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MHD Nanofluid Flow With Cattaneo-Christov Double Diffussion Model and Chemical Reaction

by

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in the

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*I dedicate my dissertation work to my **family** and dignified **teachers**. A special feeling of gratitude to, **My Father Mr. Sultan Mehmood and My Mother**, who gave me the encouragement I needed throughout my studies.*

CERTIFICATE OF APPROVAL

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Abstract

This thises numerically investigates the influence of aligned magnetic field, Cattaneo-Christov double diffussion model and chemical reaction of the flow of an electrically conducting nanofluid past a nonlinear stretching sheet through a porous medium with frictional heating. The partial differential eqution governing the flow problems are converted to ordinary differential eqution via similarity variables. The reduced equations are then solved numerically with the aid of shooting method. The influence of physical parameters such as aligned angle, magnetic field strength, dimensionless Maxwell parameter, mixed convection variable, nonlinear thermal variable, gravitational accelration, relaxation time parameter, Prandtl number Pr, porosity parameter.

Contents

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Abbreviations

IVPs	Initial value problems
MHD	Magnetohydrodynamics
ODEs	Ordineary differential equations
PDEs	Partial differential equations
RK	Runge-Kutta

Symbols

C_∞	Ambient concentration
μ	Viscosity
ρ	Density
ν	Kinematic viscosity
τ	Stress tensor
k	Thermal conductivity
α	Thermal diffisuitivity
σ	Electrical conductivity
u	x-component of fluid velocity
v	y-component of fluid velocity
B_0	Magnetic field constant
k_0	Permeability constant
a	Stretching constant
T_w	Temperature of the wall
T_∞	Ambient temperature of the nanofluid
T	Temperature
ρ_f	Density of the fluid
μ_f	Viscosity of the fluid
ν_f	Kinematic viscosity of the base fluid
ρ_{nf}	Density of the nanofluid
μ_{nf}	Viscosity of the nanofluid
q_r	Radiative heat flux
q	Heat generation constant

q_w	Heat flux
q_m	Mass flux
σ^*	Stefan Boltzmann constant
k^*	Absorption coefficient
ψ	Stream function
ξ	Similarity variable
C_f	Skin friction coefficient
Nu	Nusselt number
Nu_x	Local Nusselt number
Sh	Sherwood number
Sh_x	Local Sherwood number
Re	Reynolds number
Re_x	Local Reynolds number
ϕ	Nanoparticle volume fraction
R	Thermal radiation parameter
n	Stretching parameter
M	Magnetic parameter
K	Permeability parameter
Ec	Eckert number
Pr	Prandtl number
Q	Heat generation parameter
γ_1	Relaxation time parameter
Nb	Brownain motion parameter
Nt	Thermophoresis parameter
γ_2	Chemical reaction parameter
L_e	Lewis number
ρ_f	Density of the pure fluid
ρ_s	Density of nanoparticle
μ_f	Viscosity of the base fluid
$(\rho C_p)_f$	Heat capacity of base fluid
$(\rho C_p)_s$	Heat capacity of nanoparticle

σ_f	Electrical conductivity of the base fluid
σ_s	Electrical conductivity of the nanoparticle
k_f	Thermal conductivity of the base fluid
k_s	Thermal conductivity of the nanoparticle
f	Dimensionless velocity
θ	Dimensionless temperature
h	Dimensionless concentration
C	Concentration
C_w	Nanoparticles concentration at the stretching surface

Chapter 1

Introduction

The study of properties of fluid on various surfaces and geomateries is one of the most important topics of discussion among researchs because it includes many technological as well as industrial aspects like glass fibric generation, elastic sheets assembling, wire drawing etc [1]. Sakiadis was the first one to examine that boundary layer flow by continuous solid panel flowing with constant speed [2]. Non-Newtonian fluid flowing over the streched surface has bacome a topic of critical debate for past few decades. Non-Newtonian fluid play a huge role in many engineering and technological applications. The wide range of applications includes liquids film condensation process, aerodynamics, plasticfilms emission, copper wire thinnings etc [3]. The most common examples of non-Newtonian fluids in food industry are starch suspension, mayonnaise, yogurts, fruit juices and alcoholic beverages. All the characteristics of these fluids, structures cannot be showed by single constitutive equation. These non-Newtonian fluid models are classified as differential, rate and integral types. Rate type fluid include Maxwell model as it shows the fluid relaxation time phenomenon.

Harris and Wilkinson [4] who presented $2D$ flow of upper convected Maxwell fluid urged the researcgers to find new possibilities. Due to stretching sheet for upper convected Maxwell fluid, the transfer of heat and boundary layer flow have been discussed by many scientists. In past few decades, area of work in fluid dynamics has been heat analysis [5–7].

Heat and mass transfer have many industrial applications like meat and poultry in food industry, plastic and pipe industries, generation of glass fiber, steam generators and electronic devices, streamlined expulsion of plastic sheet, glass blowing, cloth industry etc. Developing an analytical solution and examining two-dimensional flow in the steady fluid induced by a streching sheet was first done by Crane [8]. Lin and Chen [9] explored the heat transfer features over a persistent streching area between different surface temperatures. The heat and mass transition referred as blowing over the stretching panel was studied by Laha et al. [10]. The spearheading work of Crane was carried ahead by various researchers indicating the significance of heat transfer flow [11–18].

In fluid dynamics, material which contains pores is known as porous medium. The study of porous medium has huge role in manufacturing and agricultural processes like condensers (used as heat exchange), catalytical plants (used to decrease harmful quality of depleting emanations from automobile engines), gas turbines (utilized to cool gas turbine blades), geophysics and geothermal energy system. The porous media are useful particularly in fermentation process, grain storage, contamination of ground water, generation of gasline, water motion in petroleum sources, beds of fossil fuel, power conserving areas, petroleumes reservoirs, ground water frameworks, thermal storage, depleting radioactives waste units and many others. Liquid flow through porous media in the high velocity systems is an area of great challenge for researchers. Non-parcial porous model is better and improved form of old Darcian model that merge concurrent characteristics of inertial tortuosity drag and boundary features. Henry Darcy, a french engineer, while working on fluid flow through the sand beds developed the flow of heterogeneous liquids by a porous medium in 1856. When inertia and boundary characteristic at a high flow rate are taken in account, the classical Darcy law is ineffective. I.Ullah [19] integrated a square velocity element of Darcian speed to predict the extract boundary layer flow and inertia, to evaluate the inertia and boundary features. This characteristic is credible for high Reynolds number."Forchheimer phrase "was the name given to the factor by Muskat [20]. Forchheimer on a stretching panel was developed by Pal and Mondal [21].

In adequacy of Darcy-Forchheimer hydro magnetic nanofluid flow in the direction of shrinking panel and the presence of thermal stratification, second order velocity slip, Ohmic dissipation and viscous dissemination impact were inspected by Ganesh et al [22]. Thermal radiation in Non-Darcian hydrophobia medium and the hydromagnetic flow of viscous liquid with viscous dissipation were researched by Gireesha et al [23]. Heat transfer model for the magnetohydrodynamics and the streamwise Darcy-Brinkman-Forchheimer liquid flow were discovered by Rashidi et al [24]. Bejan's thermal lines were used by Ahmed [25] to analyse infused non-Darcy hydrophobia medium with natural as well as forced convection in two sided lid driven enclosure. Hayat et al. [26] examined the Darcy-Forchheimer mobility of viscoelastic nanofluids on account of non-linear stretching sheet. The boundary conditions of Neumann for a standard Darcy-Forcheimer framework was recently investigated by Kang et al. [27].

Magnetic charateristics of electrically conductive liquid, known as Magnetohydrodynamics (MHD) have great role in fluid d ynamics. H annes A lfen used t he word MHD for the very first t ime. M HD fl ow ha s a hu ge im portance in in dustry and is being used in several fields like metallurgical procedures and petroleum production. Cooling speed engaged in these processes plays a vital role in properties of the final result and by use of magnetic field and electrically conductive liquid, the required final product features can b e regulated.

Other electrically conductive liquids such as aresnic copper alloys, molten metals, enriched urranium,engine oils, biochemicalfluids and other grades etc have different features in the nonattendance as well as in magnetic field v iew [28]. Nanofluid flows with MHD impact and convective circumstances were examined bt Hayat et al. [29]. Passing a porous wedge, the elastic and viscous MHD liquid flow i n a mixed convection form was inspected by Hsiao [30].

Ramesh et al. [31] inspected the MHD mixed convection flow of viscoelastic fluid past a stretching plate with Ohmic dissipation. In the presence of the uniform magnetic field, t he n atural h eat t ransfer o f n anofluid in side a lo ngitudinal framework was investigated by Ganji and Malvandis. The effect of associated magnetic

fields on a constant two dimensional flux on a vertical stretching layer is examined by Raju et al. [32]. They found that an increase in the aligned angle lower the velocity profile and enhance the temperature of the fluid.

1.1 Thesis Contributions

In this thesis, first of all the work of Bilal and Muzma Nazeer. [33] will be reviewed. Their work has been extended by considering the Chattaneo-Christov double diffusion model, which is not discussed so far. The given PDEs will be converted into a system of ODEs by applying similarity transformations.

Furthermore, to get the numerical results of non-linear ODEs, the shooting method will be applied. The numerical results are computed by using MATLAB. The impact of different parameters on velocity, temperature, concentration profile along with skin friction, Nusselt number and sherwood number will be discussed through ghraphs and tables.

1.2 Layout of Thesis

A brief overview of the contents of the thesis is provided below.

Chapter 2 includes some basic definitions and terminologies, which are useful to understand the concepts discussed later on.

Chapter 3 provides the reviewed analytical study of numerical analysis for the non-Newtonian flow over a stratified stretching inclined sheet with the aligned magnetic field and nonlinear convection. The numerical results are derived by the shooting method.

Chapter 4 extends the flow model discussed in Chapter 3 by including the impacts of Chattaneo Cristove Double Diffusion Model and Chemical Reaction.

Chapter 5 provides the concluding remarks of the thesis.

References used in the thesis are mentioned in **Biblography**.

Chapter 2

Preliminaries

This chapter contains some basic definitions and governing laws, which will be helpful in the subsequent chapters.

2.1 Some Basic Terminologies

Definition 2.1.1 (Fluid)

"A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be." [34]

Definition 2.1.2 (Fluid Mechanics)

"Fluid mechanics is that branch of science which deals with the behavior of the fluid (or gases) at rest as well as in motion." [35]

Definition 2.1.3 (Fluid Dynamics)

"The study of fluid if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics." [35]

Definition 2.1.4 (Fluid Statics)

"The study of fluid at rest is called fluid statics." [35]

Definition 2.1.5 (Viscosity)

"Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. Mathematically,

$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}},$$

where μ is viscosity coefficient, τ is shear stress and $\frac{\partial u}{\partial y}$ represents the velocity gradient." [35]

Definition 2.1.6 (Kinematic Viscosity)

"It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by symbol ν called **'nu'**. Mathematically,

$$\nu = \frac{\mu}{\rho}." \text{ [35]}$$

Definition 2.1.7 (Thermal Diffusivity)

"The rate at which heat diffuses by conducting through a material depends on the thermal diffusivity and can be defined as,

$$\alpha = \frac{k}{\rho C_p},$$

where α is the thermal diffusivity, k is the thermal conductivity, ρ is the density and C_p is the specifc heat at constant pressure." [36]

Definition 2.1.8 (Thermal Conductivity)

"The Fourier heat conduction law states that the heat flow is proportional to the

temperature gradient. The coefficient of proportionality is a material parameter known as the thermal conductivity which may be a function of a number of variables." [37]

2.2 Types of Fluid

Definition 2.2.1 (Ideal Fluid)

"A fluid, which is incompressible and has no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity." [35]

Definition 2.2.2 (Real Fluid)

"A fluid, which possesses viscosity, is known as a real fluid. In actual practice, all the fluids are real fluids." [35]

Definition 2.2.3 (Newtonian Fluid)

"A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid." [35]

Definition 2.2.4 (Non-Newtonian Fluid)

"A real fluid in which the shear stress is not directly proportional to the rate of shear strain (or velocity gradient), is known as a non-Newtonian fluid.

$$\tau_{xy} \propto \left(\frac{du}{dy}\right)^m, \qquad m \neq 1$$

$$\tau_{xy} = \mu\left(\frac{du}{dy}\right)^m$$." [35]

Definition 2.2.5 (Magnetohydrodynamics)

"Magnetohydrodynamics(MHD) is concerned with the mutual interaction of fluid

flow and magnetic fields. The fluids in question must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionised gases (plasmas) and strong electrolytes." [38]

2.3 Types of Flow

Definition 2.3.1 (Rotational Flow)

"Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis." [35]

Definition 2.3.2 (Irrotational Flow)

"Irrotational flow is that type of flow in which the fluid particles while flowing along stream-lines, do not rotate about their own axis then this type of flow is called irrotational flow." [35]

Definition 2.3.3 (Compressible Flow)

"Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid, Mathematically,

$$\rho \neq k,$$

where k is constant." [35]

Definition 2.3.4 (Incompressible Flow)

"Incompressible flow is that type of flow in which the density is constant for the fluid. Liquids are generally incompressible while gases are compressible, Mathematically,

$$\rho = k,$$

where k is constant." [35]

Definition 2.3.5 (Steady Flow)

"If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to time, the flow is said to be steady flow. Mathematically,

$$\frac{\partial Q}{\partial t} = 0,$$

where Q is any fluid property." [35]

Definition 2.3.6 (Unsteady Flow)

"If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes with respect to time, the flow is said to be unsteady. Mathematically,

$$\frac{\partial Q}{\partial t} \neq 0,$$

where Q is any fluid property." [35]

Definition 2.3.7 (Internal Flow)

"Flows completely bounded by a solid surfaces are called internal or duct flows." [34]

Definition 2.3.8 (External Flow)

"Flows over bodies immersed in an unbounded fluid are said to be an external flow." [34]

2.4 Modes of Heat Transfer

Definition 2.4.1 (Heat Transfer)

"Heat transfer is a branch of engineering that deals with the transfer of thermal energy from one point to another within a medium or from one medium to another

due to the occurrence of a temperature difference." [37]

Definition 2.4.2 (Convection)

"Convection heat transfer is usually defined as energy transport effected by the motion of a fluid. The convection heat transfer between two dissimilar media is governed by Newton's law of cooling." [37]

Definition 2.4.3 (Conduction)

"The transfer of heat within a medium due to a diffusion process is called conduction." [37]

Definition 2.4.4 (Thermal Radiation)

"Thermal radiation is defined as radiant (electromagnetic) energy emitted by a medium and is solely to the temperature of the medium." [37]

2.5 Dimensionless Numbers

Definition 2.5.1 (Eckert Number)

"It is the dimensionless number used in continuum mechanics. It describes the relation between flows and the boundary layer enthalpy difference and it is used for characterized heat dissipation. Mathematically,

$$Ec = \frac{u^2}{C_p \nabla T}$$

where C_p denotes the specific heat." [34]

Definition 2.5.2 (Prandtl Number)

"It is the ratio between the momentum diffusivity ν and thermal diffusivity α.

Mathematically, it can be defined as

$$Pr = \frac{\nu}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{C_p \rho}} = \frac{\mu C_p}{k}$$

where μ represents the dynamic viscosity, Cp denotes the specific heat and k stands for thermal conductivity. The relative thickness of thermal and momentum boundary layer is controlled by Prandtl number. For small Pr, heat distributed rapidly corresponds to the momentum." [34]

Definition 2.5.3 (Skin Friction Coefficient)

"The steady flow of an incompressible gas or liquid in a long pipe of internal D. The mean velocity is denoted by u_w. The skin friction coefficient can be defined as

$$C_f = \frac{2\tau_0}{\rho u_w^2}$$

where τ_0 denotes the wall shear stress and ρ is the density." [39]

Definition 2.5.4 (Nusselt Number)

"The hot surface is cooled by a cold fluid stream. The heat from the hot surface, which is maintained at a constant temperature, is diffused through a boundary layer and convected away by the cold stream. Mathematically,

$$Nu = \frac{qL}{k}$$

where q stands for the convection heat transfer, L for the characteristic length and k stands for thermal conductivity." [40]

Definition 2.5.5 (Sherwood Number)

"It is the nondimensional quantity which show the ratio of the mass transport by convection to the transfer of mass by diffusion. Mathematically:

$$Sh = \frac{kL}{D}$$

here L is characteristics length, D is the mass diffusivity and k is the mass transfer" coeffcient." [41]

Definition 2.5.6 (Reynolds Number)

"It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. Mathematically,

$$Re = \frac{VL}{\nu},$$

where U denotes the free stream velocity, L is the characteristic length and ν stands for kinematic viscosity." [35]

2.6 Governing Laws

Governing Law 2.6.1 (Continuity Equation)

"The principle of conservation of mass can be stated as the time rate of change of mass is fixed volume is equal to the net rate of flow of mass across the surface. Mathematically, it can be written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0."[37]$$

Governing Law 2.6.2 (Momentum Equation)

"The momentum equation states that the time rate of change of linear momentum of a given set of particles is equal to the vector sum of all the external forces acting on the particles of the set, provided Newton's Third Law of action and reaction governs the internal forces. Mathematically, it can be written as:

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla.[(\rho \mathbf{u})\mathbf{u}] = \nabla.\mathbf{T} + \rho g."[37]$$

Governing Law 2.6.3 (Energy Equation)

"The law of conservation of energy states that the time rate of change of the total energy is equal to the sum of the rate of work done by the applied forces and

change of heat content per unit time.

$$\frac{\partial \rho}{\partial t} + \nabla.\rho\mathbf{u} = -\nabla.\mathbf{q} + Q + \phi,$$

where ϕ is the dissipation function." [37]

2.7 Shooting Method

To explain the shooting method, consider the following nonlinear boundary value problem.

$$\left.\begin{aligned} f''(x) &= f(x)f'(x) + 4f(x) \\ f(0) &= 0, \qquad f(C) = D. \end{aligned}\right\} \tag{2.1}$$

To decrease the order of the above boundary value problem, establish the following notations.

$$f = Z_1 \qquad f' = Z_1' = Z_2 \qquad f'' = Z_2'. \tag{2.2}$$

As a result, (2.1) is converted into the following system of first order ODEs.

$$Z_1' = Z_2, \qquad Z_1(0) = 0, \tag{2.3}$$

$$Z_2' = Z_1 Z_2 + 4Z_1, \qquad Z_2(0) = s, \tag{2.4}$$

where s is the missing initial condition which will be guessed.
The above IVP will be numerically solved by the RK-4 method. The missing condition s is to be chosen such that.

$$Z_1(C, s) = D. \tag{2.5}$$

For convenience, now onward $Z_1(C, s)$ will be denoted by $Z_1(s)$.
Let us further denote $Z_1(s) - D$ by $H(s)$, so that

$$H(s) = 0. \tag{2.6}$$

The above equation can be solved by using Newton's method with the following iterative formula.

$$s^{n+1} = s^n - \frac{H(s^n)}{\frac{\partial H(s^n)}{\partial s}},$$

or

$$s^{n+1} = s^n - \frac{Z_1(s^n) - D}{\frac{\partial Z_1(s^n)}{\partial s}}. \tag{2.7}$$

To find $\frac{\partial Z_1(s^n)}{\partial s}$, introduce the following notations.

$$\frac{\partial Z_1}{\partial s^n} = Z_3, \quad \frac{\partial Z_2}{\partial s^n} = Z_4. \tag{2.8}$$

As a result of these new notations the Newton's iterative scheme, will then get the form.

$$s^{n+1} = s^n - \frac{Z_1(s) - D}{Z_3(s)}. \tag{2.9}$$

Now differentiating the system of two first order ODEs (2.3)-(2.4) with respect to s, we get another system of ODEs, as follows.

$$Z_3' = Z_4, \qquad Z_3(0) = 0. \tag{2.10}$$

$$Z_4' = Z_3 Z_2 + Z_1 Z_4 + 4Z_3, \qquad Z_4(0) = 1. \tag{2.11}$$

Writing all the four ODEs (2.3), (2.4), (2.10) and (2.11) together, we have the following initial value problem.

$$\begin{aligned} Z_1' &= Z_2, & Z_1(0) &= 0. \\ Z_2' &= Z_1 Z_2 + 4Z_1, & Z_2(0) &= s. \\ Z_3' &= Z_4, & Z_3(0) &= 0. \\ Z_4' &= Z_3 Z_2 + Z_1 Z_4 + 4Z_3, & Z_4(0) &= 1. \end{aligned}$$

The above system together will be solved numerically by Runge-Kutta method of order four.

The stopping criteria for the Newton's technique is set as,

$$| Z_1(s) - D | < \epsilon,$$

where $\epsilon > 0$ is an arbitrarily small positive number.

Chapter 3

A Non-Newtonian Flow Over Stratified Stretching/Shrinking Inclined Sheet

3.1 Introduction

In this chapter, the numerical analysis of upper-convected Maxwell fluid flow over nonlinear shrinking inclined sheet with inclination angle α is discussed. The nonlinear partial differential equations are converted into dimensionless ODEs by utilizing similiarity transformation. To solve these ODEs, shooting method is used. The well known software MATLAB is adopted for numerical computations. At the end, tables and graphs are displyed to show the numerical results of ODEs.

3.2 Mathematical Modeling

Consider a non-Newtonian fluid flowing over an inclined shrinking sheet along with inclination α, aligned magnetic field B_0 with angle γ and velocity $u_w = a\,x$ has been considered. The temperature on the wall is $T_f = T_0 + a_1 x$ and $T_\infty = T_0 + d_1 x$ is the temperature far from the wall.

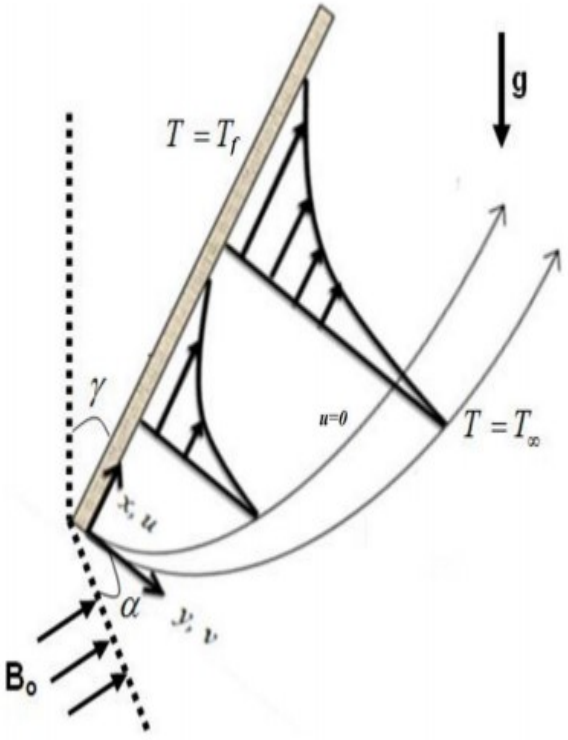

FIGURE 3.1: Geometry for the flow under discussion.

The set of partial differential equations describing the non-Newtonian flow are as follow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$\begin{aligned} &u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) \\ &= \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{\rho}\sin^2(\gamma)\left(u + \lambda v\frac{\partial u}{\partial y}\right) + g\big(\beta_1(T - T_\infty) \\ &\quad + \beta_2(T - T_\infty)^2\big)\cos(\alpha) - \frac{v}{k}u - Fu^2, \end{aligned} \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}. \tag{3.3}$$

The corresponding BCs have been given as:

$$\left.\begin{aligned} &u = \epsilon u_w(x), \quad v = v_o, \quad T = T_f = T_o + a_1 x, \quad at \quad y = 0. \\ &u \to 0, \quad T \to T_\infty = T_o + d_1 x, \quad as \quad y \to \infty. \end{aligned}\right\} \tag{3.4}$$

For the conversion of the mathematical model (3.1)-(3.4) into the system of ODEs, the following similarity transformation has been used.

$$\left.\begin{aligned} &\omega(x,y) = \sqrt{\nu x u_w(x)} f(\xi) = \sqrt{a\nu} x f(\xi), \qquad \theta(\xi) = \frac{T - T_\infty}{T_f - T_o}, \\ &\xi = \sqrt{\frac{u_w(x)}{\nu x}} y = \left(\frac{a}{\nu}\right)^{\frac{1}{2}} y. \end{aligned}\right\} \tag{3.5}$$

The detailed procedure for the conversion of (3.1)-(3.3) into the dimensionless form has been discussed below.

$$\begin{aligned} u &= \frac{\partial \omega}{\partial y} \\ \frac{\partial \omega}{\partial y} &= \frac{\partial}{\partial y}\left(\sqrt{a\nu} x f\right) \\ &= \sqrt{a\nu} x f' \frac{\partial \xi}{\partial y}. \\ \frac{\partial \xi}{\partial y} &= \sqrt{\frac{a}{\nu}} \\ \frac{\partial \omega}{\partial y} &= a x f' \\ u &= a x f'. \end{aligned} \tag{3.6}$$

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial}{\partial x}\left(a x f'\right) \\ &= a \frac{\partial}{\partial x}\left(x f'(\xi)\right) \\ &= a\left(f' + a x f''.0\right) \\ &= a f'. \end{aligned} \tag{3.7}$$

$$\begin{aligned} v &= -\frac{\partial \omega}{\partial x} \\ &= -\frac{\partial}{\partial x}\left(\sqrt{a\nu} x f\right) \\ &= -\sqrt{a\nu} \frac{\partial}{\partial x}(x f) \\ &= -\sqrt{a\nu}(f + x f'.0) \end{aligned}$$

$$v = -\sqrt{a\nu} f \tag{3.8}$$

$$\begin{aligned}\frac{\partial v}{\partial y} &= -\sqrt{a\nu} f' \frac{\partial \xi}{\partial y} \\ &= -\sqrt{a\nu} f' \left(\frac{a}{\nu}\right)^{\frac{1}{2}} \\ &= -af'. \end{aligned} \tag{3.9}$$

Adding (3.7) and (3.9)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = af' - af' = 0. \tag{3.10}$$

Now, the conversion of the momentum equation to get its dimensionless form, will be conducted through the following steps.

$$\begin{aligned}\frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}\left(axf'\right) \\ &= axf''\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \\ &= \frac{a^{\frac{3}{2}}x}{\nu^{\frac{1}{2}}} f''. \end{aligned} \tag{3.11}$$

$$\begin{aligned}u\frac{\partial u}{\partial x} &= axf'\left(af'\right) \\ &= a^2 x f'^2. \end{aligned} \tag{3.12}$$

$$\begin{aligned}v\frac{\partial u}{\partial y} &= -\sqrt{a\nu} f\left(\frac{a^{\frac{3}{2}}x}{\nu^{\frac{1}{2}}} f''\right) \\ &= -a^2 x f f''. \end{aligned} \tag{3.13}$$

$$\begin{aligned}\frac{\partial^2 u}{\partial x^2} &= \frac{\partial}{\partial x}\left(af'\right) \\ &= af''0 \\ &= 0. \end{aligned} \tag{3.14}$$

$$\begin{aligned}\frac{\partial^2 u}{\partial x \partial y} &= af''\left(\frac{a}{\nu}\right)^{\frac{1}{2}} \\ &= \frac{a^{\frac{3}{2}}}{\nu^{\frac{1}{2}}} f''. \end{aligned} \tag{3.15}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{a^{\frac{3}{2}}}{\nu^{\frac{1}{2}}} x f''' \left(\frac{a}{\nu}\right)^{\frac{1}{2}}$$

$$= \frac{a^2}{\nu} f'''. \tag{3.16}$$

Putting the above derivatives in the left hand side of (3.2),

$$\begin{aligned}
&\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) \\
&= a^2 x f'^2 - a^2 x f f'' + \lambda\big(a^2x^2 f'^2.0 \\
&\quad + (a\nu f^2)\left(\frac{a^2}{\nu} x f'''\right) + 2(axf')(-\sqrt{a}\nu f)\left(\frac{a^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right) f''\big).
\end{aligned} \tag{3.17}$$

Now the substitution of derivative in right hand side of (3.2):

$$\begin{aligned}
&\nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_o^2}{\rho}\sin^2(\gamma)\left(u + \lambda v\frac{\partial u}{\partial y}\right) + g\big(\beta_1(T - T_\infty) + \beta_2(T - T_\infty)^2\big)\cos(\alpha) \\
&\quad - \frac{v}{k}u - Fu^2 \\
&= \nu\big(\frac{a^2}{\nu} x f'''\big) - \frac{\sigma\beta_o^2}{\rho}\sin^2(\gamma)\big(axf'\big) - \lambda(a^2 x f f'') + g(\beta_1(T - T_\infty) \\
&\quad + \beta_2(T - T_\infty)^2)\cos(\alpha) - \frac{\nu}{k}(axf') - F(a^2x^2f'^2) \\
&= a^2 x f''' - \frac{\sigma B_0^2}{\rho}\sin^2(\gamma)\big(axf' - \lambda(a^2 x f f'')\big) + g\big(\beta_1(T_f - T_o) \\
&\quad + \beta_2(T_f - T_o)^2\big)\cos(\alpha) - \frac{\nu}{ka} x f' - Fa^2x^2f'^2
\end{aligned} \tag{3.18}$$

Now comparing the right hand side and left hand side of momentum equation (3.2), we get dimensionless form as below .

$$\begin{aligned}
&a^2 x(f'^2 - f f'' + \lambda(a f^2 f''') - \lambda(2a f f' f'')) \\
&= a^2 x(f''' - \frac{\sigma B_o^2}{\rho}\sin^2(\gamma)(f' - \beta f f'') + g\frac{\beta_1}{a^2 x}(T_f - T_o)(1 + \beta_t\theta)\theta\cos(\alpha) \\
&\quad - \frac{\nu}{ka} f' - Fx f'^2)
\end{aligned}$$

$$
\begin{aligned}
&= f''' + ff'' + \beta\big(2ff'f'' - f^2f'''\big) + M\sin^2(\gamma)\big(\beta ff'' - f'\big) \\
&\quad + g\frac{\beta_1}{a^2x}(T_f - T_o)(1+\beta_t\theta)\theta cos(\alpha) - \lambda_1 f' - (1+Fx)f'^2 \\
\Rightarrow & f''' + ff'' + \beta\big(2ff'f'' - f^2f'''\big) + M\sin^2(\gamma)\big(\beta ff'' - f'\big) \\
&\quad + \delta(1+\beta_t\theta)\theta\cos(\alpha) - \lambda_1 f' - (1+Fr)f'^2 = 0. \qquad (3.19)
\end{aligned}
$$

Next, to find the dimensionless form of temperature equation, the procedure is as follow .

$$
\begin{aligned}
\theta(\xi) &= \frac{T - T_\infty}{T_f - T_0} \\
\Rightarrow \quad T &= (T_f - T_0)\theta + T_\infty \\
&= (T_f - T_0)\theta + (T_0 + d_1x) \\
&= (a_1x\theta) + (T_0 + d_1x) \\
\frac{\partial T}{\partial x} &= a_1\theta + d_1 \\
u\frac{\partial T}{\partial x} &= axf'(a_1\theta + d_1) \\
&= aa_1x\theta f' + axf'd_1. \qquad (3.20) \\
\frac{\partial T}{\partial y} &= a_1x\theta'(\frac{a}{\nu})^{\frac{1}{2}} \\
v\frac{\partial T}{\partial y} &= -\sqrt{a\nu}f\left(a_1x\theta'\left(\frac{a}{\nu}\right)^{\frac{1}{2}}\right) \\
&= -aa_1xf\theta'. \qquad (3.21) \\
\frac{\partial^2 T}{\partial y^2} &= a_1x\theta''\left(\frac{a}{\nu}\right). \qquad (3.22)
\end{aligned}
$$

Using (3.20) and (3.21) for the conversion of left hand side of (3.3),

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = aa_1x\theta f' + axf'd_1 - aa_1xf\theta'. \qquad (3.23)
$$

Now using (3.22) in right hand side of (3.3),

we get

$$\begin{aligned}\frac{\partial^2 T}{\partial y^2} &= \frac{k}{\rho c_p}\left(a_1 x\theta''\frac{a}{\nu}\right)\\ &= \frac{k}{\rho c_p \nu} a a_1 x\theta''\\ &= \frac{k}{\rho \nu c_p} a a_1 x\theta''\\ &= \frac{k}{\mu c_p} a a_1 x\theta''\\ &= \frac{1}{Pr} a a_1 x\theta''. \end{aligned} \tag{3.24}$$

Then comparing (3.23) and (3.24) for the dimensionless form of (3.3), we get

$$\begin{aligned} & a a_1 x\theta f' + a x f' d_1 - a a_1 x f\theta' = \frac{1}{Pr} a a_1 x\theta''.\\ \Rightarrow\ & a_1\theta f' + f' - a_1 f\theta' = \frac{1}{Pr} a_1\theta''.\\ \Rightarrow\ & \theta f' + \frac{d_1}{a_1} f' - f\theta' = \frac{1}{Pr}\theta''.\\ \Rightarrow\ & \theta f' + S_1 f' - f\theta' = \frac{1}{Pr}\theta''.\\ \Rightarrow\ & \frac{1}{Pr}\theta'' + f\theta' - f'\theta - f' S_1 = 0. \end{aligned} \tag{3.25}$$

The boundary conditions are converted into dimensionless form through the following steps:

- $u = \epsilon u_w$ $\quad at \quad y = 0.$

$\Rightarrow a x f'(\xi) = \epsilon a x \quad at \quad \xi = 0.$

$\Rightarrow f'(\xi) = \epsilon \quad at \quad \xi = 0.$

$\Rightarrow f'(0) = \epsilon$

- $v = v_0$ $\quad at \quad y = 0.$

$\Rightarrow -\sqrt{a}\nu f(\xi) = -\sqrt{a}\nu f(0) \quad at \quad \xi = 0.$

$\Rightarrow f(\xi) = f(0)$.

$$
\begin{aligned}
&\bullet\ T = T_f = T_0 + a_1 x && at \quad y = 0.\\
&\Rightarrow \theta(\xi)(T_w - T_\infty) + T_\infty = T_0 + a_1 x && at \quad \xi = 0.\\
&\Rightarrow \theta(\xi)(T_0 + a_1 x - T_0 - d_1 x) + T_0 + d_1 x = T_0 + a_1 x && at \quad \xi = 0.\\
&\Rightarrow \theta(\xi)(a_1 x - d_1 x) + d_1 x = a_1 x && at \quad \xi = 0.\\
&\Rightarrow \theta(0)(a_1 - d_1)x = (a_1 - d_1)x && at \quad \xi = 0.\\
&\Rightarrow \theta(0) = 1\\
&\bullet\ u \to 0 && as \quad y \to 0.\\
&\Rightarrow a x f'(\xi) \to 0 && as \quad \xi \to \infty.\\
&\Rightarrow f'(\xi) \to 0 && as \quad \xi \to \infty.\\
&\Rightarrow f'(\infty) \to 0\\
&\bullet\ T \to T_\infty = T_0 + d_1 x && as \quad y \to \infty.\\
&\Rightarrow \theta(\xi)(T_w - T_\infty) + T_\infty \to T_0 + d_1 x\\
&\Rightarrow \theta(\xi)(T_0 + a_1 x - T_0 - d_1 x) + (T_0 + d_1 x) \to T_0 + d_1 x\\
&\Rightarrow \theta(\xi)(a_1 x - d_1 x) \to 0\\
&\Rightarrow \theta(\xi) \to 0 && as \quad \xi \to \infty.\\
&\Rightarrow \theta(\infty) \to 0 && as \quad \xi \to \infty.
\end{aligned}
$$

The converted BCs are in the following form.

$$
\left.
\begin{aligned}
&f'(0) = \epsilon, \qquad f(0) = S, \qquad \theta(0) = 1 - S_1 \quad at \quad \xi = 0,\\
&f'(\xi) \to 0, \quad \theta(\xi) \to 0, \qquad as \quad \xi \to \infty.
\end{aligned}
\right\} \tag{3.26}
$$

The dimensionless forms of (3.2) and (3.3) are:

$$
\begin{aligned}
&f''' + f(\xi) f'' + \beta\big(2 f f' f'' - f^2 f'''\big)\\
&+ M sin^2(\gamma)\big(\beta f f'' - f'\big) + \delta(1 + \beta_t \theta)\theta cos(\alpha)\\
&- \lambda_1 f' - (1 + Fr) f'^2 = 0.
\end{aligned} \tag{3.27}
$$

$$
\frac{1}{Pr}\theta'' + f\theta' - f'\theta - f' S_1 = 0. \tag{3.28}
$$

In (3.27) and (3.28), different parameters are used that are formulated as:

$$\delta = \frac{Gr_x}{Re_x^2}, \quad \beta_t = \frac{\beta_2(T_f - T_0)}{\beta_1}, \quad \lambda_1 = \frac{\nu}{Ka}, \quad F_r = \frac{C_p}{\sqrt{K}}x,$$
$$Gr_x = g\beta_1\frac{(T_f - T_0)x^3}{\nu^2}, \quad Re_x = \frac{xu_w}{\nu}, \quad u_w = ax. \quad Pr = \frac{\mu c_p}{k},$$
$$\beta = \lambda a, \quad M = \sigma\frac{\beta_o^2}{\rho a}, \quad \mu = \rho\nu, \quad S_1 = \frac{d_1}{a_1}.$$

The skin fraction coeffiecient is written as:

$$C_f = \frac{(\tau_w)_{y=0}}{\rho u_w^2(x)/2}, \tag{3.29}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}.$$

To get non-dimensional form of skin friction the following steps are adopted.

$$\begin{aligned} Cf_x &= \frac{\mu\frac{a^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}xf''(0)}{\rho a^2x^2/2} \\ &= \frac{\mu\sqrt{\frac{ax^2}{\nu}}af''(0)}{\rho a^2x^2/2} \\ &= \frac{\mu Re_x^{\frac{1}{2}}f''(0)}{\rho\frac{ax^2}{2}} \\ &= \frac{\rho\nu Re_x^{\frac{1}{2}}f''0)}{\rho u_w x/2} \end{aligned}$$

$$\begin{aligned} &= \frac{\nu Re_x^{\frac{1}{2}}f''0)}{u_w x/2} \\ &= \frac{Re_x^{\frac{1}{2}}f''(0)}{\frac{Re_x}{2}}. \\ \Rightarrow \ &\frac{1}{2}Re_x^{\frac{1}{2}}Cf_x = f''(0). \end{aligned} \tag{3.30}$$

Here Reynolds number is given as $Re = \frac{u_w x}{\nu}$.

The local Nusselt number Nu_x is defined as:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}. \tag{3.31}$$

The conversion of the Nusselt number into dimensionless form, has been explained through the following procedure.

$$\begin{aligned} Nu_x &= \frac{xq_w}{k(T_w - T_\infty)} \\ &= \frac{x(-k\frac{\partial T}{\partial y})}{k(T_w - T_\infty)} \\ &= \frac{-x(T_w - T_\infty)\sqrt{\frac{a}{\nu}}\theta'(0)}{(T_w - T_\infty)} \\ &= -\sqrt{\frac{ax^2}{\nu}}\theta'(0) \\ &= -Re_x^{\frac{1}{2}}\theta'(0) \\ \Rightarrow \frac{Nu_x}{Re_x^{\frac{1}{2}}} &= -\theta'(0). \end{aligned} \tag{3.32}$$

3.3 Solution Methodology

In this section, shooting method has been used to obtain the approximate solution of the ordinary differential equation (3.27) and (3.28) along with the boundary conditions. First of all, we need to convert these equations into system of first order ODEs. Let us consider the following notations:

$$\begin{aligned} &f = z_1, \quad f' = z_1' = z_2, \quad f'' = z_1'' = z_2' = z_3, \quad f''' = z_3', \\ &\theta = z_4, \quad \theta' = z_4' = z_5, \quad \theta'' = z_5. \end{aligned}$$

Using these notations, the system (3.28) and (3.29) with BCs (3.27) are transformed into the following system of five first order ODEs.

(3.33)

$$
\begin{aligned}
z_1' &= z_2, && z_1(0) = S,\\
z_2' &= z_3, && z_2(0) = c,\\
z_3' &= (1-\beta z_1^2)^{-1}\big(- z_1 z_3 - 2\beta z_1 z_2 z_3 - M sin^2(\gamma)(\beta z_1 z_3 - z_2) - \delta(1+\beta_t z_4) z_4 cos(\alpha)\\
&\quad + \lambda_1 z_2 + (1+Fr) z_2^2\big) && z_3(0) = p,\\
z_4' &= z_5, && z_4(0) = 1 - S_1,\\
z_5' &= (z_2 z_4 + z_2 S_1 - z_1 z_5) Pr, && z_5(0) = q,
\end{aligned}
$$

where p and q are the initial gusses. We take domain $[0, \xi_\infty]$ instead of $[0, \infty)$, where ξ_∞ is a positive real number which is chosen such that there are no noticeable variations in the solution after $\xi = \xi_\infty$. We need to meet the following two conditions.

$$z_2(\xi_\infty, p, q) = 0, \tag{3.34}$$

$$z_4(\xi_\infty, p, q) = 0. \tag{3.35}$$

Newton's method will be used to find p and q. This method has the following iterative scheme.

$$\begin{bmatrix} p \\ q \end{bmatrix}_{n+1} = \begin{bmatrix} p \\ q \end{bmatrix}_{n} - \begin{bmatrix} \frac{\partial z_2}{\partial p} & \frac{\partial z_2}{\partial q} \\ \frac{\partial z_4}{\partial p} & \frac{\partial z_4}{\partial q} \end{bmatrix}_{n}^{-1} \begin{bmatrix} z_2 \\ z_4 \end{bmatrix}_{n}.$$

Now intorduce some new notations:

$$\frac{\partial z_1}{\partial p} = z_6, \quad \frac{\partial z_2}{\partial p} = z_7, \quad \frac{\partial z_3}{\partial p} = z_8, \quad \frac{\partial z_4}{\partial p} = z_9, \quad \frac{\partial z_5}{\partial p} = z_{10},$$
$$\frac{\partial z_1}{\partial q} = z_{11}, \quad \frac{\partial z_2}{\partial q} = z_{12}, \quad \frac{\partial z_3}{\partial q} = z_{13}, \quad \frac{\partial z_4}{\partial q} = z_{14}, \quad \frac{\partial z_5}{\partial q} = z_{15}.$$

Using the above notations, we get

$$\begin{bmatrix} p \\ q \end{bmatrix}_{n+1} = \begin{bmatrix} p \\ q \end{bmatrix}_{n} - \begin{bmatrix} z_7 & z_{12} \\ z_9 & z_{14} \end{bmatrix}_{n}^{-1} \begin{bmatrix} z_2 \\ z_4 \end{bmatrix}_{n}.$$

The above itrative process will be continued untill the following criteria is fulfilled,

$$max\{|z_2(\xi_\infty, p_n, q_n)|, |z_4(\xi_\infty, p_n, q_n)|\} < \chi, \tag{3.36}$$

where $\chi > 0$ is tolerance which we set as $\chi = 10^{-5}$.

Now to attain another sytem of ODEs, differentiating the last system of five equations w.r.t p and q, we get

$$z_6' = z_7, \qquad z_6(0) = 0,$$
$$z_7' = z_8, \qquad z_7(0) = 0,$$
$$\begin{aligned} z_8' = (1-\beta z_1^2)^{-1}\big(& - z_1 z_8 - z_6 z_3 - 2\beta z_1 z_2 z_8 - 2\beta z_1 z_7 z_3 - 2\beta z_6 z_2 z_3 \\ & - M\sin^2(\gamma)(\beta z_1 z_8 + \beta z_6 z_3 - z_7) - \delta(1+\beta_t z_9) z_4 \cos(\alpha) - \delta(1+\beta_t z_4) z_9 \cos(\alpha) \\ & + \lambda_1 z_7 + 2 z_2 z_7 (1+Fr)\big) + \big(- z_1 z_3 - 2\beta z_1 z_2 z_3 - M\sin^2(\gamma)(\beta z_1 z_3 - z_2) \\ & - \delta(1+\beta_t z_4) z_4 \cos(\alpha) + \lambda_1 z_2 + (1+Fr) z_2^2\big)((1-\beta {z_1}^2)^{-2}(2\beta z_1 z_6)), \quad z_8(0) = 1, \end{aligned}$$
$$z_9' = z_{10}, \qquad z_9(0) = 0,$$
$$z_{10} = Pr\big(z_7 z_4 + z_2 z_9 + S_1 z_7 - z_6 z_5 - z_1 z_{10}\big), \qquad z_{10}(0) = 0,$$
$$z_{11}' = z_{12}, \qquad z_{11}(0) = 0,$$
$$z_{12}' = z_{13}, \qquad z_{12}(0) = 0,$$

$$z'_{13} = (1-\beta z_1^2)^{-1}\big(- z_1 z_{13} - z_{11} z_3 - 2\beta z_1 z_2 z_{13} - 2\beta z_1 z_{12} z_3 - 2\beta z_{11} z_2 z_3$$
$$- M\sin^2(\gamma)(\beta z_1 z_{13} + \beta z_{11} z_3 - z_{12}) - \delta(1+\beta_t z_{14}) z_4 \cos(\alpha)$$
$$- \delta(1+\beta_t z_4) z_{14}\cos(\alpha) + \lambda_1 z_{12} + 2 z_2 z_{12}(1+Fr)\big) + \big(- z_1 z_3 - 2\beta z_1 z_2 z_3$$
$$- M\sin^2(\gamma)\big(\beta z_1 z_3 - z_2\big) - \delta(1+\beta_t z_4) z_4\cos(\alpha) + \lambda_1 z_2 + (1+Fr) z_2^2\big)$$
$$((1-\beta z_1^2)^{-2}(2\beta z_1 z_{11})), \qquad z_{13}(0) = 0,$$

$$z'_{14} = z_{15}, \qquad z_{14}(0) = 0,$$
$$z_{15} = Pr\big(z_{12} z_4 + z_2 z_{14} + S_1 z_{12} - z_{11} z_5 - z_1 z_{15}\big), \qquad z_{15}(0) = 1.$$

3.4 Results and Discussions

In this section, we analyze the effect of different parameters on the velocity profile and temperature profile by using tables and graphs. In TABLE 3.1 and TABLE 3.2, different parameters are used to calculate numerical values of skin friction $(Re_x)^{\frac{1}{2}} C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}} Nu_x$. It is observed that when magnatic parameter M and the injection parameter S are increased, the skin friction and local Nusselt number are increased. By rising, nonlinear thermal variable β_t, Maxwell parameter β, Prandtl number P_r, aligned angle γ, mixed convection variable δ and porosity parameter λ_1, skin friction $(Re_x)^{\frac{1}{2}} C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}} Nu_x$ increase. In these tables, I_f and I_θ are the intervals from which the missing conditions can be chosen.

However, an increase in the thermal stratification variable S_1 and intertia coefficient F_r causes a decrement in the skin friction $(Re_x)^{\frac{1}{2}} C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}} Nu_x$. For the deceasing value of the shrinking parameter ϵ, the skin friction increases while the Nusselt number decrease. FIGURE 3.2 shows the behaviour of velocity for various values of γ. By rising the value of γ, the fluid velocity $f'(\xi)$ is devastated. FIGURE 3.3 shows the impact of aligned angle γ on the velocity profile. By increasing the value of γ, the velocity profile escalates.

It can be observed in FIGURE 3.4 that boosting the value of Maxwell parameter β, $f'(\xi)$ is found to demolish. FIGURE 3.5 shows the impact of β on velocity. Inceasing the value β causes an increment in $f'(\xi)$. FIGURE 3.6 shows that when mixed convection parameter δ escalates, the temperature profile $\theta(\xi)$ is demolished. It can be observed from FIGURE 3.7 that when δ increases, $f'(\xi)$ increases drastically. FIGURE 3.8 indicates the effect of F_r on velocity. The higher value of F_r reduces the value of $f'(\xi)$.

FIGURE 3.9 shows that when B_t is boosted, the velocity $f'(\xi)$ increased. FIGURE 3.10 indicates the effect of Prandtl number P_r on thermal profile $\theta(\xi)$. When P_r escalates, $\theta(\xi)$ is reduced. FIGURE 3.11 observes that increasing in λ_1, the temperature profil $\theta(\xi)$ increases. FIGURE 3.12 reveals that when the angle of inclination α is boosted, $f'(\xi)$ decreases. FIGURE 3.13 observes that thermal stratification parameter S_1 is boosted, $\theta(\xi)$ declines. FIGURE 3.14 shows that the impact of shrinking parameter ϵ on the temperature $\theta(\xi)$.When shrinking parameter c is increased, the temperature profile decreases. FIGURE 3.15 indicates that an increment in the injection parameter S causes a decrement in $\theta(\xi)$. FIGURE 3.16 shows the effect of the thermal stratification S_1 and Prandtl number Pr on the Nusselt number. It is observed that the heigher Pr increses the Nusselt number. When S_1 increases, the Nusselt number is decreased.

TABLE 3.1:

Results of $(Re_x)^{\frac{1}{2}}C_f$ for $S = 2$ and other various parameters

γ	Pr	M	β	Fr	β_t	λ_1	S_1	δ	c	$(Re_x)^{\frac{1}{2}}C_f$	I_f
$\frac{\pi}{6}$	0.7	1	0.1	0.3	0.5	0.3	0.1	0.2	-1	2.728905	[2.1,3.34]
$\frac{\pi}{4}$										2.951310	[2.4,3.42]
$\frac{\pi}{3}$										3.155816	[2.7,3.53]
$\frac{\pi}{2}$										3.347068	[2.9,3.64]
	0.5									2.819168	[2.3,3.43]
	0.9									2.663871	[2.0,3.28]
	1.1									2.616171	[2.0,3.27]
		0								2.480468	[2.0,3.27]
		2								2.951310	[2.4,3.42]
		3								3.155816	[2.7,3.53]
			0							1.898974	[1.0,8.08]
			0.05							2.209428	[1.6,3.04]
			0.15							3.771375	[3.1,4.26]
				0						2.821788	[2.1,3.44]
				0.6						2.631038	[2.1,3.23]
				0.9						2.527297	[2.0,3.12]
					0					2.662509	[2.1,3.21]
					1.0					2.793957	[2.2,3.41]
					1.5					2.857773	[2.2,3.47]
						0.1				2.603259	[1.9,3.37]
						0.5				2.844406	[2.3,3.33]
						0.7				2.951852	[2.5,3.33]
							0.3			2.648688	[1.9,3.29]
							0.5			2.573386	[1.7,3.25]
							0.7			2.503105	[1.5,3.16]
								0		2.370199	[1.7,3.0]
								0.4		3.040205	[2.6,3.68]
								0.6		3.322332	[2.9,3.73]
									-0.4	1.530050	[1.0,2.01]
									-0.6	2.041396	[1.5,2.57]
									-0.8	2.445679	[1.9,3.09]

TABLE 3.2:

Results of $(Re_x)^{\frac{-1}{2}} Nx_u$ for $S = 2$ and other various parameters

γ	Pr	M	β	Fr	β_t	λ_1	S_1	δ	c	$(Re_x)^{\frac{-1}{2}} Nx_u$	I_θ
$\frac{\pi}{6}$	0.7	1	0.1	0.3	0.5	0.3	0.1	0.2	-1	0.926772	[-1.71,-0.4]
$\frac{\pi}{4}$										0.945807	[-1.93,-0.5]
$\frac{\pi}{3}$										0.961403	[-2.04,-0.6]
$\frac{\pi}{2}$										0.974594	[-2.2,-0.8]
	0.5									0.671698	[-0.60,-0.3]
	0.9									1.200992	[-2.09,-0.7]
	1.1									1.489144	[-2.48,-0.9]
		0								0.902379	[-1.60,-0.4]
		2								0.945807	[-1.09,-0.5]
		3								0.961403	[-2.03,-0.6]
			0							0.805105	[-1.84,-0.02]
			0.05							0.860453	[1.82,-0.4]
			0.15							1.009624	[-1.72,-0.4]
				0						0.932396	[-1.93,-0.5]
				0.6						0.920609	[-1.64,-0.3]
				0.9						0.913791	[-1.55,-0.3]
					0					0.919236	[-1.89,-0.3]
					1.0					0.933991	[-1.65,-0.2]
					1.5					0.940923	[-1.74,-0.6]
						0.1				0.915208	[-2.81,-0.5]
						0.5				0.936715	[-1.86,-0.5]
						0.7				0.945428	[-1.96,-0.6]
							0.3			0.655559	[-1.41,-0.2]
							0.5			0.383464	[-100,-0.85]
							0.7			0.110480	[-1.00,-1.84]
								0		0.869786	[-100,-1.85]
								0.4		0.969597	[-2.01,-0.6]
								0.6		1.004387	[-2.0,-0.5]
									-0.4	1.530050	[-2.0,-0.7]
									-0.6	1.099834	[-1.8,-0.5]
									-0.8	1.021087	[-1.8,-0.5]

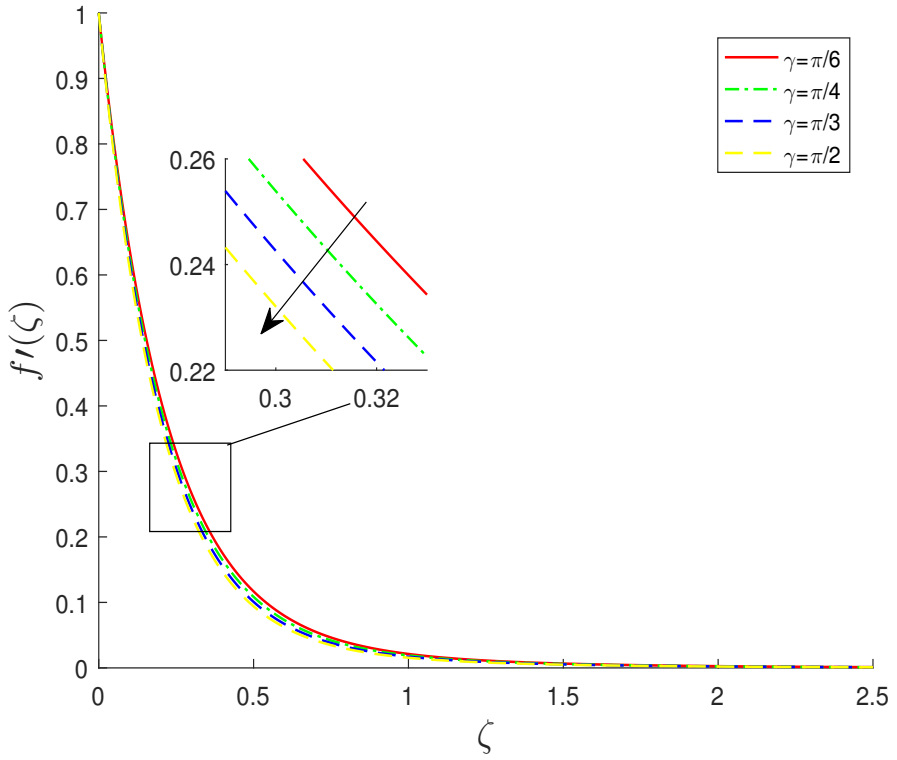

FIGURE 3.2: Impact of γ on $f'(\xi)$ for $\epsilon = 1$.

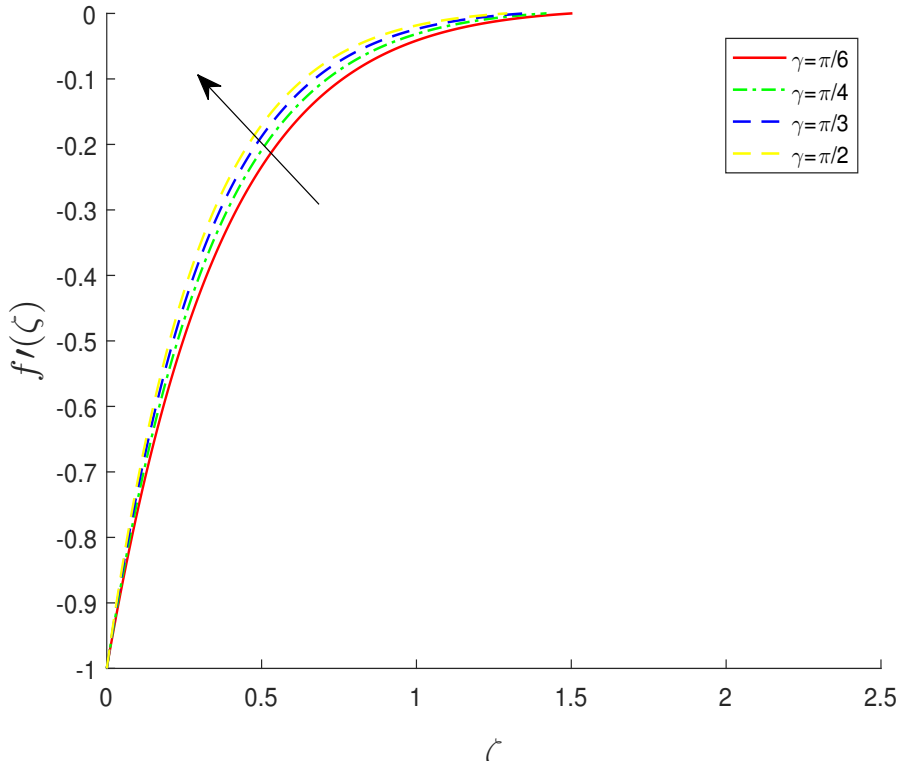

FIGURE 3.3: Impact of γ on $f'(\xi)$ for $\epsilon = -1$.

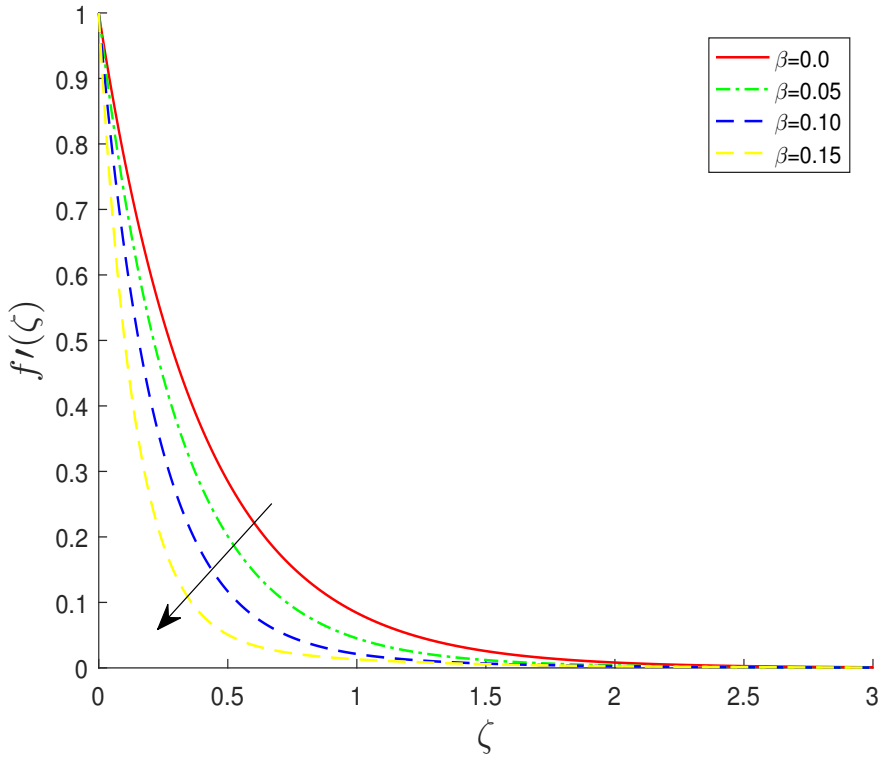

FIGURE 3.4: Impact of β on $f'(\xi)$ for $\epsilon = 1$.

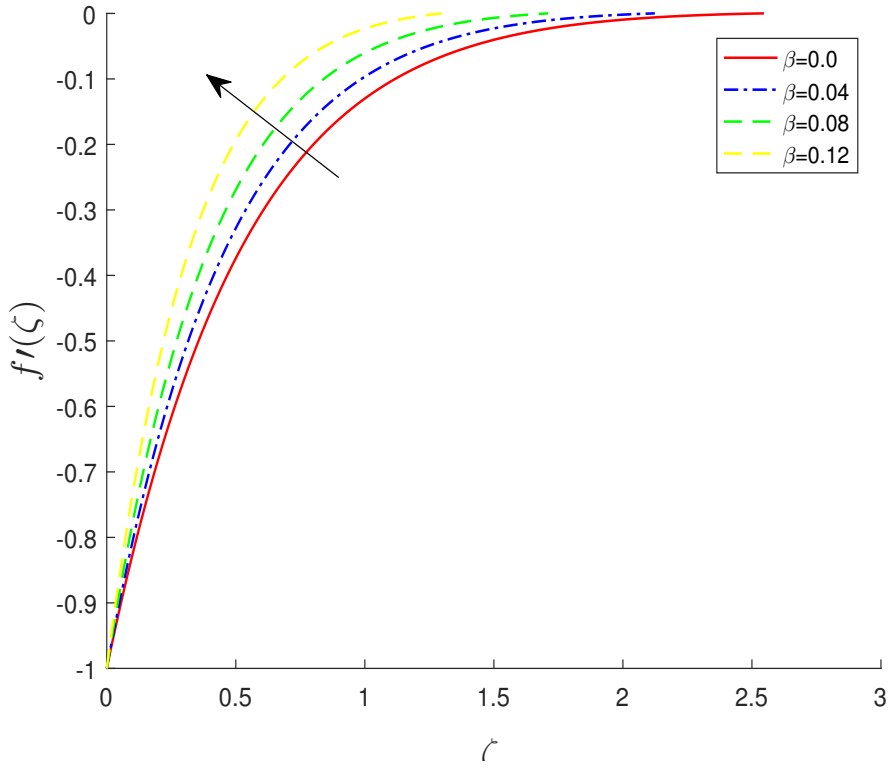

FIGURE 3.5: Impact of β on f' for $\epsilon = -1$.

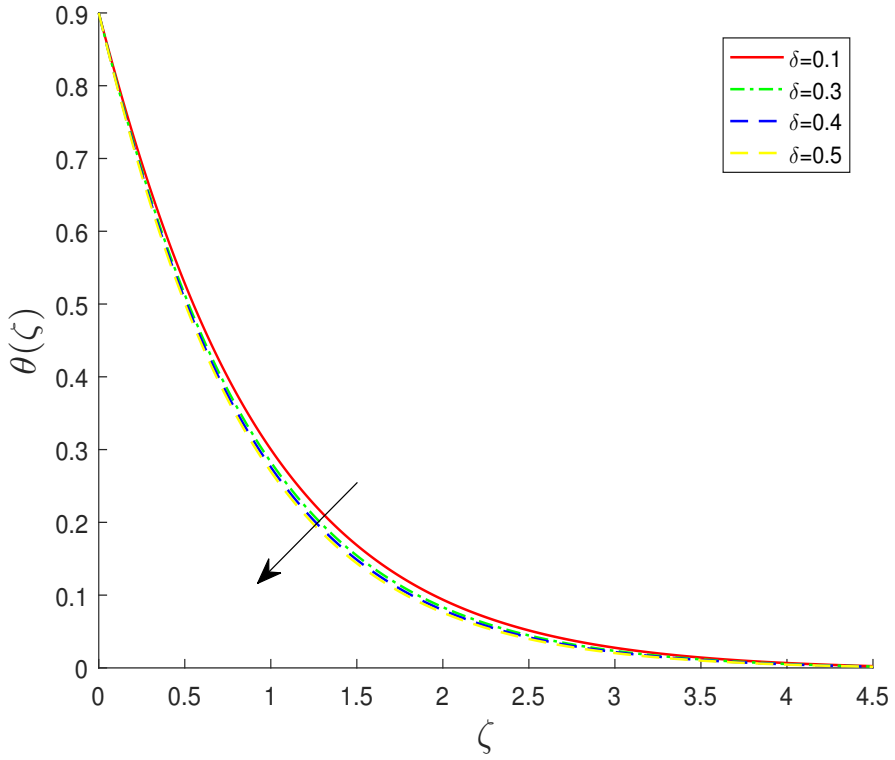

FIGURE 3.6: Impact of δ on temperature

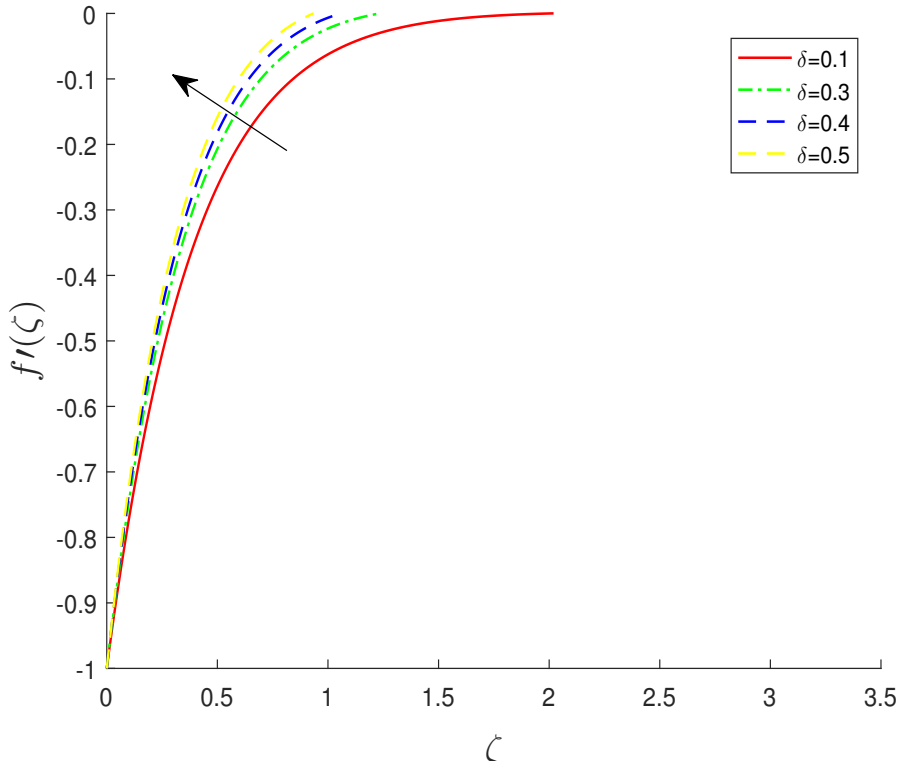

FIGURE 3.7: Impact of δ on velocity profile

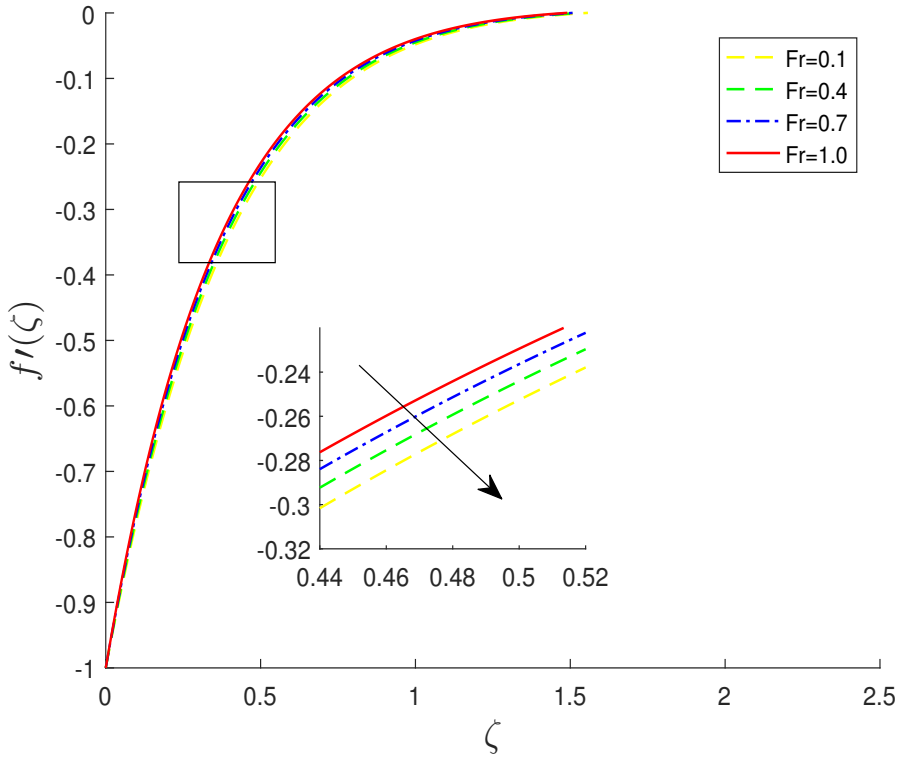

FIGURE 3.8: Impact of Fr on velocity profile

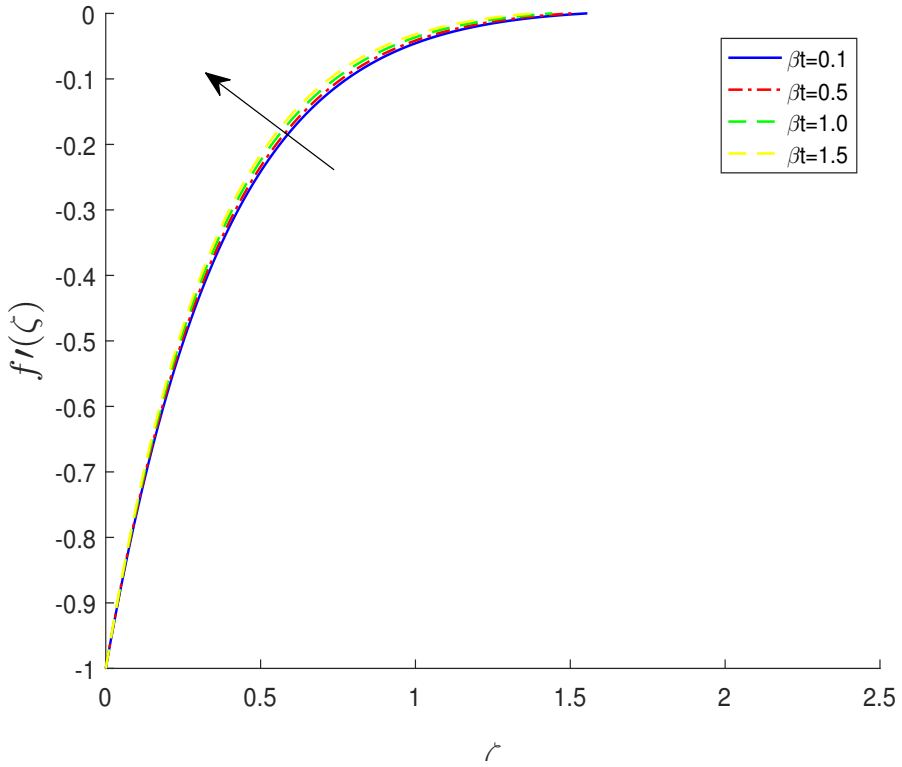

FIGURE 3.9: Impact of β_t on velocity

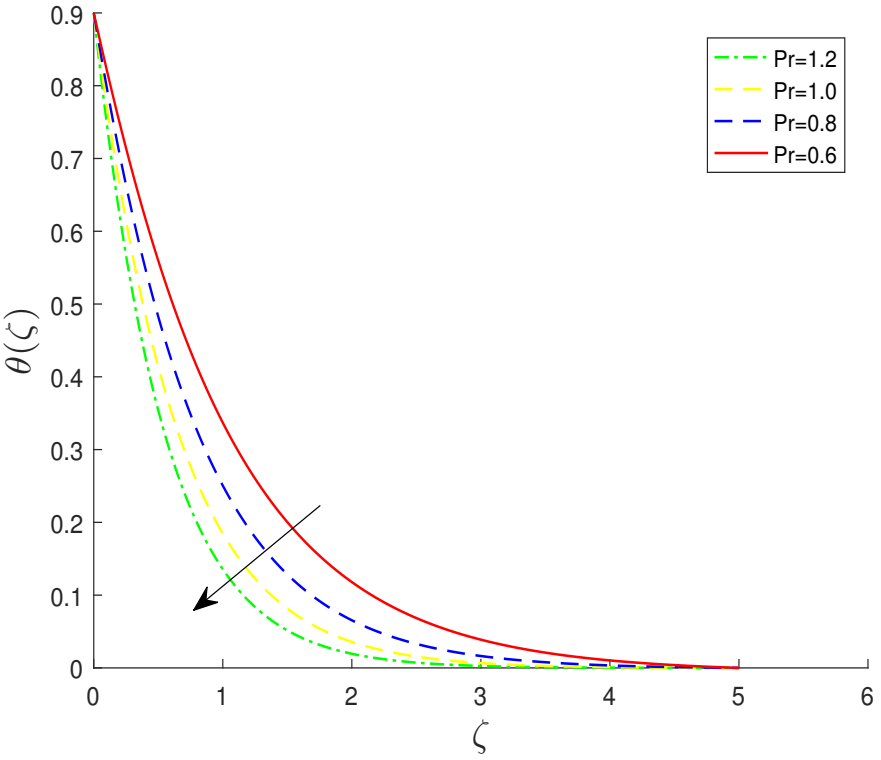

FIGURE 3.10: Impact of Pr on temperature profile

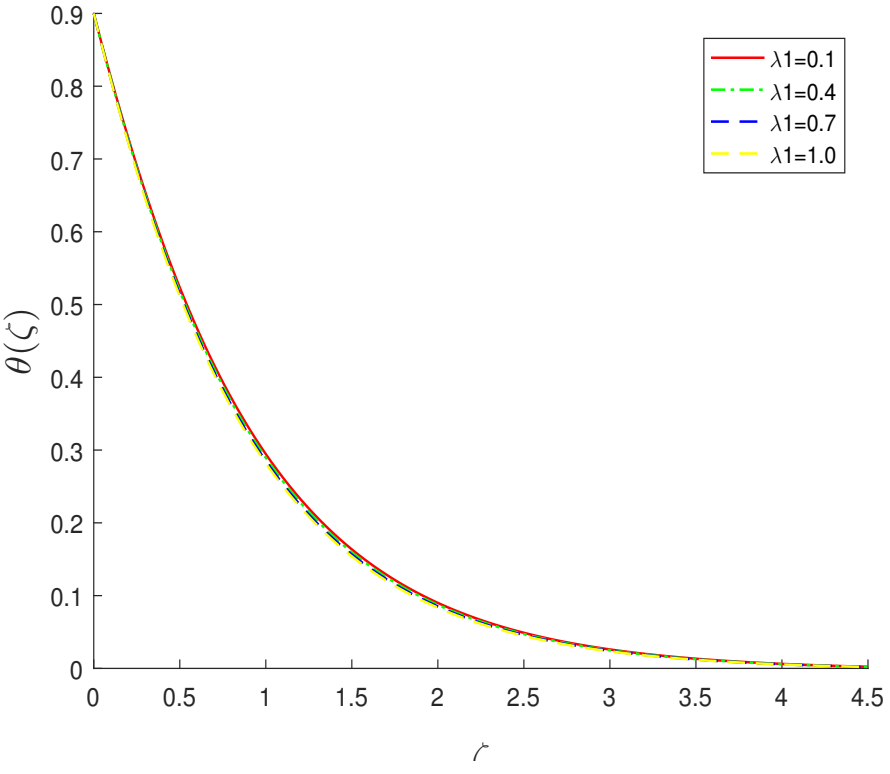

FIGURE 3.11: Impact of λ_1 on temperature

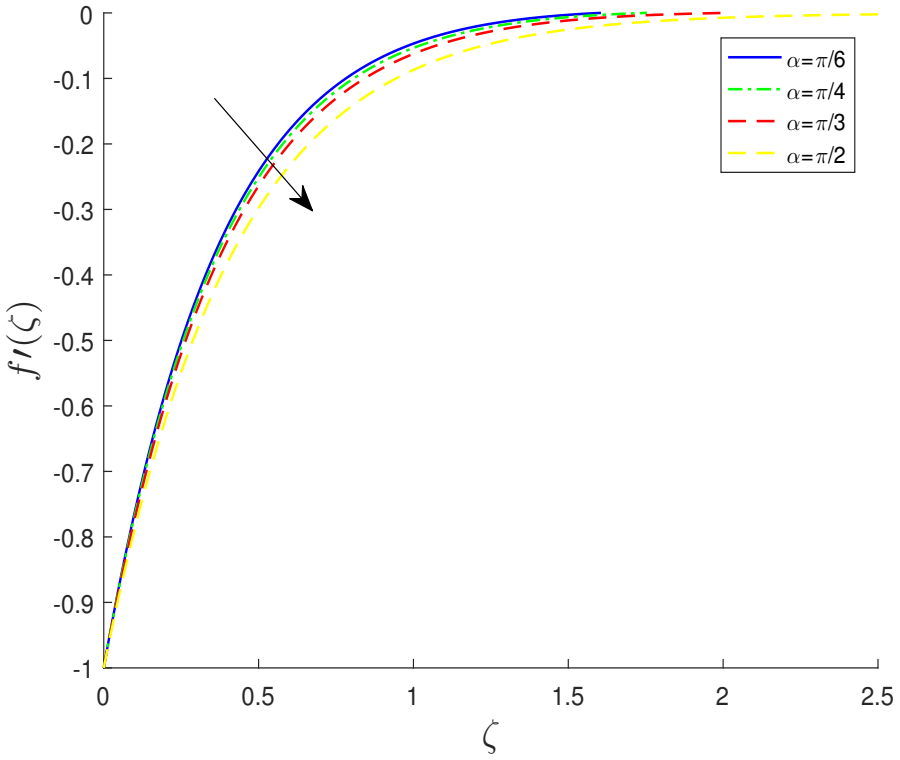

FIGURE 3.12: Impact of α on velocity

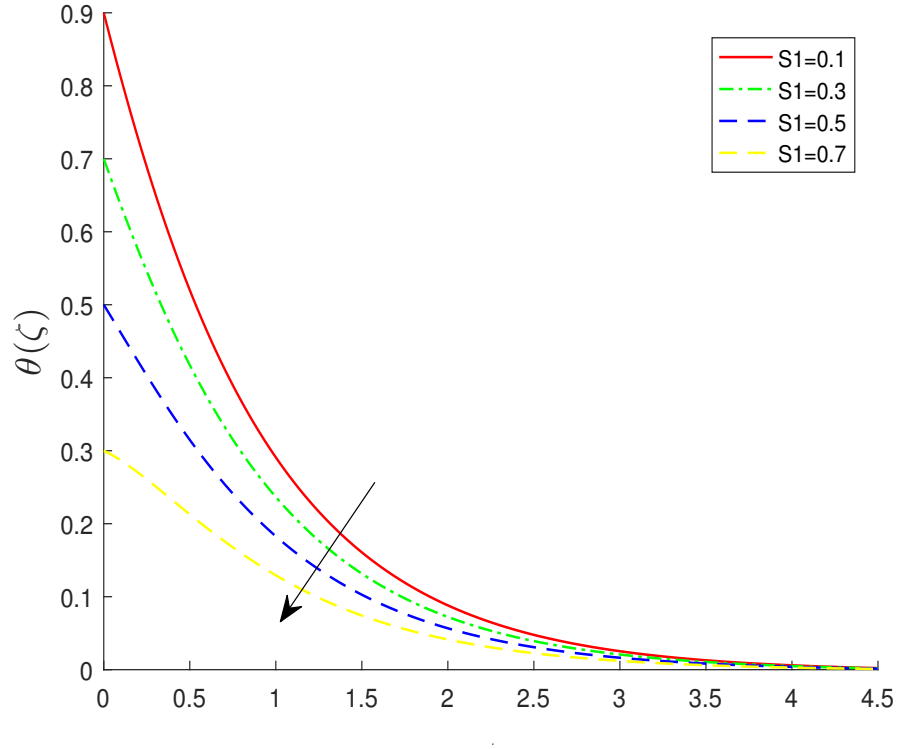

FIGURE 3.13: Impact of S_1 on temperature profile

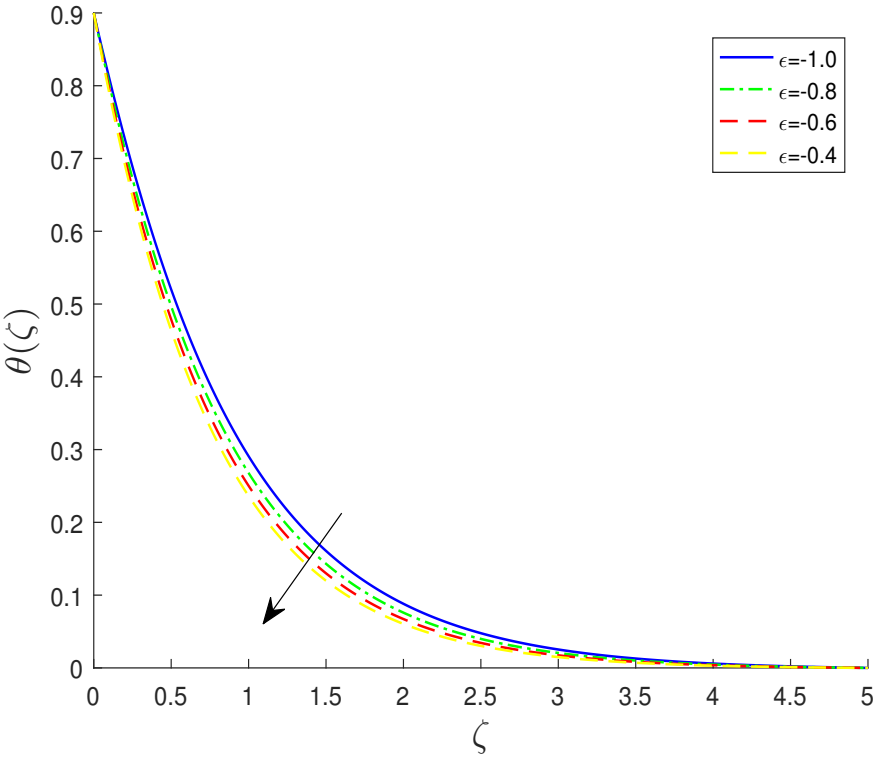

FIGURE 3.14: Impact of c on temperature

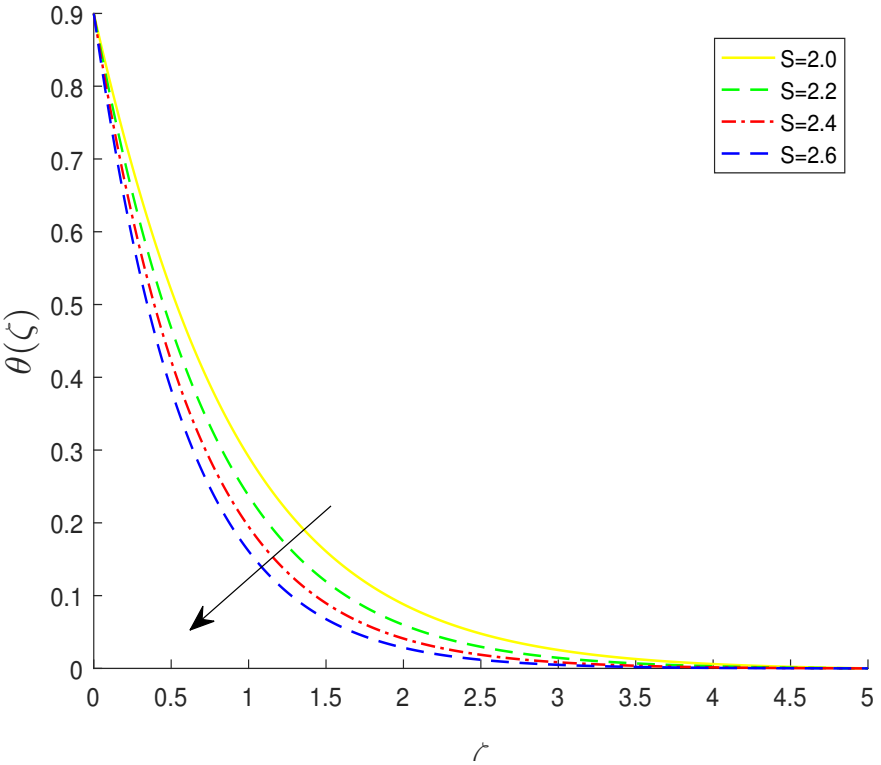

FIGURE 3.15: Impact of S on temperature

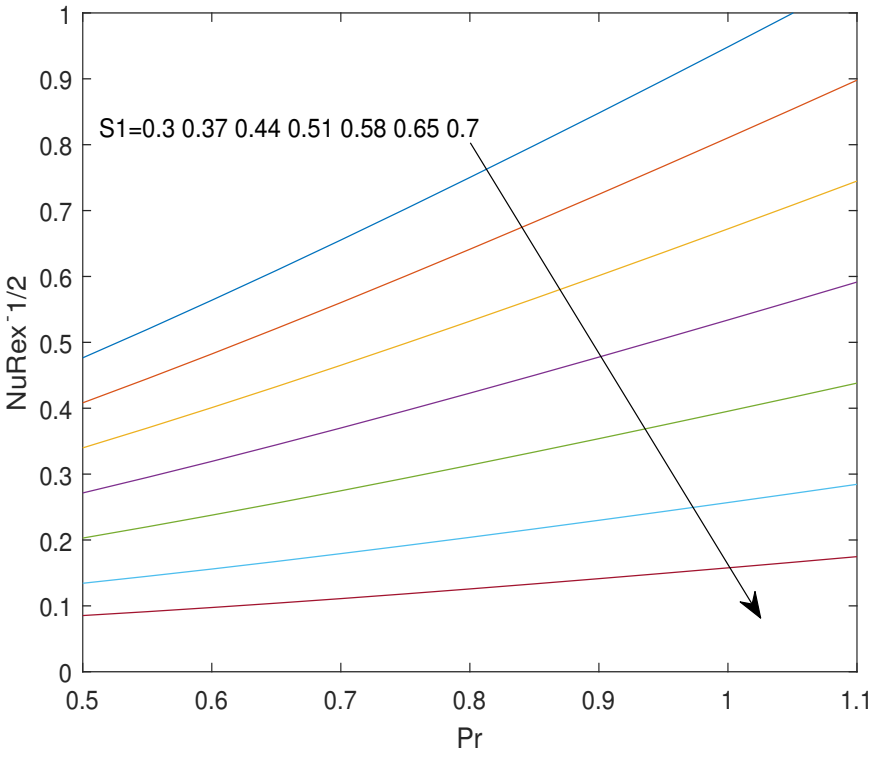

FIGURE 3.16: Impact of Pr and S_1 on Nusselt number

Chapter 4

Cattaneo-Christov Model For a Non-Newtonian Flow Over Stratified Stretching/Shrinking Inclined Sheet

4.1 Introduction

In this chapter, an extension of the model discussed in Chapter 3, has been proposed and analyzed. Momentum equation remains the same as that in Chapter 3 and the Cattaneo-Christov heat flux is added in the temperature equation.

Furthermore, the concentration equation is extended by adding Cattaneo-Christov mass flux and chemical reaction. The given nonlinear partial differential equations are converted into dimensionless form by utilizing the same similarity transformation. In order to solve the ordinary differential equations, shooting method is applied. For numerical computation, the computational software MATLAB is adopted. Lastly, the numerical results are discussed for different parameters affecting velocity, tempertature and concentration profiles and these results are presented through tables and graphs.

4.2 Mathematical Modeling

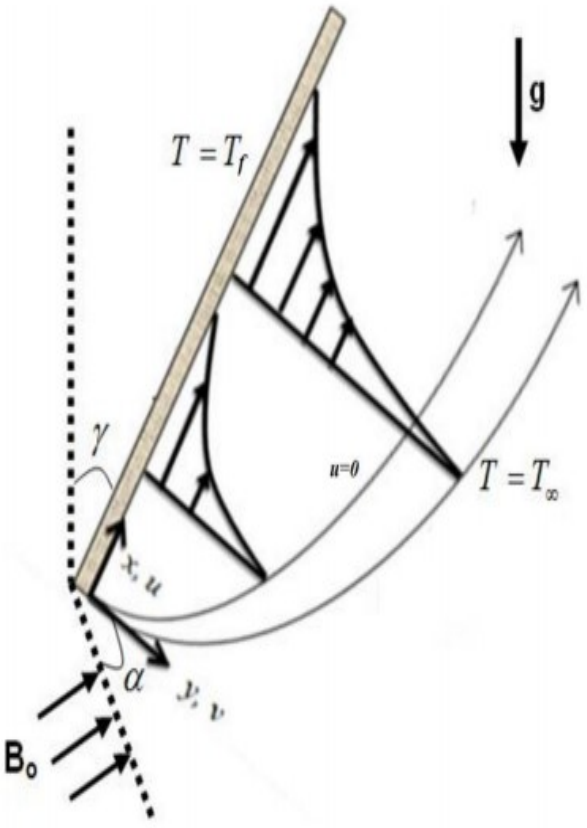

FIGURE 4.1: Geometry for the flow under discussion.

It has been aimed to examine a $2D$ upper-convected Maxwell fluid flow over shrinking sheet with aligned magnetic field.

The inclination angle is α, B_0 is the aligned magnetic field with angle γ and velocity $u_w = ax$ has been considered. The temperature on the wall is $T_f = T_0 + a_1 x$ and $T_\infty = T_0 + d_1 x$ is the temperature far from the wall. In addition, Cattaneo-Cristove double diffusion model and the concentration of fluid is also examined with the assistance of concentration equation under the effect of chemical reaction

.

By considering the above assumptions, the governing partial differential equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{4.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\nu\frac{\partial^2 u}{\partial y^2} \tag{4.2}$$
$$-\frac{\sigma B_o^2}{\rho}\sin^2(\gamma)\left(u+\lambda v\frac{\partial u}{\partial y}\right)+g\big(\beta_1(T-T_\infty)+\beta_2(T-T_\infty)^2\big)\cos(\alpha)-\frac{v}{k}u-Fu^2,$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda\Bigg(\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}$$
$$+2uv\frac{\partial^2 T}{\partial x\partial y}\Bigg)=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{4.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\delta\Bigg(\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\frac{\partial C}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\frac{\partial C}{\partial y}+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}$$
$$+2uv\frac{\partial^2 C}{\partial x\partial y}\Bigg)=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+D_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-R(C-C_\infty). \tag{4.4}$$

The associated BCs have been taken as:

$$\left.\begin{aligned}&u=\epsilon u_w(x),\quad v=v_0,\quad T=T_f=T_0+a_1x,\quad \phi=1,\quad at\quad y=0,\\&u\to 0,\quad T\to T_\infty=T_o+d_1x,\quad \phi\to 0,\quad as\quad y\to\infty.\end{aligned}\right\} \tag{4.5}$$

Following similarity transformation has been used to convert the partial differential equations (4.1)-(4.4) into a system of ordinary differential equations .

$$\left.\begin{aligned}&\omega(x,y)=\sqrt{\nu x u_w(x)},f(\xi)=\sqrt{a\nu}xf(\xi),\quad \theta(\xi)=\frac{T-T_\infty}{T_f-T_o},\\&\xi=\sqrt{\frac{u_w(x)}{\nu x}}y=\left(\frac{a}{\nu}\right)^{\frac{1}{2}}y,\quad \phi(\xi)=\frac{C-C_\infty}{C_w-C_\infty}.\end{aligned}\right\} \tag{4.6}$$

where ω denotes the stream function, ξ denotes the similarity variable, f', θ and ϕ are the dimensionless velocity, temperature and concentration. The detailed procedure for the conversion of (4.1) is discussed in Chapter 3. The complete procedure for the conversion of (4.2) discussed in Chapter 3.

For the non-dimensional form of (4.3) the following procedure is adopted.

$$\frac{\partial^2 T}{\partial x^2} = 0,$$
$$\frac{\partial^2 T}{\partial x \partial y} = a_1 \sqrt{\frac{a}{\nu}}\theta'.$$

Now, use the above derivatives to get the non-dimensional form of left hand side of (4.3).

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \lambda\Bigg(\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y} + u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2}$$
$$+ 2uv\frac{\partial^2 T}{\partial x \partial y}\Bigg) = \theta f' + S_1\Big(f' + \lambda_2 f'^2 - \lambda_2 f f''\Big)$$
$$+ \lambda_2\Big(f'^2\theta - f f''\theta - f f'\theta'\Big). \tag{4.7}$$

Now, convert the right hand side of (4.3) into the dimensionless form.

$$\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} = \left(\frac{1}{Pr} - \lambda_2 f^2\right)\theta''. \tag{4.8}$$

Comparing (4.7) and (4.8), we get the dimensionless form of (4.3), as follows.

$$\theta f' + S_1\Big(f' + \lambda_2 f'^2 - \lambda_2 f f''\Big) + \lambda_2\Big(f'^2\theta - f f''\theta - f f'\theta'\Big) = \left(\frac{1}{Pr} - \lambda_2 f^2\right)\theta''. \tag{4.9}$$

For the conversion of (4.4), we do the following steps:

$$\phi(\xi) = \frac{C - C_\infty}{C_w - C_\infty}.$$
$$C = \phi(C_w - C_\infty) + C_\infty.$$
$$\frac{\partial C}{\partial x} = 0.$$
$$\frac{\partial^2 C}{\partial x^2} = 0.$$
$$\frac{\partial C}{\partial y} = (C_w - C_\infty)\phi'(\xi)\sqrt{\frac{a}{\nu}}.$$
$$\frac{\partial^2 C}{\partial^2 y} = \frac{a}{\nu}(C_w - C_\infty)\phi''.$$

To get dimensionless form of left hand side of (4.4), we have following steps:

$$\begin{aligned}
&u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\delta\bigg(\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\frac{\partial C}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\frac{\partial C}{\partial y}\\
&\quad+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}+2uv\frac{\partial^2 C}{\partial x\partial y}\bigg)\\
&=-\sqrt{a\nu}(C_w-C_\infty)\phi'\sqrt{\frac{a}{\nu}}+\delta\bigg((-\sqrt{a\nu})(-af')(C_w-C_\infty)\phi'\sqrt{\frac{a}{\nu}}\bigg)\\
&\quad+(-\sqrt{a\nu}f)^2(\frac{a}{\nu}(C_w-C_\infty)\phi'')\\
&=\bigg(C_w-C_\infty\bigg)\bigg(-af\phi'+\delta\bigg(a^2ff'\phi'+a^2f^2\phi''\bigg)\bigg).
\end{aligned}\tag{4.10}$$

To get the non-dimensional form of right hand side of (4.4), we have following steps:

$$\begin{aligned}
&D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+D_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-R(C-C_\infty)\\
&=D_B\bigg(\frac{a}{\nu}(C_w-C_\infty)\phi''\bigg)+D_T\bigg(\frac{a}{\nu}(T_w-T_\infty)\theta''\bigg)-R(C_w-C_\infty)\phi\\
&=\bigg(C_w-C_\infty\bigg)\bigg(D_B\bigg(\frac{a}{\nu}\bigg)\phi''+D_T\bigg(\frac{a}{\nu}\bigg)\bigg(\frac{T_w-T_\infty}{C_w-C_\infty}\bigg)\theta''-R\phi\bigg).
\end{aligned}\tag{4.11}$$

Comparing (4.10) and (4.11), we get the dimensionless form of governing mathematical model is as follow.

$$\begin{aligned}
&(C_w-C_\infty)\big(-af\phi'+\delta\big(a^2ff'\phi'+a^2f^2\phi''\big)\big)\\
&=\bigg(C_w-C_\infty\bigg)\bigg(D_B\bigg(\frac{a}{\nu}\bigg)\phi''+D_T\bigg(\frac{a}{\nu}\bigg)\bigg(\frac{T_w-T_\infty}{C_w-C_\infty}\bigg)\theta''-R\phi\bigg).\\
&\Rightarrow -af\phi'+\delta(a^2ff'\phi'+a^2f^2\phi'')\\
&=D_B\bigg(\frac{a}{\nu}\bigg)\phi''+D_T\bigg(\frac{a}{\nu}\bigg)\bigg(\frac{T_w-T_\infty}{C_w-C_\infty}\bigg)\theta''-R\phi\\
&\Rightarrow -f\phi'+\delta a\big(ff'\phi'+f^2\phi''\big)\\
&=\frac{D_B\phi''}{\nu}+D_T\bigg(\frac{a}{\nu}\bigg)\bigg(\frac{T_w-T_\infty}{C_w-C_\infty}\bigg)\theta''-\frac{R}{a}\phi\\
&\Rightarrow -f\phi'+\lambda_3(ff'\phi'+f^2\phi'')=\frac{1}{S_c}\phi''+S_o\theta''-\epsilon\phi.
\end{aligned}\tag{4.12}$$

BCs are converted into the dimensionless form through the following steps:

$$\bullet \quad u = \epsilon u_w \qquad at \quad y = 0.$$
$$\Rightarrow axf'(\xi) = \epsilon ax \qquad at\ \xi = 0.$$
$$\Rightarrow f'(\xi) = \epsilon \qquad at\ \xi = 0.$$
$$\Rightarrow f'(0) = \epsilon$$

$$\bullet \quad v = v_0 \qquad at \quad y = 0.$$
$$\Rightarrow -\sqrt{a\nu} f = -\sqrt{a\nu} f(0) \qquad at\ \xi = 0.$$
$$\Rightarrow f(\xi) = f(0)$$

$$\bullet \quad T = T_f = T_0 + a_1 x \qquad at \quad y = 0.$$
$$\Rightarrow \theta(T_w - T_\infty) + T_\infty = T_0 + a_1 x \qquad at\ \xi = 0.$$
$$\Rightarrow \theta(T_0 + a_1 x - T_0 - d_1 x) + T_0 + d_1 x = T_0 + a_1 x \qquad at\ \xi = 0.$$
$$\Rightarrow \theta(a_1 x - d_1 x) + d_1 x = a_1 x \qquad at\ \xi = 0.$$
$$\Rightarrow \theta(0) = 1$$
$$\Rightarrow \theta(0)(a_1 - d_1)x = (a_1 - d_1)x \qquad at\ \xi = 0.$$

$$\bullet\, C = C_w \qquad at \quad y = 0.$$
$$\Rightarrow (C_w - C_\infty)\phi + C_\infty = C_w \qquad at\ \xi = 0.$$
$$\Rightarrow (C_w - C_\infty)\phi(\xi) = (C_w - C_\infty) \qquad at\ \xi = 0.$$
$$\Rightarrow \phi(\xi) = 1 \qquad at\ \xi = 0.$$
$$\Rightarrow \phi(0) = 1$$

$$\bullet\, u \to 0 \qquad as\ y \to 0.\ as$$
$$\Rightarrow axf' \to 0 \qquad \xi \to \infty.\ as\ \xi$$
$$\Rightarrow f' \to 0 \qquad \to \infty.$$
$$\Rightarrow f'(\infty) \to 0 \qquad as\ \xi \to \infty.$$

$\bullet\, T \to T_\infty = T_0 + d_1 x \qquad as \quad y \to \infty.$

$\Rightarrow \theta(T_w - T_\infty) + T_\infty \to T_0 + d_1 x.$

$\Rightarrow \theta(T_0 + a_1 x - T_0 - d_1 x) + (T_0 + d_1 x) \to T_0 + d_1 x.$

$\Rightarrow \theta(a_1 x - d_1 x) \to 0.$

$\Rightarrow \theta \to 0 \qquad as \quad \xi \to \infty.$

$\Rightarrow \theta(\infty) \to 0 \qquad as \quad \xi \to \infty.$

$\bullet\, C \to C_\infty \qquad as \quad y \to \infty.$

$\Rightarrow C_\infty + \phi(\xi)(C_w - C_\infty) \to C_\infty \qquad as \quad \xi \to \infty.$

$\Rightarrow \phi(C_w - C_\infty) \to 0 \qquad as \quad \xi \to \infty.$

$\Rightarrow \phi \to 0 \qquad as \quad \xi \to \infty.$

Dimensionless form of the BCs:

$$\left.\begin{aligned} &f'(0) = \epsilon, \quad f(0) = S, \quad \theta(0) = 1 - S_1, \quad \phi(0) = 1, \quad at \quad \xi = 0 \\ &f'(\xi) \to 0, \quad \theta(\xi) \to 0, \qquad \phi(\xi) \to 0 \qquad as \quad \xi \to \infty. \end{aligned}\right\} \tag{4.13}$$

Parameters used in the equations are:

$$\begin{aligned} &\delta = \frac{Gr_x}{Re_x^2}, \quad \beta_t = \frac{\beta_2(T_f - T_0)}{\beta_1}, \quad \lambda_1 = \frac{\nu}{Ka}, \quad F_r = \frac{C_p}{\sqrt{K}}x, \\ &Gr_x = g\beta_1 \frac{(T_f - T_0)x^3}{\nu^2}, \quad Re_x = \frac{xu_w}{\nu}, \quad u_w = ax. \quad Pr = \frac{\mu c_p}{k}, \\ &\beta = \lambda a, \quad M = \sigma\frac{\beta_0^2}{\rho a}, \quad \mu = \rho\nu, \quad S_1 = \frac{d_1}{a_1}, \\ &\lambda_2 = \delta a, \quad S_c = \frac{\nu}{D_B}, \quad S_0 = \frac{D_T}{\nu}(\frac{T_w - T_\infty}{C_w - C_\infty}), \\ &\epsilon = \frac{R}{a}, \quad \lambda_2 = \lambda a, \quad \lambda_3 = \delta a. \end{aligned}$$

The sherwood number is expressed as:

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}. \tag{4.14}$$

Now for non-dimentional form of sherwood number ,following steps are taken:

$$
\begin{aligned}
q_m &= -D_B\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (4.15)\\
Sh_x &= \frac{x - D_B\left(\frac{\partial T}{\partial y}\right)_{y=0}}{D_B(C_w - C_\infty)}\\
&= -x((C_w - C_\infty)\sqrt{\frac{a}{\nu}}\phi'(0))\\
&= -\sqrt{\frac{ax^2}{\nu}}\phi'(0)\\
&= -\sqrt{Re_x}\phi'(0)\\
\Rightarrow \frac{sh_x}{\sqrt{Re_x}} &= \phi'(0).
\end{aligned}
$$

where $Re = \frac{xu_x(x)}{\nu_f}$.

4.3 Solution Methodology

In this section, shooting method has been used to obtain the approximate solution of the ordinary differential equations (3.27), (4.9) and (4.12) along with the boundary conditions (4.13).Firstly, we solve the coupled ordinary differential equations (3.27) and (4.9).

Let us consider the following notations:

$$
\begin{aligned}
&f = z_1, \quad f' = z_1' = z_2, \quad f'' = z_1'' = z_2' = z_3, \quad f''' = z_3',\\
&\theta = z_4, \quad \theta' = z_4' = z_5, \quad \theta'' = z_5.
\end{aligned}
$$

Convert equation into first order ODEs by using notations:

$$
\begin{aligned}
z_1' &= z_2, & z_1(0) &= S,\\
z_2' &= z_3, & z_2(0) &= c,
\end{aligned}
$$

$$z_3' = \big(- z_1 z_3 - 2\beta z_1 z_2 z_3 - M\sin^2(\gamma)[\beta z_1 z_3 - z_2] - \delta(1+\beta_t z_4) z_4 \cos(\alpha)$$
$$+ \lambda_1 z_2 + (1+Fr) z_2^2 \big)\big((1-\beta z_1^2)^{-1}\big), \qquad z_3(0) = MC_1,$$
$$z_4' = z_5, \qquad z_4(0) = 1 - S_1,$$
$$z_5' = \left(\frac{1}{Pr} - \lambda_2 z_1^2\right)^{-1}\left(z_2 z_5 + S_1(z_2 + \lambda_2 z_2^2 - \lambda_2 z_1 z_3) + \lambda_2(z_2^2 z_4 - z_1 z_3 z_4 - z_1 z_2 z_5)\right),$$
$$z_5(0) = MC_2.$$

In the above IVP, the missing conditions MC_1 and MC_2, are chosen to satisfy the following relation.

$$(z_2(MC_1, MC_2))_{\xi=\xi_\infty} = 0, \qquad (z_4(MC_1, MC_2)_{\xi=\xi_\infty} = 0.$$

Now apply Newton's method to solve the above algebric equations, by using following formula.

$$\begin{bmatrix} MC_1 \\ MC_2 \end{bmatrix}_{n+1} = \begin{bmatrix} MC_1 \\ MC_2 \end{bmatrix}_{n} - \begin{bmatrix} \frac{\partial z_2}{\partial MC_1} & \frac{\partial Z_2}{\partial MC_2} \\ \frac{\partial z_4}{\partial MC_1} & \frac{\partial z_4}{\partial MC_2} \end{bmatrix}_{n}^{-1} \begin{bmatrix} z_2 \\ z_4 \end{bmatrix}_{n}.$$

Now intorduce the following new notations:

$$\frac{\partial z_1}{\partial MC_1} = z_6, \quad \frac{\partial z_2}{\partial MC_1} = z_7, \quad \frac{\partial z_3}{\partial MC_1} = z_8, \quad \frac{\partial z_4}{\partial MC_1} = z_9, \quad \frac{\partial z_5}{\partial MC_1} = z_{10},$$
$$\frac{\partial z_1}{\partial MC_2} = z_{11}, \quad \frac{\partial z_2}{\partial MC_2} = z_{12}, \quad \frac{\partial z_3}{\partial MC_2} = z_{13}, \quad \frac{\partial z_4}{\partial MC_2} = z_{14}, \quad \frac{\partial z_5}{\partial MC_2} = z_{15}.$$

Newton iterrative scheme get form by using new notations

$$\begin{bmatrix} MC_1 \\ MC_2 \end{bmatrix}_{n+1} = \begin{bmatrix} MC_1 \\ MC_2 \end{bmatrix}_{n} - \begin{bmatrix} z_7 & z_{12} \\ z_9 & z_{14} \end{bmatrix}_{n}^{-1} \begin{bmatrix} z_2 \\ z_4 \end{bmatrix}_{n}$$

The above itrative process will be continued untill the following criteria is fulfilled.

$$max\{|z_2(\xi_\infty, MC_1 n, MC_2 n)|, |z_4(\xi_\infty, MC_1 n, MC_2 n)|\} < \chi,$$

where χ has been taken as 10^{-5}.

Differentiating the last system of first order ODEs w.r.t MC_1 and MC_2, we get:

$$
\begin{aligned}
z_6' &= z_7, && z_6(0) = 0,\\
z_7' &= z_8, && z_7(0) = 0,\\
z_8' &= (1-\beta z_1^2)^{-1}\big(- z_1 z_8 - z_6 z_3 - 2\beta z_1 z_2 z_8 - 2\beta z_1 z_7 z_3 - 2\beta z_6 z_2 z_3\\
&\quad - M sin^2(\gamma)\big(\beta z_1 z_8 + \beta z_6 z_3 - z_7\big) - \delta(1+\beta_t z_9) z_4 \cos(\alpha)\\
&\quad - \delta(1+\beta_t z_4) z_9 cos(\alpha) + \lambda_1 z_7 + 2 z_2 z_7 (1+Fr)\big)\\
&\quad + \big(- z_1 z_3 - 2\beta z_1 z_2 z_3 - M\sin^2(\gamma)\big(\beta z_1 z_3 - z_2\big)\\
&\quad - \delta(1+\beta_t z_4) z_4 \cos(\alpha) - \lambda_1 z_2 + (1+Fr) z_2^2\big)((1-\beta z_1^2)^{-2}(2\beta z_1 z_6)),\\
&&& z_8(0) = 1,\\
z_9' &= z_{10}, && z_9(0) = 0,\\
z_{10} &= \left(\frac{1}{Pr} - \lambda_2 z_1^2\right)^{-2}(2\lambda_2 z_1 z_6)(z_2 z_5 + S_1(z_2 + \lambda_2 z_2^2 - \lambda_2 z_1 z_3)\\
&\quad + \lambda_2(z_2^2 z_4 - z_1 z_3 z_4 - z_1 z_2 z_5)) + \left(\frac{1}{Pr} - \lambda_2 z_1^2\right)^{-1}(z_7 z_5 + z_2 z_{10}\\
&\quad + S_1(z_7 + 2\lambda_2 z_2 z_7 - \lambda_2(z_6 z_3 + z_1 z_8)) + \lambda_2(2 z_2 z_4 z_7 + z_2^2 z_9 - z_6 z_3 z_4\\
&\quad - z_1 z_8 z_4 - z_1 z_3 z_9 - z_6 z_2 z_5 - z_1 z_7 z_5 - z_1 z_2 z_{10})),\\
&&& z_{10}(0) = 0,\\
z_{11}' &= z_{12}, && z_{11}(0) = 0,\\
z_{12}' &= z_{13}, && z_{12}(0) = 0,
\end{aligned}
$$

$$
\begin{aligned}
z_{13}' &= \big((1-\beta z_1^2)^{-1}\big)\big(- z_1 z_{13} - z_{11} z_3 - 2\beta z_1 z_2 z_{13} - 2\beta z_1 z_{12} z_3 - 2\beta z_{11} z_2 z_3\\
&\quad - M\sin^2(\gamma)\big(\beta z_1 z_{13} + \beta z_{11} z_3 - z_{12}\big) - \delta(1+\beta_t z_{14}) z_4 \cos(\alpha)\\
&\quad - \delta(1+\beta_t z_4) z_{14} cos(\alpha) + \lambda_1 z_{12} + 2 z_2 z_{12}(1+Fr)\big)\\
&\quad + \big(- z_1 z_3 - 2\beta z_1 z_2 z_3 - M\sin^2(\gamma)\big(\beta z_1 z_3 - z_2\big)\\
&\quad - \delta(1+\beta_t z_4) z_4 \cos(\alpha) + \lambda_1 z_2 + (1+Fr) z_2^2\big)\big((1-\beta z_1^2)^{-2}(2\beta z_1 z_{11})\big),\\
&&& z_{13}(0) = 0,\\
z_{14}' &= z_{15}, && z_{14}(0) = 0,
\end{aligned}
$$

$$z_{10}' = \left(\frac{1}{Pr} - \lambda_2 z_1^2\right)^{-2}(2\lambda_2 z_1 z_{11})(z_2 z_5 + S_1(z_2 + \lambda_2 z_2^2 - \lambda_2 z_1 z_3)$$
$$+ \lambda_2(z_2^2 z_4 - z_1 z_3 z_4 - z_1 z_2 z_5)) + \left(\frac{1}{Pr} - \lambda_2 z_1^2\right)^{-1}(z_{12} z_5 + z_2 z_{15}$$
$$+ S_1(z_{12} + 2\lambda_2 z_2 z_{12} - \lambda_2(z_{11} z_3 + z_1 z_{13})) + \lambda_2(2z_2 z_4 z_{12} + z_2^2 z_{14}$$
$$- z_{11} z_3 z_4 - z_1 z_{13} z_4 - z_1 z_3 z_{14} - z_{11} z_2 z_5 - z_1 z_{12} z_5 - z_1 z_2 z_{15})),$$
$$z_{15}(0) = 1.$$

Now, solve the equation (4.12).Let us consider following notations:

$$\phi = e_1, \quad \phi' = e_1' = e_2, \quad \phi'' = e_2'.$$

Convert equations into first order ODEs by using notations:

$$e_1' = e_2, \qquad e_1(0) = 1,$$
$$e_2' = \left(\frac{1}{S_c} - \lambda_3 c_1^2\right)^{-1}\left(-c_1 e_2 + \lambda_3 c_1 c_2 e_2 - S_0\left(\frac{-c_1 c_3 + \lambda_2 c_1 c_2 c_3}{1/Pr - \lambda_2 c_1^2}\right) + \epsilon e_1\right),$$
$$e_2(0) = w.$$

In the above IVP, the missing condition w is chosen to satisfy the following relation.

$$e_1(\xi_\infty)_w = 0$$

Now apply Newton's method to solve algebric equations by using following formula.

$$w_{n+1} = w_n - \frac{(e_1(\xi_\infty))_{w=w_n}}{\left(\frac{\partial e_1(\xi_\infty)}{\partial w}\right)}$$

Now, intorduce the following new notations:

$$\frac{\partial e_1}{\partial w} = e_3, \quad \frac{\partial e_2}{\partial w} = e_4.$$

Newton iterrative scheme get form by using new notations

$$w_{n+1} = w_n - \frac{(e_1(\xi_\infty))_{w=w_n}}{(e_3(\xi_\infty))}$$

The above iterrative procedure will be continued untill the following criteria is fulfilled.

$$(e_1(\xi_\infty))_{w=w_n} < \chi$$

where χ has been taken 10^{-5}.

Differentiate the system of first order ODEs w.r.t w, we get

$$\begin{aligned} &e_3 = e_4, && e_3(0) = 0, \\ &e_4 = \left(\frac{1}{S_c} - \lambda_3 c_1^2\right)^{-1}\left(- c_1 e_4 + \lambda_3 c_1 c_2 e_4 + \epsilon e_3\right), && e_4(0) = 1. \end{aligned}$$

4.4 Results and Discussions

In this section, the effect of different parameters on the velocity, temperature and concentration distributions will be analyzed by using table and graph. TABLE 4.1 and TABLE 4.2 show the impact of different parameters on skin friction $(Re_x)^{\frac{1}{2}}C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}}Nu_x$ repectively. Increasing the value of Pr, M, β, β_t, λ_1 and δ causes a gain in the skin friction $(Re_x)^{\frac{1}{2}}C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}}Nu_x$. However Fr and S_1 have inverse relation with skin friction $(Re_x)^{\frac{1}{2}}C_f$ and local Nusselt number $(Re_x)^{-\frac{1}{2}}Nu_x$. Moreover, when γ is increased, the skin friction $(Re_x)^{\frac{1}{2}}C_f$ is decreased but an increment in the local Nusselt number $(Re_x)^{-\frac{1}{2}}Nu_x$ can be observed. Similarly when c is boosted, the in skin friction $(Re_x)^{\frac{1}{2}}C_f$ increased and local Nusselt number $(Re_x)^{-\frac{1}{2}}Nu_x$ decreased. TABLE 4.3 analyze the impact of different parameters on Sherwood number $(Re_x)^{-\frac{1}{2}}sh_x$. By

rising γ, M, β, β_t, δ, λ_2, Sc, Sr, ϵ, Sherwood number $(Re_x)^{-\frac{1}{2}}sh_x$ is increased. But when Pr, Fr, λ_1, S_1, c are increased, Sherwood number $(Re_x)^{-\frac{1}{2}}sh_x$ decreased.In these tables, I_f and I_θ are the intervals from which the missing conditions can be chosen.

FIGURE 4.2 elaborates the impact of γ on velocity profile. By increaing the value of γ, the velocity profile is increased. FIGURE 4.3 shows that when β is increased, the velocity profile declined. FIGURE 4.4 indicates the effect of Fr on velocity profile. Increasing the value of Fr causes increment in velocity profile. FIGURE 4.5 observes that when α increases, velocity decreases drastically. FIGURE 4.6 shows that when δ increases, velocity is decreased. FIGURE 4.7 elaborates the impact of B_t on velocity profile. When B_t increases, velocity is increased.

FIGURE 4.8 shows the impact of δ on temperature profile. It can be observed that when δ increases, the temperature profile is increased. FIGURE 4.9 tells about the impact of Pr on the temperature profile. It can be recongnised that by increasing the value of Pr, the temperature profile demolished. FIGURE 4.10 observes that when λ_1 is boosted, temperature decreases. FIGURE 4.11 shows the effect of S_1 on the temperature profile. Here by rising the value of S_1 causes an decrement in temperature profile.

FIGURE 4.12 shows that when the value ϵ is increased, concentration profile decreases. FIGURE 4.13 shows the relationship between λ_3 and concentration profile. For higher values of λ_3 reduces concetration profile. FIGURE 4.14 shows the impact of Sc on concentration. Increaseing the value of Sc causes decrement in concentration profile. FIGURE 4.15 indicates that rising the value of Sr causes decline in concentration domain. FIGURE 4.16 indicates the effect of λ_2 on the concentration profile. It can be observed that when λ_2 increases, concentration profile goes up.

TABLE 4.1:

Results of $(Re_x)^{\frac{1}{2}}C_f$ for $S = 2$ and other various parameters

γ	Pr	M	β	Fr	β_t	λ_1	λ_2	S_1	δ	c	I_f	I_θ	$(Re_x)^{\frac{1}{2}}C_f$
$\frac{\pi}{6}$	0.7	1	0.1	0.3	0.5	0.3	0.3	0.1	0.2	-1	[2.0,2.5]	[-5.5,-5.0]	2.473228
$\frac{\pi}{4}$											[2.4,2.9]	[-5.5,-5.0]	2.730991
$\frac{\pi}{3}$											[2.7,3.1]	[-5.5,-5.0]	2.959223
$\frac{\pi}{2}$											[2.9,3.3]	[-5.5,-5.0]	3.167758
	0.4										[0.1,0.5]	[-0.9,-0.2]	-0.291145
	0.5										[0.1,0.5]	[-0.9,-0.5]	-0.259397
	0.6										[0.1,0.5]	[-0.9,-0.5]	-0.212756
		0									[2.0,2.5]	[-5.5,-5.0]	2.163861
		2									[2.5,2.9]	[-5.5,-5.0]	2.730994
		3									[2.6,2.9]	[-5.5,-5.0]	2.959229
			0								[1.5,1.9]	[-5.5,-5.0]	1.712874
			0.005								[1.6,1.9]	[-5.5,-5.0]	1.735966
			0.05								[1.6,1.9]	[-5.5,-5.0]	1.996800
				0							[1.9,2.3]	[-5.5,-5.0]	2.582383
				0.6							[1.9,2.3]	[-5.5,-5.0]	2.355702
				0.9							[1.9,2.3]	[-5.5,-5.0]	2.227542
					0						[2.3,2.6]	[-5.5,-5.0]	2.457678
					1.0						[2.0,2.6]	[-5.5,-5.0]	2.488740
					1.5						[2.0,2.6]	[-5.5,-5.0]	2.504217
						0.09					[2.6,2.8]	[-4.7,-4.2]	2.836244
						0.1					[2.3,2.6]	[-4.7,-4.3]	2.842350
						0.2					[2.0,2.4]	[-5.0,-4.6]	2.901979
							0.09				[2.0,2.5]	[-6.5,-6.0]	3.102011
							0.1				[2.0.2.5]	[-6.5,-6.0]	3.096768
							0.2				[2.3,2.5]	[-5.7,-5.3]	3.037842
								0.2			[2.0,2.6]	[-7.5,-6.5]	2.460423
								0.3			[2.0,2.5]	[-6.9,-6.5]	2.447942
								0.4			[2.3,2.6]	[-6.5,-6.0]	2.435804
									0		[1.6,1.9]	[-5.7,-5.2]	2.370204
									0.4		[1.9,2.3]	[-5.7,-5.3]	2.572868
									0.6		[1.9,2.3]	[-5.7,-5.3]	2.669556
										-0.4	[2.3,2.6]	[-1.9,-1.5]	2.040746
										-0.6	[2.3,2.6]	[-1.9,-1.5]	2.218710
										-0.8	[2.3,2.6]	[-2.9,-2.5]	2.364065

TABLE 4.2:

Results of $(Re_x)^{-\frac{1}{2}}Nu_x$ for for $S = 2$ and other various parameters

γ	Pr	M	β	Fr	β_t	λ_1	λ_2	S_1	δ	c	I_f	I_θ	$(Re_x)^{-\frac{1}{2}}Nu_x$
$\frac{\pi}{6}$	0.7	1	0.1	0.3	0.5	0.3	0.3	0.1	0.2	-1	[2.0,2.5]	[-5.5,-5.0]	5.861098
$\frac{\pi}{4}$											[2.4,2.9]	[-5.5,-5.0]	5.920942
$\frac{\pi}{3}$											[2.7,3.1]	[-5.5,-5.0]	5.969606
$\frac{\pi}{2}$											[2.9,3.3]	[-5.5,-5.0]	6.011046
	0.4										[0.1,0.5]	[-0.9,-0.2]	0.584707
	0.5										[0.1,0.5]	[-0.9,-0.5]	0.931997
	0.6										[0.1,0.5]	[-0.9,-0.5]	1.713615
		0									[2.0,2.5]	[-5.5,-5.0]	5.780898
		2									[2.5,2.9]	[-5.5,-5.0]	5.920943
		3									[2.6,2.9]	[-5.5,-5.0]	5.969608
			0								[1.5,1.9]	[-5.5,-5.0]	5.633143
			0.005								[1.6,1.9]	[-5.5,-5.0]	5.641548
			0.05								[1.6,1.9]	[-5.5,-5.0]	5.728825
				0							[1.9,2.3]	[-5.5,-5.0]	5.881269
				0.6							[1.9,2.3]	[-5.5,-5.0]	5.838610
				0.9							[1.9,2.3]	[-5.5,-5.0]	5.813097
					0						[2.3,2.6]	[-5.5,-5.0]	5.858799
					1.0						[2.0,2.6]	[-5.5,-5.0]	5.863386
					1.5						[2.0,2.6]	[-5.5,-5.0]	5.865664
						0.09					[2.6,2.8]	[-4.7,-4.2]	5.943639
						0.1					[2.3,2.6]	[-4.7,-4.3]	5.944954
						0.2					[2.0,2.4]	[-5.0,-4.6]	5.957651
							0.09				[2.0,2.5]	[-1.9,-1.5]	1.462495
							0.1				[2.0,2.5]	[-1.9,-1.5]	1.513975
							0.2				[2.0,2.4]	[-2.9,-2.5]	2.367066
								0.2			[2.0,2.5]	[-6.5,-6.0]	5.207551
								0.3			[2.0.2.5]	[-6.5,-6.0]	4.554608
								0.4			[2.3,2.5]	[-5.7,-5.3]	3.902265
									0		[2.0,2.6]	[-7.5,-6.5]	5.839112
									0.4		[2.0,2.5]	[-6.9,-6.5]	5.881840
									0.6		[2.3,2.6]	[-6.5,-6.0]	5.901508
										-0.4	[1.6,1.9]	[-5.7,-5.2]	6.490795
										-0.6	[1.9,2.3]	[-5.7,-5.3]	6.286028
										-0.8	[1.9,2.3]	[-5.7,-5.3]	6.076781

TABLE 4.3:

Results of $(Re_x)^{\frac{1}{2}}Sh_x$ for $S = 2$ and other various parameters

γ	Pr	M	β	Fr	β_t	λ_1	λ_2	S_1	δ	c	λ_3	S_c	S_r	ϵ	$(Re_x)^{-\frac{1}{2}}Sh_x$
$\frac{\pi}{6}$	0.7	1	0.1	0.3	0.5	0.3	0.3	0.1	0.2	-1	0.2	0.3	0.1	0.2	0.959256
$\frac{\pi}{4}$															0,975188
$\frac{\pi}{3}$															0.987858
$\frac{\pi}{2}$															0.998420
	0.4														0.549587
	0.5														0.556657
	0.6														0.567433
		0													0.937417
		2													0.975189
		3													0.987859
			0												0.895778
			0.005												0.898154
			0.05												0.922756
				0											0.964297
				0.6											0.953602
				0.9											0.947147
					0										0.958698
					1.0										0.959809
					1.5										0.960361
						0.09									0.947206
						0.1									0.947856
						0.2									0.953901
							0.09								0.557339
							0.1								0.558008
							0.2								0.566855
								0.2							0.958576
								0.3							0.957905
								0.4							0.957244
									0						0.953470
									0.4						0.964689
									0.6						0.969817
										-0.7					0.967776
										-0.8					0.967595
										-0.9					0.965095
											0.1				0.857018
											0.3				1.096633
											0.4				1.291429
												0.1			0.392016
												0.2			0.641148
												0.4			1.362916
													0.2		1.152611
													0.3		1.345966
													0.4		1.539322
														0.1	0.916263
														0.3	0.999247
														0.4	1.036703

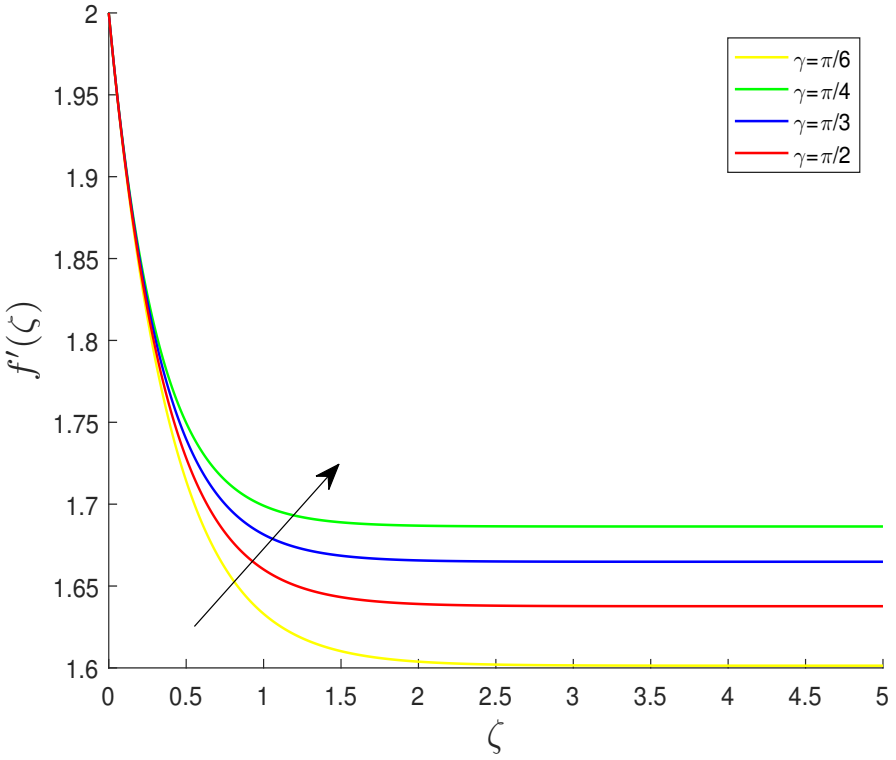

FIGURE 4.2: Impact of γ on the velocity profile.

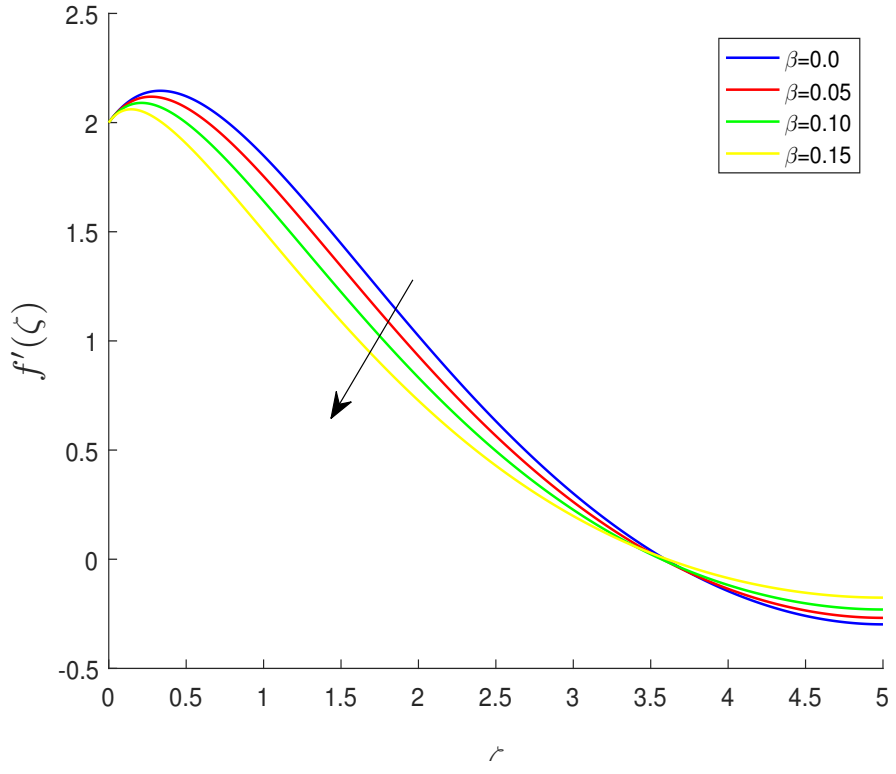

FIGURE 4.3: Impact of β on the velocity profile.

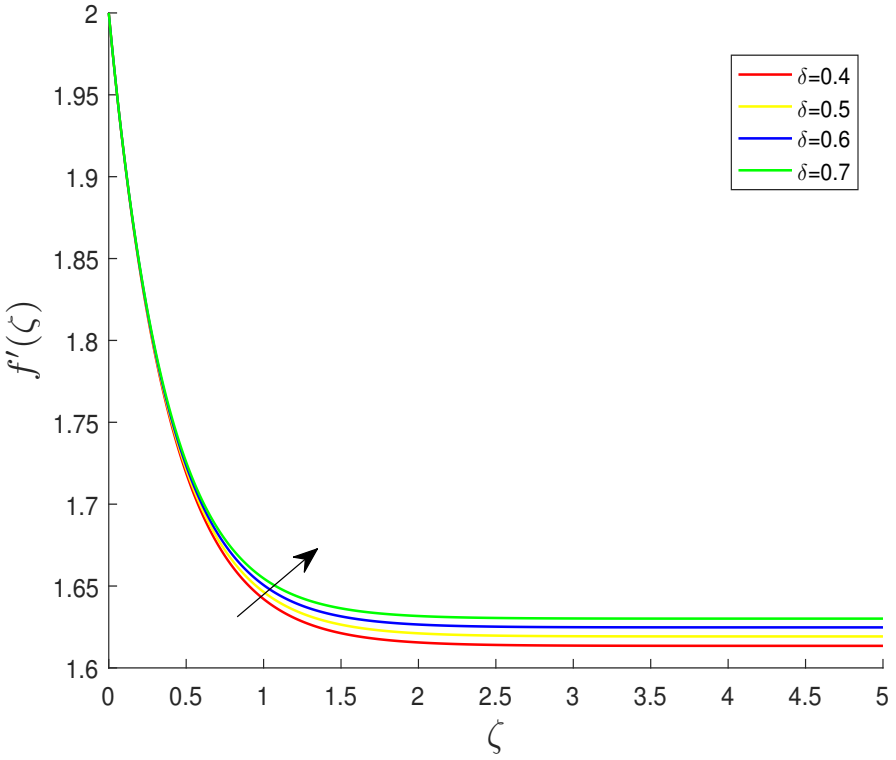

FIGURE 4.4: Impact of δ on the velocity profile.

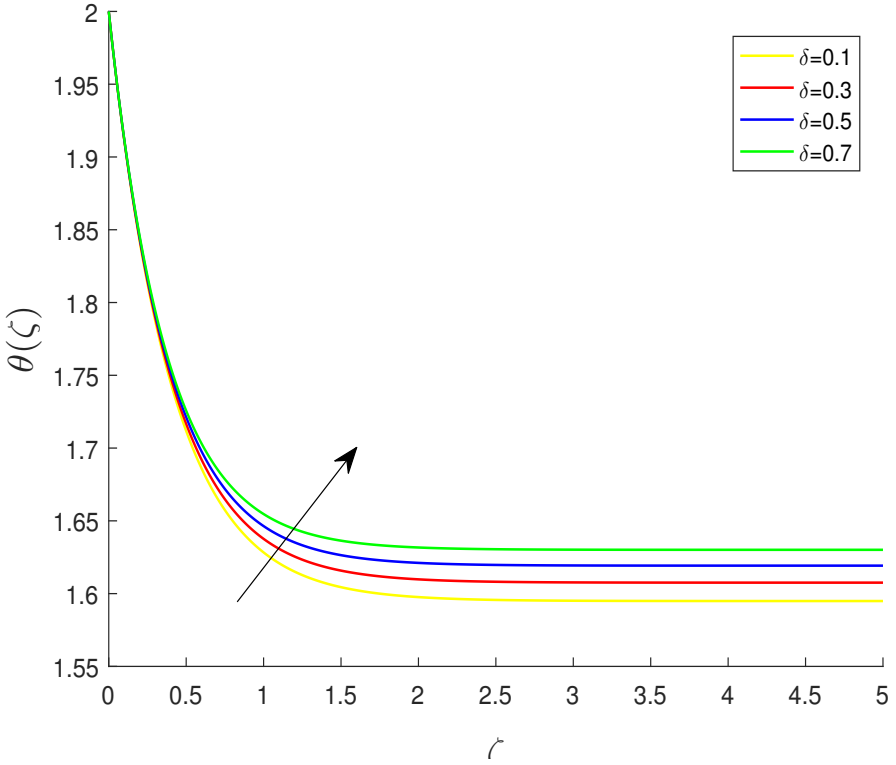

FIGURE 4.5: Impact of δ on the temperature profile.

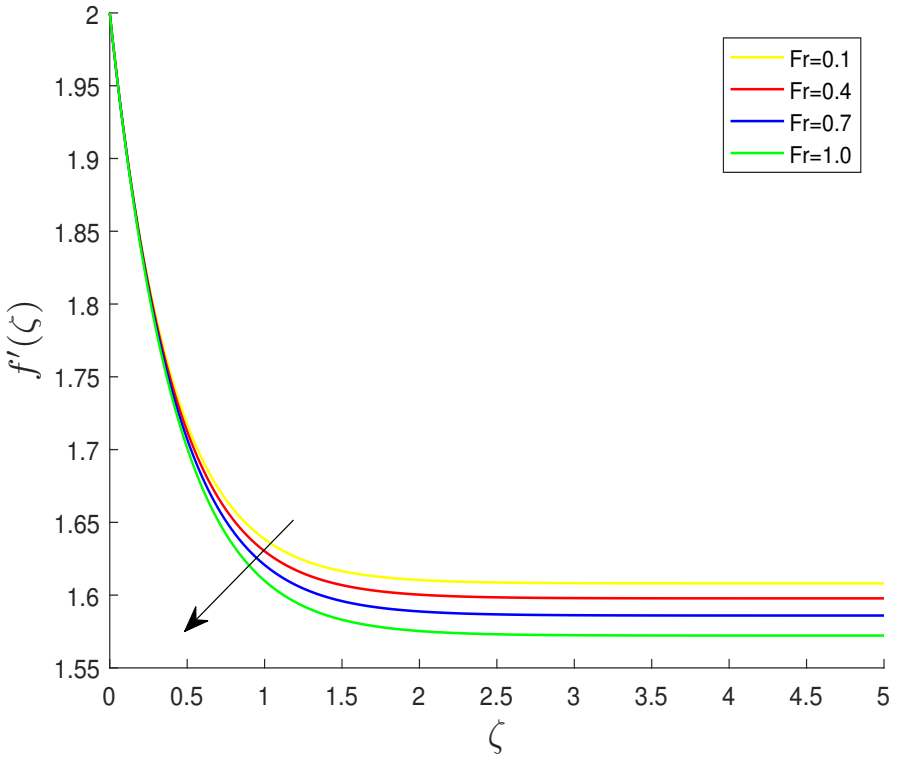

FIGURE 4.6: Impact of Fr on the velocity profile.

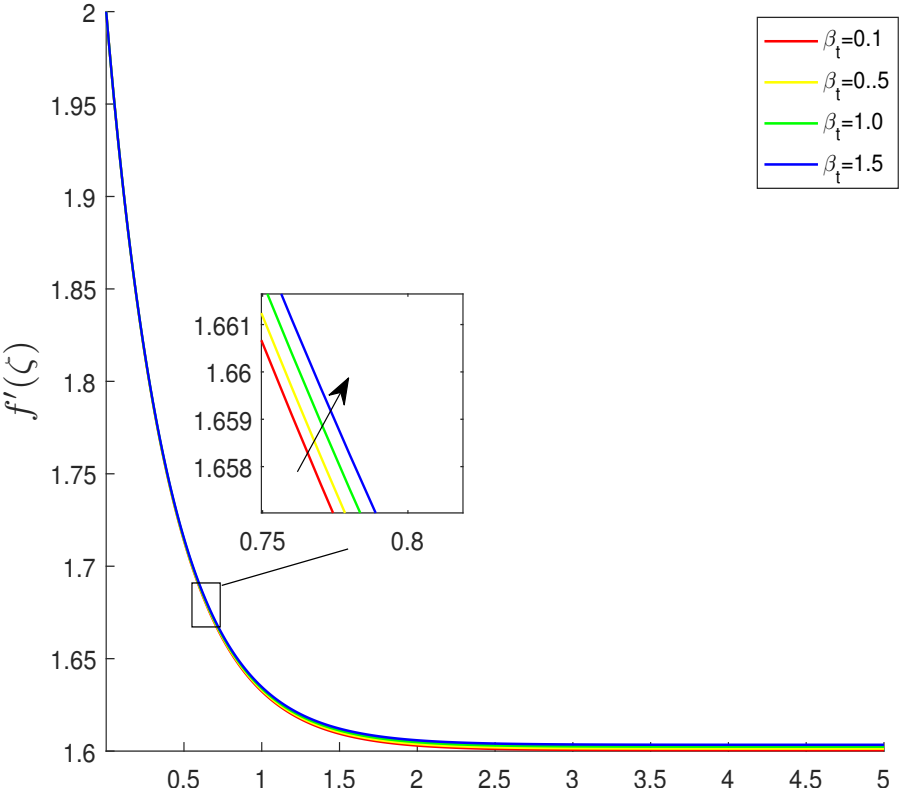

FIGURE 4.7: Impact of B_t on the velocity.

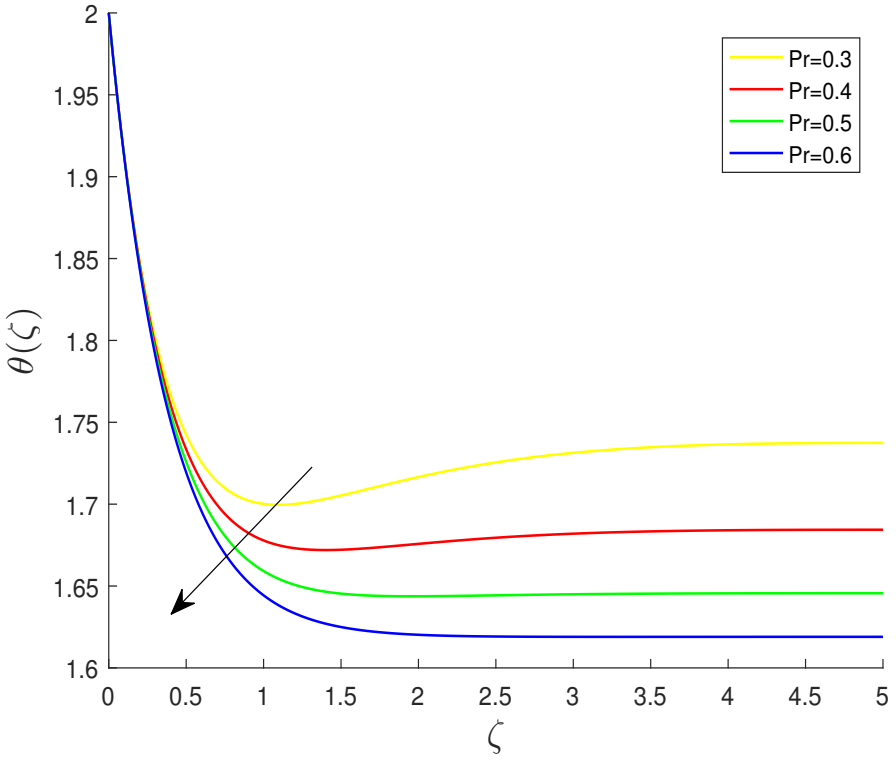

FIGURE 4.8: Impact of Pr on the temperature profile.

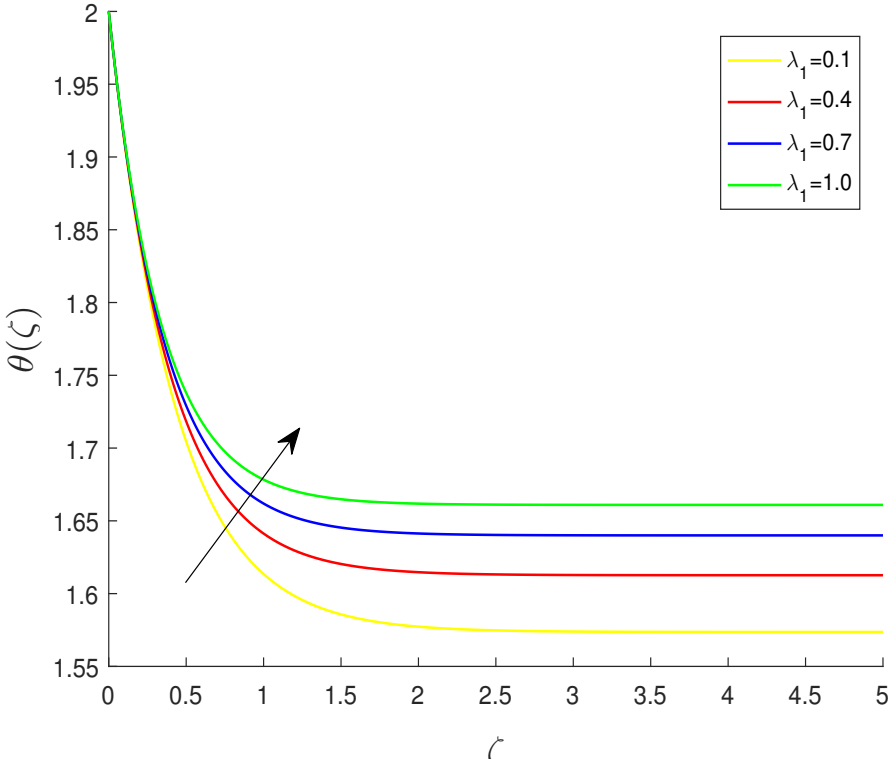

FIGURE 4.9: Impact of λ_1 on the temperature profile.

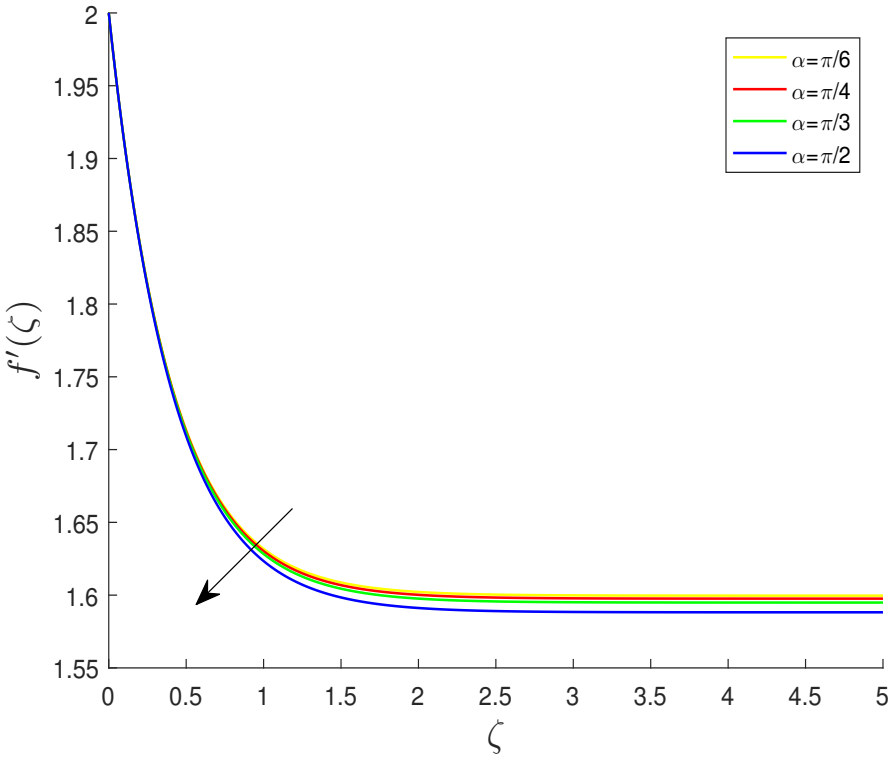

FIGURE 4.10: Impact of α on the velocity profile.

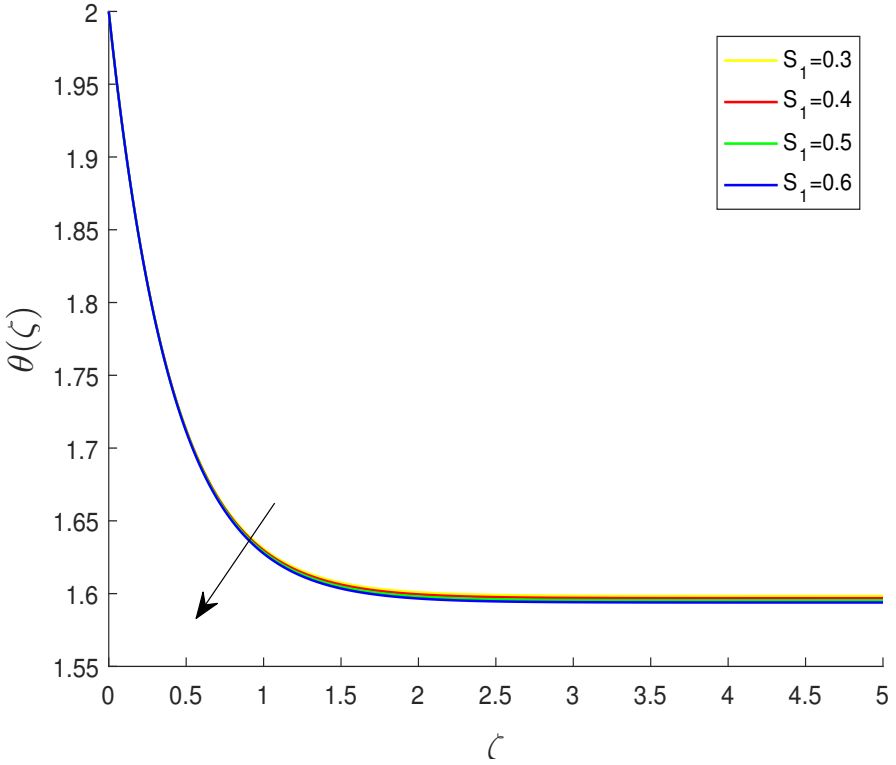

FIGURE 4.11: Impact of S_1 on the temperature profile.

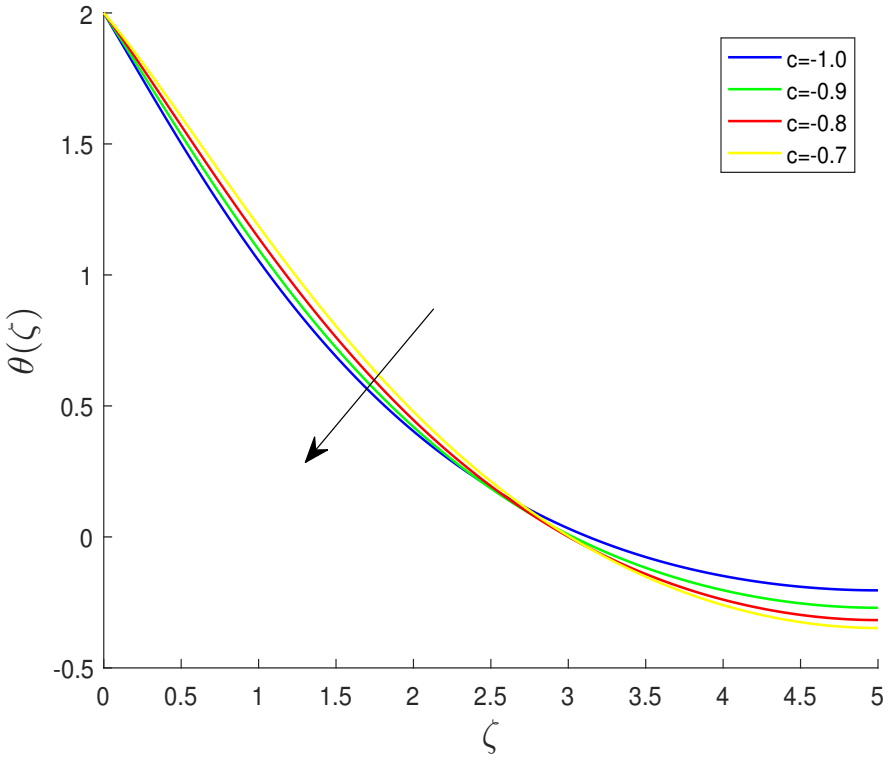

FIGURE 4.12: Impact of c on the temperature profile.

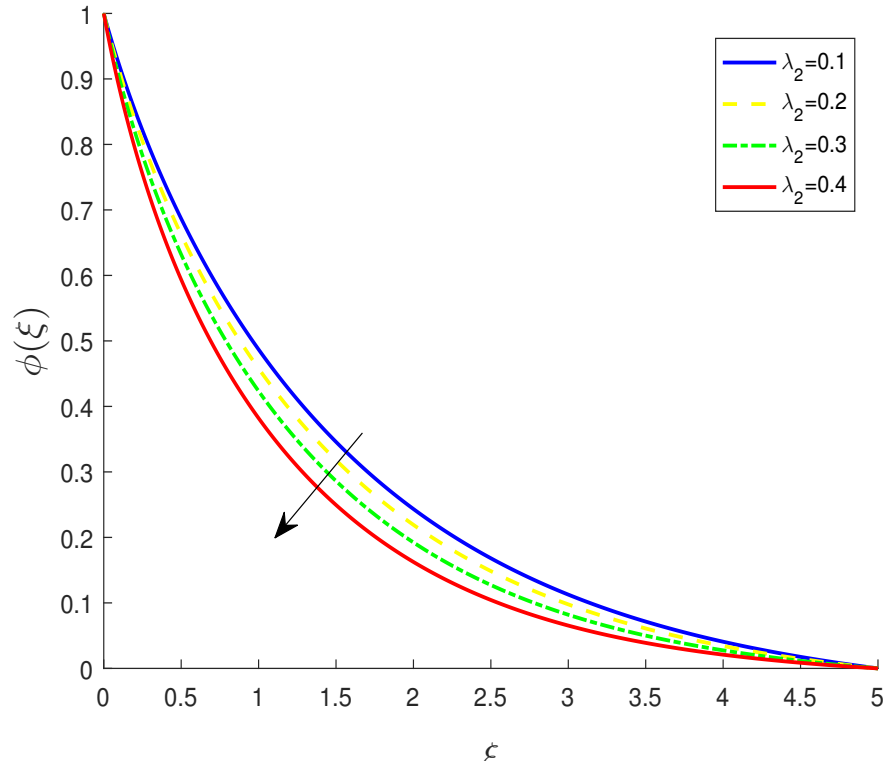

FIGURE 4.13: Impact of λ_2 on the concentration profile.

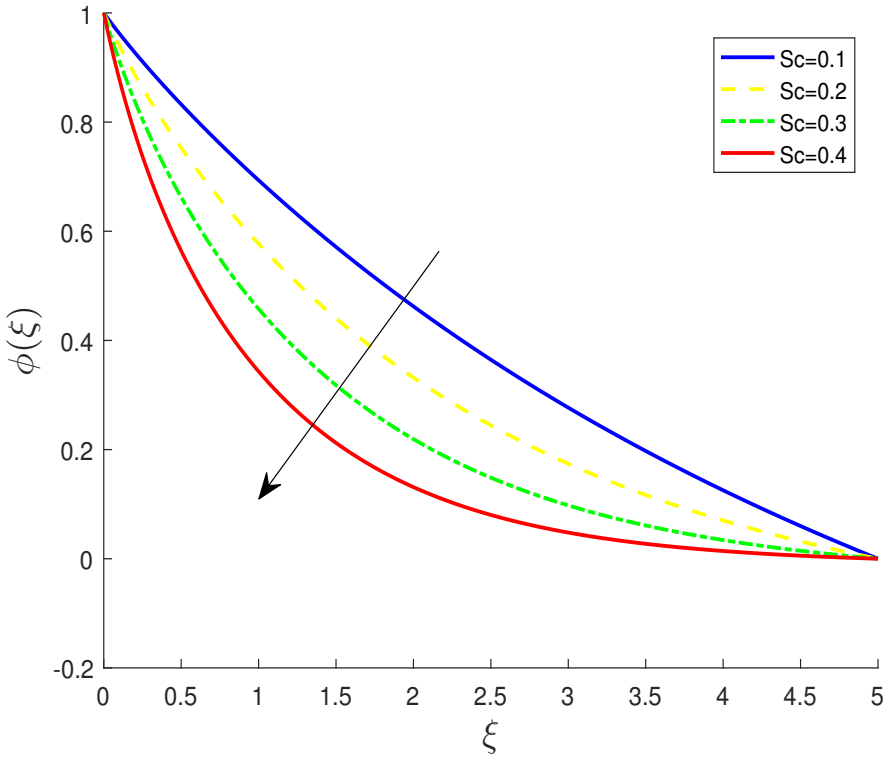

FIGURE 4.14: Impact of Sc on the concentration profile.

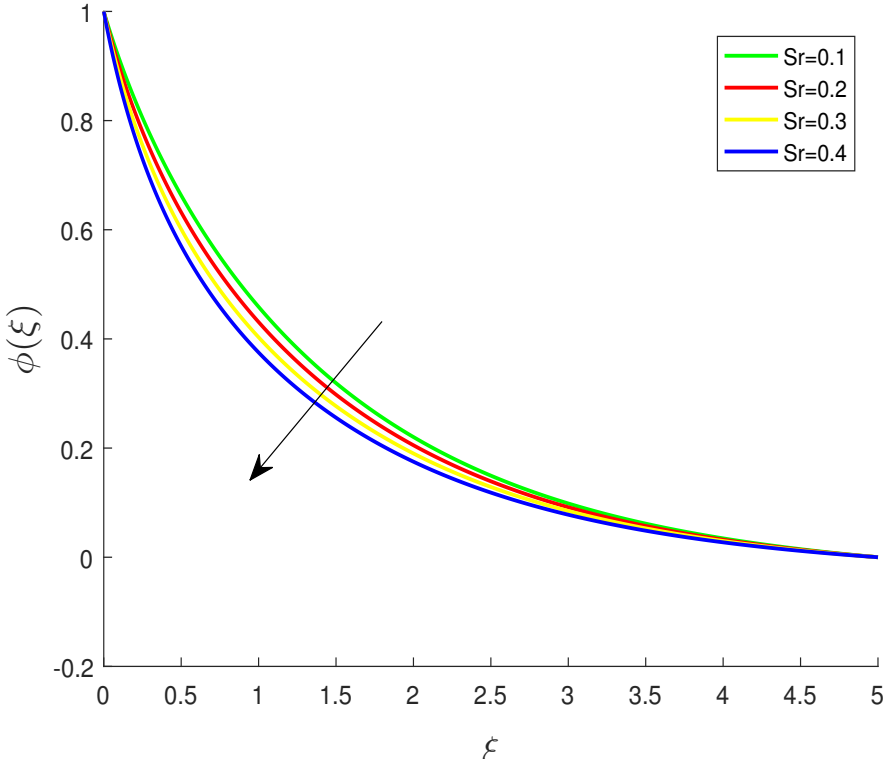

FIGURE 4.15: Impact of Sr on the concentration profile.

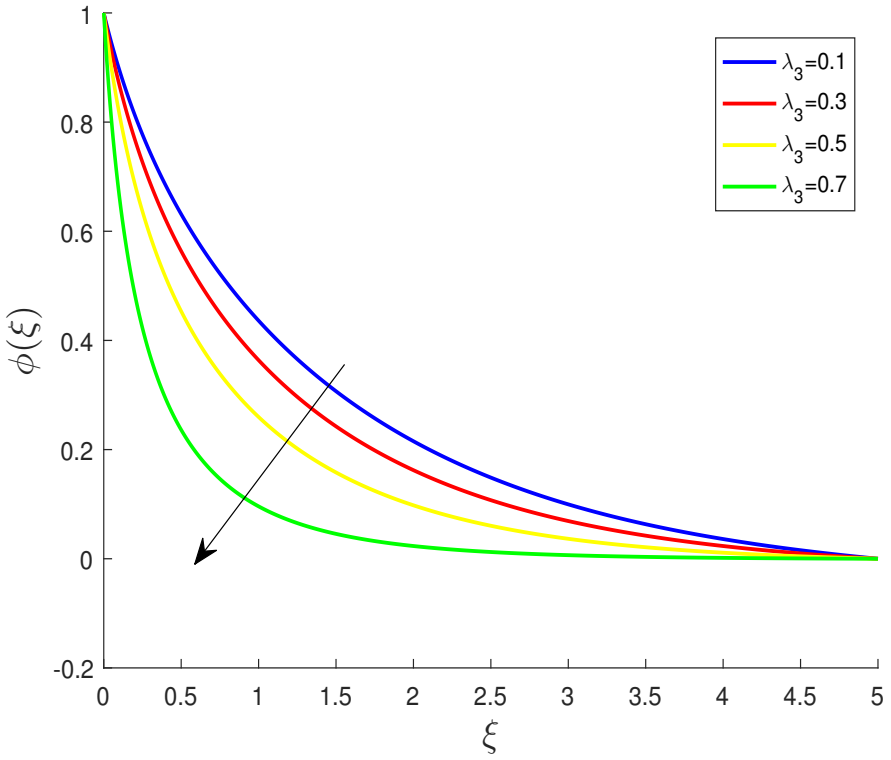

FIGURE 4.16: Impact of λ_3 on the concentration profiles.

Chapter 5

Conclusion

In this thesis, the work of Bilal et al. [33] is reviewed and extended by including the impact of Cattaneo-Christov, double diffusion and chemical reaction. First of all, momentum, temperature and concentration equations are converted into the ordinary differential equations by using similarity transformations. Numerical solution of ODEs has been obtained by using the shooting method. The results are represented through graphs and tables by using different parameters for velocity, temperature and concentration profiles. The achievements of the current research can be summarized as below:

- Increasing the values of the aligned angle γ, the velocity profile is decreased but the temperature and concentration profiles are increased.
- Due to rising the values of the Prandtl number Pr, the velocity and temperature profiles increase but concetration profile decrease .
- By rising the values of the magnetic parameter M causes an increase in the velocity, temperature and concentration profiles.
- An enhancement in the dimensionless Maxwell parameter β, the velocity, temperature and concentration profiles are increased.
- By boosting the values of the local inertia coefficient Fr, the velocity, temperature and concentration profiles are decreased

- The velocity, temperature and concenration profiles are increased due to an increment in the nonlinear thermal variable β_t.
- Increasing the value of the porosity parameter λ_1 causes an increase in velocity and temperature profiles but a decrement in the concentration profile
- By rising the values of the thermal stratification variable S_1, the velocity, temperature and concentration profiles are decreased
- Increasing the mixed convection variable δ causes an increment in the velocity, temperature and concentration profiles.
- By increasing the shrinking parameter c, the velocity profile is increased but the temperature and concentration profiles are decreased.
- Expanding the Schmidt number Sc, Soret number Sr and ϵ causes an increase in the concentration profile.

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