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MHD Flow of Nanofluid with Joule Heating and Arrhenius Activation Energy

by

Muhammad Adnan Manzoor

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This thesis is dedicated to my beloved, **family** specially,

My Mother.

A determined and aristocratic embodiment who educate me to belief in ALLAH, believe in hard work and that so much could be done little.

Manzoor Ahmed Khan

For my father I quote the remarkable words of Hadith,

“A father gives his child nothing better than an education.”

CAPITAL UNIVERSITY OF SCIENCE & TECHNOLOGY
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Abstract

The main objective of this thesis is to focus on numerical study of chemical reaction and heat generation/absorption in a porous medium on boundary layer flow of Carreau nanofluid over a nonlinearly stretching sheet. Meanwhile, the ordinary differential equations are obtained by applying the similarity appropriate transformation on the governing partial differential equations. The resulting system of ordinary differential is solved numerically by using shooting method and obtained the numerical results. This dissertation investigates the effects of physical parameters like heat source ($\lambda > 0$), chemical reaction parameter (γ_1), porosity parameter (K_1), the sink parameter ($\lambda < 0$) and Biot number (Bi) on the flow velocity, nanofluid volume fraction and temperature. Effect of MHD, Joule heating and Arrhenius activation energy are also discussed. The numerical values of skin friction coefficient, Nusselt number and Sherwood number are also computed.

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Abbreviations

FEM	Finite Element Method
FVM	Finite Volume Method
GFEM	Galerkin Finite Element Method
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations

Symbols

b	Constant
Bi	Biot Number
C	Nanoparticle Volume Fraction
C_p	Specific Heat (j/kgK)
C_w	Concentration at the Surface (moles/kg)
C_∞	Ambient Volume Concentration
C_{fx}	Skin Friction Coefficient
D_B	Brownian Diffusion $(\frac{m^2}{s})$
D_T	Thermophoresis $(\frac{m^2}{s})$
f	Dimensionaless Stream Funcation $(\frac{W}{m^2K})$
h_f	Heat Transfer Coefficient $(\frac{W}{m^2K})$
K	Permeability of Porous Medium
K_1	Porosity Parameter
k	Thermal Conductivity $\frac{W}{mK}$
Le	Lewis Number
m	Stretching Parameter
n	Power Law Index
N_b	Brownian Motion Parameter
N_t	Thermophoresis Parameter
Pr	Prandtl Number
Q_0	Heat Absorbtion/Generation Coefficient
R_1	Chemical Reaction
Re_a	Local Reynolds Number

T	Temperature of Fluid
T_∞	Ambient Temperature (K)
T_w	Temperature at the Surface (K)
u, v	Velocity Components
u_w	Stretching Sheet Velocity $(\frac{m}{s})$
W_e	Local Weissenberg Number
x, y	Space Coordinates (m)
M	Magnetic Field Parameter
Sh_x	Local Sherwood Number
Nu_x	Local Nusselt Number
Ec	Eckert Number
E	Activation Energy

Greek symbols

α	Thermal Diffusivity $(m^2 s^{-1})$
λ	Heat Sourse $(\lambda > 0)$ or Sink $(\lambda < 0)$
γ_1	Chemical Reaction Parameter
θ	Nondimensional Temperature $\left(\frac{T-T_c}{T_h-T_c}\right)$
ρ	Density (kgm^{-3})
σ	Electrical Conductivity
ν	Kinematic Viscosity $(m^2 s^{-1})$
ΔT	Temperature Gradient (K)
γ_2	Temperature Difference Parameter
ρ	Density $(\frac{kg}{m^3})$
Γ	Relaxation Parameter
(ρc_f)	Heat Capacity of the Base Fluid $(\frac{j}{m^3 K})$
(ρc_p)	Heat Capacity of the Nanoparticle $(\frac{j}{m^3 K})$
μ	Viscosity (Pas)
μ_0	Zero Shear Viscosity (Pas)
μ_∞	Infinity Shear Viscosity (Pas)
ψ	Stream Function
η	Dimensionless Similarity Variable

θ	Dimensionless Temperature
ϕ	Dimensionless Concentration
τ	The Ratio of Heat Capacities
τ_w	Surface Shear Stress (Pa)
Subscripts	
∞	Ambient Condition
w	Condition on Surface
f	Base Fluid

Chapter 1

Introduction

A substance in the gas or liquid phase is named as the fluid. Flow of fluid has all types of aspects, compressible and incompressible, steady and unsteady, viscous and inviscid, uniform and non uniform, rotational and irrational, Meir [1]. The fluid flow analysis on a stretching surface is one of the essential problems of the modern period as it exists in various engineering and technology processes. Metal, extrusion, spining, wired drawing, manufacturing of rubber sheets, food manufacturing and cooling of vast metallic plates like electrolyte are the common examples. Sakialdis [2] was the first who initiated the problem of boundary layer approximations over stretching surface. He analyzed the non Newtonian Maxwell fluid with nano materials over exponentially stretched surface. The flow caused by the stretching sheet was investigated by Crane [3]. He examined the behaviour of boundary layer on the continuous surface. In recent time, numerious researchers such as Shehzad et al. [4], Zheng et al. [5] and Gireesha et al. [6] have been involved in investigating the phenomenon of the fluid flow through stretching surface. They analyzed the impact of magnetic field parameter and predicted to reduce the velocity of the fluid. The fluids play a vital role in heat transfer. The small solid particle is termed as nanoparticle, the range of such nanoparticles is from 1-100 nanometer in size. The homogenous mixture of the base fluid and nanoparticles is called nanofluid. In 1995, Choi [7] intiated the term of nanofluid in his pioneering work. Due to its prospective engineering application, detailed work on this topic is carried out by various researchers. The nanotechnology has a vast range of application in the fields of science and technology in modern developments. Recently, the improvement in nanotechnology has increased exponentially. Malvandi and Ganji [8] observed the impact of nanoparticles movement on the forced convection in a channel for Alumina. They analyzed that the suction from the surface enhances the rate of heat transfer, while the blowing reduce the rate of heat transfer. The critical observation of characterstics of nanofluids was carried out by Khanafer and Vafai [9]. It was found that the effective viscosity of nanofluids increased with an increment in the volume fraction and decreased with temperature increment. Furthermore, copper water nanofluid in porous plates is investigated by Sureshkumar and Muthtamilselvan [10]. They observed that with the growth of solid volume fraction in porous cavity, the average Nusselt number increases. Nagarajana and Akbar [11] debated the heat transfer improvement of copper water nanofluid flow in a porous square enclosure driven on moving plate. The analysis for the driven cavity flow with various properties of heat exchange in nanofluid can be noticed in [12, 13]. Ho et al. [14] explored the numerical solutions utilizing finite volume and finite difference method for convective heat transfer in nanofluids. The numerical simulation is achived for nanofluid, they studied the impact of thermal conductivity and viscosity in nanofluids. Recentlly, the nanofluid flow through various shaped geometries has acquired the attention in different fields. Hsiao [15] carried out the electrical magnetohydrodynamics Carreau and micropolar nanofluid flow with impact of different parameters. Hydromagnetic flow problem with magnetic and viscous dissipation effects of micropolar nanofluids over a stretching surface as an applied thermal system for heat and mass transfer and energy management was studied by [16]. An experiment was performed to investigate the issue of convective boundary layer flow through a nonlinear stretched surface in the existence of yield stress in a porous media [17, 18]. The analysis of peristaltic of a Carreau fluid with chemical reaction has motivated the interest of several researchers due to its broad uses in science and engineering like Biochemistry, diagnostic therapy, neurology and treatment for cancer. The Carreau model lies in the category of non Newtonian fluid models with low and high shear levels for which the constituent relationship accumulate. Because of this fact it has achieved extensive acceptance. For the explanation of the non Newtonian fluids several experimental terms have been proposed, based on their various characteristics obtained by Bird et al. [19]. Among these the rheological model of Carreau is a subordinate category of generalized Newtonian fluids [20]. Due to distinct application of Carreau model in engineering and technology, various researchers have worked on properties of such kinds of model. Hayat et al. [21] illustrated the flow properties of Careau fluid along a stretching sheet. Additionally, different researchers have been investigating the Carreau fluid model for various flow problems and present analysis deals with the study of Carreau fluid in two-dimensional MHD flow over a stretched sheet component [22, 23]. Laterly, Martins et al. [24] investigated the numerical analysis of shear thinning axisymmetric flow impacts of a Carreau fluid. Olajuwon [25] numerically illustrated the heat and mass exchange in a hydro magnetic Carreau fluid with radiation and thermal diffusion. Tshehla [26] analyzed the free surface of the Carreau fluid flowing down in the inclined plane. Recentlly, MHD flow of viscoelastic fluids with and without heat exchange over a stretching sheet has been addressed by various researchers [27, 28]. Bhattacharyya et al. [29] examined the convection flow of boundary layer force and transfer of heat past a porous plate with velocity and temperature slip affect. Das [30] investigated the effect of partial slip, chemical reaction, thermal radiation and temperature dependent fluid characteristics with constant heat flux over a premeable plate and non uniform heat source/sink. The impact of partial slip, heat generation thermal buoyancy and heat transfer of nanofluid past a stretching sheet was examined by Das [30]. Amineraza et al. [31] analysed the impact of partial slip on flow and transfer of heat of nanofluid past a stretching sheet. Zheng et al. [5] examined the impact of velocity slip on MHD flow and transfer of heat over a porous sheet. In recent time, the influence of partial slip flow and heat transfer over a nanofluid stretched sheet was analyzed by Sharma et al. [32]. The existing study focuses on quantitative research in the presence of chemical reaction, inner heat generation and heat absorption of Carreau nanofluid flow over nonlinearly stretching sheet saturated with a porous medium. The govering equations contain thermal energy, momentum and nanoparticles concentration are eased with the help of suitable similarity transformation. The govering PDEs are transformed to ODEs before being numerically solved using Shooting method together with RK4 technique. Tables and graphs are used to analyze the impacts of the porosity parameter, heat source, Biot number, chemically reaction parameters and heat sink on the temperature, velocity and concentration distribution aside from the transfer of mass and heat rates.

1.1 Thesis Contribution

The main purpose of the research work is to explore the numerical investigation of mass and heat transfer in MHD flow with Joule heating effects and Arrhenius activation energy. The nonlinear system of PDEs is converted into an ODEs system and solved by using shooting method. The impact of govering parameters on the temperature, velocity and the concentration distribution are illustrated graphically.

1.2 Thesis Outline

This thesis is further composed of the following chapters:
Chapter 2 demonstrates some important definitions, laws and concepts which are useful in understanding upcoming work.

Chapter 3 provides the details of numerical analysis of research paper by Eid et al. [33]. By using simiarity transformation we reduce the set of nonlinear PDEs into set of nonlinear ODEs and then numerically solved. Results are discussed through graphs and tables.

Chapter 4 extend the work of Eid et al. [33] by considering the additional impact of MHD, activation energy and Joule heating.

Chapter 5 summarizes the overall analysis performed in this dissertation.

Chapter 2

Fundamental Concepts and Governing Equations

Some definitions, basic laws and terminologies would be discussed in the current chapter, which would be used in next chapters.

2.1 Basic Definitions

Definition 2.1.1. (**Fluid**)
"A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be." [34]

Definition 2.1.2. (**Fluid Mechanics**)
"Fluid mechanics is that branch of science which deals with the behavior of the fluid (liquids or gases) at rest as well as in motion." [35]

Definition 2.1.3. (**Fluid Statics**)
"The study of fluid at rest is called fluid statics." [35]

Definition 2.1.4. (**Fluid Dynamics**)
"The study of fluid if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics." [35]

Definition 2.1.5. (**Viscosity**)

"Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. Mathematically,

$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}},$$

where μ is viscosity coefficient, τ is shear stress and $\frac{\partial u}{\partial y}$ represents the velocity gradient." [35]

Definition 2.1.6. (**Kinematic Viscosity**)

"It is defined as the ratio between the dynamic viscosity and density of fluid. Mathematically,

$$\nu = \frac{\mu}{\rho}.$$

It is denoted by symbol ν called **'nu'.**" [35]

Definition 2.1.7. (**Ideal Fluid**)

"A fluid which is incompressible and has no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity." [35]

Definition 2.1.8. (**Real Fluid**)

"A fluid which possesses viscosity, is known as a real fluid. In actual practice, all the fluids are real fluids." [35]

Definition 2.1.9. (**Newtonian Fluid**)

"A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid." [35]

Definition 2.1.10. (**Non-Newtonian Fluid**)

"A real fluid in which the shear stress is not directly proportional to the rate of shear strain (or velocity gradient), is known as a Non-Newtonian fluid." [35]

Definition 2.1.11. (**Hydrodynamics**)

"The study of the motion of fluids that are practically incompressible such as liquids, especially water and gases at low speeds is usually referred to as hydrodynamics." [36]

Definition 2.1.12. (**Magnetohydrodynamics**)
"Magnetohydrodynamics (MHD) is concerned with the flow of electrically conducting fluids in the presence of magnetic fields, either externally applied or generated within the fluid by inductive action." [37]

2.2 Classification of Fluids

Definition 2.2.1. (**Laminar Flow**)
"Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel." [35]

Definition 2.2.2. (**Turbulent Flow**)
"Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way." [35]

Definition 2.2.3. (**Compressible Flow**)
"Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid, Mathematically,

$$\rho \neq b,$$

where b is constant." [35]

Definition 2.2.4. (**Incompressible Flow**)
"Incompressible flow is that type of flow in which the density is constant for the fluid. Liquids are generally incompressible while gases are compressible, Mathematically,

$$\rho = b,$$

where b is constant." [35]

Definition 2.2.5. (**Steady Flow**)

"If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow donot change with respect to time, the flow is said to be steady flow. Mathematically,

$$\frac{\partial Q}{\partial t} = 0,$$

where Q is the rate of flow." [35]

Definition 2.2.6. (**Unsteady Flow**)

"If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes with respect to time, the flow is said to be unsteady. Mathematically,

$$\frac{\partial Q}{\partial t} \neq 0,$$

where Q is the rate of flow." [35]

2.3 Heat Transfer Mechanism and Properties

Definition 2.3.1. (**Radiation**)

"Radiation is the energy transfer due to the release of photons or electromagnetic waves from a surface volume. Radiation doesn't require any medium to transfer heat. The energy produced by radiation is transformed by electromagnetic waves." [34]

Definition 2.3.2. (**Boundary Layer**)

"Viscous effects are particularly important near the solid surfaces, where the strong interaction of the molecules of the fluid with molecules of the solid causes the relative velocity between the fluid and the solid to become almost exactly zero for a stationary surface. Therefore, the fluid velocity in the region near the wall must reduce to zero. This is called no slip condition. In that condition there is no relative motion between the fluid and the solid surface at their point of contact. It follows that the flow velocity varies with distance from the wall; from zero at the wall to its full value some distance away, so that significant velocity gradients are established close to the wall. In most cases this region is thin (compared to the typical body dimension), and it is called a boundary layer." [38]

2.4 Some Important Definitions of Heat Transfer

Definition 2.4.1. (**Heat Transfer**)
"Heat transfer is a branch of engineering that deals with the transfer of thermal energy from one point to another within a medium or from one medium to another due to the occurrence of a temperature difference." [39]

Definition 2.4.2. (**Conduction**)
"The transfer of heat within a medium due to a diffusion process is called conduction." [39]

Definition 2.4.3. (**Convection**)
"Convection heat transfer is usually defined as energy transport effected by the motion of a fluid. The convection heat transfer between two dissimilar media is governed by Newton's law of cooling." [39]

Definition 2.4.4. (**Force Convection**)
"Forced convection heat transfer is induced by forcing a liquid, or gas, over a hot body or surface." [40]

Definition 2.4.5. (**Natural Convection**)
"Natural convection is generated by the density difference induced by the temperature differences within a fluid system and the small density variations present in these types of flows." [40]

Definition 2.4.6. (**Thermal Conductivity**)
"The Fourier heat conduction law states that the heat flow is proportional to the temperature gradient. The coefficient of proportionality is a material parameter known as the thermal conductivity which may be a function of a number of variables." [39]

Definition 2.4.7. (**Thermal Diffusivity**)

"The rate at which heat diffuses by conducting through a material depends on the thermal diffusivity and can be defined as:

$$\alpha = \frac{\kappa}{\rho C_p},$$

where α is the thermal diffusivity, κ is the thermal conductivity, ρ is the density and C_p is the specific heat at constant pressure." [39]

2.4.1 Dimensionless Quantities

Definition 2.4.8. (**Nusselt Number** Nu_x)

"It is the ratio of the convective to the conductive heat transfer at a boundary in a fluid. Mathematically,

$$Nu_x = \frac{hL}{k},$$

where h stands for convective heat transfer, L for the characteristics length and k stands for the thermal conductivity." [34]

Definition 2.4.9. (**Sherwood Number** Nu_x)

"It is the nondimensional quantity which show the ratio of the mass transport by convection to the transfer of mass by diffusion. Mathematically:

$$Sh_x = \frac{KL}{D},$$

here L is characteristics length, D is the mass diffusivity and K is the mass transfer coefficient."

Definition 2.4.10. (**Skin Friction Coefficient** C_{fx})

"The steady flow of an incompressible gas or liquid in a long pipe of internal D. The mean velocity is denoted by u_w. The skin friction coefficient can be defined as

$$C_f = \frac{2\tau_0}{\rho u_w^2},$$

where τ_0 denotes the wall shear stress and ρ is the density." [34]

Definition 2.4.11. (**Prandtl Number** Pr)

"It is the ratio between the momentum diffusivity (ν) and thermal diffusivity (α). Mathematically, it can be defined as

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/c_p} = \frac{\mu C_p}{k},$$

where μ represents the dynamic viscosity, C_p denotes the specific heat and k stands for thermal conductivity. The relative thickness of thermal and momentum boundary layer is controlled by Prandtl number. For small Pr, heat distributed rapidly corresponds to the momentum." [34]

Definition 2.4.12. (**Eckert Number** Ec)

"It is the dimensionless number used in continuum mechanics. It describes the relation between flows and the boundary layer enthalpy difference and it is used for characterized heat dissipation. Mathematically,

$$Ec = \frac{u^2}{c_p \nabla T}," \text{ [34]}$$

Definition 2.4.13. (**Weissenberg Number** We)

"The dimensionless Weissenberg number, formulated by German physicist Karl Weissenberg, is defined as

$$We = \frac{\rho u^2}{\tau}, \tag{2.1}$$

where ρ is the fluid density, u denotes the flow velocity and τ stands for the shear stress. This number expresses the characteristic material time (relaxation time) and the shear velocity. It characterizes the velocity and time relations in rheological processes in viscoelastic shear flow. Furthermore, it also expresses the ratio of the dynamic viscoelastic force to the viscous force." [41]

Definition 2.4.14. (**Biot Number** Bi)

"The Biot number is a dimensionless quantity used in heat transfer calculation. It gives a simple index of the ratio of heat transfer resistance inside of and at the surface of a body. The Biot number is defines as:

$$Bi = \frac{h_h L}{k}, \tag{2.2}$$

where h_h represents the heat transfer coefficient, L denotes the characteristic length and k is the thermal conductivity." [41]

Definition 2.4.15. (**Thermophoresis Parameter** N_t)
"In a temperature gradient, small particles are pushed towards the lower temperature because of the asymmetry of molecular impacts. The resulting force which drives the particles along a temperature gradient towards the lower temperature, is called thermophoretic force and the mechanism thermophoresis." [41]

2.5 Fundamental Equations and Conservation Laws

2.5.1 Law Conservation of Mass

"The principle of conservation of mass can be stated as the time rate of change of mass is fixed volume is equal to the net rate of flow of mass across the surface. Mathematically, it can be written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0.\text{" [39]} \tag{2.3}$$

2.5.2 Law of Conservation of Momentum

"The momentum equation states that the time rate of change of linear momentum of a given set of particles is equal to the vector sum of all the external forces acting on the particles of the set, provided Newton's Third Law of action and reaction governs the internal forces. Mathematically, it can be written as:

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla.[(\rho \mathbf{u})\mathbf{u}] = \boldsymbol{\nabla}.\mathbf{T} + \rho g.\text{" [39]} \tag{2.4}$$

2.5.3 Law of Conservation of Energy

"Energy can neither created nor destroyed, it can be transformed from one form to another form but total amount of an isolated system remains constant. For example energy is conserved over time. It is the fundamental law of physics which is also known as the first law of thermodynamics.
The mathematical form of energy equation in two-dimensional for fluid can be written as,

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{\rho C_p}\Phi^* \tag{2.5}$$

where Φ^* is dissipation function." [39]

2.6 Solution Methodology

"In this method, the differential equation is kept in its nonlinear form and the missing slope is found systematically by Newton's method. This method provides quadratic convergence of the iteration and is far better than the usual cut and try methods. Consider the second-order differential equation

$$y''(x) = f(x, y, y'(x)) \tag{2.6}$$

subject to the boundary conditions

$$y(0) = 0, \; y(L) = C \tag{2.7}$$

By denoting y by y_1 and y_1' by y_2, Eq. (2.6) can be written in the form of following system of first order equations.

$$\left.\begin{aligned} y_1' &= y_2, & y_1(0) &= 0, \\ y_2' &= f(x, y_1, y_2), & y_1(L) &= C. \end{aligned}\right\} \tag{2.8}$$

Denote the missing initial condition by $y_2(0) = k$, to have

$$\left.\begin{aligned} y_1' &= y_2, & y_1(0) &= 0, \\ y_2' &= f(x, y_1, y_2), & y_2(0) &= k \end{aligned}\right\}. \tag{2.9}$$

Now the problem is to find s such that the solution of the IVP (2.9) satisfies the boundary condition $y(L) = C$. In other words, if the solutions of the initial value problem (2.9) are denoted by $y_1(x, k)$ and $y_2(x, k)$, one should search for that value of s which is an approximate root the equation.

$$y_1(L, k) - C = \phi(k_n) = 0. \tag{2.10}$$

To find an approximate root of the Eq. (2.10) by the Newton's method, the iteration formula is given by

$$k_{n+1} = k_n - \frac{\phi(k_n)}{d\phi(k_n)/dk}, \tag{2.11}$$

$$k_{n+1} = k_n - \frac{y_1(L, k_n) - C}{dy_1(L, k_n))/dk}. \tag{2.12}$$

To find the derivatives of y_1 with respect of s, differentiate (2.9) with respect to s. For simplification, use the following notations

$$\frac{dy_1}{dk} = y_3, \quad \frac{dy_2}{dk} = y_4 \tag{2.13}$$

$$\left.\begin{aligned} y_3' &= y_4, & y_3(0) &= 0, \\ y_4' &= \frac{\partial f}{\partial y_1} y_3 + \frac{\partial f}{\partial y_2} y_4, & y_4(0) &= 1. \end{aligned}\right\} \tag{2.14}$$

Now, solving the IVP Eq. (2.14), the value of y_3 at L can be computed. This value is actually the derivative of y_1 with respect to s computed at L. Using the value of $y_3(L, k)$ in Eq. (2.12), the modified value of k can be achieved. This new value of k is used to solve the Eq. (2.9) and the process is repeated until the value of k is within a described degree of accuracy." [42]

Chapter 3

Numerical Study for Carreau Nanofluid with Chemically Species over Convectively Heated Nonlinear Stretching Surface

3.1 Introduction

The numerical analysis of Carreau nanofluid flow towards nonlinear stretching surface with chemically reactive species has been performed in this chapter. Furthermore by using shooting technique, the solution of ODEs is obtained. At the end of this chapter the numerical outcomes against various parameters have been discussed. The detailed review study of Eid et al. [33] is explained in this unit.

3.2 Mathematical Modeling

We consider a 2D steady flow and heat exchange of Carreau nanofluid flow in porous medium $y > 0$ moving over a nonlinear stretching surface under the impact of chemical reaction. The sheet is stretching along x-axis with speed $u_w = bx$ where as m is stretching parameter and b is a stretching constant, the surface sheet temperature is T_w, convective fluid temperature T_f, h_f gives heat transfer coefficient. At nonlinear stretching sheet surface, the nanoparticles fraction C and temperature T are represented by C_w and T_w. At $y \to \infty$ the ambient values C and T are symbolized by C_∞ and T_∞.

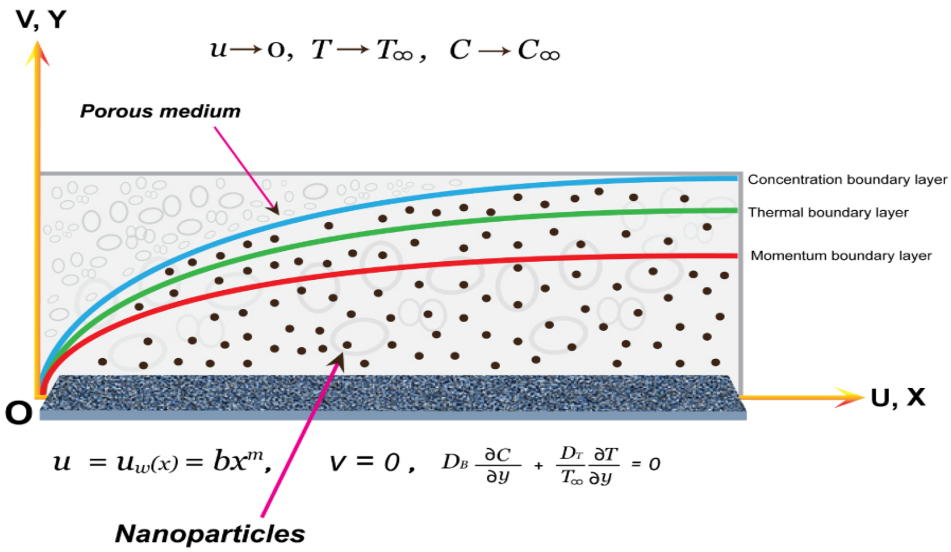

Figure 3.1: Flow configration and coordinate system

3.3 The Governing Equations

The flow is explained by considering the $2D$ governing equations containing the continuity, momentum, energy and concentration are as follow:

- Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3.1}$$

- Momentum Equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2}\left[1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-1}{2}}-\left(\frac{v}{k}\right)u$$
$$+v(n-1)\Gamma^2\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2\left[1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-3}{2}}. \tag{3.2}$$

- Temperature Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y})\right)^2\right]+\frac{Q_0}{(\rho c_p)_f}\left(T-T_\infty\right). \tag{3.3}$$

- Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-R_1(C-C_\infty). \tag{3.4}$$

Boundary Conditions

$$\left.\begin{array}{l} u=u_w(x)=bx^m,\ v=0,\ k\dfrac{\partial T}{\partial y}=-h_f(T_w-T),\ D_B\dfrac{\partial C}{\partial y}+\dfrac{D_T}{T_\infty}\dfrac{\partial T}{\partial y}=0 \text{ at } y=0, \\ u\to 0, T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty. \end{array}\right\}$$

The velocity component along x and y-axis are denoted by u and v, ν denotes kinematic viscosity, the base fluid density is denoted by ρ, permeability of porous medium is denoted by K, material constant is denoted by γ, power law index is denoted by n, $\alpha=\frac{k}{\rho c_p}$ denotes thermal diffusivity, the specific heat is denoted by c_p, thermal conductivity of fluid is denoted by k, dimensional heat generation coefficient is denoted by Q_0, thermophoresis diffusion coefficient is denoted by D_T, τ denotes the ratio of heat capacities.

Following similarity transformation has been introduced for transforming the mathematical model Eq. (3.1) to Eq. (3.4) into nondimensional form [33]

$$\left.\begin{aligned} &\psi(x,y)=\sqrt{\frac{2\nu b}{m+1}}x^{\frac{m+1}{2}}f(\eta), \quad \eta=y\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}},\\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\\ &u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}. \end{aligned}\right\} \tag{3.5}$$

The velocity components are composed as:

$$u=\frac{\partial\psi}{\partial y}=bx^m f', v=-\frac{\partial\psi}{\partial x}=-\sqrt{\frac{\nu(m+1)}{2}})bx^{\frac{m-1}{2}}\left[f'\left(\frac{m-1}{m+1}\right)\eta+f\right]. \tag{3.6}$$

We differentiate the above Eq. (3.6) w.r.t 'x' and 'y',

$$\frac{\partial u}{\partial x}=b\left[mx^{m-1}f'(\eta)+x^m f''(\eta)\frac{\partial\eta}{\partial x}\right],$$

$$\frac{\partial u}{\partial x}=b\left[x^m f''(\eta)\frac{\partial\eta}{\partial x}+mx^{m-1}f'(\eta)\right],$$

$$\frac{\partial u}{\partial x}=b\left[x^m f''(\eta)\left(\frac{m-1}{2}\right)\eta x^{-1}+mx^{m-1}f'(\eta)\right],$$

$$\frac{\partial u}{\partial x}=bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right]. \tag{3.7}$$

Similarly, we have

$$\frac{\partial v}{\partial y}=-\sqrt{\frac{(m+1)\nu b}{2}}x^{\frac{m-1}{2}}\left[f'(\eta)\frac{\partial\eta}{\partial y}+\left(\frac{m-1}{m+1}\right)f''(\eta)\eta\frac{\partial\eta}{\partial y}\eta+\left(\frac{m-1}{m+1}\right)f'(\eta)\frac{\partial\eta}{\partial y}\right],$$

$$\frac{\partial v}{\partial y}=-\frac{b(m+1)}{2}x^{m-1}\left[f'(\eta)+\left(\frac{m-1}{m+1}\right)f''(\eta)\eta+\left(\frac{m-1}{m+1}\right)f'(\eta)\right].$$

$$\frac{\partial v}{\partial y}=-bx^{m-1}\left(\frac{m+1}{2}\right)\left[\frac{(m+1)f'\eta+(m-1)f''(\eta)\eta+(m-1)f'(\eta)}{m+1}\right],$$

$$\frac{\partial v}{\partial y}=-bx^{m-1}mf'(\eta)-\frac{b(m-1)}{2}x^{m-1}\eta f''(\eta),$$

$$\frac{\partial v}{\partial y}=-bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right]. \tag{3.8}$$

From Eq. (3.7) and Eq. (3.8), we satisfy the continuity equation,

$$bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right]-bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right]=0. \tag{3.9}$$

Now we include the procedure for the conversion of Eq. (3.2) and Eq. (3.3) into dimensionless form.

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= v\frac{\partial^2 u}{\partial y^2}\left[1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-1}{2}}-\left(\frac{v}{k}\right)u \\ &\quad +v(n-1)\Gamma^2\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2\left[1+\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-3}{2}} \end{aligned} \tag{3.10}$$

In order to transform Eq. (3.2) and Eq. (3.3). We differentiate the above Eq. (3.6) w.r.t 'x'

$$\begin{aligned} \frac{\partial u}{\partial x} &= b\left[x^m f''(\eta)\frac{\partial \eta}{\partial x}+mx^{m-1}f'(\eta)\right], \\ \frac{\partial u}{\partial x} &= b\left[x^m f''(\eta)\left(\frac{m-1}{2}\right)\eta x^{-1}+mx^{m-1}f'(\eta)\right], \\ \frac{\partial u}{\partial x} &= bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right]. \end{aligned} \tag{3.11}$$

Similarly differentiate Eq. (3.6) w.r.t 'y', we have

$$\begin{aligned} \frac{\partial v}{\partial y} &= -bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta+mf'(\eta)\right], \\ \frac{\partial u}{\partial y} &= bx^m f''(\eta)\frac{\partial \eta}{\partial y}, \\ \frac{\partial u}{\partial y} &= bx^m\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}f''(\eta), \end{aligned} \tag{3.12}$$

again differenting (3.13)

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= bx^m\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}f'''(\eta)\frac{\partial \eta}{\partial y}, \\ \frac{\partial^2 u}{\partial y^2} &= bx^m\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}f'''(\eta)\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}, \end{aligned}$$

$$\begin{aligned}\frac{\partial^2 u}{\partial y^2} &= b^2 x^{2m-1}\left(\frac{m+1}{2\nu}\right) f'''(\eta),\\ \frac{\partial^2 u}{\partial y^2} &= b^2 x^{2m-1}\left(\frac{2m+1}{2\nu}\right) f'''(\eta).\end{aligned} \tag{3.14}$$

Now,

$$\begin{aligned}u\frac{\nu}{K} &= \frac{\nu}{K} b f' x^m,\\ u\frac{\nu}{K} &= b^2 x^{2m-1}\left(\frac{\nu b x^m f'}{K b^2 x^{2m-1}}\right),\\ u\frac{\nu}{K} &= b^2 x^{2m-1}\left(\frac{\nu f'}{K b x^{m-1}}\right),\\ u\frac{\nu}{K} &= b^2 x^{2m-1}(K_1 f').\\ K_1 &= \frac{\nu}{K(b u_w^{m-1})^{1/m}}.\end{aligned} \tag{3.15}$$

Using Eq. (3.11) to Eq. (3.15) in Eq. (3.10)

$$\left(\frac{m+1}{2}\right)\left[1+nWe^2(f'')^2\right]\left[1+We^2(f'')^2\right]^{\frac{n-3}{2}} f''' - m(f')^2 + \left(\frac{m+1}{2}\right) f''f \\ -K_1 f' = 0,$$

multiplying with $\left(\frac{2}{m+1}\right)$, we get

$$\left[1+nWe^2(f'')^2\right]\left[1+We^2(f'')^2\right]^{\frac{n-3}{2}} f''' + f''f - \left(\frac{2}{m+1}\right)\left[m(f')^2 + K_1 f'\right] = 0,$$

$$f''' = -\frac{ff'' - \left(\frac{2}{m+1}\right)(mf'^2 + K_1 f')}{\left[1+nWe^2(f'')^2\right]\left[1+We^2(f'')\right]^{\frac{n-3}{2}}}. \tag{3.16}$$

Now for Eq. (3.3), we proceed as follows (3.17)

$$\theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \tag{3.18}$$

$$\phi(\eta) = \frac{C-C_\infty}{C_w - C_\infty}, \tag{3.19}$$

$$C = C_\infty + (C_w - C_\infty)\,\phi, \tag{3.20}$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta). \tag{3.21}$$

Differentiating Eq. (3.21) w.r.t 'x' and 'y' respectively,

$$\begin{aligned}
\frac{\partial T}{\partial x} &= (T_w - T_\infty)\theta' \frac{\partial \eta}{\partial x}, \\
\frac{\partial T}{\partial x} &= (T_w - T_\infty)\theta' y\sqrt{\frac{b(m+1)}{2\nu}} x^{\frac{m-1}{2}} x^{-1}, \\
\frac{\partial T}{\partial x} &= (T_w - T_\infty)\theta' \left(\frac{m-1}{2x}\right)\eta. \qquad (3.22)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial T}{\partial y} &= (T_w - T_\infty)\theta' \frac{\partial \eta}{\partial y}, \\
\frac{\partial T}{\partial y} &= \theta'(\eta)(T_w - T_\infty)\sqrt{\frac{b(m+1)}{2\nu}} x^{\frac{m-1}{2}}. \qquad (3.23)
\end{aligned}$$

$$\frac{\partial^2 T}{\partial y^2} = \theta'' \sqrt{\frac{b(m+1)}{2\nu}} x^{\frac{m-1}{2}} \sqrt{\frac{b(m+1)}{2\nu}} x^{\frac{m-1}{2}} (T_w - T_\infty). \qquad (3.24)$$

Similarly for equation Eq. (3.20), we have

$$\frac{\partial C}{\partial x} = y\sqrt{\frac{b(m+1)}{2\nu}} \left(\frac{m-1}{2}\right) x^{\frac{m-3}{2}} (C_w - C_\infty)\phi'(\eta). \qquad (3.25)$$

$$\frac{\partial C}{\partial y} = x^{\frac{m-1}{2}} (C_w - C_\infty)\sqrt{\frac{b(m+1)}{2\nu}} \phi'(\eta). \qquad (3.26)$$

$$\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)\phi''(\eta)\left(\frac{b(m+1)}{2\nu}\right) x^{m-1}. \qquad (3.27)$$

From Eq. (3.3), we get

$$\begin{aligned}
&- b\left(\frac{m+1}{2}\right)(T_w - T_\infty) f\theta' x^{m-1} = \alpha (T_w - T_\infty)\theta'' \frac{b(m+1)}{2\nu} \\
&+ \tau\Bigg[D_B((C_w - C_\infty))(T_w - T_\infty)\theta'\phi' b(\frac{m+1}{2\nu})x^{m-1} + \frac{D_T}{T_\infty}(T_w - T_\infty)^2 \\
&b\left(\frac{m+1}{2\nu}\right) x^{m-1}(\theta')^2\Bigg] + \frac{Q_0}{(\rho c_p)_f}\frac{(T_w - T_\infty)\theta}{x^{m-1}},
\end{aligned}$$

dividing by $\frac{2}{b(m+1)(T_w-T_\infty)(x^{m-1})}$, we have

$$\begin{aligned}
-f\theta' &= \frac{\alpha}{\nu}\theta'' + \frac{\tau D_B(C_w-C_\infty)}{\nu}\theta'\phi' + \tau\frac{D_T}{T_\infty}\frac{(T_w-T_\infty)\theta'^2}{\nu} \\
&+ \frac{Q_0}{(\rho c_p)_f}\frac{2\theta}{(m+1)x^{m-1}},
\end{aligned}$$

$$\frac{\theta''}{Pr} + f\theta' + N_b\phi'\theta' + N_t\theta'^2 + \frac{Q_0}{(\rho c_p)_f}\frac{2\theta}{(m+1)x^{m-1}} = 0,$$

$$\frac{\theta''}{Pr} + f\theta' + N_b\phi'\theta' + N_t\theta'^2 + \frac{Q_0}{(\rho c_p)_f(u_w)^{m-1}}\frac{2\theta}{(m+1)} = 0,$$

$$\frac{\theta''}{Pr} + f\theta' + N_t\theta'^2 + \frac{2\theta\lambda}{(m+1)} + N_b\phi'\theta' = 0. \tag{3.28}$$

Now using Eq. (3.20) to Eq. (3.27) in Eq. (3.4), we have

$$\begin{aligned}
&bx^m f'(\eta)y\sqrt{\frac{b(m+1)}{2\nu}}\left(\frac{m-1}{2}\right)x^{\frac{m-3}{2}}(C_w-C_\infty)\phi'(\eta) \\
&+\left[-\sqrt{\frac{b(m+1)\nu x^{m-1}}{2}}\left(\frac{m-1}{m+1}\right)\eta f' - \sqrt{\frac{b(m+1)\nu x^{m-1}}{2}}f\right] \\
&\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}(C_w-C_\infty)\phi'(\eta) = \frac{D_B(C_w-C_\infty)}{2\nu} \\
&x^{m-1}b(m+1)\phi''(\eta) + \frac{D_T(T_w-T_\infty)}{T_\infty}\frac{b(m+1)}{2\nu}x^{m-1}\theta''(\eta) - R_1(C_w-C_\infty)\phi(\eta),
\end{aligned}$$

$$\begin{aligned}
&y\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}\left(\frac{m-1}{2}\right)x^{m-1}f'\phi' - \eta(C_w-C_\infty)f'\phi'\frac{b(m+1)}{2}x^{m-1}\left(\frac{m-1}{m+1}\right) \\
&-\frac{b(m+1)}{2}x^{m-1}(C_w-C_\infty)f\phi' = \frac{b(m+1)}{2}(C_w-C_\infty)x^{m-1}\left[\frac{D_B}{\nu}\phi''(\eta)\right. \\
&\left.+\frac{D_T(T_w-T_\infty)}{\gamma T_\infty(C_w-C_\infty)}\theta'' - R_1\frac{2}{b(m+1)x^{m-1}}\phi(\eta)\right],
\end{aligned}$$

$$\begin{aligned}
&-\frac{b(m+1)}{2}(C_w-C_\infty)x^{m-1}f\phi' = \frac{b(m+1)}{2}(C_w-C_\infty)x^{m-1}\left[\frac{D_B}{\gamma}\phi''(\eta)\right. \\
&\left.+\frac{D_T}{\gamma T_\infty}\frac{T_w-T_\infty}{C_w-C_\infty}\theta'' - \frac{2R_1}{b(m+1)x^{m-1}}\phi(\eta)\right],
\end{aligned}$$

$$\frac{D_B}{\gamma}\phi''(\eta)+\left(\frac{T_w-T_\infty}{C_w-C_\infty}\right)\frac{D_T}{T_\infty}\frac{\theta''}{\gamma}-\frac{2}{m+1}\frac{R_1}{bx^{m-1}}\phi+f\phi'=0,$$

$$\frac{1}{LePr}\phi''+\frac{N_t}{N_b}\theta''-\frac{2}{m+1}\gamma_1\phi+f\phi'=0,$$

$$\phi''+\frac{N_t}{N_b}\theta''-\frac{2}{m+1}\gamma_1\phi LePrf\phi'=0. \tag{3.29}$$

where porosity parameter is K_1, heat source is $(\lambda>0)$ or sink parameter $(\lambda<0)$, local Weissenberg number is We, Prandtl number is Pr, Brownian motion parameter is N_b, thermophoresis parameter is N_t, Lewis number is Le, chemical reaction paramer is γ_1, Biot number is Bi, these parameter have the following values:

$$\left.\begin{array}{l} K_1=\dfrac{\nu}{K(bu_w^{m-1})^{\frac{1}{m}}}, We^2=\dfrac{b^3(m+1)\Gamma^2}{2\nu}x^{3m-1}, \lambda=\dfrac{Q_0}{(\rho c_p)_f((bu_w^{m-1})^{\frac{1}{m}})},\\ Pr=\dfrac{\nu}{\alpha}, N_b=\tau\dfrac{D_B}{\nu}(C_w-C_\infty), N_t=\tau\dfrac{D_T}{T_\infty\nu}(T_w-T_\infty),\\ Le=\dfrac{\alpha}{D_B}, \gamma_1=\dfrac{\nu R_1}{D_B((bu_w^{m-1})^{\frac{1}{m}})}, Bi=\left(\dfrac{h_f}{x^{\frac{m-1}{2}}K\sqrt{\frac{b(m+1)}{2\nu}}}\right). \end{array}\right\}$$

Now convert given boundary conditions into dimensionless boundary conditions:

- $u=bx^m \quad at \quad y=0$

$$u=bx^m f'(\eta),$$

$$\Rightarrow bx^m=bx^m f'(\eta),$$

$$\Rightarrow f'(0)=1.$$

- $v=0,$

$$v=-\sqrt{\frac{b(m+1)}{2}}x^{\frac{m-1}{2}}\left[f'\left(\frac{m-1}{m+1}\right)\eta+f\right],$$

$$\Rightarrow f'\left(\frac{m-1}{m+1}\right)\eta=f(\eta),$$

$$\Rightarrow f(0)=0.$$

- $K\dfrac{\partial T}{\partial y} = -h_f(T_w - T),$

$$\theta'(\eta) = \frac{-h_f}{K\sqrt{\frac{bx^{(m-1)}(m+1)}{2\nu}}}\frac{(T_w - T_\infty)(1+\theta(\eta))}{T_w - T_\infty},$$

$$\theta'(0) = -Bi(1+\theta(\eta)). \tag{3.30}$$

- $D_B\dfrac{\partial C}{\partial y} + \dfrac{DT}{T_\infty}\dfrac{\partial T}{\partial y} = 0,$

$$\Rightarrow D_B(C_w - C_\infty)\phi'(\eta)\sqrt{\frac{bx^{m-1}(m+1)}{2\nu}} + \frac{D_T}{T_\infty}(T_w - T_\infty)\theta'(\eta) = 0,$$

$$\sqrt{\frac{bx^{m-1}(m+1)}{2\nu}} = 0,$$

$$N_b\phi'(0) + N_t\theta'(0) = 0.$$

- $u \to 0,$

$$\Rightarrow u = bx^m f'(\eta), \quad bx^m \to 0,$$

$$f'(\eta) \to 0, \quad f'(\infty) \to 0, at\ y \to \infty.$$

- $T \to T_\infty, at\ y \to \infty$

$$\Rightarrow T = T_\infty + (T_w - T_\infty)\theta(\eta),$$

$$\Rightarrow T_\infty + (T_w - T_\infty)\theta(\eta) \to T_\infty,$$

$$\Rightarrow (T_w - T_\infty)\theta(\eta) \to 0, \tag{3.31}$$

$$\theta(\eta) \to 0.$$

- $C \to C_\infty,$

$$\Rightarrow C = C_\infty + (C_w - C_\infty)\phi(\eta), y \to \infty,$$

$$\Rightarrow C_\infty + (C_w - C_\infty)\phi(\eta) \to C_\infty,$$

$$\Rightarrow (C_w - C_\infty)\phi(\infty) \to 0,$$

$$(C_w - C_\infty) \nrightarrow 0, \phi(\infty) \to 0.$$

3.4 Physical Quantities of Interest

Mathematically the shear stress coefficient, local Nusselt and Sherwood numbers are written as

$$Cf_x = \frac{\tau_w}{\rho u_w^2}, \tag{3.32}$$

$$Nu_x = \frac{xq_m}{k(T_w - T_\infty)}, \tag{3.33}$$

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{3.34}$$

where τ_w represent shear stress at rest, q_m the heat and q_w mass flux written as:

$$\tau_w = \mu_0 \frac{\partial u}{\partial y}\left[1 + \Gamma^2 \left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-1}{2}}\Bigg|_{y=0}, \tag{3.35}$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)\Bigg|_{y=0}, \quad q_m = -D_B\left(\frac{\partial C}{\partial y}\right)\Bigg|_{y=0}. \tag{3.36}$$

Using Eq. (3.35) and Eq. (3.40) in Eq. (3.32) to Eq. (3.34), we get the dimensionless form, as follows

$$Cf_x = \frac{\tau_w}{\rho u_w^2},$$

$$= \frac{\mu_0 b x^m \sqrt{\frac{b(m+1)}{2\nu}} x^{\frac{m-1}{2}} f''(0)\left[1 + \Gamma^2 b^2 x^{3m-1}(\frac{b(m+1)}{2\nu}) f''^2(0)\right]^{\frac{n-1}{2}}}{\frac{\mu_0}{\upsilon} b^2 x^{2m}},$$

$$= \frac{\sqrt{\nu}\sqrt{\frac{m+1}{2}} f''(0)[1 + We^2 f''^2(0)]^{\frac{n-1}{2}}}{b^{2-\frac{3}{2}} x^{\frac{m+1}{2}}}, \qquad \therefore \frac{\mu}{\rho} = \nu$$

$$= \sqrt{\frac{\nu}{bx^{m+1}}}\sqrt{\frac{m+1}{2}} f''(0)\left[1 + We^2 (f''(0))^2\right]^{\frac{n-1}{2}},$$

$$Cf_x\sqrt{\frac{bx^{m+1}}{\nu}} = \sqrt{\frac{m+1}{2}} f''(0)\left[1 + We^2 (f''(0))^2\right]^{\frac{n-1}{2}},$$

$$R_e^{\frac{1}{2}} Cf_x = \sqrt{\frac{m+1}{2}} f''(0)\left[1 + We^2 (f''(0))^2\right]^{\frac{n-1}{2}}. \tag{3.37}$$

The Nusselt number can be written by:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)},$$

$$Nu_x = \frac{-kx(\frac{\partial T}{\partial y})}{k\frac{T-T_\infty}{\theta(\eta)}}, y = 0$$

$$Nu_x = \frac{-(T_w - T_\infty)\theta'(0)\sqrt{\frac{bx^{\frac{m+1}{2}}(m+1)}{2\nu}}}{\frac{T-T_\infty}{\theta(0)}},$$

$$Nu_x = \frac{-\theta(0)\left(\frac{(T-T_\infty)}{\theta(0)}\right)\theta'(0)\sqrt{\frac{b(m+1)}{2\nu}x^{m+1}}}{(T - T_\infty)},$$

$$Nu_x = -\sqrt{\frac{m+1}{2}}\sqrt{\frac{x^{m+1}b}{\nu}}\theta'(0),$$

$$R_e^{-\frac{1}{2}}Nu_x = -\sqrt{\frac{m+1}{2}}\theta'(0). \tag{3.38}$$

The Sherwood number is described as:

$$Sh_x = \frac{xq_m}{(C_w - C_\infty)D_B},$$

$$= \frac{-xD_B\frac{\partial C}{\partial y}}{(C_w - C_\infty)D_B},$$

$$= \frac{-x(C_w - C_\infty)\sqrt{\frac{b(m+1)}{2\nu}x^{m-1}}\phi'(\eta)}{(C_w - C_\infty)},$$

$$= -\phi'(0)\sqrt{\frac{m+1}{2}}\sqrt{\frac{bx^{m+1}}{\nu}},$$

$$= -\phi'(0)\sqrt{\frac{m+1}{2}}R_e^{\frac{1}{2}},$$

$$R_e^{-\frac{1}{2}}Sh_x = -\phi'(0)\sqrt{\frac{m+1}{2}}. \tag{3.39}$$

3.5 Solution Methodology

Consider the third order ODE with associated boundary conditions

$$f''' = -\frac{ff'' - (mf'^2 + K_1 f')\left(\frac{2}{m+1}\right)}{[1 + nWe^2(f'')^2]\,[1 + We^2(f'')]^{\frac{n-3}{2}}}. \tag{3.40}$$

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) \to 0. \tag{3.41}$$

We use shooting method for solving above equation with dimensionless boundary conditions. First of all we covert higher order PDEs into the system of first order ODEs. We used the RK4 method to solve IVP and assume the missing condition. Following notations are used:

$$f = y_1,$$
$$f' = y_2,$$
$$f'' = y_3.$$

The associated IVP takes the form:

$$y_1' = y_2; \qquad y_1(0) = 0, \tag{3.42}$$
$$y_2' = y_3; \qquad y_2(0) = 1, \tag{3.43}$$

$$y_3' = -\frac{y_1 y_3 - \left(\frac{2}{m+1}\right)(my_2^2 + K_1 y_2)}{[1 + nWe^2(y_3)^2]\,[1 + We^2(y_3)]^{\frac{n-3}{2}}}; \qquad y_3(0) = \xi, \tag{3.44}$$

Missing condition ξ is assumed to satisfy following relation;

$$y_2(\eta_\infty, \xi) = 0,$$

where η_∞ is positive real number.

Now we use Newton's method

$$\xi^{(k+1)} = \xi^{(k)} - \frac{Y(\xi)}{Y'(\xi)}, \tag{3.45}$$

$$\text{where } Y(\xi) = y_2(\eta_\infty, \xi).$$

In order to calculate the Eq. (3.44) Eq. (3.42) and Eq. (3.43) following notations are used

$$\frac{\partial y_1}{\partial \xi} = y_4, \qquad \frac{\partial y_2}{\partial \xi} = y_5, \qquad \frac{\partial y_3}{\partial \xi} = y_6.$$

$$y_4' = y_5; \qquad\qquad y_4(0) = 0,$$

$$y_5' = y_6; \qquad\qquad y_5(0) = 0,$$

$$y_6' = \frac{1}{(1+nWe^2y_3^2)^2(1+We^2y_3^2)^{n-3}}[-(y_4y_3 + y_1y_6) + (\frac{2}{m+1})(2my_2y_5 + K_1y_5)$$
$$(1+nWe^2y_3^2)(1+We^2y_3^2)^{\frac{n-3}{2}} - [(y_1y_3 + \frac{2}{m+1}(my_2^2 + K_1y_2)](2mWe^2y_3y_6)$$
$$(1+We^2y_3^2)^{\frac{n-3}{2}} + (\frac{n-3}{2})(1+nWe^2y_3^2)^{\frac{n-5}{2}}(2nWe^2y_3y_6)]; \qquad y_6(0) = 1.$$

The RK4 method is used to solve IVP for some suitable choice ξ. The missing condition ξ is updated by Newton's method and process will be continued until the following criteria is met.

$$|y_2(\eta_\infty, \xi) - 0| < \epsilon.$$

where $\epsilon = 10^{-8}$ is set for numerical calculation.

Now for Eq. (3.28) and Eq. (3.29),

$$\theta'' = [-f\theta' - N_b\phi'\theta' - N_t\theta'^2 - (\frac{2}{m+1})\lambda\theta]Pr = 0. \tag{3.46}$$

$$\phi'' = -PrLef\phi' - \frac{N_t}{N_b}\theta'' + \frac{2PrLe\gamma_1\phi}{m+1} = 0. \tag{3.47}$$

with boundary conditions:

$$\theta'(0) = -Bi + Bi\theta(0), \quad N_b\phi'(0) + N_t\theta'(0) = 0,$$
$$\theta(\infty) \to 0, \quad \phi(\infty) \to 0. \tag{3.48}$$

Following notations are used:

$$\theta = z_1, \quad \theta' = z_2.$$
$$\phi = z_3, \quad \phi' = z_4.$$

Rewritting equation:

$$z_1' = z_2; \qquad z_1(0) = p, \tag{3.49}$$

$$z_2' = Pr[-z_2y_1 - N_bz_4z_2 - N_tz_2^2 - (\frac{2}{m+1})\lambda z_1]; \qquad z_2(0) = -Bi(1-p), \tag{3.50}$$

$$z_3' = z_4; \qquad z_3(0) = q, \tag{3.51}$$

$$z_4' = PrLey_1z_4 - \frac{N_t}{N_b}Pr\Bigg[- z_2y_1 - N_bz_4z_2 - N_tz_2^2 - \left(\frac{2}{m+1}\right)\lambda z_1\Bigg] + \frac{2PrLe\gamma_1 z_3}{m+1}; \quad z_4(0) = \frac{N_tBi(1-p)}{N_b}. \tag{3.52}$$

Missing conditions 'p' and 'q' are assumed to satisfy the following relation:

$$z_1(\eta_\infty, p, q) = 0,$$
$$z_3(\eta_\infty, p, q) = 0.$$

We have to solve the above equations by applying the Newton's method,

$$\begin{bmatrix} p^{(k+1)} \\ q^{(k+1)} \end{bmatrix} = \begin{bmatrix} p^{(k)} \\ q^{(k)} \end{bmatrix} - \begin{bmatrix} \frac{\partial z_1}{\partial p} & \frac{\partial z_1}{\partial q} \\ \frac{\partial z_3}{\partial p} & \frac{\partial z_3}{\partial q} \end{bmatrix}^{-1}_{(p=p^{(k)},q=q^{(k)})} \begin{bmatrix} z_1 \\ z_3 \end{bmatrix}. \tag{3.53}$$

We use following notations:

$$\frac{\partial z_1}{\partial p} = z_5, \qquad \frac{\partial z_2}{\partial p} = z_6, \qquad \frac{\partial z_3}{\partial p} = z_7, \qquad \frac{\partial z_4}{\partial p} = z_8,$$
$$\frac{\partial z_1}{\partial q} = z_9, \qquad \frac{\partial z_2}{\partial q} = z_{10}, \qquad \frac{\partial z_3}{\partial q} = z_{11}, \qquad \frac{\partial z_4}{\partial q} = z_{12}. \tag{3.54}$$

Using above notations in Eq. (3.53), we have

$$\begin{bmatrix} p^{(k+1)} \\ q^{(k+1)} \end{bmatrix} = \begin{bmatrix} p^{(k)} \\ q^{(k)} \end{bmatrix} - \begin{bmatrix} z_5 & z_9 \\ z_7 & z_{11} \end{bmatrix}^{-1}_{(p=p^{(k)},q=q^{(k)})} \begin{bmatrix} z_1 \\ z_3 \end{bmatrix}$$

Differentiating Eq. (3.49) to Eq. (3.52) and using above notations from (3.54), we have

$$z_5' = z_6; \qquad z_5(0) = 1,$$

$$z_6' = Pr\left[-z_6 y_1 - N_b(z_6 z_4 + z_8 z_2) - 2N_t z_2 z_6 - \left(\frac{2}{m+1}\right)\lambda z_5\right]; \qquad z_6(0) = Bi,$$

$$z_7' = z_8; \qquad z_7(0) = 0,$$

$$z_8' = -PrLez_8 y_1 - \frac{N_t}{N_b}Pr\left[-z_6 y_1 - N_b(z_6 z_4 + z_8 z_2) - 2N_t z_2 z_6 - \left(\frac{2}{m+1}\right)\lambda z_5\right] + \frac{2}{m+1}PrLe\gamma_1 z_7; \qquad z_8(0) = -\frac{N_t Bi}{N_b},$$

$$z_9' = z_{10}; \qquad z_9(0) = 0,$$

$$z_{10}' = Pr\left[-z_{10} y_1 - N_b(z_4 z_{10} + z_2 z_{12}) - 2N_t z_2 N_{10} - \left(\frac{2}{m+1}\right)\lambda z_9\right]; \; z_{10}(0) = 0,$$

$$z_{11}' = z_{12}; \qquad z_{11}(0) = 1,$$

$$z_{12'} = -PrLez_{12} y_1 - \frac{N_t Pr}{N_b}[-y_1 z_{10} - N_b(z_4 z_{10} + z_2 z_{12}) - 2N_t z_2 z_{10}, - \left(\frac{2}{m+1}\right)\lambda z_9] + \left[\left(\frac{2}{m+1}\right)PrLe\gamma_1 z_{11}\right]; \qquad z_{12}(0) = 0.$$

The RK4 method is used to solve the IVP and initial values are choosen arbitrarily. During the execution of iterations these initial guesses shall be updated by the Newton's method and the process will be repeated untill the following stoping criteria is met,

$$max(|z_1(\eta_\infty, p, q)|, |z_3(\eta_\infty, p, q)|) < \epsilon,$$

where $\epsilon = 10^{-8}$ is set for numerical calculation.

3.6 Results and Discussions

The mathematical outcomes of the equations are discussed in this unit by using tables and graphs. The impact of various parameters such as heat generation/absorption coeffcient K_1, porosity parameter λ, chemical reaction effect γ_1, skin friction coefficient, Sherwood and Nusselt number, thermophoresis parameter N_t, Prandtl number is observed graphically. These physical parameters have a direct impact on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. The heat and mass transfer rate for fixed values of We, m, N, Pr, Le and N_b are analyzed numerically as shown in figures. In ordered to evaluate the exactitude of current results and the validity of numerical codes are analyzed in the absence of heat generation/absorption, chemical reaction parameter and porosity parameter in Table 3.1. The results of heat transfer rate at $Re^{-\frac{1}{2}}Nu_x$ surface are matched by the published results of Hashim and Khan [33] for different values of n, N, Pr and also m when $We = 3$, $N_b = 0.5$ and $Bi \rightarrow \infty$. An excellent agreement is found, that provides us assurance in our numerical outcome. Table 3.2 represents the different values of $-f''(0)$, $-\theta(0)$ and $-\phi(0)$ for various values of the govering parameters (Pr, Bi, K_1, n, γ). It is observed that n and K_1 affects Skin friction significantly. The skin friction is decreased by the increament in porosity parameter as well as power law index.

Figure 3.2 exhibits the effect of porosity parameter K_1 on velocity profile for the fixed parameters such as $Pr = 2$, $Le = 1$, $m = 2$, $N_b = 0.5$, $\lambda = 0.1$, $N_t = 0.1$, $\gamma_1 = 0.1$ and $Bi = 1$. The velocity profile for pseudoplastic $n = 0.5$ and dilatant nanofluids $n = 1.5$ increases by increasing the value of porosity parameter K_1. Figures 3.3-3.6 illustrate the impact of porosity parameter K_1 and Biot number on the temperature distribution $\theta(\eta)$. It can be seen that the temperature profile reduces by enlarging the porosity parameter. Due to the fact that the suction of the wall of the wedge imparts an other affects to the fluid flow process, which causes the fluid to move at a retarding rate with reduced temperature. These behaviour are shown in figures. Furthermore, the porosity of the boundary does no affect of the fluid motion as we move away from the boundary surface. On conflicting of Biot number and heat source parameter rise the temperature distribution. It is demonstrated that by increasing the heat source and Biot number, the thickness of thermal layer enhanced. Figures 3.4 and 3.5 represent the impact of heat source/sink and the ratio of outer radius to inner radius on the temperature profiles. The impact of ratio of outer radius to inner radius is to rise the temperature distribution, due to shrinking the thickness of thermal boundary layer. The heat sink/source contribute more thermal energy into the boundary layer flow, generation of some more thermal energy improve the temperature distribution of the flow. In Figure 3.6, the impact of Biot number on $\theta(\eta)$ is shown. The enhancement of Biot number increases the temperature profile. Generally, Biot number can be expressed as, the heat transfer by convection in a body surface to the heat transfer by conduction on a body surface. The reason is that, the convective heat transfer at the surface will rise the boundary layer thickness, therefore the nanofluid with convective boundary condition is further effected as constrat to the static surface temperature. Figures 3.7-3.11 clarify the infiuence of λ, Bi, K_1, and γ_1 on concentration distribution for $n = 1.5$ and $n = 0.5$, when the other parameters are static. It is observed that from Figure 3.7 and Figure 3.11 the boundary layer thickness of concentration profile shrink by the enhancement of K_1 and γ_1 while increases by rising the value of λ and Bi. It is noticed that enlarging values of γ_1, reduces the thickness of concentration. Due to the fact that the γ_1 in this system results in chemical dissipation and therefore results decrease in concentration profile. The most important effect is that the chemical reaction in the concentration distribution and its associated boundary layer appears to decrease the overshoot.

In Figures 3.8-3.10, it is observed that in the case of shear thinning nanofluids the concentration boundary layer thickness is higher than the shear thinning nanofluids case. Figure 3.12 $(a-b)$ illustrates the deviation of Nusselt number with K_1 for different values of γ_1, for the case of $n = 1.5$ and $n = 0.5$ while all other parameters are constant. It is detected that an enhancement in both γ_1 and K_1 causes the increment in the local Nusselt number. Figure 3.13 $(a-b)$ and Figure 3.14 $(a-b)$ show the variation of local Nusselt number with K_1 for different values of λ and n, while other parameters are fixed. It is noted to see that the local Nusselt number upswing with λ. Figure 3.16 $(a-b)$ and Figure 3.17 $(a-b)$ represent the difference of Sherwood number with K_1 for distinct values of λ in case of $n = 0.5$ and $n = 1.5$. It is noted that local Sherwood number decreases in both cases by upturn in λ.

Table 3.1: Calculated values for $-\theta(0)$ for different value of Pr, N_t, Le when $We = 3, \lambda = \gamma = K_1 = 0$

				$-\theta'(0)$			
Pr	N_t	Le	m	Ref. [22]		Present	
				$n = 0.5$	$n = 1.5$	$n = 0.5$	$n = 1.5$
1	0.1	1	2	0.6140	0.7354	0.6142	0.7054
3				1.2440	1.4198	1.2167	1.0900
5				1.6635	1.8615	1.2079	1.3849
2	0.3			0.9215	1.0758	0.7571	0.8347
	0.5			0.8727	1.0243	0.8716	0.6855
	0.7			0.8252	0.9738	0.8626	0.6493
	0.1	0.5		0.9808	1.1379	0.8377	0.9032
		1.5		0.9649	1.1209	0.8178	0.8712
		2.5		0.9563	1.1114	0.7983	0.8521
		1	1	0.8144	0.9314	0.8210	0.8919
			2	0.9713	1.1295	0.9665	1.1565
			0.5	1.3406	1.5785	1.1704	1.1855

Table 3.2: Calculated values for Skin friction coefficient, mass transfer rate and heat transfer rate for different values of various parameter given below

							Ref. [33]			Present	
Bi	Pr	n	γ_1	λ	K_1	$-f''(0)$	-$\theta'(0)$	$-\phi'(0)$	$-f''(0)$	-$\theta'(0)$	$-\phi'(0)$
0.1	0.01	0.5	0.1	-0.1	0.5		0.07176	-0.07176	1.08334	0.07160	-0.07160
0.2							0.11191	-0.11191	1.08334	0.11160	-0.11160
				0.2			0.11139	-0.11138	1.08334	0.11210	-0.11210
	0.1					1.0823	0.11395	-0.11396	1.08334	0.11560	-0.11560
	10						0.18096	-0.18096	1.08334	0.18470	-0.18470
							0.18078	-0.18078	1.08334	0.19030	-0.19030
	50						0.19069	-0.19069	1.08334	0.19770	-0.19770
					0.9	0.8850	0.19082	-0.19082	0.91993	0.19770	-0.19770
0.3	1	1.5	0.5		0.5	0.8626	0.18512	-0.18513	1.20239	0.21580	-0.21580
			0.5		0.9	0.7470	0.18914	-0.18914	0.99939	0.21700	-0.21700

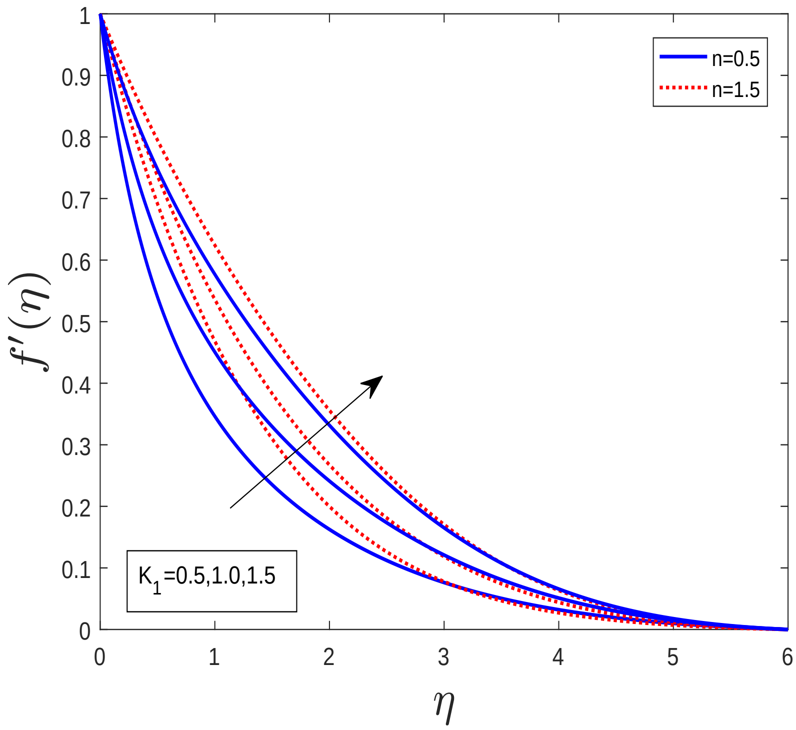

Figure 3.2: Effect of velocity via porosity parameter.

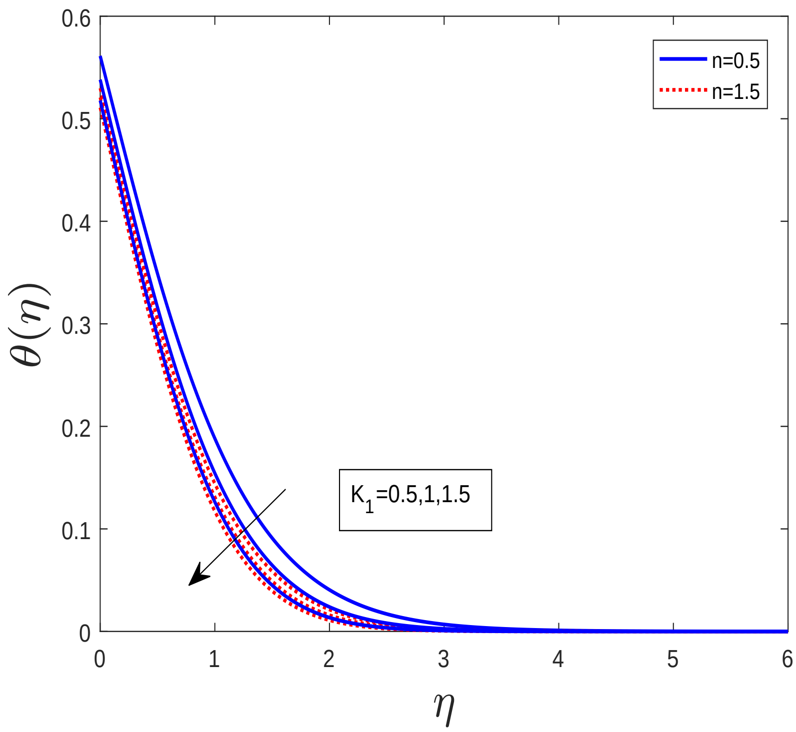

Figure 3.3: Effect of temperature via heat generation.

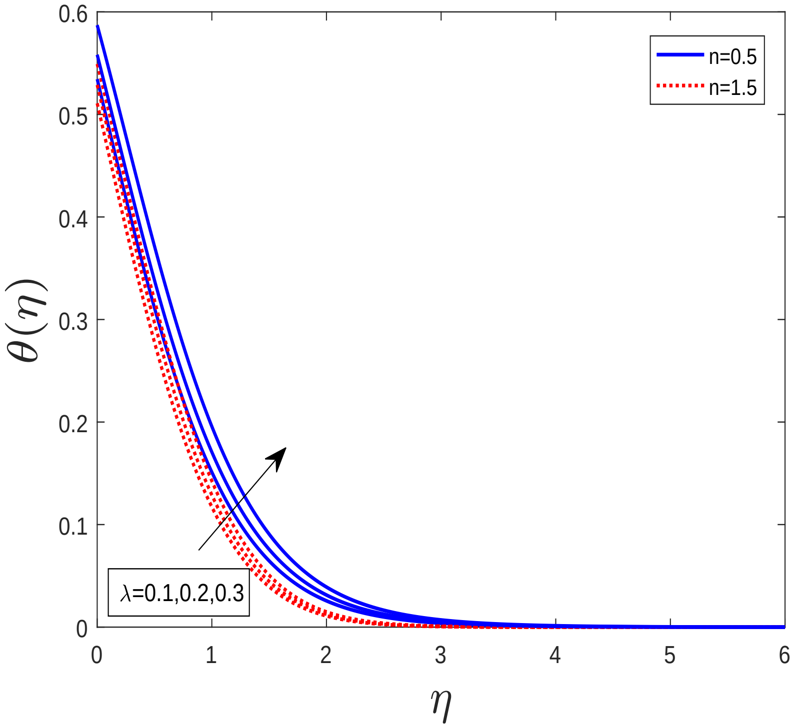

Figure 3.4: Effect of temperature via heat source.

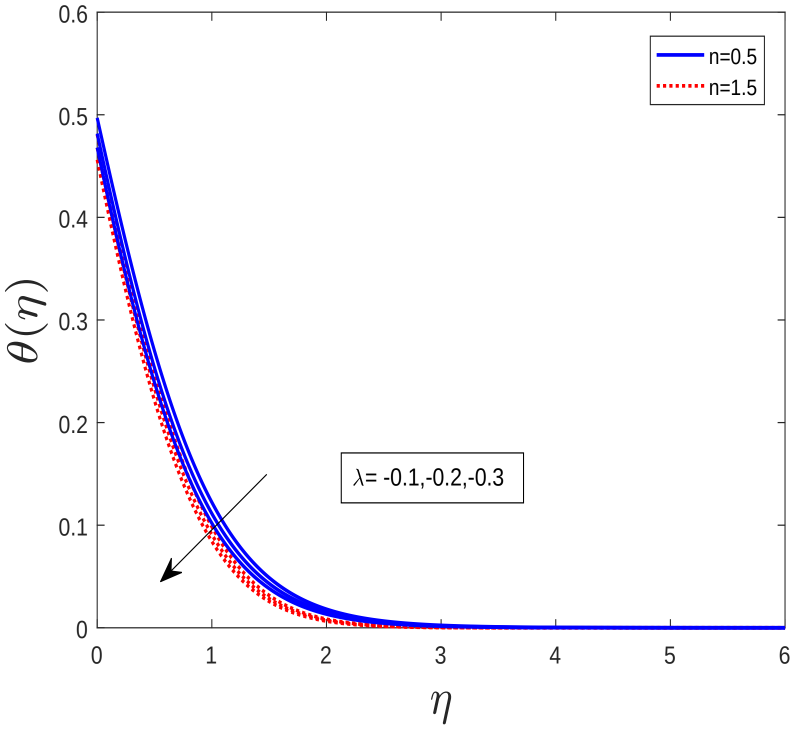

Figure 3.5: Effect of temperature via sink parameter.

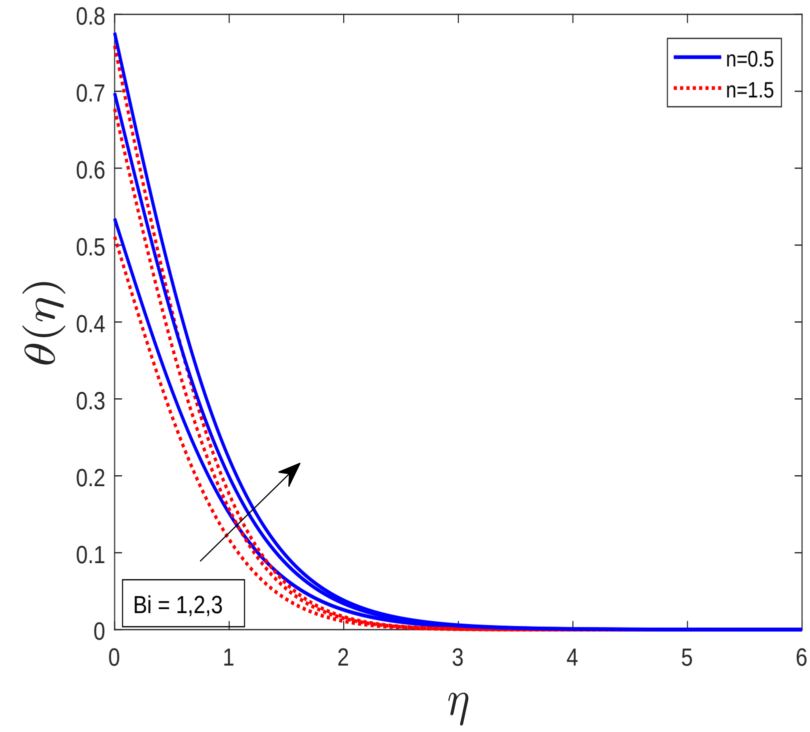

Figure 3.6: Effect of temperature via Biot number.

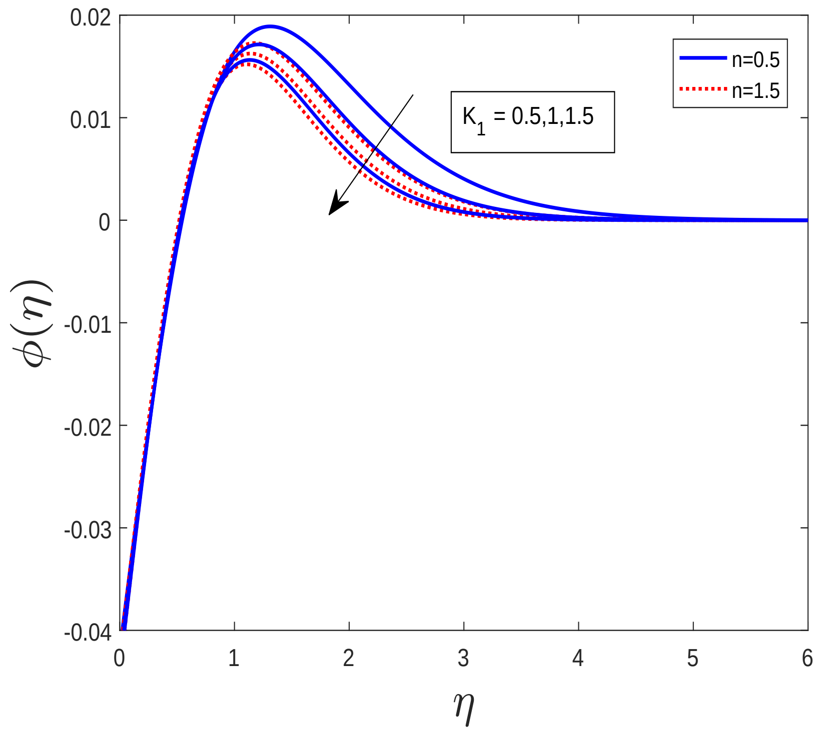

Figure 3.7: Effect of nanoparticles concentration via heat generation.

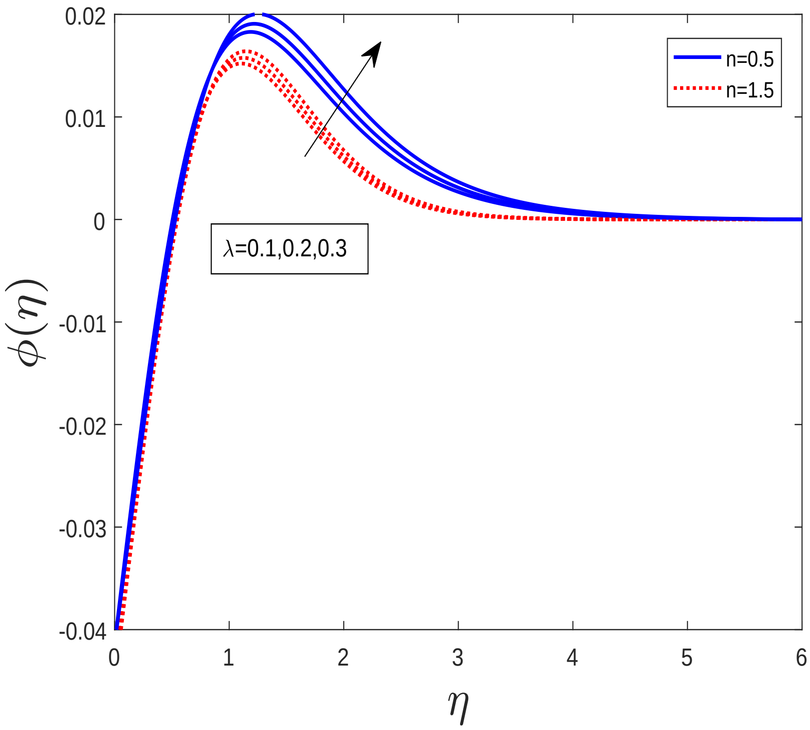

Figure 3.8: Effect of nanoparticles concentration via heat source.

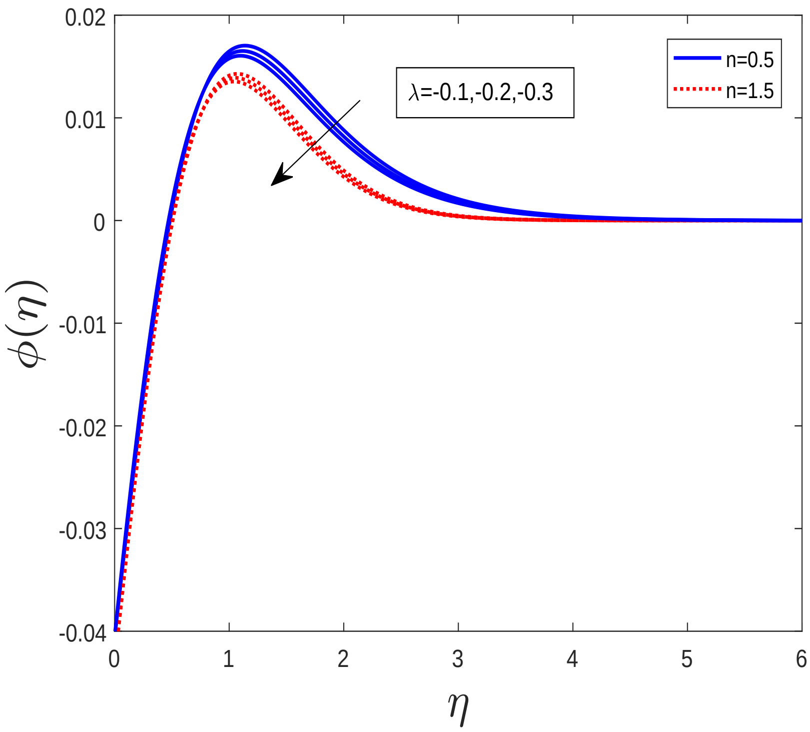

Figure 3.9: Effect of nanoparticles concentration via sink parameter.

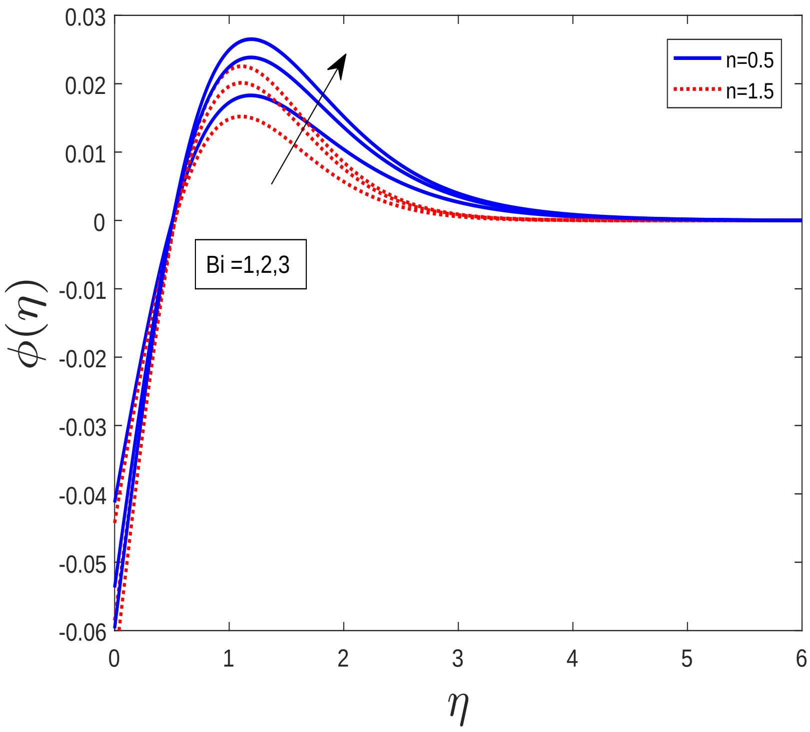

Figure 3.10: Effect of nanoparticles concentration via Biot number.

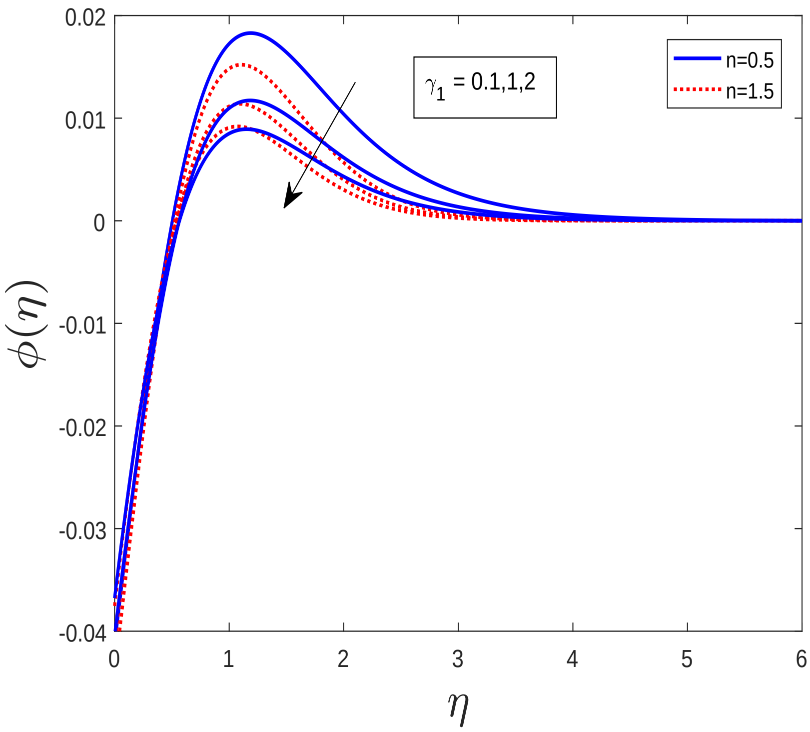

Figure 3.11: Effect of nanoparticles concentration via chemical reaction parameter.

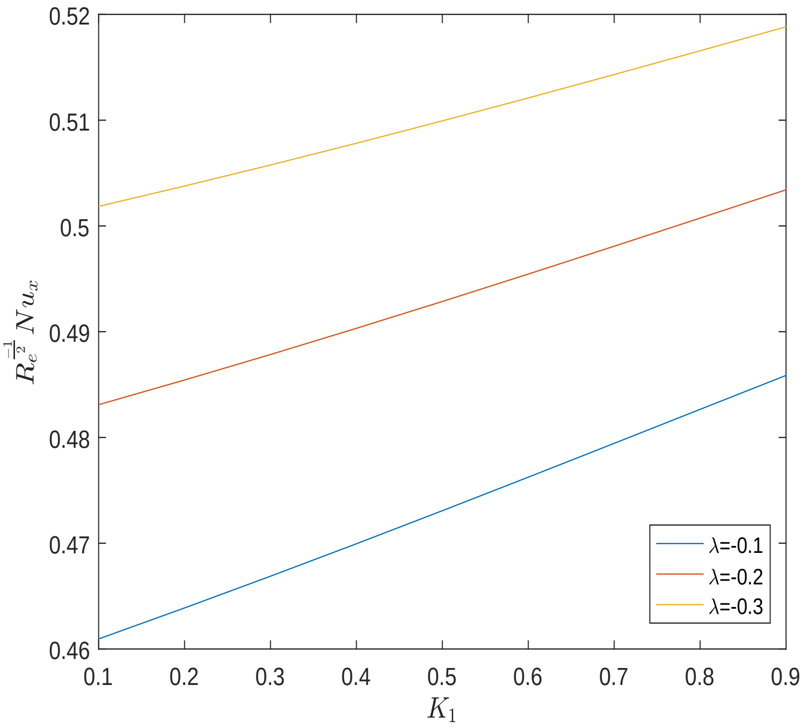

Figure 3.12: Effect of sink parameter on Nusslet number.

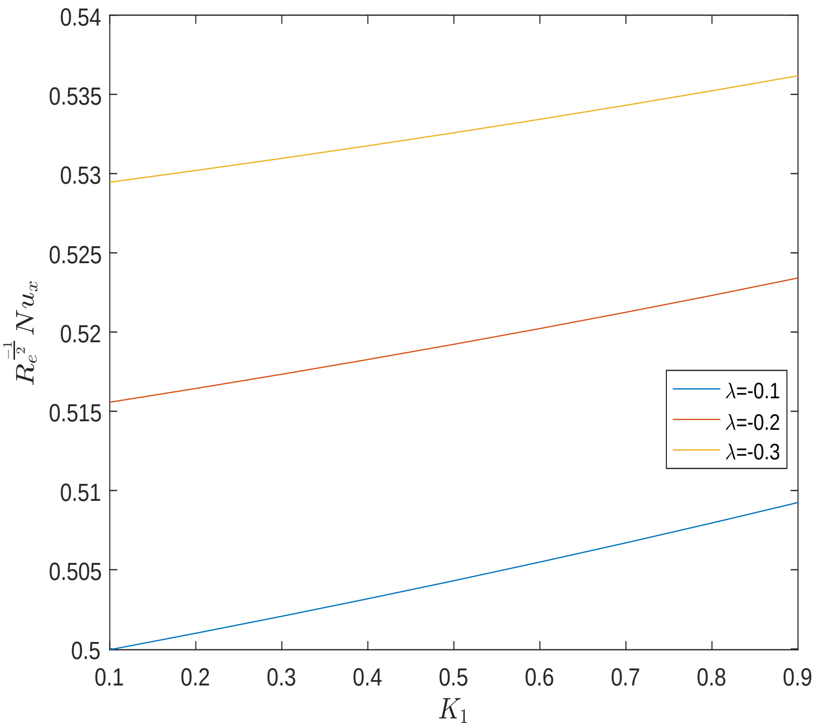

Figure 3.13: Effect of sink parameter on Nusslet number.

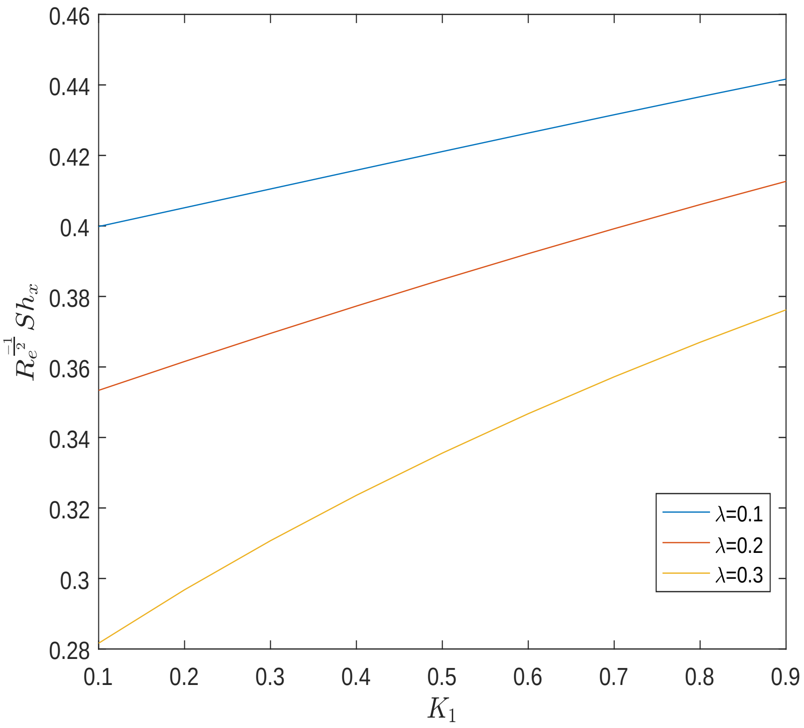

Figure 3.14: Effect of heat source on Sherwood number.

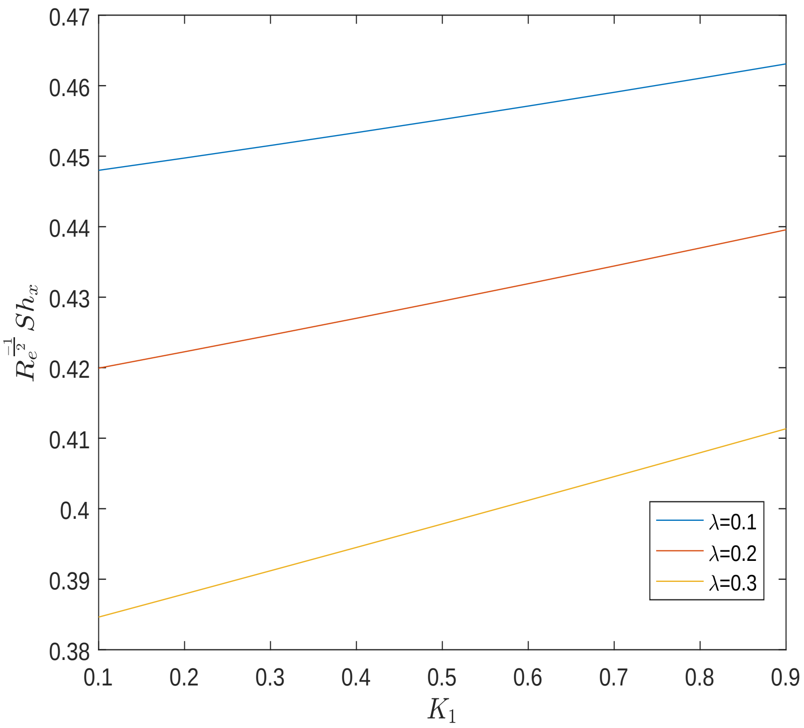

Figure 3.15: Effect of heat source on Sherwood number.

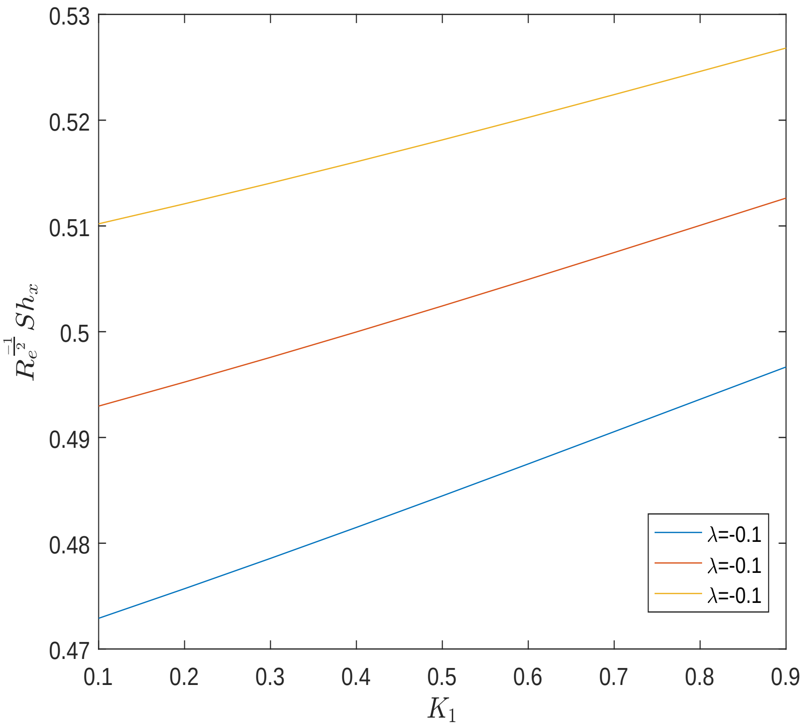

Figure 3.16: Effect of sink parameter on Sherwood number.

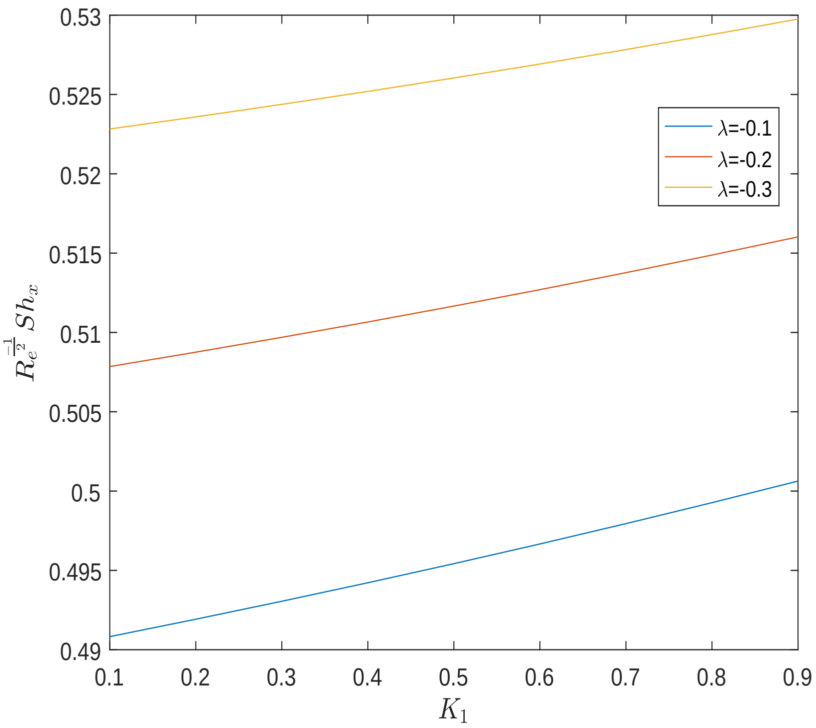

Figure 3.17: Effect of sink parameter on Sherwood number.

Chapter 4

MHD Flow of Nanofluid with Joule Heating and Arrhenius Activation Energy

4.1 Introduction

The flow model of Eid et al. [33] has been extended in this chapter by including additional impact of MHD, Joule heating and Arrhenius activation energy. Additionally, we converted the nonlinear PDEs of concentration, temperature and momentum into set of ODEs by using the similarity transformation. We will use the familiar shooting technique for the computation of the numerical solution of these model ODEs. The effects of different parameters on the velocity, temperature and concentration will be discussed in detail in the result and discussion part.

4.2 Mathematical Modeling

The problem is formed in the following way. We consider a $2D$ steady incompressible flow and transfer of heat by using stretching sheet. Sheet is displaced in such a way that flow is constrained as in the plane $y > 0$. The sheet has been stretched with velocity $u_w(x) = bx^m$ with free stream velocity, where b is constant which is positive and m is stretching parameter, the sheet surface temperature is T_w and convecting fluid temperature is T_f. For extremely large value of y the nanoparticles temperature and concentration will be represented by C_∞ and T_∞ respectively.

4.3 The Governing Equations:

The flow is explained by considering the $2D$ governing equations containing the continuity, momentum,energy and concentration are as follow:

- Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{4.1}$$

- Momentum Equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2}\left[1 + \Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-1}{2}}$$
$$+ v(n-1)\Gamma^2\frac{\partial^2 u}{\partial y^2}\left(\frac{\partial u}{\partial y}\right)^2\left[1 + \Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\right]^{\frac{n-3}{2}} - \left(\frac{v}{k}\right)u - \frac{\sigma B_0^2 u}{\rho}. \tag{4.2}$$

- Temperature Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y})\right)^2\right] + \frac{Q_0}{(\rho c_p)_f}(T - T_\infty)$$
$$+ \frac{\sigma B_0^2 u^2}{\rho c_p}. \tag{4.3}$$

- Concentration Equation:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - R_1(C - C_\infty)\left(\frac{T}{T_\infty}\right)^m exp\left(-\frac{E^*}{K^* T}\right). \tag{4.4}$$

The boundary conditions are:

$$\left.\begin{aligned} &u = u_w(x) = bx^m,\ v = 0,\ k\frac{\partial T}{\partial y} = -h_f(T_W - T),\ D_B\frac{\partial C}{\partial y} + \ \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \text{ at } y = 0,\\ &u \to 0, T \to T_\infty,\ C \to C_\infty \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{as } y \to \infty. \end{aligned}\right\}$$

We convert PDEs and these boundary conditions into the ODEs by adopting below similarity transformation [33].

$$\left.\begin{aligned} &u = \frac{\partial \psi}{\partial y} = bx^m f',\\ &v = -\frac{\partial \psi}{\partial x} = \sqrt{\frac{\nu(m+1)}{2}})bx^{\frac{m-1}{2}}\left[f'\left(\frac{m-1}{m+1}\right)\right]\eta + f,\\ &\psi(x,y) = \sqrt{\frac{2\nu b}{m+1}}x^{\frac{m+1}{2}}f(\eta),\quad \eta = y\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}},\\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \end{aligned}\right\} \tag{4.5}$$

Complete procedure for confirmation of continuity Eq. (4.1) has been discussed Chapter 3. Now we apply same process for conversation of Eq. (4.2) into dimensionless form, differentiating 'u' w.r.t 'x', we have

$$\frac{\partial u}{\partial x} = bx^{m-1}\left[\left(\frac{m-1}{2}\right)f''(\eta)\eta + mf'(\eta)\right]. \tag{4.6}$$

Similarly differentiating 'u' w.r.t 'y'

$$\frac{\partial u}{\partial y} = bx^m\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}f''(\eta), \tag{4.7}$$

$$\frac{\partial^2 u}{\partial y^2} = b^2x^{2m-1}\left(\frac{2m+1}{2\nu}\right)f'''(\eta). \tag{4.8}$$

We write the L.H.S of Eq. (4.2) as,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = b^2x^{2m-1}\left[m(f'^2) - \left(\frac{m+1}{2}\right)f''f\right]. \tag{4.9}$$

Using all above Eqs. (4.6) to (4.9) in Eq. (4.2). Multiplying with $\left(\frac{2}{m+1}\right)$ and dividing with b^2x^{2m-1} with Eq. (4.2), we get dimensionless form,

$$\left[1+nWe^2(f'')^2\right]\left[1+We^2(f'')^2\right]^{\frac{n-3}{2}} f''' + f''f - \left(\frac{2}{m+1}\right)\left[m(f')^2+K_1f'\right]$$

$$-\frac{\sigma}{\rho}B_0^2bx^mf'\left(\frac{2}{m+1}\right)\frac{1}{b^2x^{2m-1}} = 0,$$

$$\left[1+nWe^2(f'')^2\right]\left[1+We^2(f'')^2\right]^{\frac{n-3}{2}} f''' + f''f - \left(\frac{2}{m+1}\right)\left[m(f')^2+K_1f'\right]$$

$$-Mf' = 0, \qquad \therefore M = \frac{2\sigma B_0^2}{\rho(m+1)bx^{m-1}}. \tag{4.10}$$

The same process is used to converts Eq. (4.3) into dimensionless form,

$T = T_\infty + (T_w - T_\infty)\theta(\eta).$

$$\frac{\partial T}{\partial x} = (T_w - T_\infty)\theta'\left(\frac{m-1}{2x}\right)\eta, \tag{4.11}$$

$$\frac{\partial T}{\partial y} = (T_w - T_\infty)\theta'\frac{\partial \eta}{\partial y}, \tag{4.12}$$

$$\frac{\partial^2 T}{\partial y^2} = (T_w - T_\infty)\theta''\frac{b(m+1)}{2\nu}x^{m-1}. \tag{4.13}$$

$$C = C_\infty + (C_w - C_\infty)\phi(\eta),$$

$$\frac{\partial C}{\partial y} = x^{\frac{m-1}{2}}(C_w - C_\infty)\frac{b(m+1)}{2\nu}\phi'(\eta). \tag{4.14}$$

Using Eq. (4.11) to Eq. (4.14) in Eq. (4.3), we get dimensionless form:

$$-b\left(\frac{m+1}{2}\right)(T_w - T_\infty)f\theta'x^{m-1} = \alpha(T_w - T_\infty)\theta''\frac{b(m+1)}{2\nu}$$

$$+\tau[D_B((C_w - C_\infty))(T_w - T_\infty)\theta'\phi'b(\frac{m+1}{2\nu})x^{m-1} + \frac{D_T}{T_\infty}(T_w - T_\infty)^2$$

$$b\left(\frac{m+1}{2\nu}\right)x^{m-1}(\theta')^2] + \frac{Q_0}{(\rho c_p)f}\frac{(T_w - T_\infty)\theta}{x^{m-1}} + \frac{\sigma B_0^2b^2x^{2m}f'^2}{\rho c_p},$$

multiplying whole equation with $\frac{2}{b(m+1)(T_w-T_\infty)}$ and dividing with x^{m-1}, we have

$$-f\theta' = \frac{\alpha}{\nu}\theta'' + \frac{\tau D_B(C_w - C_\infty)}{\nu}\theta'\phi' + \tau\frac{D_T}{T_\infty}\frac{(T_w - T_\infty)\theta^2}{\nu} + \frac{Q_0}{(\rho c_p)f}\frac{2\theta}{(m+1)x^{m-1}}$$

$$+\left(\frac{\sigma}{\rho c_p}\right)B_0^2 b^2 x^{2m} f'^2\left(\frac{2}{bx^{m-1}(m+1)(T_w - T_\infty)}\right),$$

$$\frac{\theta''}{Pr} + N_t\theta'^2 + \frac{2\theta\lambda}{(m+1)} + \frac{2\sigma B_0^2}{(m+1)\rho bx^{m-1}}\frac{b^2x^{2m}}{(T_w - T_\infty)c_p} + f\theta' + N_b\phi'\theta' = 0,$$

$$\frac{\theta''}{Pr} + N_t\theta'^2 + N_b\phi'\theta' + \frac{2\theta\lambda}{(m+1)} + MEcf'^2 = 0 + f\theta'. \tag{4.15}$$

$$\therefore\ M = \frac{2\sigma B_0^2}{\rho(m+1)bx^{m-1}}, \qquad \therefore\ Ec = \frac{b^2x^{2m}}{T_w - T_\infty)c_p}.$$

Now we use similar process to convert Eq. (4.4) into dimensionless form:

$$u = \frac{\partial\psi}{\partial y} = bx^m f'. \tag{4.16}$$

$$v = -\frac{\partial\psi}{\partial x} = \sqrt{\frac{\nu(m+1)}{2}})bx^{\frac{m-1}{2}}\left[f'\left(\frac{m-1}{m+1}\right)\right]\eta + f. \tag{4.17}$$

$$\frac{\partial C}{\partial x} = y\sqrt{\frac{b(m+1)}{2\nu}}\left(\frac{m-1}{2}\right)x^{\frac{m-3}{2}}(C_w - C_\infty)\phi'(\eta). \tag{4.18}$$

$$\frac{\partial C}{\partial y} = x^{\frac{m-1}{2}}(C_w - C_\infty)\frac{b(m+1)}{2\nu}\phi'(\eta). \tag{4.19}$$

$$\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)\phi''(\eta)\left(\frac{b(m+1)}{2\nu}\right)x^{m-1}. \tag{4.20}$$

$$\frac{\partial^2 T}{\partial y^2} = (T_w - T_\infty)\theta''\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}\sqrt{\frac{b(m+1)}{2\nu}}x^{\frac{m-1}{2}}. \tag{4.21}$$

Using all Eqs. (4.16) to (4.21) in Eq. (4.4), we get

$$-\frac{b(m+1)}{2}x^{m-1}(C_w - C_\infty)f\phi' = \frac{b(m+1)}{2}(C_w - C_\infty)x^{m-1}\left[\frac{D_B}{\nu}\phi''(\eta)\right.$$

$$\left.+\frac{D_T(T_w - T_\infty)}{\gamma T_\infty(c_w - c_\infty)}\theta'' - \frac{2R_1}{b(m+1)x^{m-1}}\phi(\eta)\left(\frac{T}{T_\infty}\right)^m exp\left(-\frac{E^*}{K^*T}\right)\right],$$

Taking common$-\frac{b(m+1)}{2}x^{m-1}(C_w - C_\infty)$, we get,

$$\frac{D_B}{\gamma}\phi''(\eta) + \left(\frac{T_w - T_\infty}{C_w - C_\infty}\right)\frac{D_T}{T_\infty}\frac{\theta''}{\gamma} - \frac{2}{m+1}\left(\frac{R_1}{bx^{m-1}}\phi\right)\left(\frac{T}{T_\infty}\right)^m exp\left(-\frac{E^*}{TK^*}\right)$$

$$+ f\phi' = 0,$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + LePrf\phi' - \frac{2}{m+1}\gamma_1\phi LePr\left(\frac{T}{T_\infty}\right)^m exp\left(-\frac{E^*}{TK^*}\right) = 0, \tag{4.22}$$

$$let \quad A = \frac{2}{m+1}\gamma_1\phi LePr\left(\frac{T}{T_\infty}\right)^m exp\left(-\frac{E^*}{TK^*}\right).$$

$$A = \frac{2}{m+1}\gamma_1\phi LePr\left(\frac{(T_w - T_\infty)\theta(\eta) + T_\infty}{T_\infty}\right)^m exp\left(-\frac{E^*}{T_\infty K^*(T_w - T_\infty)\theta(\eta) + T_\infty}\right),$$

$$A = \frac{2}{m+1}\gamma_1\phi LePr(\gamma_2 + \theta(\eta))^m exp\left(-\frac{E^*}{T_\infty K^*(\gamma_2\theta + 1)}\right). \tag{4.23}$$

Using Eq. (4.23) into Eq. (4.22), we get

$$\phi'' + \frac{N_t}{N_b}\theta'' + LePrf\phi' - \frac{2}{m+1}\gamma_1\phi LePr(1 + \gamma_2\theta(\eta))^m exp\left(\frac{-E^*}{K^*T_\infty(1 + \gamma_2\theta)}\right),$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + LePrf\phi' - \frac{2}{m+1}\gamma_1\phi LePr(1 + \gamma_2\theta(\eta))^m exp\left(\frac{-E}{1 + \gamma_2\theta}\right). \tag{4.24}$$

$$\therefore E = \frac{E^*}{T_\infty K^*}, \qquad \gamma_2 = \frac{T_w - T_\infty}{T_\infty}.$$

4.4 Numerical Solution

Now we consider third order ODE with associated boundary condition:

$$f''' = -\frac{ff'' - \left(\frac{2}{m+1}\right)(mf'^2 + K_1 f') - Mf'}{\left[1 + nWe^2(f'')^2\right]\left[1 + We^2(f'')\right]^{\frac{n-3}{2}}}. \tag{4.25}$$

$$f(0) = 0, \qquad f'(\infty) \to 0, \qquad f'(0) = 1. \tag{4.26}$$

We solve the above equation with associated boundry conditions using by shooting method. For this, we convert third order ODE into first order ODEs and used RK4 method to solve IVP assuming the initial missing conditions. We used following notations:

$$f = g_1,$$

$$f' = g_2,$$

$$f'' = g_3.$$

IVP takes the form:

$$g_1' = g_2; \qquad g_1(0) = 0, \tag{4.27}$$

$$g_2' = g_3; \qquad g_2(0) = 1, \tag{4.28}$$

$$g_3' = -\frac{g_1 g_3 - \left(\frac{2}{m+1}\right)(m g_2^2 + K_1 g_2) - M g_2}{[1 + nWe^2(f'')^2]\,[1 + We^2(f'')]^{\frac{n-3}{2}}}; \qquad g_3(0) = \xi. \tag{4.29}$$

Missing condition 'ξ' is assumed to satisfy following relation:

$$g_2(\eta_\infty, \xi) = 0, \qquad \text{where } \eta_\infty \text{ is positive real number.}$$

Now we use Newton's method

$$\xi^{(k+1)} = \xi^{(k)} - \frac{G(\xi)}{G'(\xi)}, \qquad \text{where } G(\xi) = g_2(\eta_\infty, \xi). \tag{4.30}$$

$$\tag{4.31}$$

In order to calculate the Eq. (4.27) , Eq. (4.28) and Eq. (4.29) following notations are used:

$$\frac{\partial g_1}{\partial s} = g_4, \qquad \frac{\partial g_2}{\partial s} = g_5, \qquad \frac{\partial g_3}{\partial s} = g_6.$$

$$g_4' = g_5; \qquad g_4(0) = 0,$$

$$g_5' = g_6; \qquad g_5(0) = 0,$$

$$g_6' = \left[\frac{1}{(1+nWe^2g_3^2)^2(1+We^2g_3^2)^{n-3}}\right]\Bigg[\Bigg[-(g_4g_3+g_1g_6)+\left(\frac{2}{m+1}\right)(2my_2g_5$$
$$+K_1g_5)+Mg_5\Bigg](1+nWe^2g_3^2)(1+We^2g_3^2)^{\frac{n-3}{2}}-\Bigg[(g_1g_3+\frac{2}{m+1}(mg_2^2+K_1g_2)$$
$$+Mg_2\Bigg]\Bigg[(2mWe^2g_3g_6)(1+We^2g_3^2)^{\frac{n-3}{2}}+(\frac{n-3}{2})(1+nWe^2y_3^2)^{\frac{n-5}{2}}(2nWe^2y_3y_6)\Bigg]\Bigg];$$
$$g_6(0) = 1.$$

The RK4 method is used to solve IVP for some suitable choice ξ. The missing condition ξ is updated by Newton‘s method and process will be continued untilthe following criteria is met.

$$|g_2(\eta_\infty, \xi) - 0| < \epsilon. \tag{4.32}$$

where $\epsilon = 10^{-8}$ is set for numerical calculation. Now for Eq. (4.33) and Eq. (4.34)

$$\theta'' = [-N_b\phi'\theta' - N_t\theta'^2 - f\theta' - (\frac{2}{m+1})\lambda\theta - MEcf'^2]Pr. \tag{4.33}$$

$$\phi'' = -PrLef\phi' - \frac{N_t}{N_b}\theta'' + \frac{2PrLe\gamma_1\phi}{m+1}\Big(1+\gamma_2\theta(\eta)\Big)^m exp\Big(-\frac{E}{1+\gamma_2\theta}\Big). \tag{4.34}$$

The transformed boundary conditions are given below:

$$\theta'(0) = -Bi + Bi\theta(0), N_b\phi'(0) + N_t\theta'(0) = 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. \tag{4.35}$$

We used following notations:

$$\theta = Z_1,$$

$$\theta' = Z_2,$$

$$\phi = Z_3,$$

$$\phi' = Z_4.$$

Rewritting equations:

$$Z_1' = Z_2; \qquad `Z_1(0) = p,$$

$$(4.36)$$

$$Z_2' = Pr[-g_1 Z_2 - N_b Z_4 Z_2 - N_t Z_2^2 - (\frac{2}{m+1})\lambda Z_1 - MEcg_2^2]; \quad Z_2(0) = -Bi(1-p),$$

$$Z_3' = Z_4; \qquad Z_3(0) = q,$$

$$Z_4' = -PrLeg_1 Z_4 - \frac{N_t}{N_b} Pr[-g_1 Z_2 - N_b Z_4 Z_2 - N_t Z_2^2 - \left(\frac{2}{m+1}\right)\lambda Z_1 - MEcg_2^2]$$
$$+ \frac{2PrLe\gamma_1 Z_3}{m+1}(1+\gamma_2 Z_1)^m exp\left(-\frac{E}{1+\gamma_2 Z_1}\right); \tag{4.37}$$
$$Z_4(0) = \frac{N_t Bi(1-p)}{N_b}.$$

Missing conditions ‘p’ and ‘q’ are assumed to satisfy the following relation:

$$Z_1(\eta_\infty, p, q) = 0 \tag{4.38}$$

$$Z_3(\eta_\infty, p, q) = 0. \tag{4.39}$$

We have to solve the above equations by applying the Newton‘s method,

$$\begin{bmatrix} p^{(k+1)} \\ q^{(k+1)} \end{bmatrix} = \begin{bmatrix} p^{(k)} \\ q^{(k)} \end{bmatrix} - \begin{bmatrix} \frac{\partial Z_1}{\partial p} & \frac{\partial Z_1}{\partial q} \\ \frac{\partial Z_3}{\partial p} & \frac{\partial Z_3}{\partial q} \end{bmatrix}^{-1}_{(p=p^{(k)}, q=q^{(k)})} \begin{bmatrix} Z_1 \\ Z_3 \end{bmatrix}. \tag{4.40}$$

We use following notations:

$$\frac{\partial Z_1}{\partial p} = Z_5, \quad \frac{\partial Z_2}{\partial s} = Z_6, \quad \frac{\partial Z_3}{\partial p} = Z_7, \quad \frac{\partial Z_4}{\partial p} = Z_8,$$
$$\frac{\partial Z_1}{\partial q} = Z_9, \quad \frac{\partial Z_2}{\partial t} = Z_{10}, \quad \frac{\partial Z_3}{\partial q} = Z_{11}, \quad \frac{\partial Z_4}{\partial q} = Z_{12}.$$

. Using above notation in Eq. (4.40), we have

$$\begin{bmatrix} p^{(k+1)} \\ q^{(k+1)} \end{bmatrix} = \begin{bmatrix} p^{(k)} \\ q^{(k)} \end{bmatrix} - \begin{bmatrix} Z_5 & Z_9 \\ Z_7 & Z_{11} \end{bmatrix}^{-1}_{(p=p^{(k)},q=q^{(k)})} \begin{bmatrix} Z_1 \\ Z_3 \end{bmatrix} \tag{4.41}$$

Differiating Eq. (4.36) to (4.37) w.r.t 'p' and 'q', we have

$$Z_5' = Z_6; \qquad Z_5(0) = 1,$$

$$Z_6' = Pr\left[g_1 Z_6 - N_b(Z_6 Z_4 + Z_8 Z_2) - 2N_t Z_2 Z_6 - \left(\frac{2}{m+1}\right)\lambda Z_5 - MEcg_2^2\right];$$

$$Z_6(0) = Bi,$$

$$Z_7' = Z_8; \qquad Z_7(0) = 0,$$

$$Z_8' = -PrLeg_1 Z_8 - \frac{N_t}{N_b}Pr\Bigg[- g_1 Z_6 - N_b(Z_6 Z_4 + Z_8 Z_2) - 2N_t Z_2 Z_6$$

$$- \left(\frac{2}{m+1}\right)\lambda Z_5 - MEcg_2^2\Bigg] + \frac{2}{m+1}PrLe\gamma_1\Bigg[Z_7(1+\gamma_2 Z_1)^m$$

$$exp\left(\frac{-E}{1+\gamma_2 Z_1}\right) + mZ_3(1+\gamma_2 Z_1)^{m-1}(\gamma_2 Z_5)exp\left(\frac{-E}{1+\gamma_2 Z_1}\right) + Z_3(1+\gamma_2 Z_1)^m$$

$$exp\left(\frac{-E}{1+\gamma_2 Z_1}\right)\left(\frac{E\gamma_2 Z_5}{1+(\gamma_2 Z_1)^2}\right)\Bigg]; \qquad Z_8(0) = -\frac{N_t Bi}{N_b},$$

$$Z_9' = Z_{10}; \qquad Z_9(0) = 0,$$

$$Z_{10}' = Pr\left[-g_1 Z_{10} - N_b(Z_4 Z_{10} + Z_2 Z_{12}) - 2N_t Z_2 N_{10} - \left(\frac{2}{m+1}\right)\lambda Z_9 - MEcg_2^2\right];$$

$$Z_{10}(0) = 0,$$

$$Z_{11}' = Z_{12}; \qquad Z_{11}(0) = 1,$$

$$Z_{12'} = -PrLeg_1 Z_{12} - \frac{N_t Pr}{N_b}[-g_1 Z_{10} - N_b(Z_4 Z_{10} + Z_2 Z_{12}) - 2N_t Z_2 Z_{10}$$

$$- \left(\frac{2}{m+1}\right)\lambda Z_9 - MEcg_2^2] + \left(\frac{2}{m+1}\right)PrLe\gamma_1\Big[Z_{11}(1+\gamma_2 Z_1)^m$$

$$exp\left(\frac{-E}{1+\gamma_2 Z_1}\right) + mZ_3(1+\gamma_2 Z_1)^{m-1}(\gamma_2 Z_9)exp\left(\frac{-E}{1+\gamma_2 Z_1}\right) + Z_3(1+\gamma_2 Z_1)^m$$

$$exp\left(\frac{-E}{1+\gamma_2 Z_1}\right)^2\left(\frac{E\gamma_2 Z_9}{1+(\gamma_2 Z_1)^2}\right)\Bigg]; \qquad Z_{12}(0) = 0.$$

We have to solve above equation by applying the shooting method. The RK4 method is use to solve the IVP and initial values are choosen arbitrarily. During the execution of iterations these initial guesses shall be updated by the Newton's method and the process will be repeated untill the following stoping criteria is met,

$$max(|Z_1(\eta_\infty, p, q)|, |Z_3(\eta_\infty, p, q)|) < \epsilon,$$

where $\epsilon = 10^{-8}$ is set for numerical calculation.

4.5 Results and Discussions

In the current section, computations are achieved for definite range of physical parameter like K_1, λ, Bi and γ_1. The influence of these parameters on local Sherwood number, local Nusselt number, temperature, velocity and concentration distibutions is observed numerically which are shown through tables and graphs. To validate the correctness of these outcomes, the assessment of the result of present study with existing reported works in litrature have been accomplished and tabularize in Tables **??** and 4.2. These assessment indicate the admirable agreement. Figure 4.1 represents the influence of porosity parameter K_1 at velocity distribution for dilatant and pseudoplastic nanofluid. For constant values of the parameters ($Pr = 2$, $Le = 1$, $We = 3$, $Bi = 1$, $m = 2$, $N_t = 0.1$, $N_b = 0.5$, $\gamma_1 = 0.1$ and $\lambda = 0.1$), it is observed that an excess in the value of porosity parameter K_1, rebates the velocity distribution rate.

Figure 4.2-4.5 demonstrate the impact of porosity parameter K_1 and Biot number Bi on temperature distribution $\theta(\eta)$. It is noted that temperature enhances with rising values of porosity parameter K_1, on contrary of heat source, heat sink and Biot number enhance the temperature distribution. It is also demonstrated that thermal boundary layer thickness enhance by increasing the value of Biot number, heat source parameter and heat sink parameter. It is observed that the existence of pseudoplastic nanofluid, the thickness of thermal boundary layer is higher. Increase in Biot number, heat transfer coefficient increased and rise in temperature. The Biot number is simply the ratio between the heat transfer by convection in the body to the heat transfer by conduction at the body surface physically. This is tangent magnatohydrodynamic nanofluid because convective heat exchange along the surface increase the boundary momentum layer. Figure 4.6-4.10 sketched for the concentration distribution with varying the value of porosity parameter K_1, Biot number Bi, chemical reaction parameter γ and heat source/sink λ in both the situations $n = 0.5$ and $n = 1.5$ while some parameter are static. It is observed from Figures 4.6-4.10 show that the concentration boundary layer thickness rebates with rising values of γ_1 respectively, while the thickness of concentration boundary layer enhances by elevation of Biot number and heat source. The impacts of chemical reaction parameter for concentration profile is observed in Figure 4.10 which shows that the concentration profile reduces by increasing chemical reaction parameter and it also decrease in concentration thickness. It is because the chemical reaction in this system, results in chemical dissipation and consequently decrease in a concentration profile. The most important effect is that the chemical reaction appears to reduce the overshoot in concentration profile and their crossponding boundary layer. It is notified that concentration boundary layer in case of shear thinning nanofluid is higher than in the case of shear thickening nanofluid. Figure 4.11 $(a - b)$ represents the Nusselt number deviation with porosity parameter K_1 for distinct values of chemical reaction parameter γ_1 for both values of n while other parameters are static. It is concluded that by increasing the value of K_1 and γ_1, enhance local Nusselt number magnitude. Figure 4.12 $(a - b)$ and 4.13 $(a - b)$ show local Nusselt number diversity with K_1 for several values for λ both value of $n = 0.5$ and $n = 1.5$. It is clarify that the local Nusselt number enhances with λ. Figure 4.14 $(a - b)$ display the Sherwood number varies with K_1 for different values of γ_1. It is assumed that both K_1 and γ_1 are raised induce to decrease in the local Sherwood number, that boosts the rate of mass transfer. Figure 4.15 $(a - b)$

and 4.16 $(a - b)$ represent the Sherwood number difference with K_1 for several values of λ for both value of n. It is clear that enhance in λ cause local Sherwood numbers to decrease. Figure 4.23 demonstrates the relationship between M and dimensionless velocity distribution. We see that the fluid's velocity profile is continuously depressed by boosting the Magnetic field value. Increasing the value of M usually creates the Lorentz force and a collision force, due to which the fluid temperature increases and the velocity reduces in the boundary layer. We see the influence of M on Temperature distribution in Figure 4.19. By enhancing M induces temperature profile increases. Physically, larger the M produces an opposing force normally known as Lorentz force which actually increases the thickness of nanofluid's boundry layer and its temperature profile. Figure 4.22 investigates the effect of M on dimensionless concentration distribution. From the curve it is obvious that the increasing value of M results in increased concentration distribution. Physically, the fluid concentration and crossponding thickness of the boundary are increased by M.

Figure 4.20 shows the relationship between Arrhenius activation energy and the concentration profile. The concentration profile is high through the increasing value of Arrhenius activation energy parameter E. Accordingly, there is sign to promote the concentration of modified Arrhenius structure. Therefore the overall chemical reaction is increased. The Ec results for the velocity and temperature profiles were characterized by Figures 4.18 and 4.21 . The Ec shows the relation between the fluid particle kinetic energy and the enthalphy of the boundary layer. The fluid particle kinetic energy increases as Ec assume a high value. It is noted that by increasing value of Ec the temperature distribution enhanced. Therefore the velocity and temperature of the fluids climb slightly and the related momentum and thermal boundary layer thickness increased. Physically, the dissipation increases with increasing Ec values due to this increases in the dissipation of internal fluid energy. Figure 4.21 sketched for analysis of Ec effect on the concentration profile. The concentration profile is clearly increasing due to increase in Ec. The increasing value of Ec is due to become a cause of the increase in the fluid thermal energy.

Table 4.1: Calculated values for $-\theta(0)$ for different value of Pr, N_t, Le when $We = 3, \lambda = \gamma = K_1 = 0$, $M = 0.3$, $Ec = 0.50$ and $E = 0.20$

Pr	N_t	Le	m	Present	
				n=0.5	n=1.5
1	0.1	1	2	0.0781	0.07827
3				0.08095	0.08136
5				0.08098	0.08138
2	0.3			0.08021	0.08061
	0.5			0.08003	0.08042
	0.7			0.07983	0.08022
	0.1	0.5		0.08043	0.08083
		1.5		0.08038	0.08077
		2.5		0.08035	0.08075
		1	1	0.07932	0.07916
			2	0.08117	0.08116
			0.5	0.07576	0.07564

Table 4.2: Calculated values for Skin friction coefficient, mass transfer rate and heat transfer rate for various value of various parameters given below when $M = 0.3$, $Ec = 0.5$, $E = 0.2$

Bi	Pr	n	γ_1	λ	K_1	-$f''(0)$	-$\theta'(0)$	-$\phi'(0)$
0.1	0.01	0.5	0.1	-0.1	0.5	0.014599	0.062971	-0.012594
0.2	0.01	0.5	0.1	-0.1	0.5	0.014599	0.092117	-0.018423
0.2	0.01	0.5	0.2	-0.1	0.5	0.014599	0.090950	-0.018190
0.2	0.1	0.5	0.2	-0.1	0.5	0.014599	0.092090	-0.018418
0.2	10	0.5	0.2	-0.1	0.5	0.014599	0.131466	-0.026293
0.2	10	0.5	0.5	-0.1	0.5	0.014599	-0.065207	0.013041
0.2	50	0.5	0.5	-0.1	0.5	0.014599	0.070705	-0.014141
0.2	50	0.5	0.5	-0.1	0.9	0.014599	0.140000	-0.028000
0.3	1	1.5	0.5	-0.1	0.5	0.014599	0.837065	-0.167413
0.3	1	1.5	0.5	-0.1	0.9	0.014599	0.005336	-0.001067

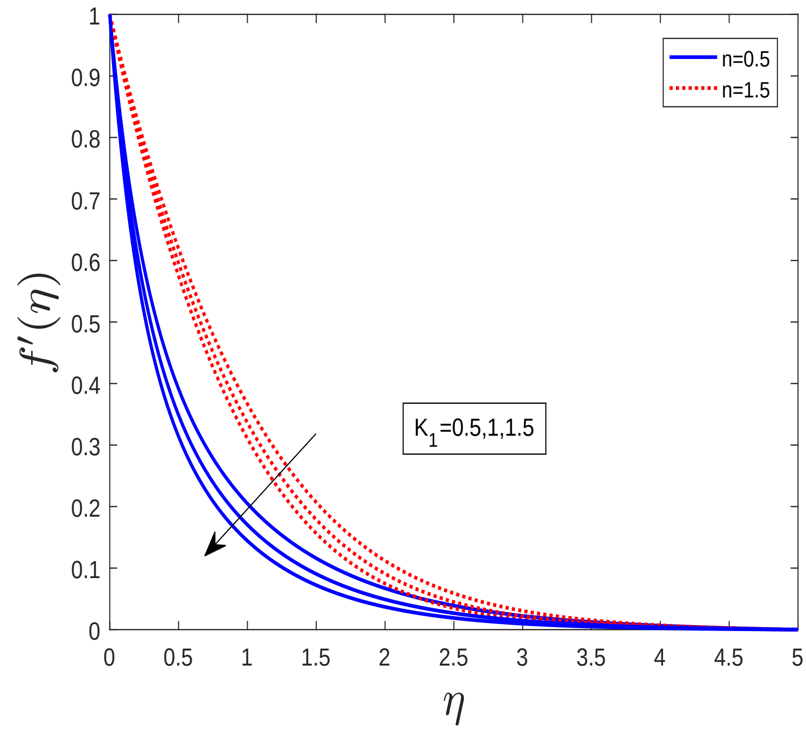

Figure 4.1: Effect of velocity via porosity parameter.

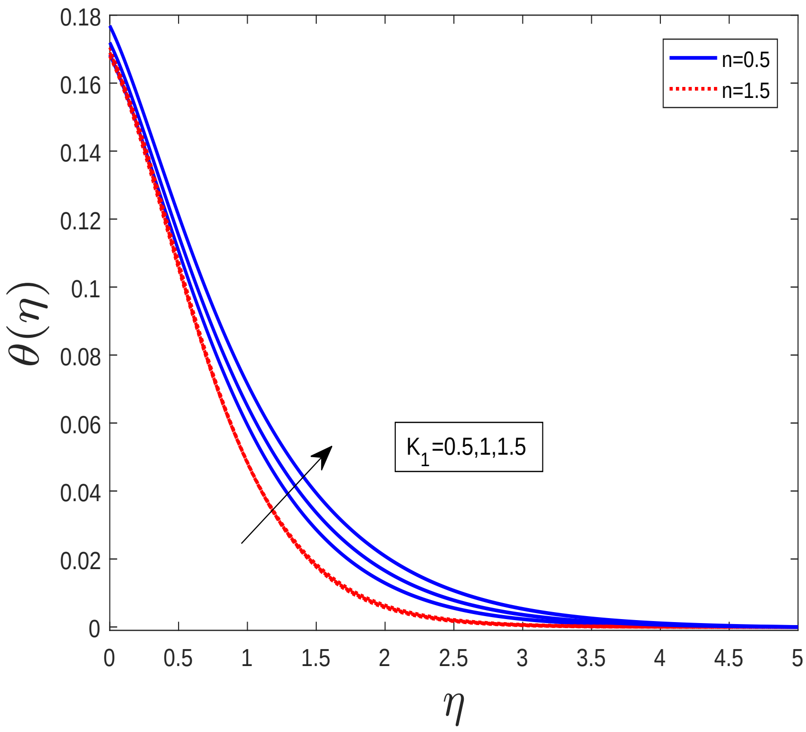

Figure 4.2: Effect of temperature via heat generation.

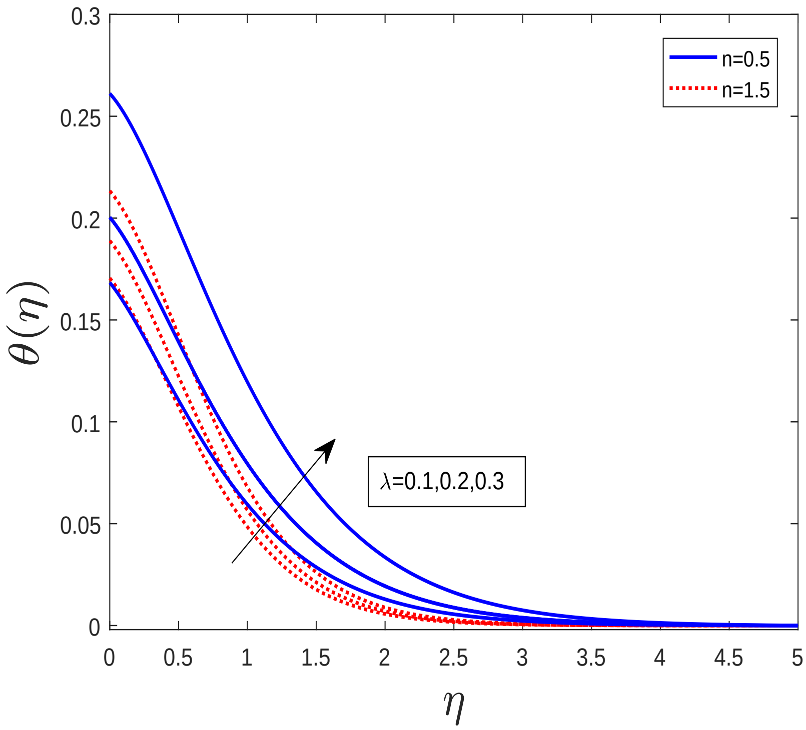

Figure 4.3: Effect of temperature via heat source.

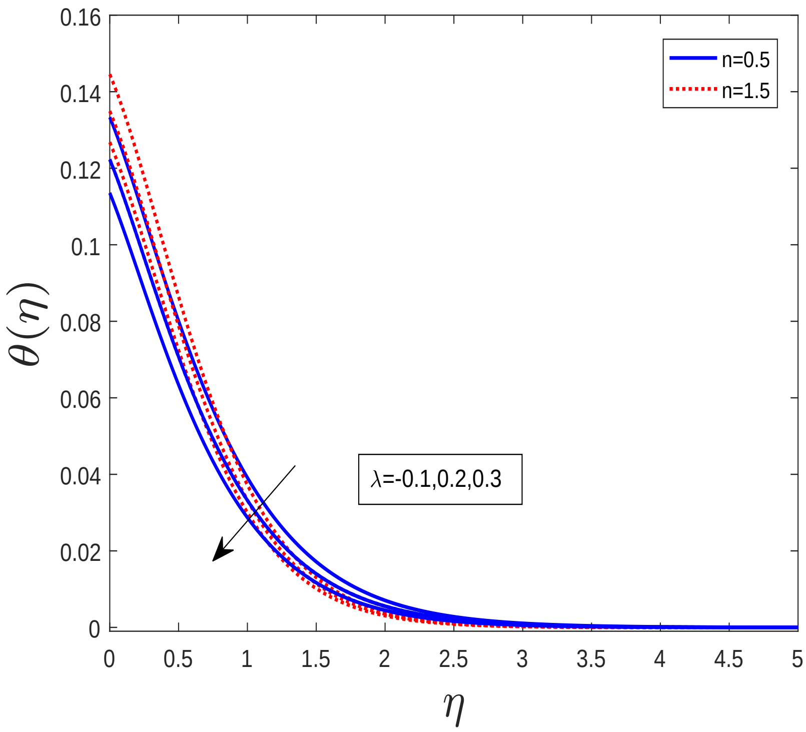

Figure 4.4: Effect of temperature via sink parameter.

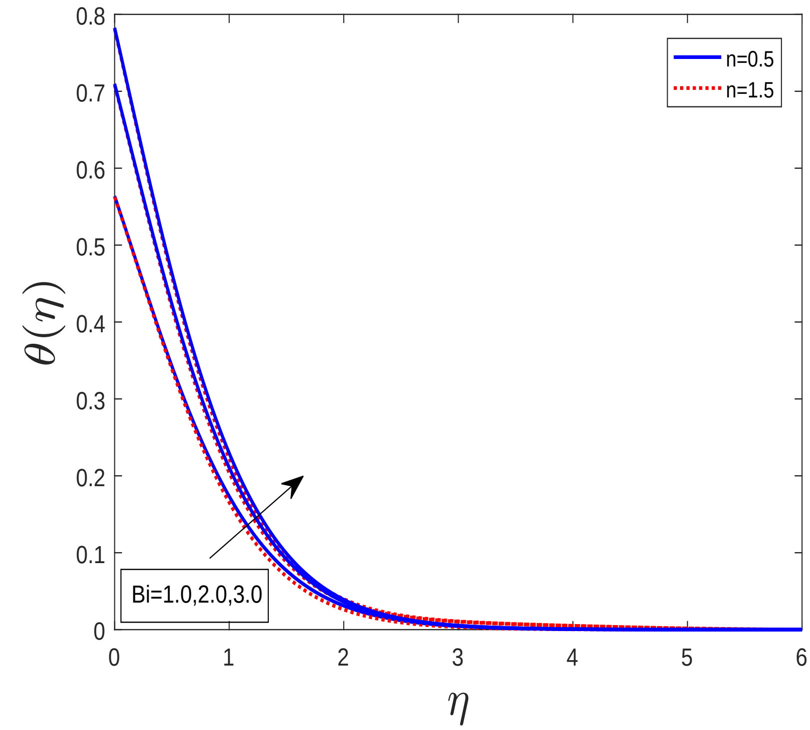

Figure 4.5: Effect of temperature via Biot number.

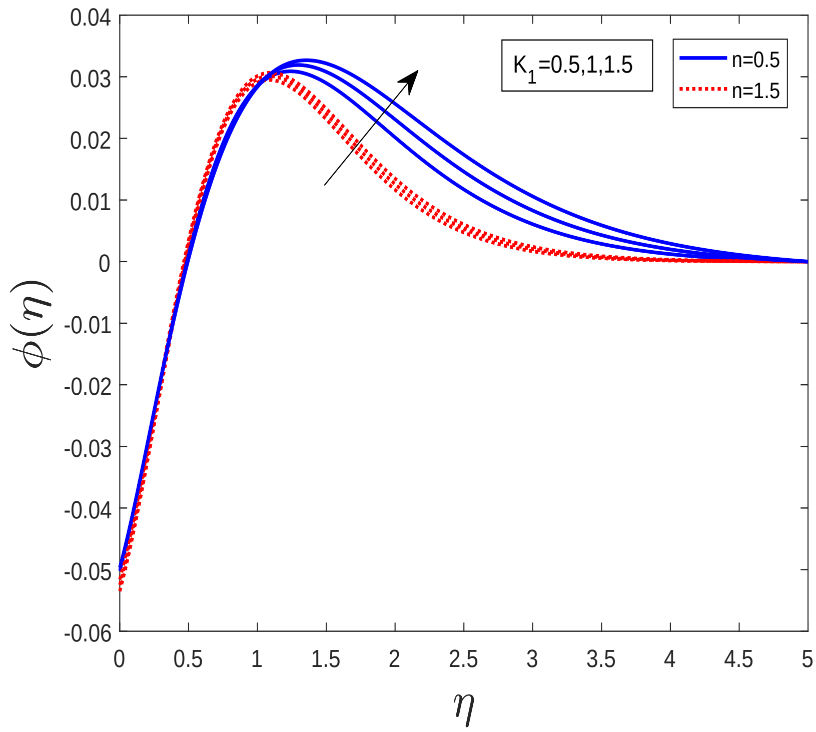

Figure 4.6: Effect of nanoparticles concentration via heat generation.

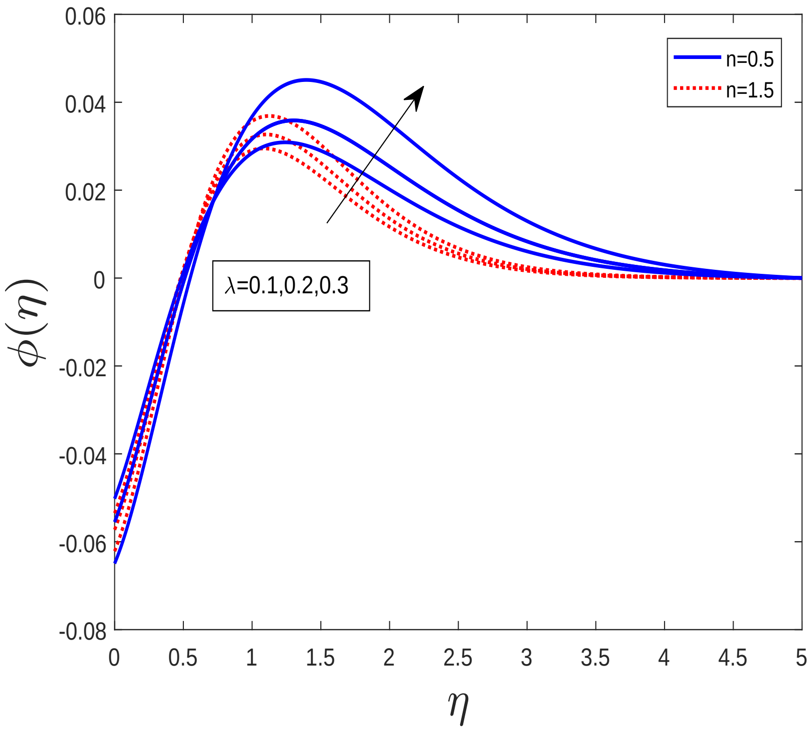

Figure 4.7: Effect of nanoparticles concentration via heat source.

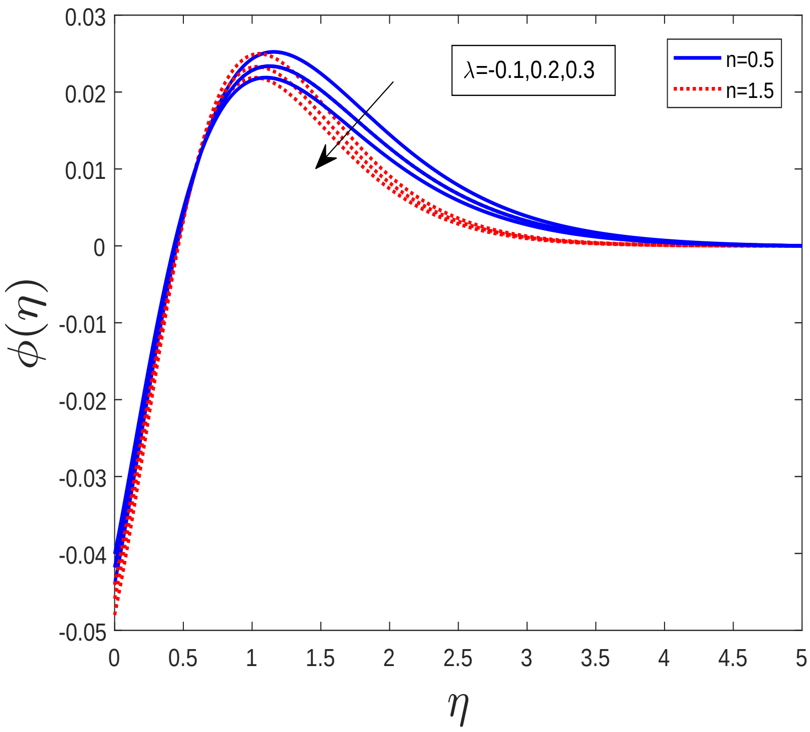

Figure 4.8: Effect of nanoparticles concentration via sink parameter.

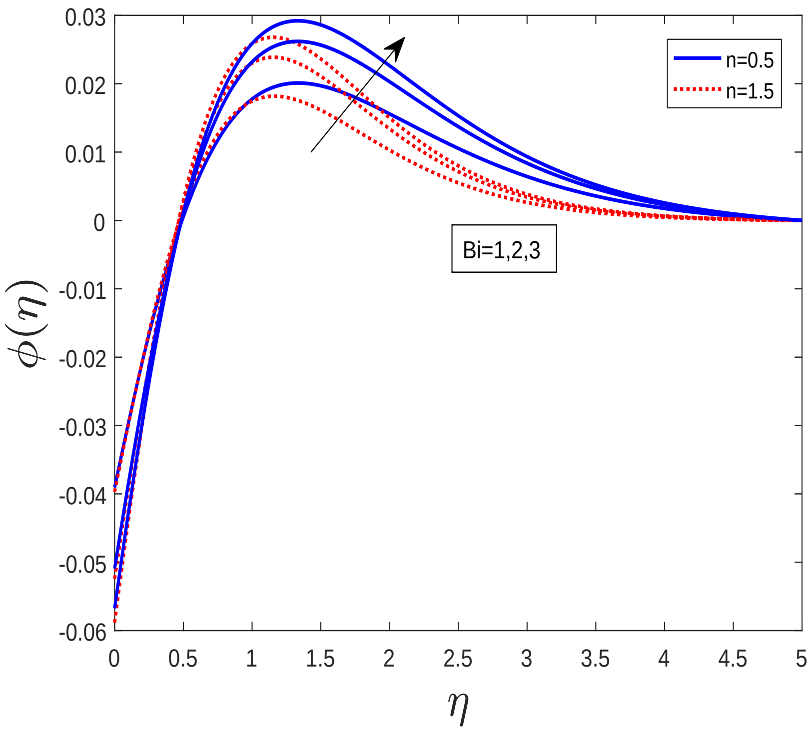

Figure 4.9: Effect of nanoparticles concentration via biot number.

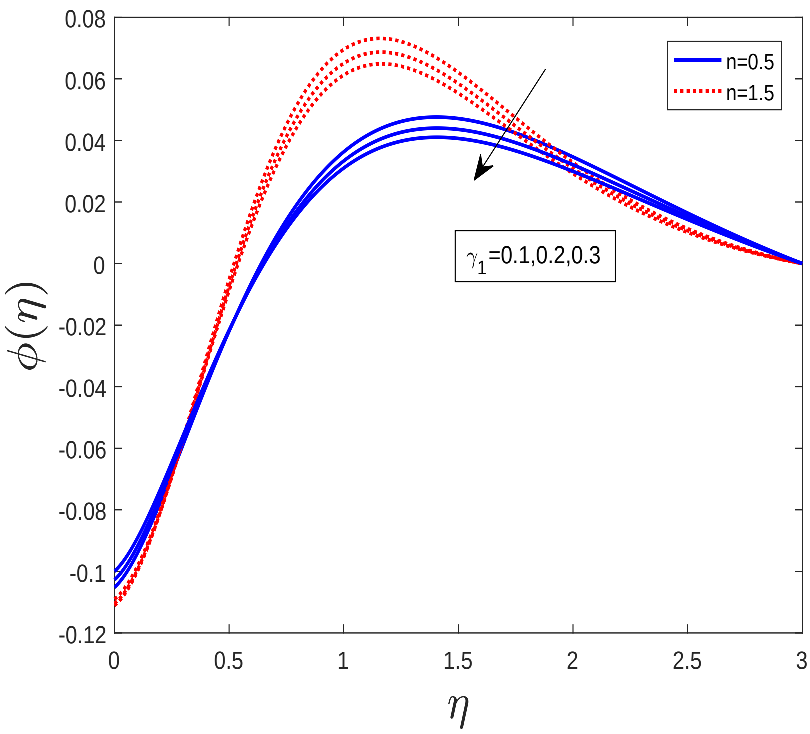

Figure 4.10: Effect of nanoparticles concentration via chemical reaction parameter.

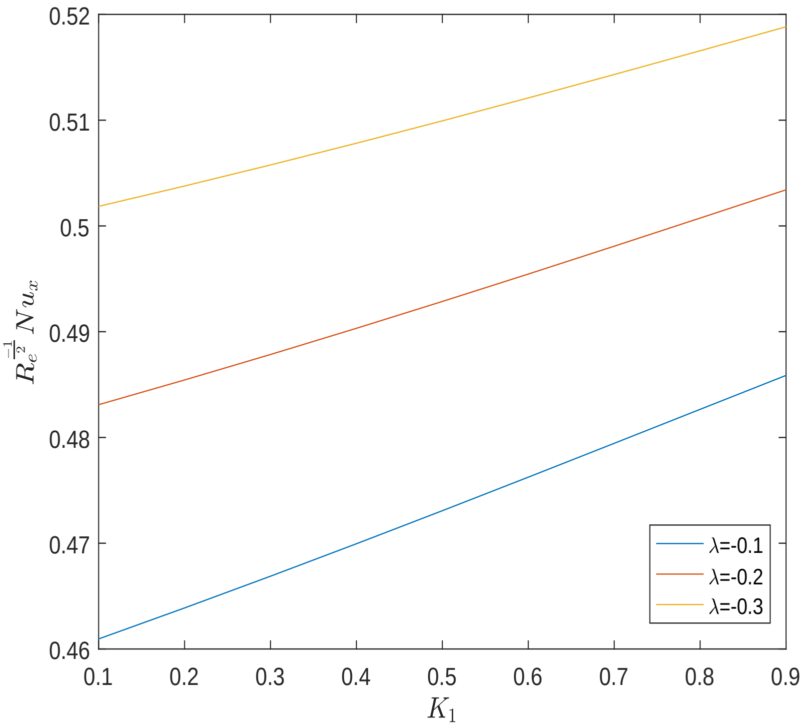

Figure 4.11: Effect of sink parameter on Nusslet number.

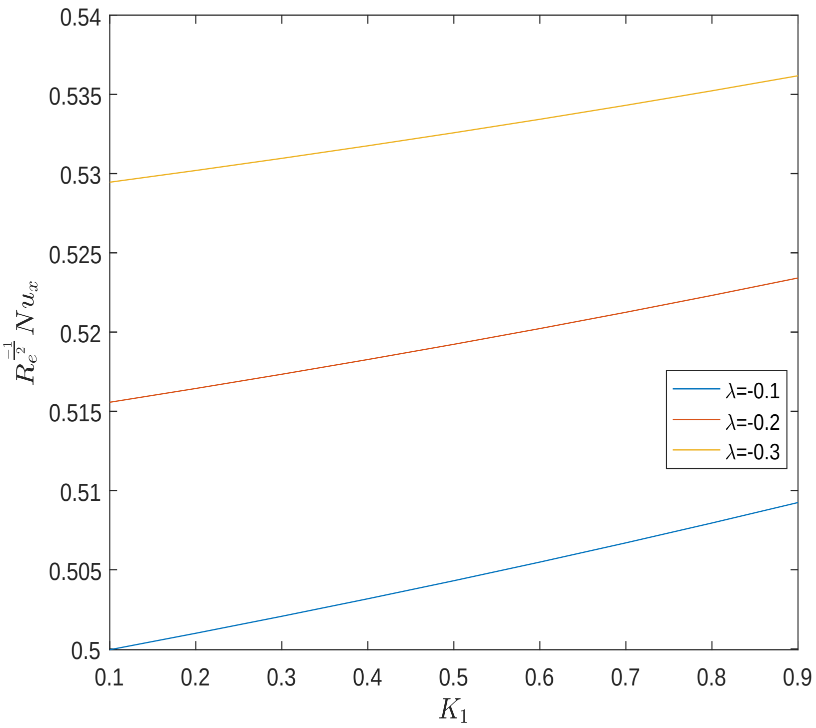

Figure 4.12: Effect of sink parameter on Nusslet number.

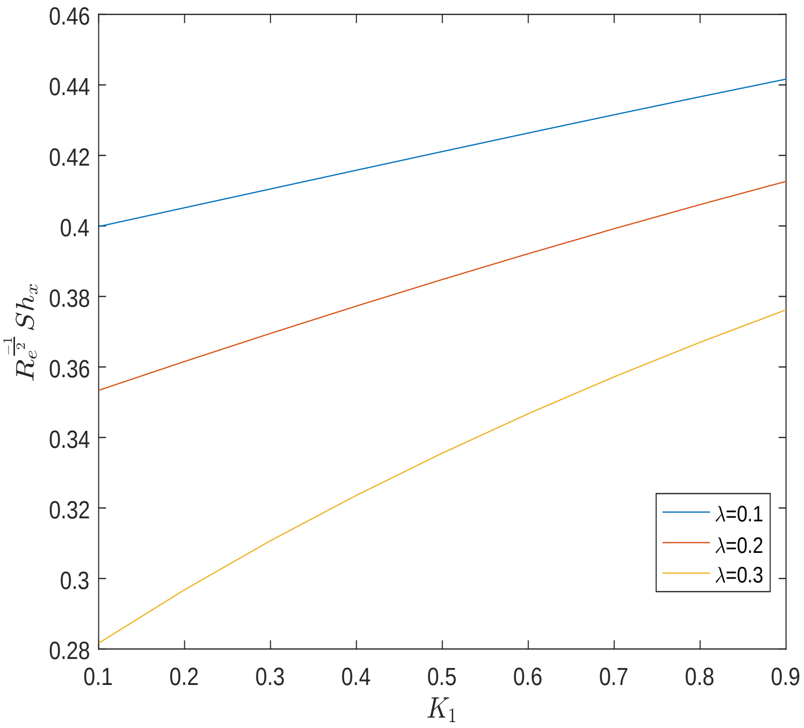

Figure 4.13: Effect of heat source on Sherwood number.

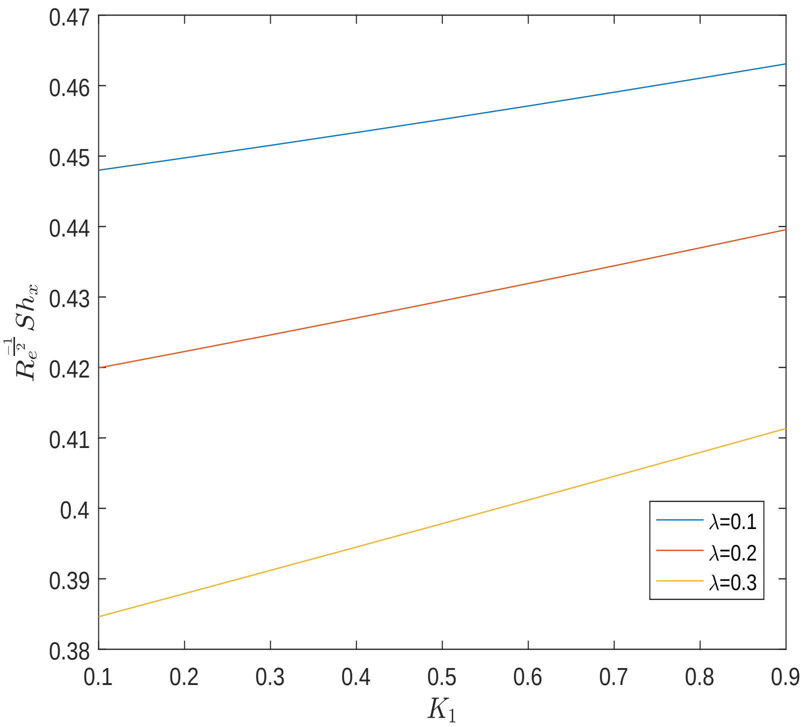

Figure 4.14: Effect of heat source on Sherwood number.

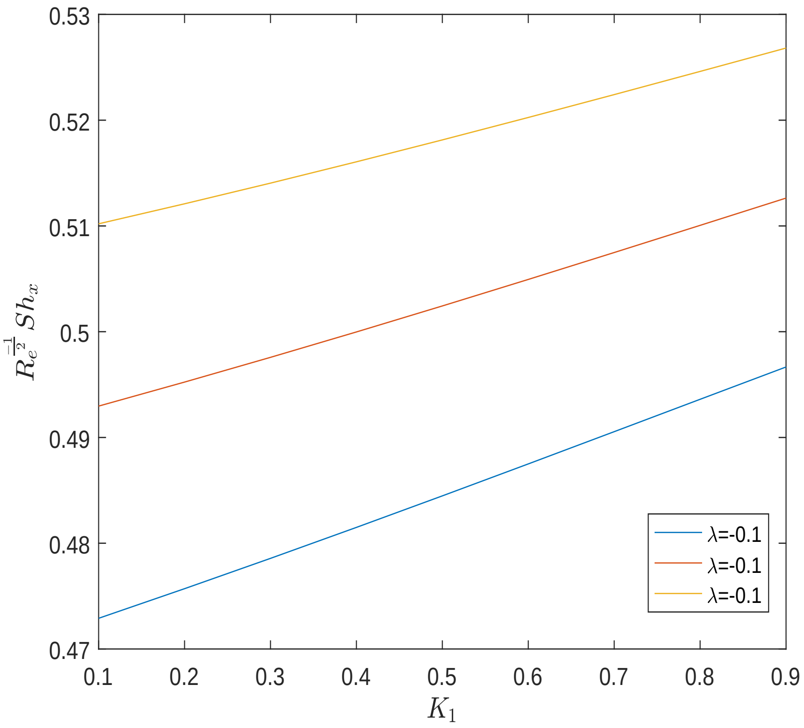

Figure 4.15: Effect of sink parameter on Sherwood number.

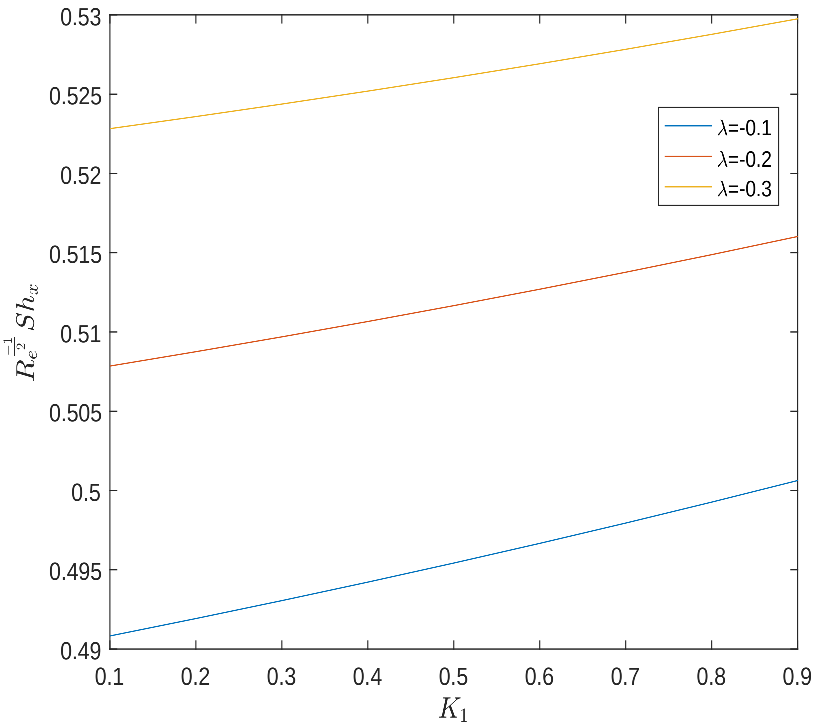

Figure 4.16: Influence of sink parameter on Sherwood number.

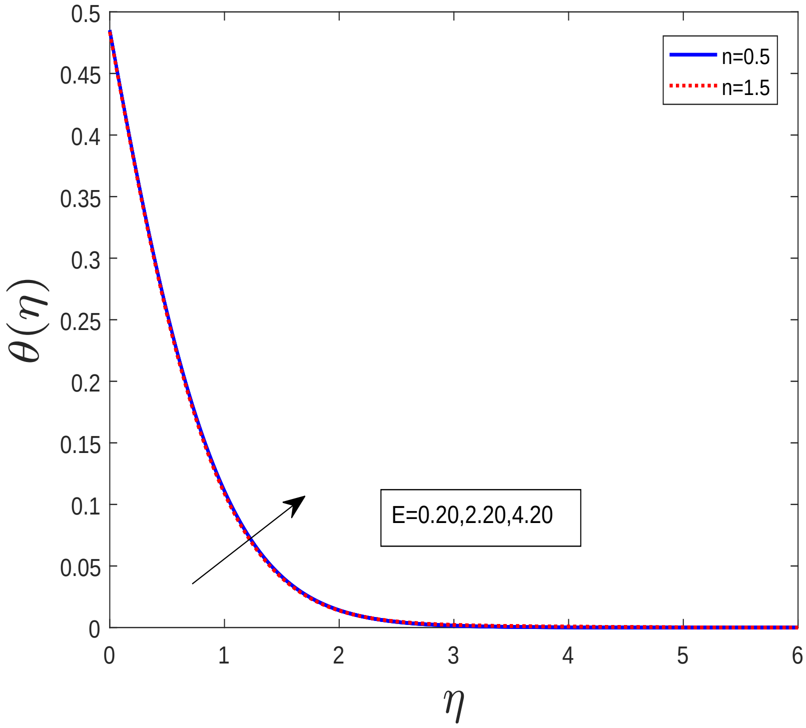

Figure 4.17: Effect of E on $\theta(\eta)$.

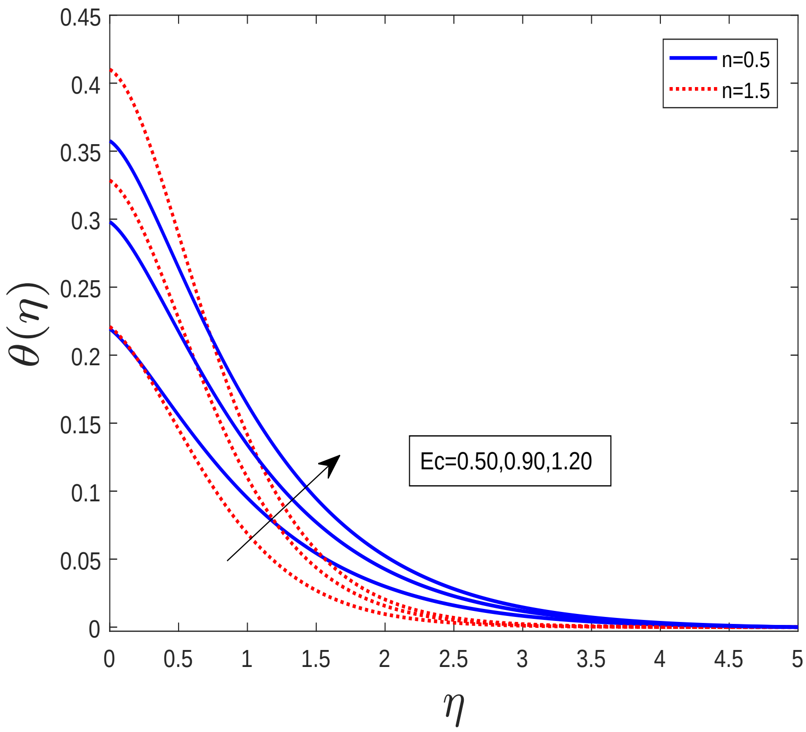

Figure 4.18: Effect of Ec on $\theta(\eta)$.

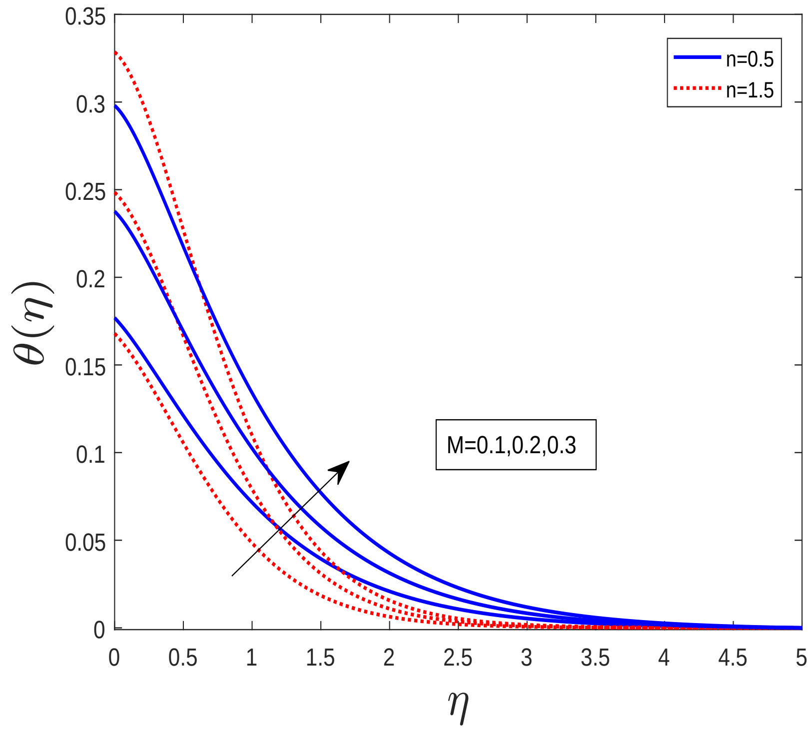

Figure 4.19: Effect of M on $\theta(\eta)$.

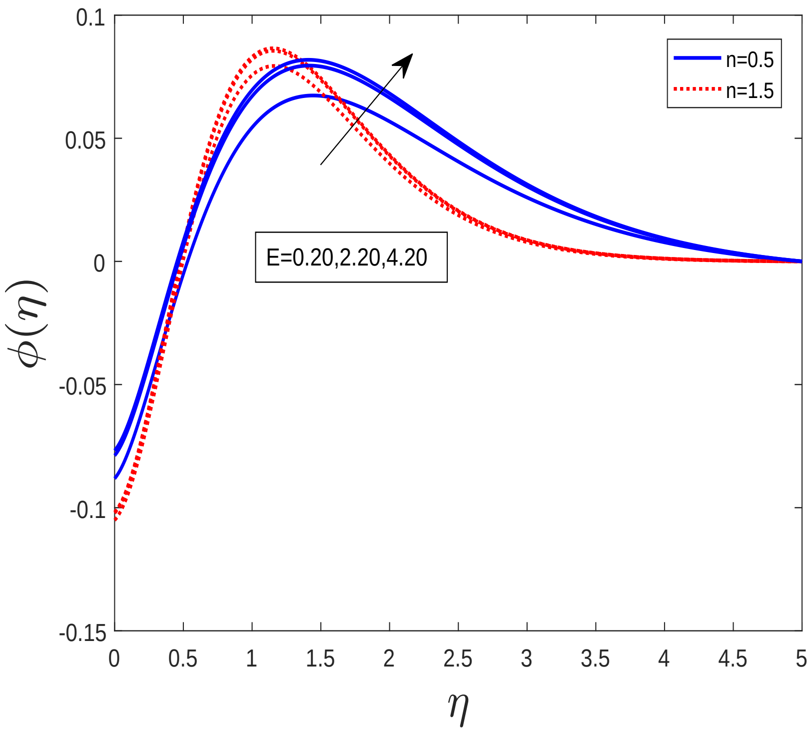

Figure 4.20: Effect of E on $\phi(\eta)$.

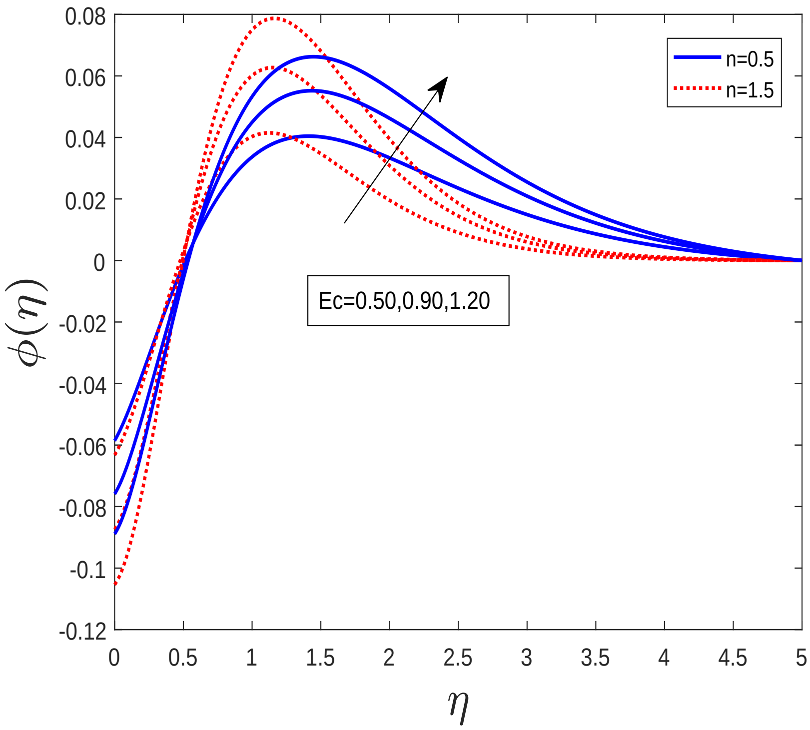

Figure 4.21: Effect of Ec on $\phi(\eta)$.

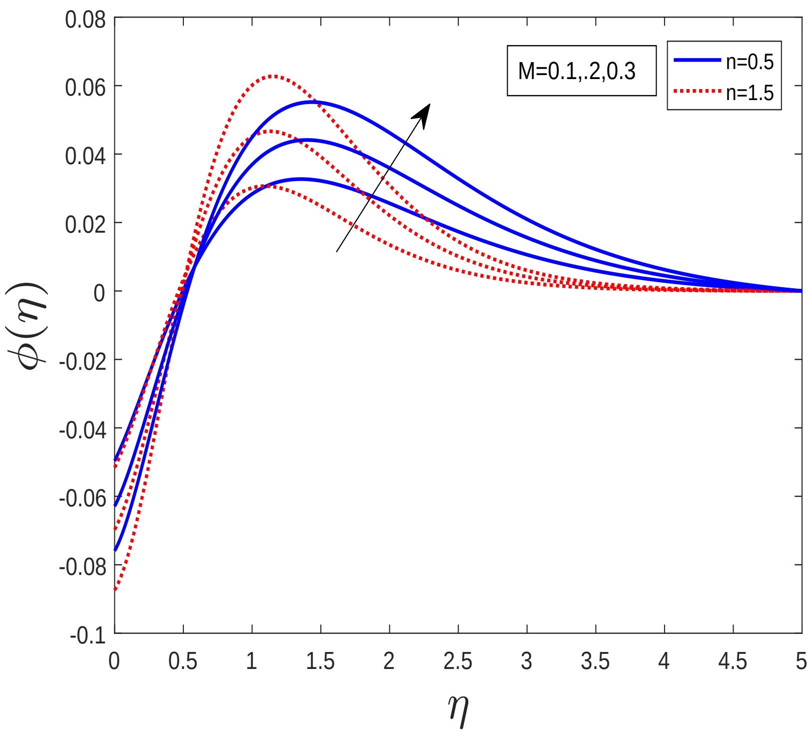

Figure 4.22: Effect of M on $\phi(\eta)$.

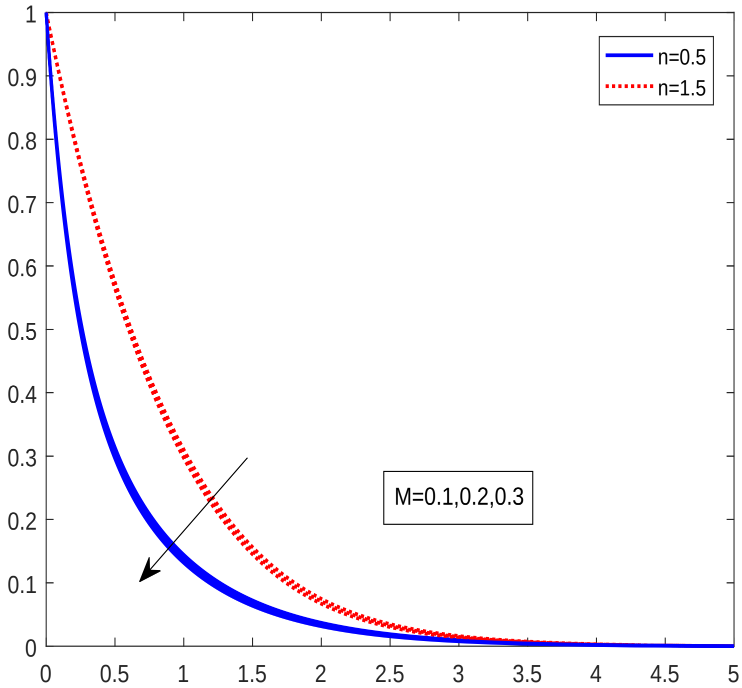

Figure 4.23: Effect of M on velocity.

Chapter 5

Conclusion

This dissertation exhibits a computational investigation for the mass and heat transfer in MHD Carreau nanofluid over porous medium with heat sink, heat source and chemical reaction. The conclusions of this dissertation are as follows:

- The velocity profile increases by increasing the values of power law index.
- A decrement is observed in the temperature and concentration because of rising values of power law index, Biot number, heat source, and heat sink.
- Increasing the chemical reaction parameter the rate of transfer of heat increases.
- The concentration profile falls for the larger estimation of chemical reaction.
- The rate of transfer of heat increases as the heat sink, heat source, power law index enhance.
- The mass transfer rate shrinks as enhances in chemical reaction, power law, heat source parameter and heat sink parameter.

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