to be in phase. If the target is displaced from the boresight axis of antenna then the relative phase is observed between the received beams. The output obtained from the phase detector is analyzed on the basis of the relative phases.

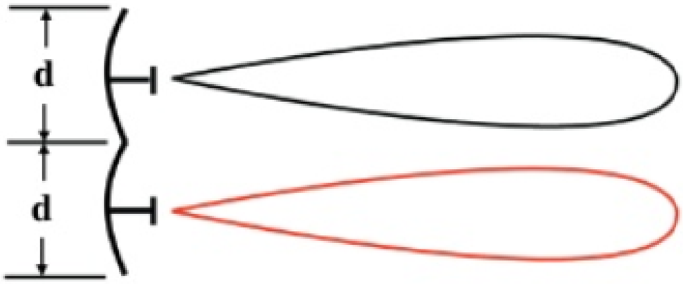

FIGURE 5.10: Overlapping Beams in Phase Comparison Monopulse [40]

Phase Comparison monopulse has the drawback over the amplitude comparison that its very tough to keep a stable boresight. There exist a long way between the antenna and the comparator used to generate the sum and difference patterns so in this type it is difficult to keep the boresight aligned to target.

5.5 Monopulse Tracking Loop

1. First of all the range of target is calculated by adding the amplitudes of all beams received simultaneously. A, B, C and D are the amplitudes of the beams received which are added and represented by the top left equation in figure 5.11.

   The sum of theses beams is then fed to the range circuit after down conversion from RF to IF. Range tracking in monopulse radar may be done using early-late gate concept.Finally the range of target is displayed on the scope.

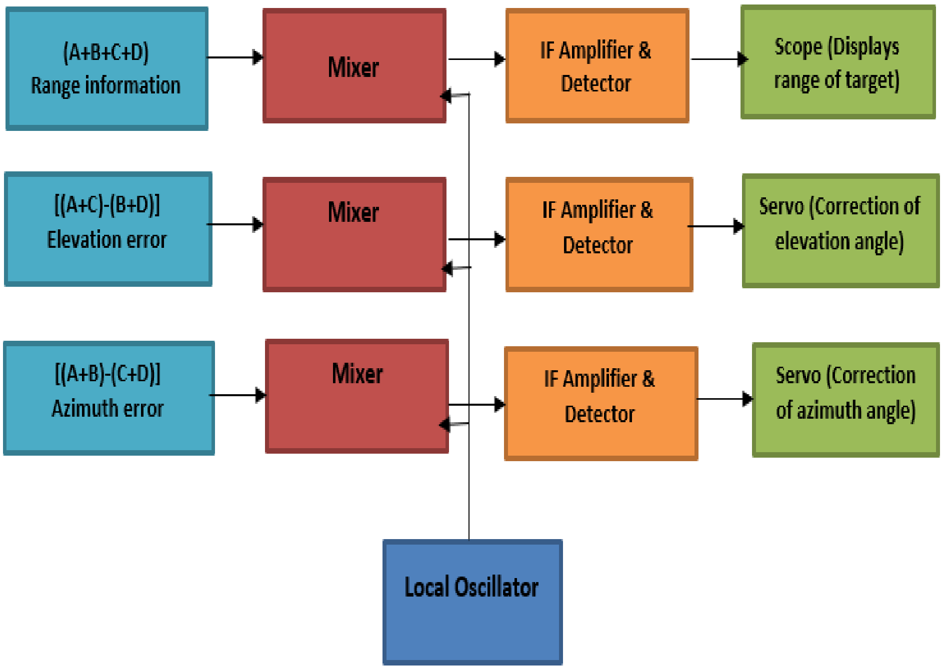

FIGURE 5.11: Monopulse Tracking Loop

2. By using the second equation shown in the figure 5.11 tracking error in the elevation is determined. Amplitude of beam A is added to beam C and similarly the amplitude of beam B is added to beam D. The sum of beam B and D is subtracted from the sum of beam A and C. After the conversion into IF,the elevation error is then fed to elevation circuit. The elevation error is also sent to servo in order to correct this error and update the target position.

3. Azimuth tracking error is determined by using the last equation shown in the Azimuth tracking error is determined by using the last equation shown in the figure 5.11 .The amplitude of two beams A and B are added similarly the amplitude of remaining two beams C and D are added. The sum of beam C and D is subtracted from the sum of beam A and B. The azimuth error is also fed to servo that ensures that the energy in the pair of both sums should remain same and tries to minimize the error.

In the range tracker, with the help of sum signal magnitude the dc voltage is produced and is then fed to all three IF amplifiers .The AGC is used that helps to prevent the receiver from saturation as the returns from target may vary over large range. AGC also tries to keep constant the gain of angle tracking loop in order to ensure good angular tracking.

Monopulse radars have many applications and some of them are listed are as follows:

1. For guiding and launching the missile.
2. Monopulse radars also find applications in military like tracking the antagonistic aircraft appearing at the long range.
3. They are also used to track the vehicles in space, satellites and some other objects in the space

5.6 Advantages of Monopulse Radar

1. In the monopulse radar, the errors that might occur in the fluctuations of amplitude of echoes are greatly reduced.
2. Information about the angles of target can be obtained in monopulse radar from each pulse rather then collecting multiple pulses.
3. Higher SNR is observed in monopulse because the range information is derived from the sum of amplitude of all received simultaneous beam. As a result, the detection of target is improved and tracking error appearing due to the presence of thermal noise is reduced.
4. In the monopulse radar, electronic beam steering is performed as compared to conical scan technique where mechanical vibrations may cause damage to the reflector used.

5. In the sequential lobing technique also known as Conical Scan ,the radar is vulnerable to inverse gain jamming. On the other hand, the transmission done is monopulse is unmodulated.

5.7 End to End Monopulse Radar

The end to end monopulse radar consist of following components:

5.7.1 Waveform

The choice of waveform plays an eminent role in the radar system as it enables the system to identify the closely spaced targets in range or angle. Therefore its necessary to examine the waveform and to clearly understand its resolution in terms of range and angle. The Phased Rectangular Waveform System object is used to create the rectangular waveform that is transmitted by the radar.The simplest form of the waveform used in the radar is Rectangular waveform also known as single frequency waveform.The parameters of waveform like sample rate, pulse width, bandwidth,PRF, etc. can be adjusted according to the design requirements.

5.7.2 Transmitter

The function of transmitter in radar system is to create the high power RF pulses having short duration that are radiated into the space by the antenna.The Phased Transmitter System Object is used to model the transmitter.

The peak transmit power is the key parameter while modeling the transmitter. Peak power is determined by assuming that probability of detection is 0.9 and false-alarm probability is 10^{-6}. Albersheim equation is used to determine the required SNR and then peak power is determined. The gain is chosen according to design parameters.

The function of transmitter is to combine the signal with the radio frequency signal known as carrier.This process is known as modulation.Radar transmitters can be divided into two main categories: Coherent and Non-coherent. In coherent transmitters the signal produced has the phase which is known prior to transmission. Non-coherent transmitters produce the signal whose phase is not already known.

5.7.3 Antenna

Antenna is used to radiate the energy in the space coming from the transmitter and at the receiver side it is used to receive the skin return and transfer it to receiver for signal processing. A single antenna can be used by multiple transmitters or multiple receivers. Monopulse operation is performed mainly at the receiver side. The main antenna types used in monopulse radar are reflector, lens and array antennas. In the monopulse radar antennas have multiple feeds for the generation of sum and difference signals. In the simulations, Phased Isotropic Antenna Element Object is used that creates an isotropic antenna element whose response is one in all directions. An antenna array is then created by using isotropic antenna elements.

5.7.4 Channel

The Phased Free Space System Object is used to model narrow-band signal propagation in the free space from one point to another point. The object applies range-dependent time delay, gain and phase shift to the input signal. Whenever there is a relative motion between radar and target, this object then accounts for doppler shift.

Free space path loss model is one of the simplest model used for the propagation of RF signals.When the signal propagates through the environment it is attenuated and the attenuation is inversely proportional to the square of distance. A free space environment is assumed to be a boundary-less medium in which speed of signal propagation is independent of position and direction.Straight line propagation is assumed from radar to target.

5.7.5 Target

In the radar system,signal is propagated towards the target, reflected back and then processed to derive useful information about the target.The Phased Radar Target system object is used to model the target which then computes the reflected signal from a target.

Mostly the electromagnetic signals are polarized, sometimes polarization is ignored and they are process as scalar signals. Different swerling models can be taken and RCS of the target can be selected according to design requirements. RCS of the target is defined as theoretical area which is detectable by the radar. If RCS is large it means that object is easily detectable by the radar. The RCS calculation does not depend upon strength of the emitter or the distance instead its the target reflectivity property.

5.7.6 Receiver

With the help of Phased Receiver Preamp System Object, the model of a receiver pre-amplifier is implemented. The receiver receives the incoming skin return and multiply them by gain of amplifier. Finally, Gaussian white noise is added to the signal. While modeling the receiver the most important thing is the thermal noise. The matched filter is used to remove this noise. The power of thermal noise and bandwidth of receiver are proportional to each other.The thermal noise can be computed using following formula:

$$P_n = KTBF$$

Here,

P_n=Noise Power at receiver

K=Boltzmann Constant

B=Receiver Bandwidth

F=Noise Figure

5.7.7 Beamformer

In radar system, for the directional transmission and reception a technique known as beamforming is used. It allows the radar to transmit and receive the signal from particular direction. Direction of arrival of target is determined using beamforming algorithms.

Beamforming can be classified as analog and digital beamforming. In analog beamforming, at the receiver side the signals from different antennas are added up before analog to digital conversion. The complex weights are multiplied to the output of each antenna and all signals are summed up to form one signal so only one ADC converter is required.

In digital beamforming, first analog to digital conversion is done then the output of antennas are multiplied with their corresponding weights and added up. Local oscillator is used to convert RF signals to IF signals. In this process, separated analog to digital converters are required and intense processing is required.

5.8 Monopulse Signal Processing

5.8.1 Range Gating

In the tracking radar, range tracking technique used is known as Early-Late gate. Range gate is mainly split into two parts called as early and late gate. The important objective of using this technique is to keep the range gate centered at the target so that the energy in both gates is equally divided. In the figure 5.12 , it is clearly visible that the target is present at the center of the range gate so its amplitude is divided equally in both gates. If early gate contains the more portion of target as compared to the late gate then in early gate the voltage measured will be more. The voltage of late gate is subtracted from that of early gate and this is termed as range discriminant or the error. The range can be tracked more precisely if this error is minimized. The function of range tracking loop is to measure the target current range and then tries to keep the range gate centered at target range [8].

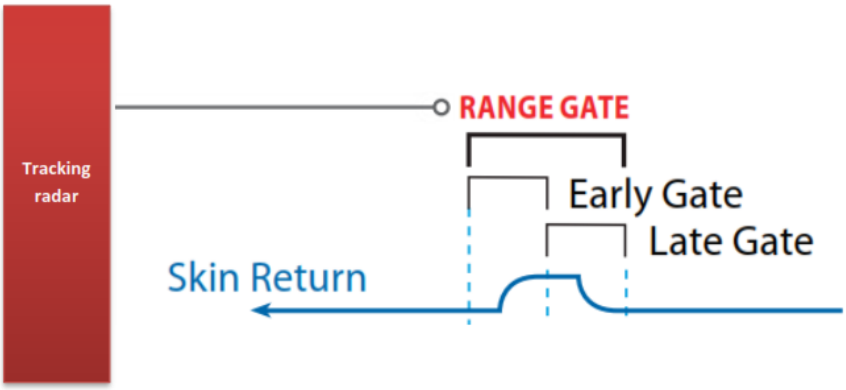

FIGURE 5.12: Early Late Gate [41]

5.8.2 AGC

In the tracking radars,as the target moves its range and cross-section also changes and as a result, amplitude of echo signal also varies. So to prevent the receiver from saturation, AGC is used at the receiver side in Monopulse tracking loop.

In case of monopulse radar with three channels at the receiver side, the AGC control voltage is used to control the gain of all the channels. The control voltage of AGC is determined with the help of output of sum channel. The AGC that averages the several pulses is used mostly in such case. The function of AGC is to provide a constant signal level to the tracker. Normally in the AGC, the detected pulse is stretched to the duration equal to the PRI.Stretching increases the average and the gain of AGC loop.

In the tracking radar, the output obtained from the sum channel is fed to AGC which is the gated signal with the gate centered at the target. The sum signal is fed to detector which then detects the range of target .Meanwhile, the sum signal is normalized by the AGC voltage to maintain the stable range tracking loop. Similarly, delta channel output is also normalized by the AGC voltage.The angle detector also knows as product detector and it gives the following output:

$$e = k\frac{\Delta S}{|S||S|}cos\theta \tag{5.4}$$

Here, e is the error signal.The phases are adjusted to get 0 or 180°.Finally the error becomes:

$$e = \pm k\frac{\Delta}{|S|} \tag{5.5}$$

The error voltage obtained is the ratio of delta signal divided by the sum signal which is the desired output obtained from the angular-error-detector.

5.8.3 Angle Tracking

The normalized errors are obtained for both dimensions (azimuth and elevation) after passing through AGC. These error are then multiplied with some gain (k) and are added to the computed azimuth and elevation angles to update their values.

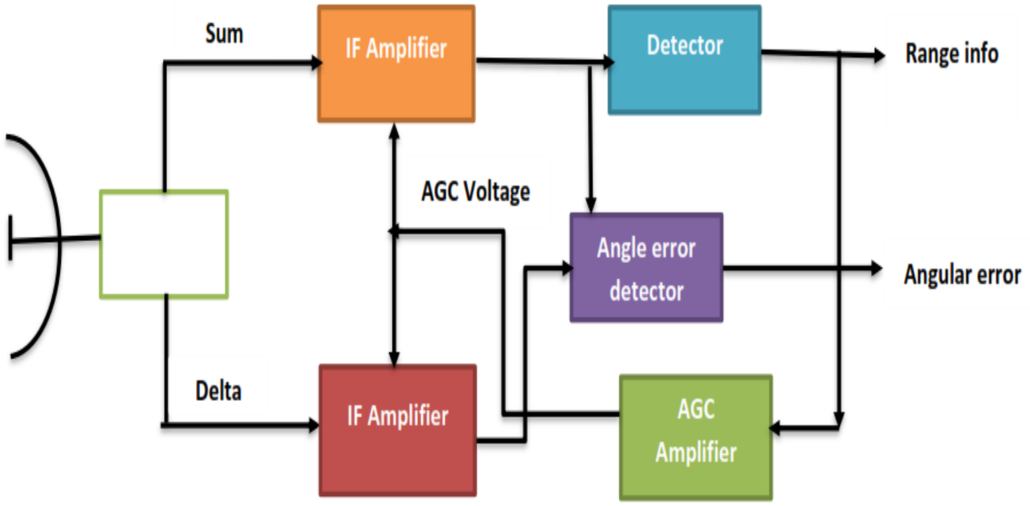

FIGURE 5.13: AGC in Tracking Loop

5.9 Monopulse Closed Loop Tracking using JPDAF-KF

In the literature, closed loop tracking using monopulse is performed in [42] but the data association algorithm is not used for tracking. In monopulse closed loop tracking with data association, first the coordinates (range,azimuth and elevation) of targets are computed by monopulse radar.The coordinates are transformed from polar to Cartesian coordinate system and fed to JPDAF-KF.Monopulse Closed loop tracking has been shown in the figure 5.14.

In JPDAF first of all gating is done to remove false detections.Then the validated matrix is generated and multiple hypothesis are generated. Posterior probabilities of multiple hypothesis are computed using Bayesian approach. Marginal association probabilities are then calculated and tracks are updated by KF based upon the weighted marginal probabilities.

The coordinates of target predicted by KF for next iteration are again transformed from Cartesian to polar coordinates.The azimuth and elevation angles predicted

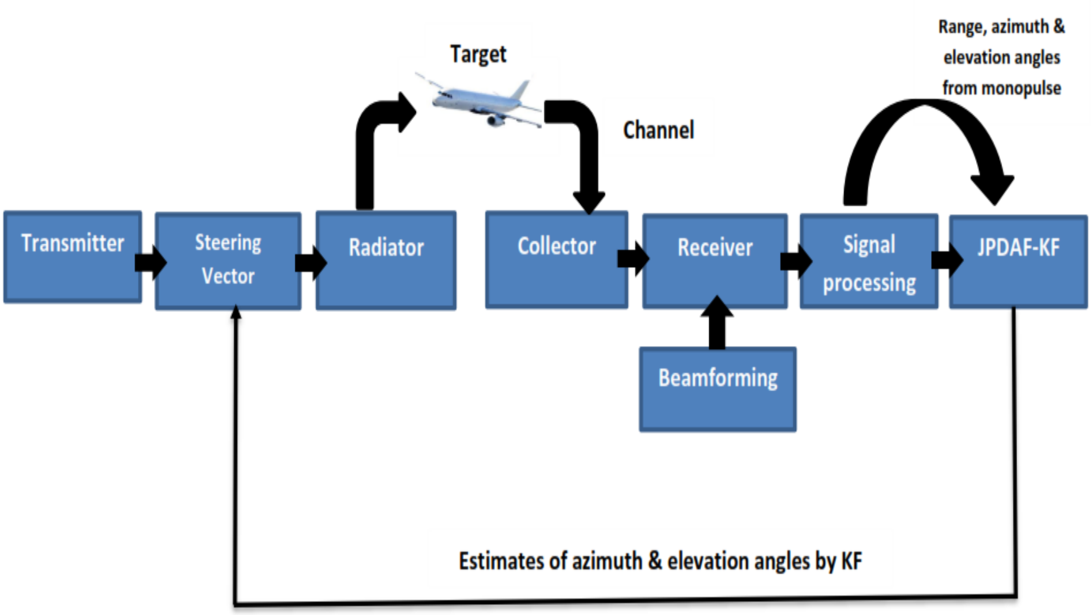

FIGURE 5.14: Monopulse Closed Loop Tracking using JPDAF-KF

by KF are then fed to steering vector to steer the radar beam according to those angles.

5.10 Chapter Summary

This chapter comprises of the explanation of closed loop 3D tracking using JPDAF-KF. In the start of the chapter, the basic difference between the pencil and fan shaped beam is discussed along with the figures. The concept of range and angle tracking is also described in detail. The advantage of monopulse radar over the conventional angle tracking techniques have been explained. The types of monopulse radar: Amplitude and Phase comparison are also the part of the chapter.This chapter concludes with the end to end working of monopulse radar.

Chapter 6

Simulation Results

This chapter covers the 2D and 3D Multitarget tracking using JPDAF algorithm. In 2D tracking, scan radar is use alongwith JPDAF-KF while 3D tracking is performed in a closed loop fashion using monopulse radar.The simulations are performed in MATLAB. Design parameters chosen for scan and monopulse radar are listed in the tables which are also the part of this chapter.

6.1 2D Tracking using TWS Radar

In the TWS Radar, phased array antenna is used that periodically scans a predefined surveillance region. The area of surveillance region is about 90° and scanning is done in the clockwise direction. Detections obtained from scan radar are then fed to JPDAF for the data association. Phased array antenna is an electronically scanned array of antennas which creates a beam that can be steered electronically in any desired direction without mechanically moving the antenna. A 1600 element rectangular array is used by the radar having the mono-static geometry.

Phased array radar deployed by China is shown in the figure 6.1.

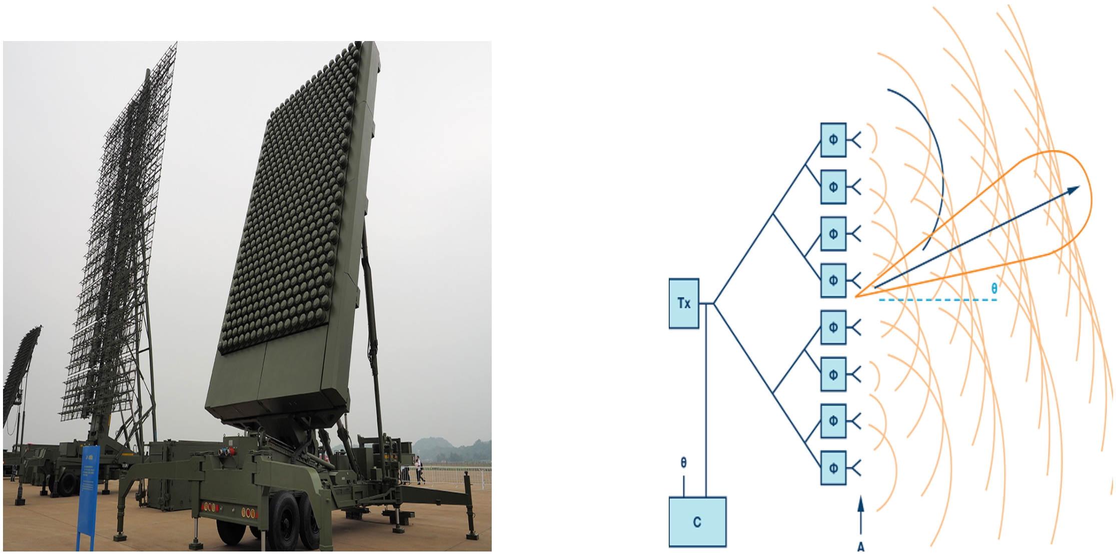

FIGURE 6.1: Phased Array Radar

6.1.1 Design Parameters

Design parameters for 2D scan radar are given in the following table along with their units.

TABLE 6.1: Design Parameters for Scan Radar

Parameters	Values	Units
P_d	0.9	unitless
P_{fa}	$1e^{-6}$	unitless
Max.range	60000	m
Range resolution	50	m
Pulse bandwidth	$3\ e^6$	Hz
Pulse width	$3.335\ e^{-7}$	sec
PRF	$2.5\ e^3$	Hz
Sampling Frequency	$6\ e^6$	Hz
Carrier Frequency	$10\ e^9$	Hz
Tx.Gain	20	dB
Rx.Gain	20	dB
Noise figure	5	dB
Array size	40X40	elements
Elements Spacing	0.015	m
Target RCS	[4 3]	m^2
Beam width	5.07	deg

6.1.2 End to End Function of Scan Radar

The rectangular pulse is generated by the waveform generator and modulated by the transmitter which is then radiated into the space by using the antenna. After passing through the channel which is assumed to be free space the pulse arrives at the target. The echo is reflected back from the target and is received at the collector which then send the pulse to the receiver for signal processing. Thresholding is performed to detect the pulse and the range estimation is done.

The 3D antenna pattern is shown in the figure 6.2. The side lobes are tapered as they might create problem in target detection.

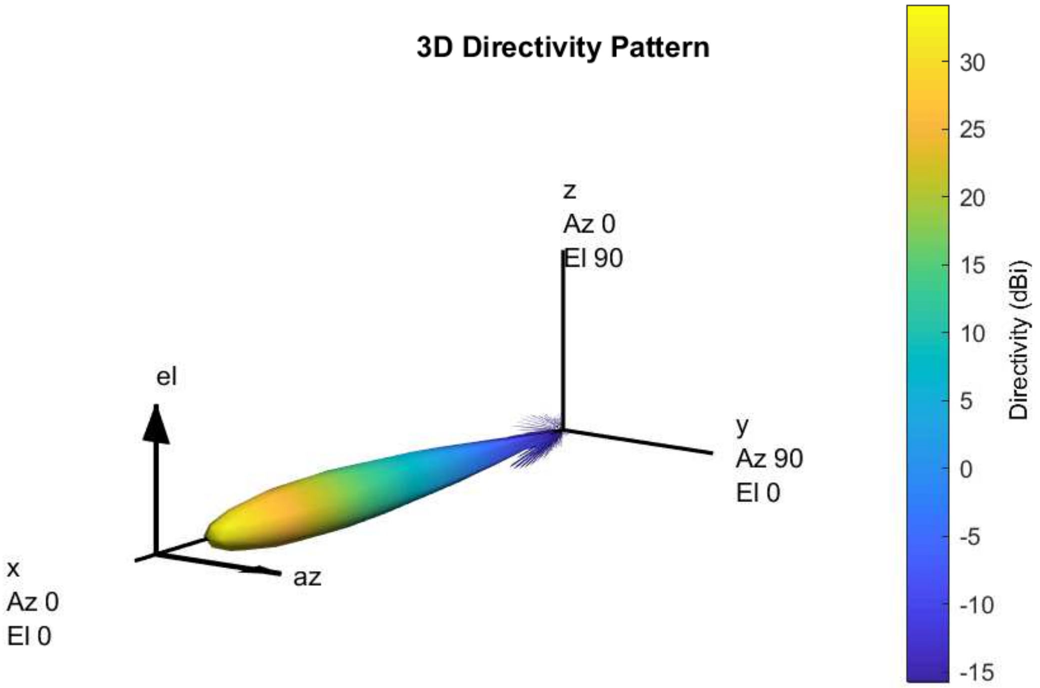

FIGURE 6.2: 3D Antenna Pattern in Scan Radar

Scan radar periodically scans a surveillance region ranging from +45° to −45° in azimuth dimension. Scanning is performed in the azimuth and elevation is ignored.The surveillance region is divided into many sectors. Each sector is of 3°.

In each sector,several pulses are transmitted and then echoes are collected and pulse integration is performed and thresholding is done for target detection.

The revisit time should be less than 1 second, meaning that the radar should revisit the same azimuth angle within 1 second.The radar equation is used to compute the peak power required for transmission of signal.Signal to Noise ratio (SNR) is computed by using the Albersheim equation.

The reflected signals are received by an array, so beamformer pointing in the steering direction is used to obtain the combined signal. Matched filters are commonly used in radar, in which a known signal is sent out, and the reflected signal is examined for common elements of the out-going signal.

To process the received signal, it is first passed through a matched filter, then integration of all pulses for each scan angle is done.To obtain an accurate estimation of the target parameters, detection threshold is applied. In order to compensate for signal power loss due to range the time varying gains are applied to the received signal. Consider the figure 6.3 in which two peaks represent the targets detected by the scan radar having max range of 60 km.

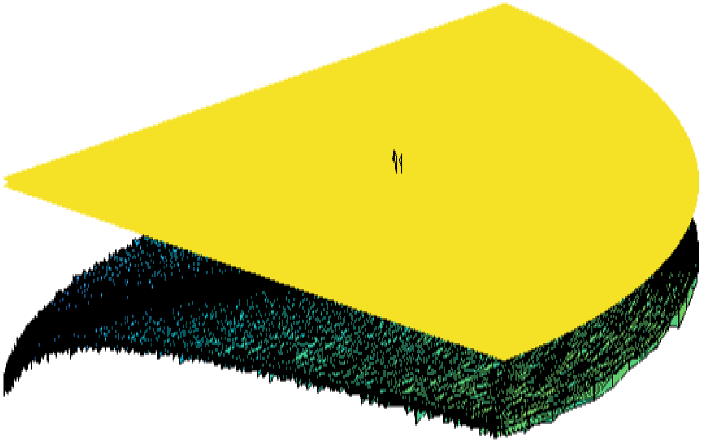

FIGURE 6.3: Detection of both targets by Scan Radar

6.1.3 Actual Trajectories of Both Targets

Here, the two targets are considered having different initial positions that are moving towards the radar. During their motion, both target also cross each other and then move ahead.

The beauty of JPDAF algorithm is that it can cope with the such situations in a good way and can track well even at the crossing point of both targets.

Initial position and velocities of both targets in (x,y) coordinates are given below:

Position of Target1=[34200,3550,0] , Position of Target2=[34400,3500,0]
Velocity of Target1=[-200,-120, 0] , Velocity of Target2=[-220,-90, 0]

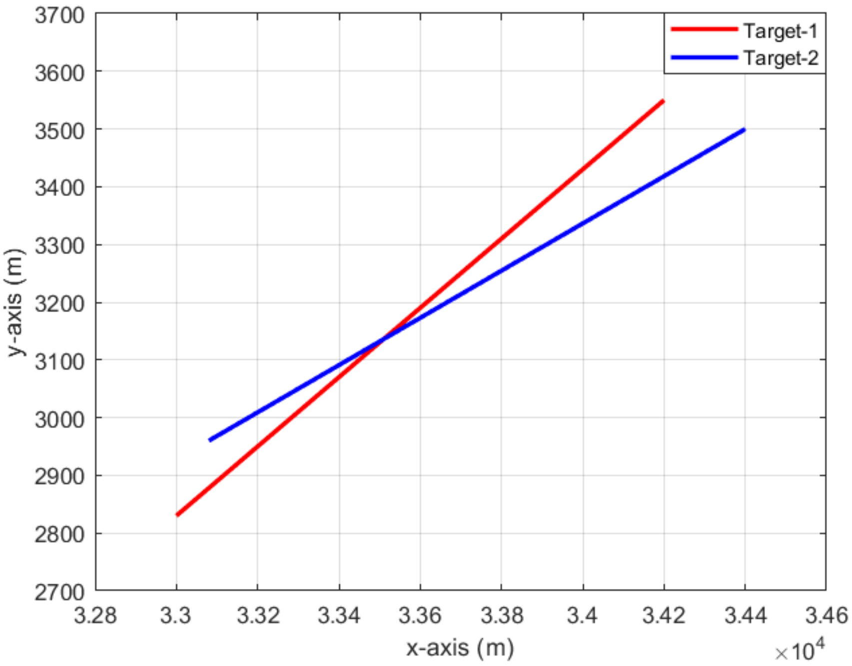

FIGURE 6.4: Actual Trajectories of Both Targets

In the figure 6.4, the actual trajectories of both targets are shown. Red line represents the trajectory of Target-1 and blue line is for Target-2. CV model is assumed for both the targets. Both the targets have different initial positions during their motion they cross at a common point and then continue their trajectories. Standard Kalman Filter is used alongwith the JPDAF to estimate the next states of the both targets.

6.1.4 Trajectories Obtained from Scan Radar and JPDAF-KF

Detections are obtained from scan radar which are then converted from polar to cartesian coordinate system and fed to filter for data association. Error is computed in KF and tracks are further updated.

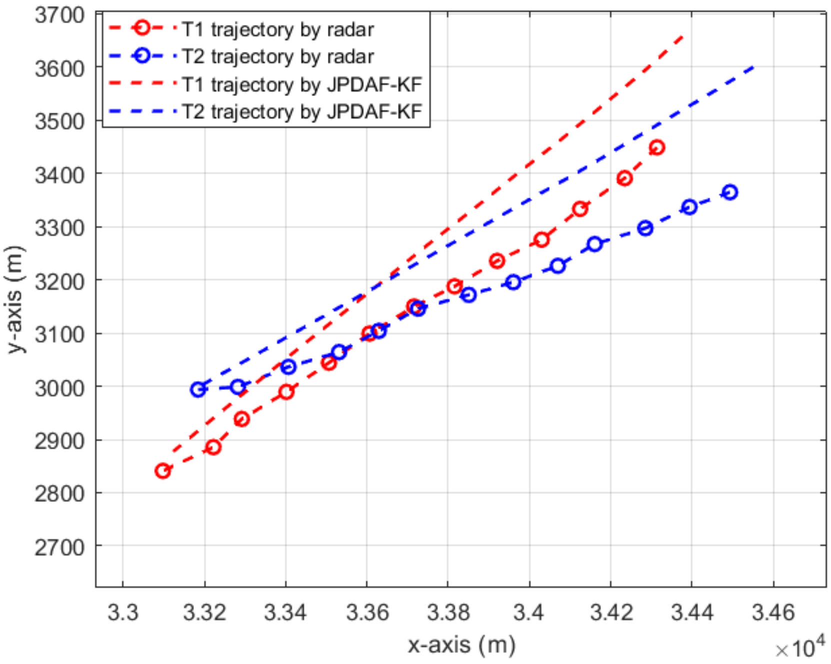

FIGURE 6.5: Trajectories Obtained from Scan Radar and JPDAF-KF

In the figure 6.5, trajectories obtained by scan radar as well as trajectories estimated by JPDAF-KF are plotted together. It is clearly visible that in the start the error between the scan and filtered trajectories is more for both targets. This error decays down after certain iterations.

6.1.5 Range Tracking

The following plots show the range tracking for both the targets.

In the figure 6.6, the range of Target-1 obtained by the scan radar (actual range) as well as estimated range by JPDAF-KF is plotted. It is visible that the JPDAF-KF is tracking the range of target very well and the error is reducing with time.

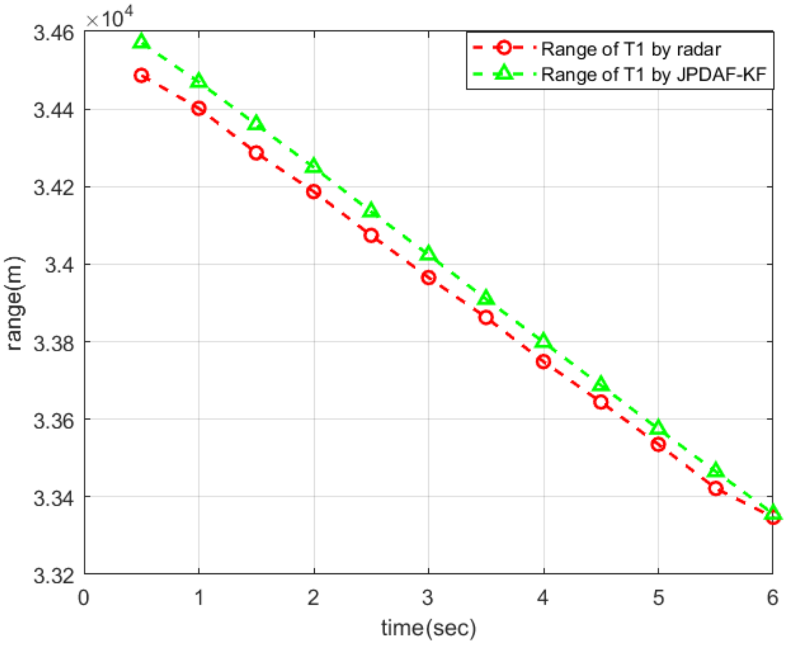

FIGURE 6.6: Range Tracking for T1

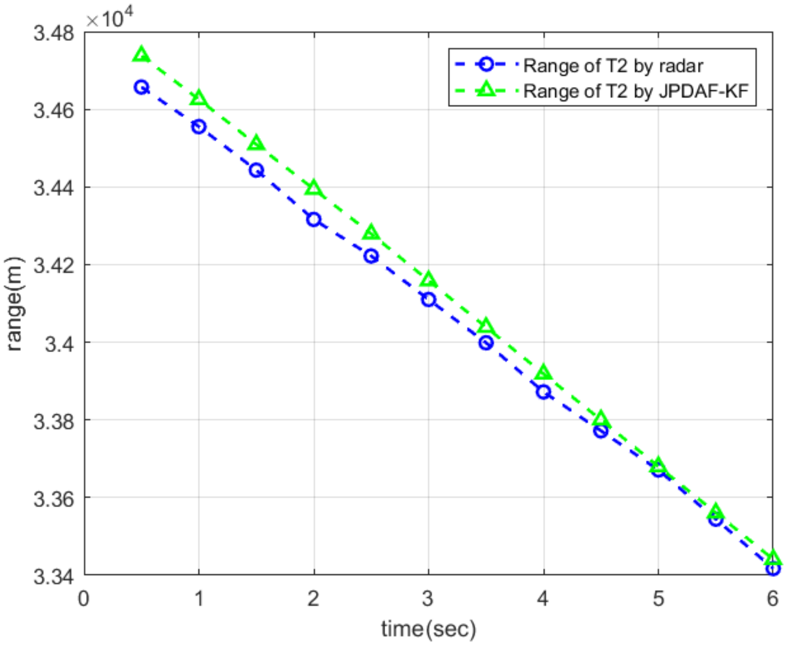

FIGURE 6.7: Range tracking for T2

Similarly, for Target2 both actual and estimated trajectories are plotted and are shown in the figure 6.7. The ranges of both targets are decreasing because both are heading towards the radar. Blue dotted line with circles give the range measured by radar and green dotted line with triangles give the measurements by the JPDAF-KF.

6.1.6 Azimuth Angle Tracking

Azimuth angle tracking is shown for both the targets.

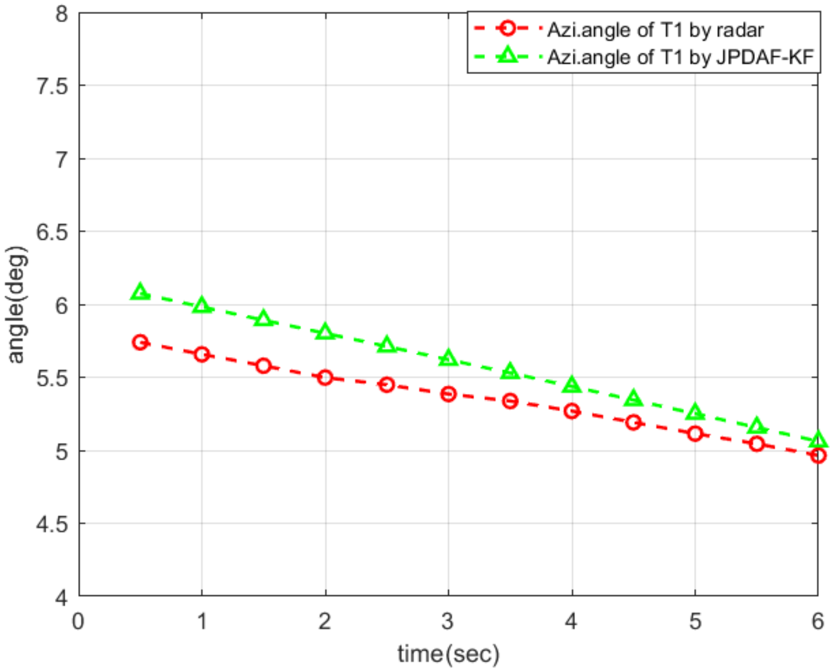

FIGURE 6.8: Azimuth Angle Tracking for T1

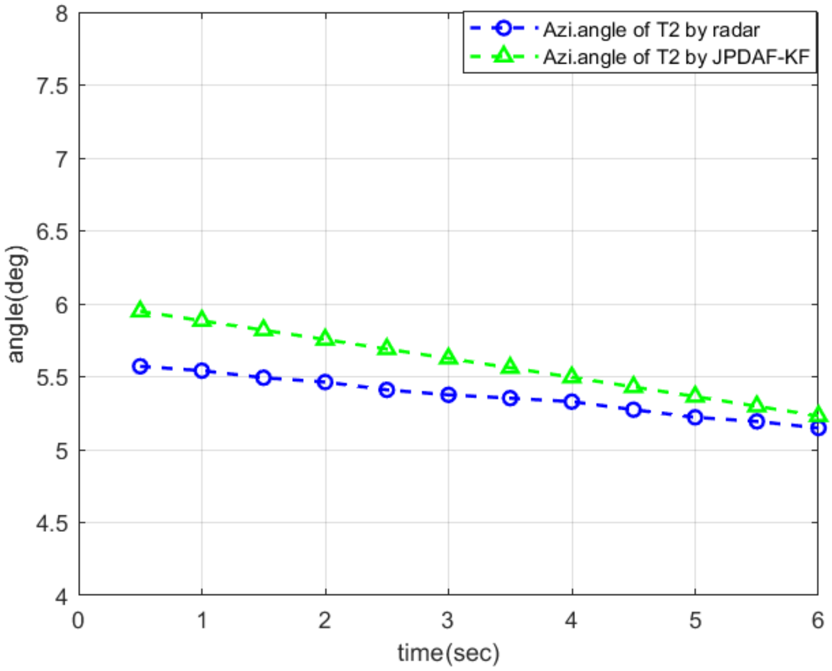

FIGURE 6.9: Azimuth Angle Tracking for T2

In the figure 6.8, the actual azimuth angle of Target-1 obtained by the scan radar as well as estimated azimuth angle by JPDAF-KF is plotted.

The red dotted line with circles represents the actual azimuth angle and green dotted line with triangles represents the estimated angle.The error between both decreases with the time.

Similarly, for Target2 both actual and estimated azimuth angle are plotted and shown in the figure 6.9.

6.1.7 Estimation Error

The estimation error is expressed by the equation.

$$\text{Estimation Error} = \sqrt{(X_{scan} - X_{est})^2 + (Y_{scan} - Y_{est})^2} \tag{6.1}$$

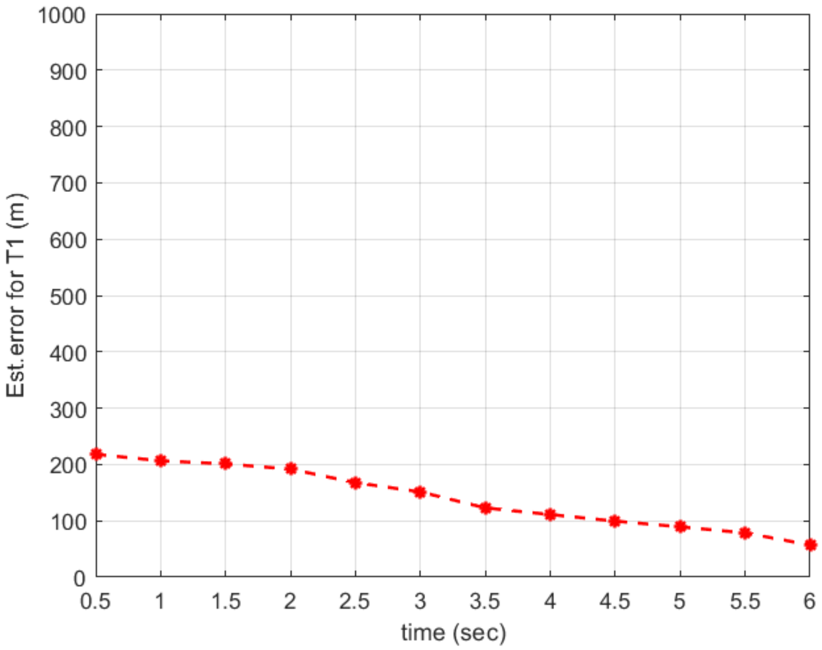

FIGURE 6.10: Estimation Error for T1

The estimation error is plotted for the trajectories of both targets shown in the figure 6.10. In the beginning,the estimation error is more but the error converges

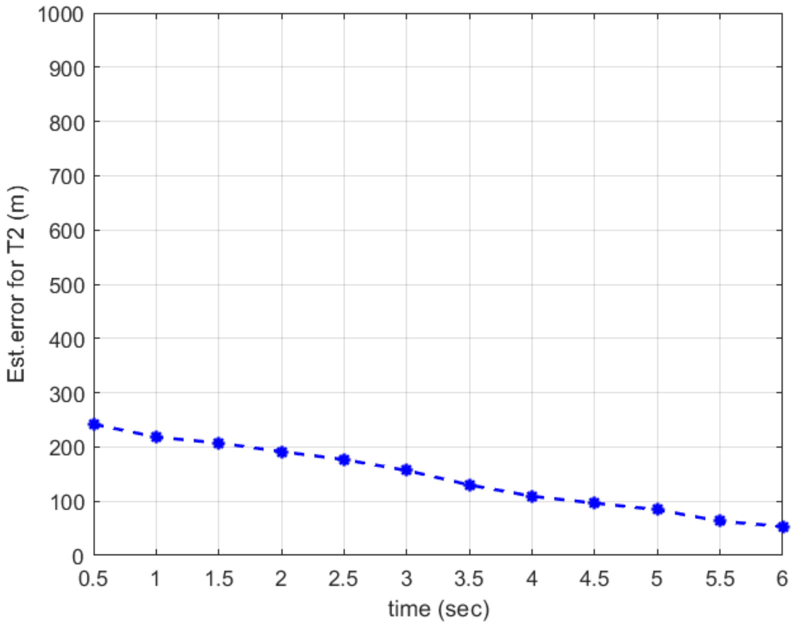

FIGURE 6.11: Estimation Error for T2

after several iterations which is clearly visible from the both graphs in figure 6.10 and 6.11.

6.2 3D Closed Loop Tracking using Monopulse and JPDAF-KF

In 3D tracking, monopulse radar is used that can track the target in range and angles. Monopulse signal processing is performed at the receiver side. Sum and delta channels are obtained,range gating and angle tracking is then performed. Monopulse radar is considered to be a good tracking radar because it makes use of narrow beam to track the actual position of target.

A rectangular array of 3600 elements is used by the radar having the mono-static geometry. Steering vector and beam former are used to steer and receive the beams in the appropriate direction.

Monopulse tracking is performed in a closed loop by using the JPDAF-KF. Monopulse radar estimates the positions of both targets. These estimates are in spherical coordinates which are converted to Cartesian coordinates and fed to JPDAF-KF for further processing.The KF estimates the next positions of both targets which are converted from Cartesian to spherical coordinates. The azimuth and elevation angle estimates are used to steer the monopulse beam in those directions for the next iteration.

The radiation pattern of antenna array used is given in the figure 6.12. It is clearly visible that the beam width was wide in scan radar as compared to that used in monopulse radar.

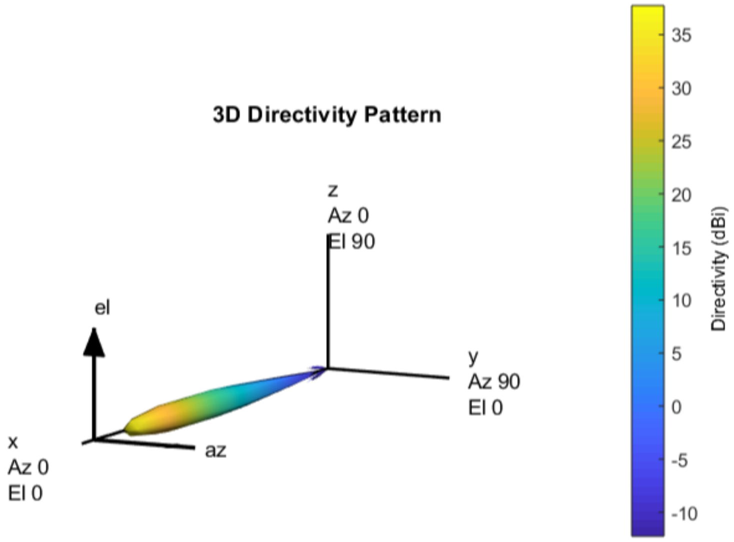

Figure 6.12: 3D Antenna Pattern in Monopulse Radar

6.2.1 Design Parameters

Design parameters for 3D monopulse radar are given in the Table 6.2 along with their units.

TABLE 6.2: Design Parameters for Monopulse Radar

Parameters	Values	Units
No of samples/range bin	6	samples
Range Gate size	4	samples
Max.range	60000	m
Range resolution	50	m
Pulse bandwidth	$3\ e^6$	Hz
Pulse width	$3.335\ e^{-7}$	sec
PRF	$2.5\ e^3$	Hz
Sampling Frequency	$1.8\ e^7$	Hz
Carrier Frequency	$10\ e^9$	Hz
Tx.Gain	20	dB
Rx.Gain	20	dB
Noise figure	5	dB
Array size	60X60	elements
Elements Spacing	0.015	m
Target RCS	[4 3]	m^2
Beam width	3.38	deg

6.2.2 Actual Trajectories of Both Targets

The two targets are considered having different initial positions that are moving towards the radar. During their motion, both target also cross each other and then move ahead. Initial position and velocities of both targets in (x,y,z) coordinates are given below:

Position of Target1=[19300,3400,1200], Position of Target2=[18700,3500,1300]
Velocity of Target1=[-400,-130,0], Velocity of Target2=[-250,-170,0]

In the figure 6.13, the actual trajectories of both targets are shown in 3D coordinate system. Red line represents the trajectory of Target-1 and blue line is for Target-2. Both the targets have different initial positions and different velocities and both are heading towards the radar.

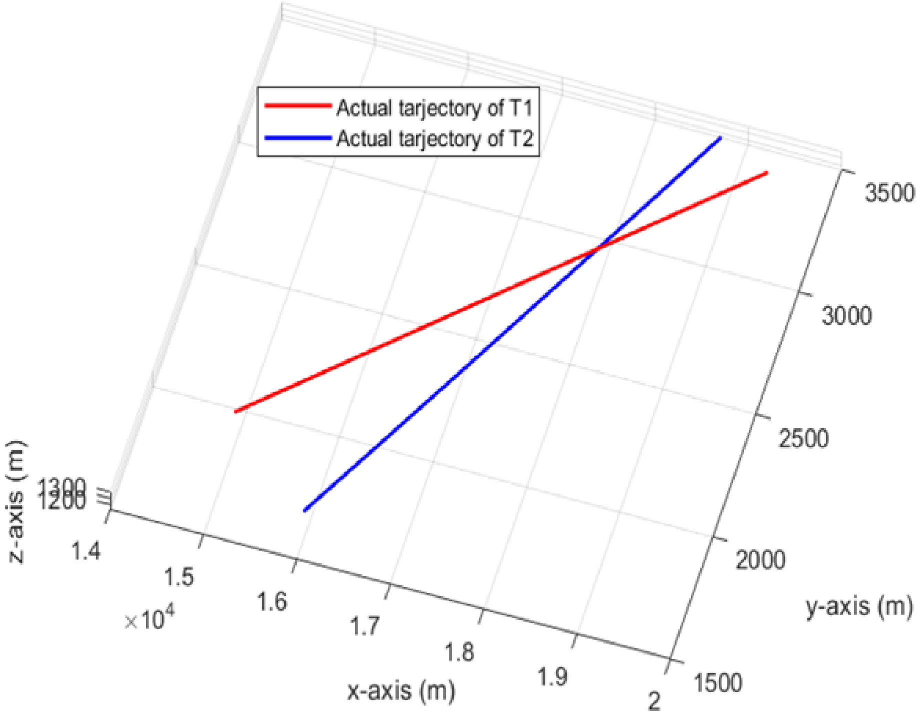

FIGURE 6.13: Actual Trajectories of Both Targets in 3D

6.2.3 Actual and Estimated Trajectories by JPDAF-KF

In the figure 6.14, actual trajectories of both targets as well as estimated trajectories obtained by JPDAF-KF are plotted together.

It is clearly visible that the filter is estimating the true trajectories very well even at the crossing point of both targets. Solid lines represent the true trajectories and dotted line represent the estimated trajectories.

6.2.4 Range Tracking

In the figure 6.15, the actual range of Target-1 and the estimated range by JPDAF-KF are plotted.

It is clearly visible that the JPDAF-KF is tracking the range of target very well and the error is reducing with the passage of time. Similarly, for Target2 both

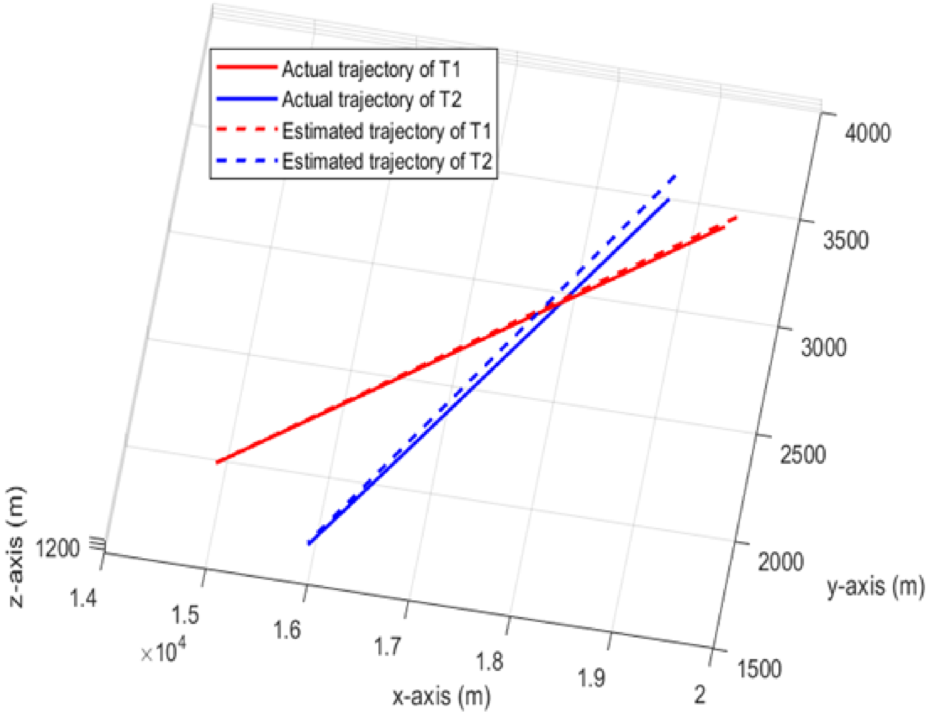

FIGURE 6.14: Actual and Estimated Trajectories by JPDAF-KF

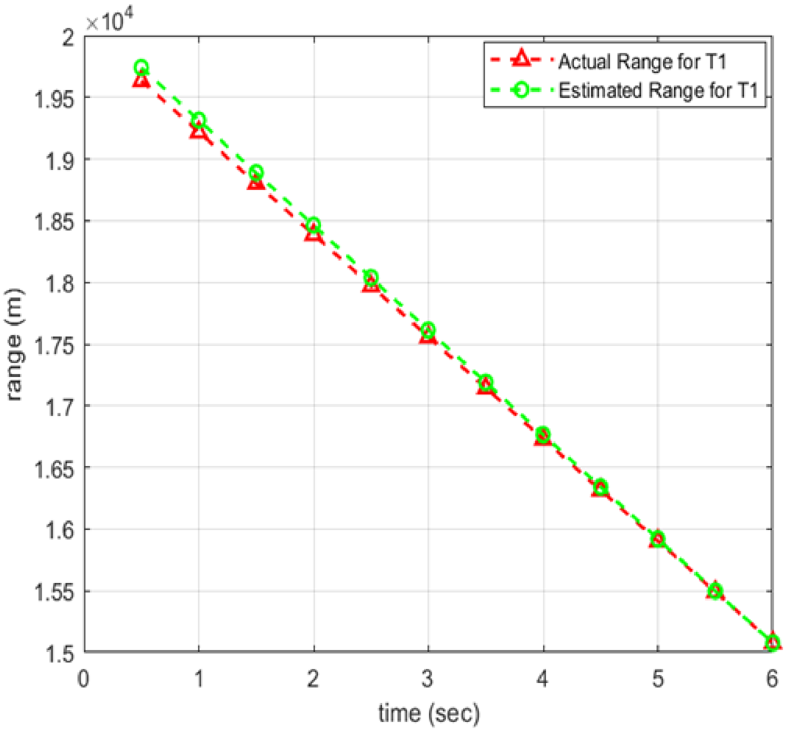

FIGURE 6.15: Range Estimation for Target1

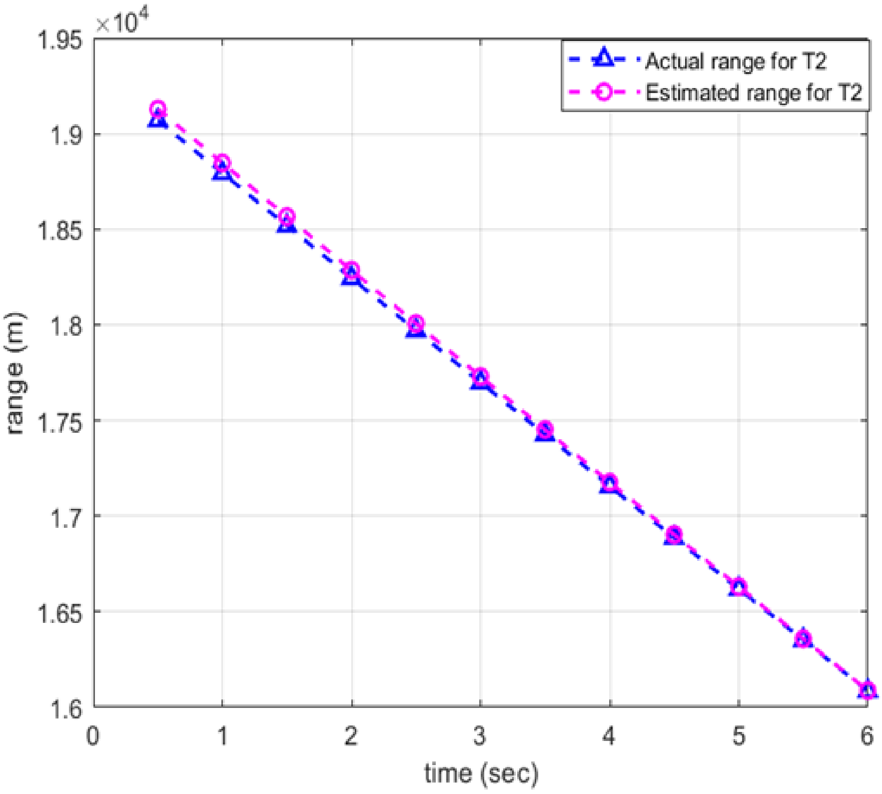

FIGURE 6.16: Range Estimation for Target2

actual and estimated ranges are plotted and are shown in the figure 6.16. The ranges of both targets are decreasing because both targets are heading towards the radar.

6.2.5 Azimuth Angle Tracking

The plots for azimuth angle estimation for both targets are given below:

The figure 6.17 and 6.18 represent the actual azimuth and estimated azimuth angles for both targets. It is clearly visible from both figures that the error between actual and estimated values is decreasing with time. The estimation error is computed in cartesian coordinate system.

The estimated parameters of target determined by JPDAF-KF are in cartesian form and are then transformed from cartesian to polar coordinate system.

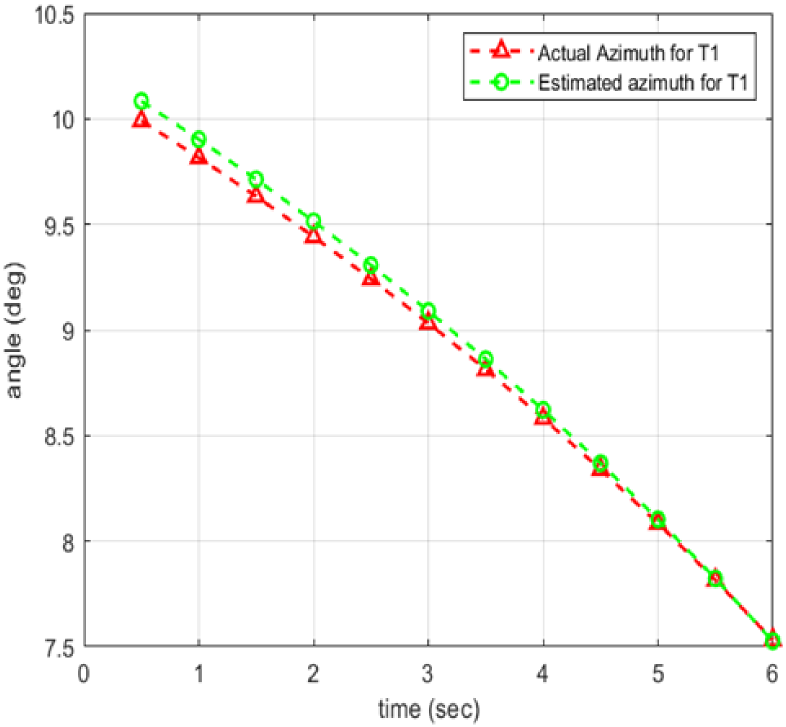

FIGURE 6.17: Azimuth Angle Estimation for Target1

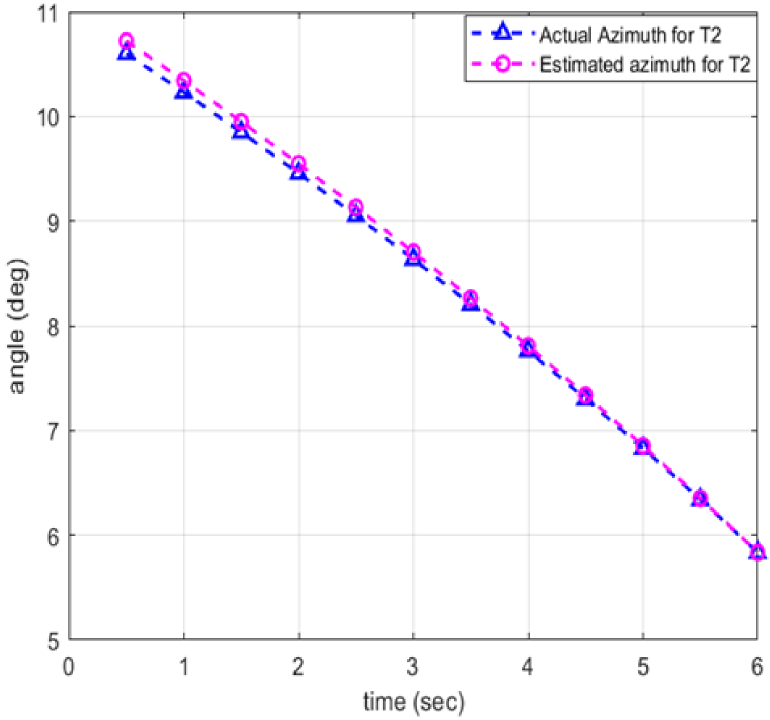

FIGURE 6.18: Azimuth Angle Estimation for Target2

6.2.6 Elevation Angle Tracking

Similarly, the plots for elevation angle estimation for both targets are given below:

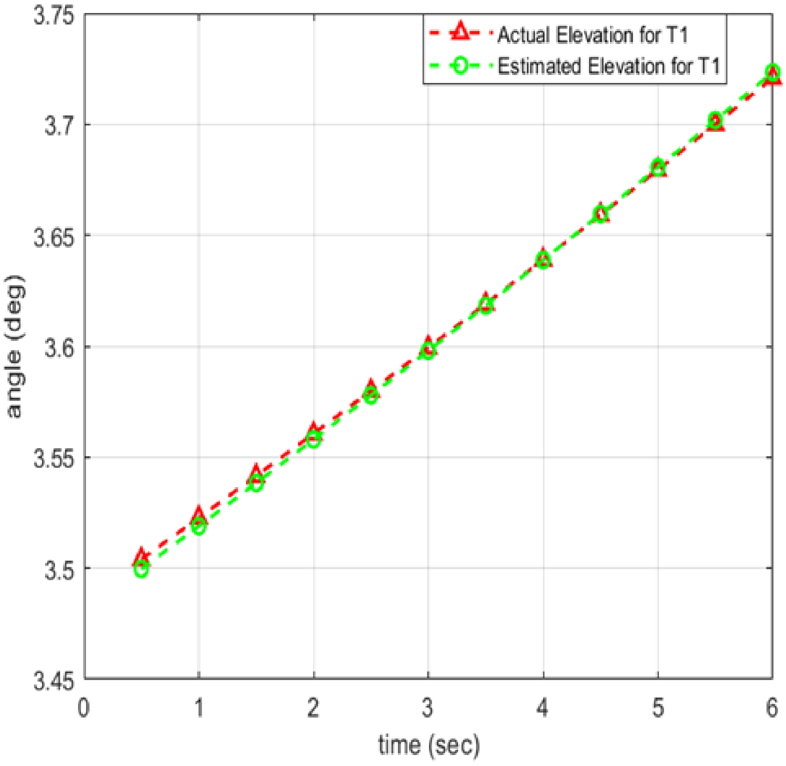

FIGURE 6.19: Elevation Angle Estimation for Target1

The actual and estimated elevation angles of both targets are shown in the figure 6.19 and 6.20 respectively. Elevation angle for both targets is increasing as they are moving towards the radar.

6.2.7 Estimation Error

The estimation error is expressed by the equation 6.2.

$$\text{Estimation Error} = \sqrt{(X_{act} - X_{est})^2 + (Y_{act} - Y_{est})^2 + (Z_{act} - Z_{est})^2} \tag{6.2}$$

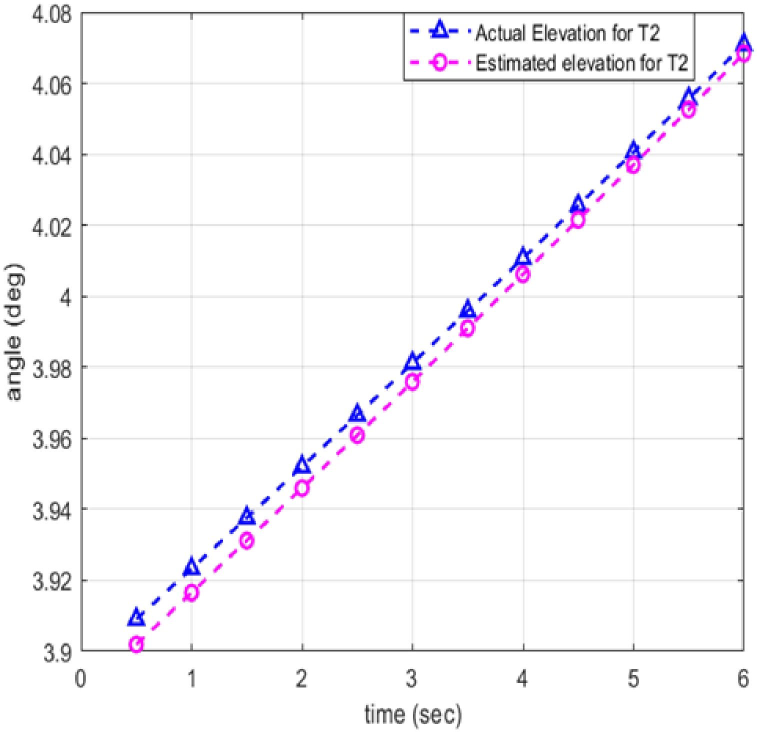

FIGURE 6.20: Elevation Angle Estimation for Target2

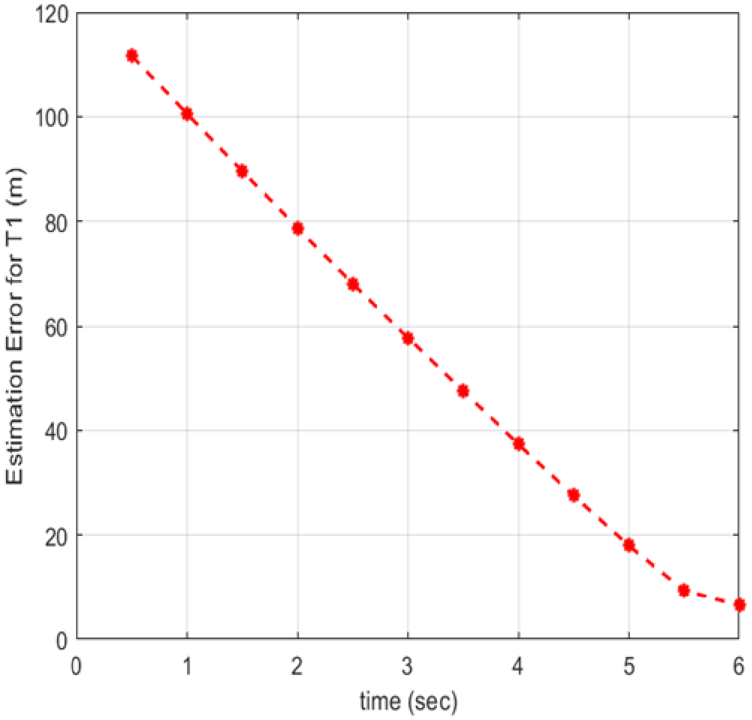

FIGURE 6.21: Estimation Error for Target1

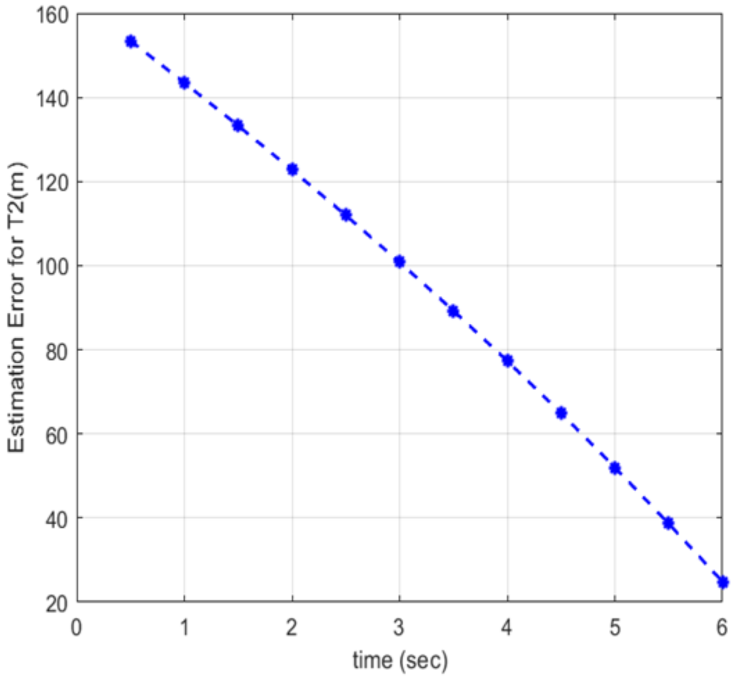

Figure 6.22: Estimation Error for Target2

In the beginning, the estimation error for both targets is more but the error converges after several iterations which is clearly visible from the both graphs in figure 6.21 and 6.22.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

In this thesis, 2D and 3D tracking is performed using different type of radars while using same data association algorithm. 2D tracking is done using TWS radar and the measurement noise covariance matrix R derived using the radar parameters is used in JPDAF algorithm. 3D tracking is performed using monopulse radar and JPDAF algorithm.

The main problem faced during tracking using the JPDAF algorithm was that there was no appropriate method to find out the measurement noise covariance matrix R used in KF. As a result, during the gating process mostly the actual detections fall outside the gates of targets. This problem then resulted in filter divergence after some time. To sort out this problem, appropriate measurement noise covariance matrix R has been derived and used in the simulations of 2D tracking using TWS radar. The desired tracking results are achieved and also the estimation error decreases with time. In case of 3D Closed Loop Tracking using Monopulse radar, measurement noise covariance matrix R is taken as a diagonal matrix and the error converges with the time. Finally the error is reduced to such an extent that it lies inside the range bin for both cases.

7.2 Future Work

For the 3D closed loop tracking using monopulse radar, measurement covariance matrix R can also be derived like the one derived for 2D TWS radar. The tracking will be performed more efficiently and performance of overall system can be improved.

Bibliography

[1] R. C. Watson Jr, *Radar Origins Worldwide: History of its evolution in 13 nations through World War II*, 1st ed. Trafford Publishing, 2009.

[2] F8f-2 Bearcat of vf-111. [Accessed: March 3, 2019]. [Online]. Available: https://commons.wikimedia.org/wiki/File:F8F-2_Bearcat_of_VF-111-on_USS_Valley_Forge_(CV-45)_on_27_April_1949_(80-G-K-9904).jpg

[3] NSSL doppler with Rainshaft. [Accessed: June 28, 2019]. [Online]. Available: https://commons.wikimedia.org/wiki/File:NSSL_Doppler_with_rainshaft_-_NOAA.jpg

[4] Synthetic Aperture Radar. [Accessed: Feb 28, 2019]. [Online]. Available: https://www.lockheedmartin.com/en-us/products/synthetic/aperture-radar.html

[5] Bistatic Radar. [Accessed: April 22, 2019]. [Online]. Available: http://www.telecomabc.com/b/bistatic-radar.html

[6] M. I. Skolnik, *Introduction to Radar Systems*, 2nd ed. New York, McGraw Hill, 1980.

[7] D. D. Howard and M. Skolnik, "Tracking Radar," *Radar handbook,McGraw-Hill*, pp. 18–43, 1990.

[8] G. Welch and G. Bishop, "An introduction to the Kalman Filter," *UNC-CH Computer Science Technical Report*, pp. 41–95, 1995.

[9] S. S. Blackman, "Multiple-target tracking with radar applications," *Dedham, MA, Artech House*, pp. 450–465, 1986.

[10] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation: Theory Algorithms and Software.* John Wiley, 2004.

[11] S. Challa, M. R. Morelande, D. Mušicki, and R. J. Evans, *Fundamentals of object tracking*, 1st ed. Cambridge University Press, 2011.

[12] Y. Bar-Shalom, F. Daum, and J. Huang, “The probabilistic data association filter,” *IEEE Control Systems Magazine*, vol. 29, no. 6, pp. 82–100, 2009.

[13] B. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne III, R. Mahler, and B. Vo, “Multitarget Tracking,” *Wiley Encyclopedia of Electrical and Electronics Engineering*, pp. 1–15, 1999.

[14] V. Vaidehi, N. Chitra, M. Chokkalingam, and C. Krishnan, “Neural Network aided Kalman Filtering for Multitarget Tracking Applications,” *Computers and Electrical Engineering*, vol. 27, no. 2, pp. 217–228, 2001.

[15] M. P. Fritz, “A Comparative Study of Kalman Filter Implementations for Relative GPS Navigation,” Ph.D. dissertation, Texas AM University, 2009.

[16] F. Orderud, “Comparison of Kalman Filter Estimation Approaches for state space models with nonlinear measurements,” *Proc. of Scandinavian Conference on Simulation and Modeling*, pp. 1–8, 2005.

[17] G. A. Einicke and L. B. White, “Robust Extended Kalman Filtering,” *IEEE Transactions on Signal Processing*, vol. 47, no. 9, pp. 2596–2599, 1999.

[18] Y. Hao, Z. Xiong, F. Sun, and X. Wang, “Comparison of Unscented Kalman Filters,” *IEEE International Conference on Mechatronics and Automation*, pp. 895–899, 2007.

[19] Y. Bar-Shalom, *Tracking and data association.* Academic Press Professional, Inc., 1987.

[20] P. Konstantinova, A. Udvarev, and T. Semerdjiev, “A Study of a Target Tracking Algorithm using Global Nearest Neighbor approach,” *Proceedings*

of the International Conference on Computer Systems and Technologies, pp. 290–295, 2003.

[21] B. K. Habtemariam, "Effective Data Association algorithms for Multitarget Tracking," Ph.D. dissertation, MC Master University Canada, 2014.

[22] J. Neira and J. D. Tardos, "Data Association in Stochastic Mapping using the Joint Compatibility Test," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 6, pp. 890–897, 2001.

[23] Target tracking. [Accessed: May 28, 2019]. [Online]. Available: http://www.control.isy.liu.se/student/graduate/targettracking/file/le4_handout.pdf

[24] B. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne III, R. Mahler, and B. Vo, "Multitarget Tracking," *Wiley Encyclopedia of Electrical and Electronics Engineering*, pp. 1–15, 1999.

[25] S. Oh, S. Russell, and S. Sastry, "Markov Chain Monte Carlo Data Association for Multi-Target Tracking," *IEEE Transactions on Automatic Control*, vol. 54, no. 3, pp. 481–497, March 2009.

[26] S. Silven, "A Neural Network for Data Association in a Multiple-target Tracking System," *IEEE Conference on Neural Networks for Ocean Engineering*, vol. 52, pp. 133–141, 1991.

[27] A. Almagbile, J. Wang, and W. Ding, "Evaluating the performances of Adaptive Kalman Filter methods in GPS integration," *Journal of Global Positioning Systems*, vol. 9, no. 1, pp. 33–40, 2010.

[28] X. R. Li and V. P. Jilkov, "Survey of Maneuvering Target Tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1333–1364, 2003.

[29] X. R. Li, "Survey of Maneuvering Target Tracking," in *Signal and Data Processing of Small Targets 2000*, vol. 4048. International Society for Optics and Photonics, 2000, pp. 212–235.

[30] P. Agarwal, D. Federman, K. Ge, T.-H. Jan, C. Jones, Y. Lou, J. Ma, S. Modi, and Shi, "Kalman Filtering for Target Tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 54, pp. 230–250, 2009.

[31] Electronic Warfare and Antenna scan types. [Accessed: April 12, 2019]. [Online]. Available: http://fullafterburner.weebly.com/next-gen-weapons/electronic-warfare-antenna-scan-types

[32] R. Poisel, *Electronic Warfare Target Location Methods*, 2nd ed. Artech House, 2012.

[33] New Images Beam. [Accessed: June 22, 2019]. [Online]. Available: http://www.krimherbarerama.com/pencil-beam-radar

[34] A. Graham, *Communications, Radar and Electronic Warfare*, 2nd ed. John Wiley and Sons, 2011.

[35] Advance Communication System Engineering. [Accessed: June 28, 2019]. [Online]. Available: https://www.ques10.com/p/36277/list-the-antenna/tracking-methods-used-in-radar-sy/

[36] Tracking Radar System and Design. [Accessed: June 30, 2019]. [Online]. Available: https://www.tutorialspoint.com/radar_systems/radar_systems_tracking_radar

[37] S. M. Sherman and D. K. Barton, *Monopulse Principles and Techniques*, 2nd ed. Artech House, 2011.

[38] P. Z. Peebles and R. Berkowitz, "Multiple-target Monopulse Radar Processing Techniques," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 30, pp. 845–854, 1968.

[39] H. Rajabloo and F. H. Kashani, "High Power seven-port Microwave duplexer for Monopulse Radar Tracking Applications based on Amplitude discriminator," *IET Microwaves, Antennas and Propagation*, vol. 12, no. 4, pp. 617–621, 2017.

[40] Radar Systems Engineering. [Accessed: June 28, 2019]. [Online]. Available: https://www.slideshare.net/Forward2025/radar-2009-a-15/parameter-estimation-and-tracking-part-1

[41] H. Griffiths and A. D. Baker, Christopher, *Stimson's Introduction to Airborne Radar*. Scitech Pub Incorporated, 2014.

[42] Y. Zheng, S.-M. Tseng, and K.-B. Yu, "Closed-form Four-channel Monopulse two-target resolution," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 3, pp. 1083–1089, 2003.