CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

Effect of Thermal Radiation and Chemical Reaction on MHD Carreau Fluid Flow over a Stretching Sheet

by

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in the

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*This thesis is dedicated to my beloved **Parents** and elegant **Teachers** whose devotions and contributions to my life are really worthless and whose deep consideration on part of my academic career, made me consolidated and inspired me as I am upto this grade now.*

CERTIFICATE OF APPROVAL

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(Syed Amir Ghazi Ali Shah)

Abstract

This thesis investigates the numerical treatment of MHD boundary layer flow of Carreau fluid with thermal radiation and chemical reaction. The thermal conductivity and viscosity is considered to vary linearly with temperature. By applying some suitable similarity transformations, the constitutive equations are transformed to a set of ordinary differential equations. The obtained problem is solved analytically by shooting method. The impact of magnetic parameter M, Prandtl number Pr, Lewis number Le, thermal radiation parameter R and chemical reaction parameter γ on the skin friction coefficient and Nusselt number are studied and presented in tabular forms. An increment in the chemical reaction parameter leads to a decrement in the concentration profile. The effect of different fluid parameters such as the suction parameter S, Prandtl number Pr and Lewis number Le on the velocity, temperature and concentration profiles are presented graphically.

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Abbreviations

IVP	Initial value problem
MHD	Magnetohydrodynamics
ODE	Ordinary differential equation
PDE	Partial differential equation
RK	Runge-Kutta

Symbols

u, v	Components of velocity(unit:ms^{-1})
u_w	Stretching velocity(unit:ms^{-1})
x, y	Spatial Cartesian coordinates(unit:m)
B	Stretching parameter
P	Pressure(unit:Nm^{-2})
μ	Viscosity(unit:m^2s^{-1})
ν	Kinematic viscosity(unit:(unit:m^2s^{-1})
ρ	Fluid density(unit:$J.kgm^{-3}$)
n	Power law index
σ	Electrical conductivity(unit:$m^{-2}s$)
ψ	Stream function
u_e	External velocity(unit:ms^{-1})
Pr	Prandtl number
J_0	Magnetic field
S	Suction parameter
T	Fluid temperature(unit:K
D	Diffusion coefficient
q_w	Surface heat flux
m	Nonlinearity Stretching parameter
k	Thermal conductivity
a, b	Constants
v_w	Suction velocity(unit:S.m^{-1})
M	Magnetic parameter

C_p	Specific heat capacity
T_∞	The free stream temperature(unit:K)
T_w	Wall temperature
C	Concentration
C_∞	Ambient concentration
C_w	Nanoparticles concentration at the stretching sheet
We	Weissenbery number
C_f	Skin friction coefficient
$R_e(x)$	Local Reynolds number
η	Similarity variable
Nu_x	local Nusselt number
Le	Lewis number
τ_w	Shear stress Pa
ϵ	Thermal conductivity parameter
ξ	Viscous parameter
R	Thermal radiation
Kr	Chemical reaction coefficient
γ	Chemical reaction parameter

Chapter 1

Introduction

Over the past few years, study of heat transfer and flow over a stretchable sheet has gained considerable importance in the scientists' community. Many scientists have given much attention to this field due to its wide range of applications in industrial and engineering process. Production of rubber, colloidal suspension of fluid, glass production, spinning of metal, use of geothermal energy and plasma studies are some examples of its applications. Likewise, nanofluids are a new class of fluids manufactured by dispersion of (nanoparticles, nanofibers, nanotubes, nanowires, nanorods, nanosheets, or droplets) of nanometer-sized materials into base fluids. In other words, nanofluids are nanoscale colloidal suspensions containing condensed nanomaterials.

They are two-phase systems with one phase (solid phase) in another (liquid phase). Nanofluids have been found to possess enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective rate of heat transfer coefficients compared to those of base fluids like oil or water. It has demonstrated great potential applications in many fields. For a two-phase system, there are some important issues we have to face. One of the most challenging issues is the stability of nanofluids.

For this purpose, Hiemenz [1] studied a two dimensional stagnation flow to a stationary flat plate in the case of an orthogonal flow. Erickson et al. [2] explained the boundary layer incompressible fluid flow over an inextensible at surface with uniform velocity. Olajuwan [3] examined the magnetohydrodynamics flow of Carreau fluid with transfer of mass and heat through a porous medium. Hayat et al. [4] solved the MHD peristaltic flow of Carreau liquid in a channel of various waveforms. Akbar et al. [5] studied the numerical simulation of 2-D tangent hyperbolic flow of fluid through an extending sheet placed in a magnetic field. Suneetha and Gangadhar [6] extended the work of Akbar et al. [5] with convective boundary conditions and MHD effect. Statistical analysis of heat and mass transfer to the Carreau MHD flow by the non-daracy decomposition and chemical reactions was observed by Abou Zeid [7]. The MHD peristaltic flow of Carreau nanoliquid in an asymmetric channel was researched by Akram et al. [8].

Akbar and Nadeem [9] also investigate the peristaltic flow of Carreau fluid in a uniform tube, taking into account the long wavelengths in the presence of heat and mass transfer. Nandeppanavar and his colleagues [10] studied, in the presence of partial slip, the flow and heat transfer of MHD fluids to the impermeable stretching surface with variable thermal conductivity and non-uniform heat sources or immersion. Cortell [11] dissected the heat transfer rate and in- compressible viscid flow on a nonlinear stretchable surface numerically. Vyas [12] investigated the impact of thermal radiation and dissipation on the flow of the boundary layer of the MHD and the transfer of heat through the nonlinear stretching surface. Ali [13] demonstrated the effect of thermal boundary layer suction and injection on the stretching surface of the power law. Sandeep et al. [14] studied the radiative and chemical reaction through semiconductor vertical porous panels in an unstable flow with the properties of heat transfer. Shen et al. [15] investigate the problem of MHD flow of heat from one place to another place close to the stagnation point with respect to the leaky leaf on the grid at a slip speed. Analytical solutions using the spectral Galerkin Legend process throughout the viscous fluid over a nonlinear stretchable layer, velocity and temperature fields are determined by Akyildiz and Siginer. [16]. The effect of partial slips on the boundary layer and the stagnation point flow of incompressible liquids with the rate of heat transfer near the shrinking surface was studied by Bhattacharyya et al. [17]. Chen [18] considered the impact of viscous dissipation on rate of heat transfer in a non-Newtonian liquid foil over an unstable stretching surface. Abdou and El-Zahar [19] demonstrated the impact of heat dependent viscosity on heat transfer over a continuous moving surface with variable internal heat generation in micropolar fluids. The impact of thermal conductivity and variable viscosity on the micropolar fluid problem in the presence of suction or injection was investigated by Salem and Odda [20]. The effect of thermal radiation on a volatile flow of the board layer has been numerically resolved [21] by Uwanta and Usman in the presence of a magnetic chip by means of a variable viscosity and a thermic conductivity of the micropolitan liquid. Abd El-Hakiem et al. have studied the effect of variable viscousness on natural micro polar fluid convection of MHD [22] as a linear temperature feature and assumes the viscosity of fluid. The influence of different types of fluid flow over a stretching sheet in various geometries was studied in [23–36].

Observed the impacts of thermo diffusion over stagnation point flow of nanofluid in the presence of stretchable sheet with applied magnetic field parameter by using similarity transformations. These are very recent experiments with heat and mass transfers in fluid dynamics with steady and unsteady flows over thin film. The purpose of this thesis is to model and analyze the MHD Carreau fluid through a stretching surface with variable viscosity and thermal conductivity in the presence of heat transfer rates under convective boundary conditions. Devi and Kandasmy [37] analyzed the impact of homogenous chemical reaction with heat and mass transfer laminar flow along with semi-infinite horizontal plate.

Chamkha and Rashad [38] talked about the impact of chemical reaction on MHD flow in the presence of heat generation or absorption of uniform vertical permeable surface. Mabood at al. [39] presented MHD heat flow and mass transfer of nanofluids with radiation, viscous dissipation and chemical reaction in the porous medium. Raptis and Perdikis [40] observed the viscid flow on a nonlinear stretchable sheet in the presence of magnetic field parameter by applying shooting technique. The impact of slip boundary condition on heat transfer rate were investigated by Das et al. [41]. Through their study, they found that in the presence of thermal slip condition or hydrodynamic suction, injection parameter has large impact on surface temperature of plate.The thermal radiation effect becomes intensified at high absolute temperature levels due to basic difference between radiation , convection and conduction energyexchange mechanisms.

1.1 Thesis Contributions

The major objective of this research work is to execute the impacts of Thermal Radiation and Chemical Reaction on MHD Carreau Fluid Flow over a Stretching Sheet. The set of nonlinear partial differential equations are transformed into ODEs and numerically solved by utilizing shooting method. Impacts of distinct parameters on the velocity, temperature and concentration distributions are expressed in tables and graphs.

1.2 Thesis Outlines

This thesis is classified into four main chapters:

Chapter 2 contains the basic definitions and terminologies, which are useful to understand the concepts discussed later on.

Chapter 3 contains the complete review of [42] which considers the analysis of MHD Carreau fluid flow over a stretching permeable sheet with variable viscosity and thermal conductivity.

Chapter 4 is an extension of the model discussed in [42] by including the impacts of thermal radiation and chemical reaction.

Chapter 5 includes the summary of the entire study.

All the references used in this thesis are listed in **Bibliography**.

Chapter 2

Basic Terminologies and Governing Equations

In this chapter, we will discuss some basic definitions, terminologies, basic laws, and dimensionless numbers, which will be helpful in conducting the work for the next chapters.
This chapter contains few essentials definitions and laws of fluid dynamics which will be used in the upcoming discussions.

2.1 Basic Concepts

Definition 2.1.1 (Fluid)
"A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be." [43]

Definition 2.1.2 (Fluid Mechanics)
"Fluid mechanics is that branch of science which deals with the behavior of the fluid (liquids or gases) at rest as well as in motion." [44]

Definition 2.1.3 (Fluid Statics)

"The study of fluid at rest is called fluid statics." [44]

Definition 2.1.4 (Fluid Dynamics)

"The study of fluid if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics." [44]

Definition 2.1.5 (Viscosity)

"Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
Mathematically,

$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}},$$

where μ is viscosity coefficient, τ is shear stress and $\frac{\partial u}{\partial y}$ represents the velocity gradient." [44]

Definition 2.1.6 (Kinematic Viscosity)

"It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by symbol ν called **'nu'.**
Mathematically,

$$\nu = \frac{\mu}{\rho}.\text{"} \ [44]$$

Definition 2.1.7 (Ideal Fluid)

"A fluid which is incompressible and has no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity." [44]

Definition 2.1.8 (Real Fluid)

"A fluid which possesses viscosity, is known as a real fluid. In actual practice, all the fluids are real fluids." [44]

Definition 2.1.9 (Newtonian Fluid)

"A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid." [44]

Definition 2.1.10 (Non-Newtonian Fluid)

"A real fluid in which the shear stress is not directly proportional to the rate of shear strain (or velocity gradient), is known as a Non-Newtonian fluid." [44]

Definition 2.1.11 (Magnetohydrodynamics)

"Magnetohydrodynamics is concerned with the mutual interaction of fluid flow and magnetic fields. The fluids in question must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionised gases (plasmas) and strong electrolytes." [45]

2.2 Types of Flow

Definition 2.2.1 (Laminar Flow)

"Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel." [44]

Definition 2.2.2 (Turbulent Flow)

"Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way." [44]

Definition 2.2.3 (Compressible Flow)

"Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid, Mathematically,

$$\rho \neq k,$$

where k is constant." [44]

Definition 2.2.4 (Incompressible Flow)

"Incompressible flow is that type of flow in which the density is constant for the fluid. Liquids are generally incompressible while gases are compressible, Mathematically,

$$\rho = k,$$

where k is constant." [44]

Definition 2.2.5 (Internal Flow)

"Flows completely bounded by solid surfaces are called internal or duct flows." [43]

Definition 2.2.6 (External Flow)

"Flows over bodies immersed in an unbounded fluid are said to be an external flow." [43]

Definition 2.2.7 (Steady Flow)

"If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow donot change with respect to time, the flow is said to be steady flow. Mathematically,

$$\frac{\partial Q}{\partial t} = 0,$$

where Q is any fluid property." [44]

Definition 2.2.8 (Unsteady Flow)

"If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes with respect to time, the flow is said to be unsteady. Mathematically,

$$\frac{\partial Q}{\partial t} \neq 0,$$

where Q is any fluid property." [44]

2.3 Heat Transfer

Definition 2.3.1 (Heat Transfer)
"Heat transfer is a branch of engineering that deals with the transfer of thermal energy from one point to another within a medium or from one medium to another due to the occurrence of a temperature difference." [46]

Definition 2.3.2 (Conduction)
"The transfer of heat within a medium due to a diffusion process is called conduction." [46]

Definition 2.3.3 (Convection) "Convection heat transfer is usually defined as energy transport effected by the motion of a fluid. The convection heat transfer between two dissimilar media is governed by Newton's law of cooling." [46]

Definition 2.3.4 (Force Convection)
"Forced convection heat transfer is induced by forcing a liquid, or gas, over a hot body or surface." [47]

Definition 2.3.5 (Natural Convection)
"Natural convection is generated by the density difference induced by the temperature differences within a fluid system and the small density variations present in

these types of flows." [47]

Definition 2.3.6 (Thermal Radiation)

"Thermal radiation is defined as radiant (electromagnetic) energy emitted by a medium and is solely to the temperature of the medium." [46]

Definition 2.3.7 (Thermal Conductivity)

"The Fourier heat conduction law states that the heat flow is proportional to the temperature gradient. The coefficient of proportionality is a material parameter known as the thermal conductivity which may be a function of a number of variables." [46]

Definition 2.3.8 (Thermal Diffusivity)

"The rate at which heat diffuses by conducting through a material depends on the thermal diffusivity and can be defined as:

$$\alpha = \frac{\kappa}{\rho C_p},$$

where α is the thermal diffusivity, κ is the thermal conductivity, ρ is the density and C_p is the specific heat at constant pressure." [48]

2.4 Dimensionless Numbers

Definition 2.4.1 (Prandtl Number)

"The Prandtl number is the connecting link between the velocity field and the temperature field. The Prandtl number is dimensionless Mathematically,

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{\kappa/\rho C_p} = \frac{\mu C_p}{\kappa}.\text{"} \ [49]$$

Definition 2.4.2 (Skin Friction Coefficient)

"The steady flow of an incompressible gas or liquid in a long pipe of internal D. The mean velocity is denoted by u_w. The skin friction coefficient can be defined as

$$C_f = \frac{2\tau_0}{\rho u_w^2},$$

where τ_0 denotes the wall shear stress and ρ is the density." [50]

Definition 2.4.3 (Nusselt Number)

"The hot surface is cooled by a cold fluid stream. The heat from the hot surface, which is maintained at a constant temperature, is diffused through a boundary layer and convected away by the cold stream. Mathematically,

$$Nu = \frac{qL}{\kappa},$$

where q stands for the convection heat transfer, L for the characteristic length and κ stands for thermal conductivity." [47]

Definition 2.4.4 (Reynolds Number)

"It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid . Mathematically,

$$Re = \frac{LV}{\nu},$$

where V denotes the free stream velocity, L is the characteristics length and ν is kinematic viscosity." [44]

2.5 Governings Law

Definition 2.5.1 (Continuity Equation)

"The principle of conservation of mass can be stated as the time rate of change of mass is fixed volume is equal to the net rate of flow of mass across the surface. Mathematically, it can be written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0.\text{''} \ [46]$$

Definition 2.5.2 (Conservation of Momentum)

"The momentum equation states that the time rate of change of linear momentum of a given set of particles is equal to the vector sum of all the external forces acting on the particles of the set, provided Newton's Third Law of action and reaction governs the internal forces. Mathematically, it can be written as:

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla.[(\rho \mathbf{u})\mathbf{u}] = \boldsymbol{\nabla}.\mathbf{T} + \rho g.\text{''} \ [46]$$

Definition 2.5.3 (Law of Conservation of Energy)

"The law of conservation of energy states that the time rate of change of the total energy is equal to the sum of the rate of work done by the applied forces and change of heat content per unit time.

$$\frac{\partial \rho}{\partial t} + \nabla.\rho \mathbf{u} = -\nabla.\mathbf{q} + Q + \phi,$$

where ϕ is a dissipation function." [46]

Definition 2.5.4 (Newton's Law of Viscosity)

"It states that the shear stress τ on a fluid element layer is proportional to the rate of shear strain. The constant of proportionally is called coefficient of viscosity. Mathematically it is expressed as

$$\tau = \mu \frac{du}{dy},$$

Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called non-Newtonian fluids." [46]

Chapter 3

Analysis of MHD Carreau Fluid Flow over a Stretching Permeable Sheet with Variable Viscosity and Thermal Conductivity

3.1 Introduction

In this chapter, the observations on MHD incompressible viscous laminar fluid flow on a nonlinearly stretchable sheet with the impacts of velocity and thermal wall slip parameters have been accomplished. The set of equations for energy, momentum and concentration are attained by utilizing the boundary layer approximation. Furthermore, the governing coupled nonlinear PDEs are transmuted into ODEs by using the appropriate transformations. A numerical technique based on the shooting method is used for the solution of first order ODEs. At the end of this chapter, the numerical solution for different parameters are considered. The impact of these parameters on the skin friction coefficient, Nusselt and sherwood numbers, is also analyzed. The tables and graphs are shown which are obtained through this investigation. This chapter presents a detailed review of [42]

3.2 Mathematical Modeling

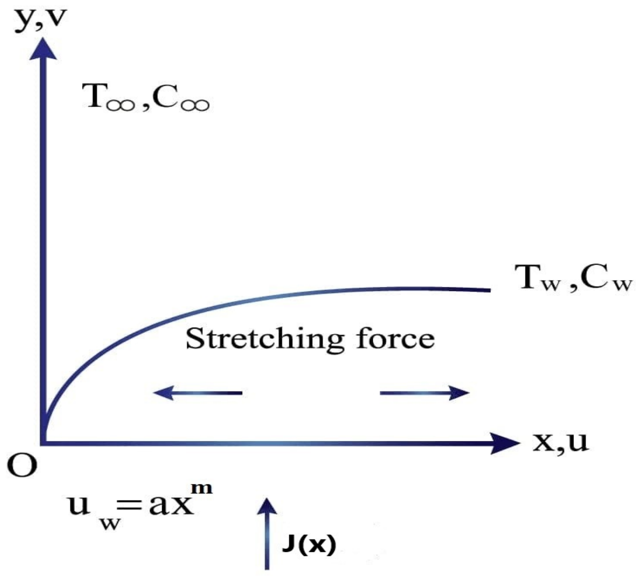

FIGURE 3.1: Systematic representation of physical model.

Assume a uniform 2-D incompressible viscid flow of an electrically conducting fluid on a nonlinearly stretchable sheet. Meanwhile, the plate has been stretched with the velocity $u_w = ax^m$ along x-direction. Here T_w is the wall temperature and C_w is the nanoparticles concentration at the stretching sheet, T_∞ is the free stream temperature and C_∞ is the Ambient concentration. The flow is explained by considering the two dimensional governing equations comprising of the continuity, momentum, energy and concentration equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + 3\nu\frac{n-1}{2}\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} + \frac{\sigma J^2}{\rho}(u_e - u) + u_e\frac{\partial u_e}{\partial x}, \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right), \tag{3.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{3.4}$$

where μ stand for the dynamic viscosity and k for thermal conductivity. Moreover σ, ν and ρ denote the electrical conductivity, kinematic viscosity and fluid density respectively. The acceleration gravity is g. C_p stand for specific heat capacity. T stand for fluid temperature, T_∞ denotes the free stream temperature, D stand for diffusion coefficient, Γ stand for time constant, n stand for power law index and J stand for magnetic field.

$$\mu(T) = \mu^*[N_1 + h_1(T_\infty - T)],\ k(T) = k^*[N_2 + h_2(T - T_w)]. \tag{3.5}$$

In the above equation, μ^* and k^* represent the effect of viscosity and thermal conductivity, whereas h_1, h_2, N_1 and N_2 are some positive constants. In addition, N_1 and N_2 are assigned the value 1.

The associated boundary conditions are taken as:

$$\left.\begin{aligned} &u = u_w(x) = ax^m,\ v = v_w(x),\ \frac{\partial T}{\partial y} = -\frac{q_w(x)}{k},\ C = C_w,\ at\ y = 0,\\ &u \to u_e(x) = bx^m,\ T \to T_\infty,\ C \to C_\infty, \qquad\qquad at\ y \to \infty, \end{aligned}\right\} \tag{3.6}$$

Here ν_w stand for injection/suction velocity and $q_w(x)$ is the surface heat flux. The following similarity transformation.

$$\left.\begin{aligned} \theta(\zeta) &= \frac{T - T_w}{T_\infty - T_w},\\ \phi(\zeta) &= \frac{C - C_\infty}{C_w - C_\infty},\\ \psi &= (bv)^{\frac{1}{2}} x^{\frac{m+1}{2}} f(\zeta),\\ \zeta &= \left(\frac{b}{v}\right)^{\frac{1}{2}} yx^{\frac{m-1}{2}}, \end{aligned}\right\} \tag{3.7}$$

where ψ stand for the stream function. The complete procedure for the conversion of (3.1)-(3.4) into the dimensionless form has been discussed below.

- $$\zeta = \left(\frac{b}{\nu}\right)^{\frac{1}{2}} yx^{\frac{m-1}{2}}. \tag{3.8}$$

- $$\frac{\partial \zeta}{\partial x} = \frac{\partial}{\partial x}\left(\frac{b}{\nu}\right)^{\frac{1}{2}} yx^{\frac{m-1}{2}}.$$
$$= \left(\frac{b}{\nu}\right)^{\frac{1}{2}} \frac{m-1}{2} x^{\frac{m-3}{2}} y. \tag{3.9}$$
- $$\frac{\partial \zeta}{\partial y} = \left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}. \tag{3.10}$$
- $$u = \frac{\partial \psi}{\partial y},$$
$$= \frac{\partial}{\partial y}\left((b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f(\zeta)\right),$$
$$= \left((b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f'(\zeta)\right)\frac{\partial \zeta}{\partial y},$$
$$= \left((b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f'(\zeta)\right)\frac{\partial}{\partial y}\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}} y,$$
$$= \left((b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f'(\zeta)\right)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}},$$
$$= bx^m f'(\zeta). \tag{3.11}$$
- $$v = -\frac{\partial \psi}{\partial x},$$
$$= -\frac{\partial}{\partial x}\left((b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f(\zeta)\right),$$
$$= -(b\nu)^{\frac{1}{2}} x^{\frac{m+1}{2}} f'(\zeta)\frac{m-1}{2}\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-3}{2}} y - (b\nu)^{\frac{1}{2}}\frac{m+1}{2} f(\zeta) x^{\frac{m-1}{2}},$$
$$= -(b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left[\frac{m-1}{2}(\zeta) f'(\zeta) + \frac{m+1}{2} f(\zeta)\right]. \tag{3.12}$$
- $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(bx^m f'(\zeta)\right),$$
$$= bx^m f''(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} \frac{m-1}{2} x^{\frac{m-3}{2}} y + bmx^{m-1} f'(\zeta),$$
$$= \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2} x^{\frac{3m-3}{2}} y f''(\zeta) + bmx^{m-1} f'(\zeta). \tag{3.13}$$
- $$\frac{\partial v}{\partial y} = -\frac{\partial}{\partial y}(b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left[\frac{m-1}{2}(\zeta) f'(\zeta) + \frac{m+1}{2} f(\zeta)\right],$$
$$= -(b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left(\frac{m+1}{2} f'(\zeta)(\frac{b}{\nu})^{\frac{1}{2}} x^{\frac{m-1}{2}}\right)$$
$$- (b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}} \frac{m-1}{2}\left(\frac{b}{\nu}\right)^{\frac{1}{2}} yx^{\frac{m-1}{2}} f''(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}$$
$$- (b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}} \frac{m-1}{2} f'(\zeta)\left(\frac{b}{\nu}^{\frac{1}{2}}\right) x^{\frac{m-1}{2}},$$

$$= -b\frac{m+1}{2}x^{m-1}f'(\zeta) - \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta)$$
$$- b\frac{m-1}{2}x^{m-1}f'(\zeta),$$
$$= -bx^{m-1}f'(\zeta)\left(\frac{m+1}{2}+\frac{m-1}{2}\right) - \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta),$$
$$= -bmx^{m-1}f'(\zeta) - \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta). \tag{3.14}$$

Equation (3.1) is very easily satisfied by using the equations (3.13)-(3.14), as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta) + bmx^{m-1}f'(\zeta)$$
$$- bmx^{m-1}f'(\zeta) - \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta),$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0.$$

Now, for the momentum equation (3.2) the following derivatives are required.

- $$u\frac{\partial u}{\partial x} = (bx^m f'(\zeta))\left(\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta) + bmx^{m-1}f'(\zeta)\right),$$
$$= \left(\frac{b^{\frac{5}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{5m-3}{2}}y\frac{m-1}{2}f'(\zeta)f''(\zeta) + b^2mx^{2m-1}f'^2(\zeta). \tag{3.15}$$
- $$\frac{\partial u}{\partial y} = \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta).$$
- $$v\frac{\partial u}{\partial y} = -(b\nu)^{\frac{1}{2}}x^{\frac{m-1}{2}}\left[\frac{m-1}{2}(\zeta)f'(\zeta) + \frac{m+1}{2}f(\zeta)\right],$$
$$= -(b\nu)^{\frac{1}{2}}x^{\frac{m-1}{2}}\left[\frac{m-1}{2}\left(\frac{b}{\nu}\right)^{\frac{1}{2}}yx^{\frac{m-1}{2}}f'(\zeta) + \frac{m+1}{2}f(\zeta)\right]$$
$$\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}x^{\frac{3m-1}{2}}f''(\zeta),$$
$$= -b^2\frac{m+1}{2}x^{2m-1}f(\zeta)f''(\zeta) - \frac{b^{\frac{5}{2}}}{\nu^{\frac{1}{2}}}\frac{m-1}{2}x^{\frac{5m-3}{2}}yf'(\zeta)f''(\zeta). \tag{3.16}$$
- $\mu(T) = \mu^*[N_1 + h_1(T_\infty - T)].$

  $\mu(T) = \mu^* N_1 + \mu^* h_1(T_\infty - T).$

- $$\mu\frac{\partial u}{\partial y} = \mu^*[N_1 + h_1(T_\infty - T)]\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta)\ .$$
- $$\theta(\zeta) = \frac{T - T_w}{T_\infty - T_w}.$$
  $$T = (T_\infty - T_w)\theta(\zeta) + T_w.$$
- $$\begin{aligned}\mu\frac{\partial u}{\partial y} &= (\mu^* N_1 + \mu^* h_1(T_\infty - T_m)(1-\theta(\zeta)))\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta), \\ &= \mu^* N_1\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta) \\ &\quad + \mu^* h_1(T_\infty - T_w)(1-\theta(\zeta))\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta).\end{aligned}$$
- $$\begin{aligned}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) &= \frac{\partial}{\partial y}\left(\mu^* N_1\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta)\right) \\ &\quad + \frac{\partial}{\partial y}\left(\mu^* h_1(T_\infty - T_w)(1-\theta(\zeta))\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f''(\zeta)\right), \\ &= \mu^* N_1\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f'''(\zeta)\left(\frac{b^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{m-1}{2}} \\ &\quad + \mu^* h_1(T_\infty - T_w)\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}f'''(\zeta)\left(\frac{b^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{m-1}{2}} \\ &\quad - \mu^* h_1(T_\infty - T_w)\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\left(\frac{b^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-1}{2}}x^{\frac{m-1}{2}}(f'''(\zeta)\theta(\zeta) + f''(\zeta)\theta'(\zeta)), \\ &= \mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}f'''(\zeta) + \mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1}f'''(\zeta) \\ &\quad - \mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1}(f'''(\zeta)\theta(\zeta) + f''(\zeta)\theta'(\zeta)).\end{aligned}$$
- $$\begin{aligned}\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) &= \mu^* N_1\left(\frac{b^2}{\nu\rho}\right)x^{2m-1}f'''(\zeta) + \mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu\rho}\right)x^{2m-1} \\ & f'''(\zeta) - \mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu\rho}\right)x^{2m-1}(f'''(\zeta)\theta(\zeta) + f''(\zeta)\theta'(\zeta)).\end{aligned} \tag{3.17}$$
- $$\left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{b^3}{\nu}\right)x^{3m-1}f''^2(\zeta).$$
  $$\begin{aligned}\frac{\partial^2 u}{\partial y^2} &= \frac{\partial^2}{\partial y^2}(bx^m f'(\zeta)), \\ &= \frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}x^{\frac{3m-1}{2}}f'''(\zeta)\frac{b^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}x^{\frac{m-1}{2}}, \\ &= \left(\frac{b^2}{\nu}\right)x^{2m-1}f'''(\zeta).\end{aligned}$$

- $$\left(\frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{b^5}{\nu^2}\right) x^{5m-2} f''^2(\zeta) f'''(\zeta).$$

$$3\nu\frac{n-1}{2}\Gamma^2\left(\frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial u}{\partial y}\right)^2 = 3\nu\frac{n-1}{2}\Gamma^2\left(\frac{b^5}{\nu^2}\right) x^{5m-2} f''^2(\zeta) f'''(\zeta). \quad (3.18)$$

- $u_e = bx^m.$

$$u_e - u = bx^m - bx^m f'(\zeta),$$

$$u_e - u = bx^m(1 - f'(\zeta)).$$

- $$\frac{\sigma(J^2)}{\rho}(u_e - u) = \frac{\sigma(J^2)}{\rho} bx^m(1 - f'(\zeta)). \quad (3.19)$$

- $$\frac{\partial u_e}{\partial x} = \frac{\partial}{\partial x}(bx^m), \quad = bmx^{m-1}.$$

- $$u_e\frac{\partial u_e}{\partial x} = bx^m bmx^{m-1}, \quad = b^2 m x^{2m-1}. \quad (3.20)$$

Using equations (3.15)-(3.20) in equation (3.2), we get

$$\left(\frac{b^{\frac{5}{2}}}{\nu^{\frac{1}{2}}}\right) x^{\frac{5m-3}{2}} y\frac{m-1}{2} f'(\zeta)f''(\zeta) + b^2 m x^{2m-1} f'^2(\zeta) - b^2\frac{m+1}{2}x^{2m-1} f(\zeta)f''(\zeta)$$
$$-\left(\frac{b^{\frac{5}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{5m-3}{2}} y f'(\zeta)f''(\zeta) = \left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1} f'''(\zeta)$$
$$+\mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1} f'''(\zeta) - \mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1}$$
$$(f'''(\zeta)\theta(\zeta) + f''(\zeta)\theta'(\zeta)) + \left(3\nu\frac{n-1}{2}\Gamma^2\left(\frac{b^5}{\nu^2}\right)x^{5m-2} f''^2(\zeta)f'''(\zeta)\right)$$
$$+\left(\frac{\sigma(J^2)}{\rho} bx^m(1 - f'(\zeta))\right) + b^2 m x^{2m-1},$$

$$\frac{b^2 m x^{2m-1}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}} f'^2(\zeta) - \frac{b^2\frac{m+1}{2}x^{2m-1}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}} f(\zeta)f''(\zeta) = f'''(\zeta)$$
$$+\frac{\mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}} f'''(\zeta) - \frac{\mu^* h_1(T_\infty - T_w)\left(\frac{b^2}{\nu}\right)x^{2m-1}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}}$$
$$(f'''(\zeta)\theta(\zeta) + f''(\zeta)\theta'(\zeta)) + \frac{3\nu\frac{n-1}{2}\Gamma^2\left(\frac{b^5}{\nu^2}\right)x^{5m-2}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}} f'''(\zeta)f''^2(\zeta)$$

$$+\frac{\frac{\sigma(J^2)}{\rho}bx^m}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}}(1-f'(\zeta))+\frac{b^2mx^{2m-1}}{\left(\frac{1}{\rho}\right)\mu^* N_1\left(\frac{b^2}{\nu}\right)x^{2m-1}},$$

$$f'''(\zeta)-(\rho\nu)\frac{b^2mx^{2m-1}}{\mu^*N_1b^2x^{2m-1}}f'^2(\zeta)+\frac{(\rho\nu)b^2\frac{m+1}{2}x^{2m-1}}{\mu^*N_1b^2x^{2m-1}}f(\zeta)f''(\zeta)$$
$$+\frac{(\rho\nu)\mu^*h_1(T_\infty-T_w)b^2x^{2m-1}}{(\rho\nu)\mu^*N_1b^2x^{2m-1}}f'''(\zeta)$$
$$-\frac{(\rho\nu)\mu^*h_1(T_\infty-T_w)b^2x^{2m-1}}{(\rho\nu)\mu^*N_1b^2x^{2m-1}}(f'''(\zeta)\theta(\zeta)+f''(\zeta)\theta'(\zeta))$$
$$+\frac{(3\nu^2\rho)(n-1)\Gamma^2b^5x^{5m-2}}{2\nu^2\mu^*N_1b^2x^{2m-1}}f''^2(\zeta)f'''(\zeta)$$
$$+\frac{(\rho\nu)\sigma J^2bx^m}{\rho\mu^*N_1b^2x^{2m-1}}(1-f'(\zeta))+\frac{(\rho\nu)b^2mx^{2m-1}}{\mu^*N_1b^2x^{2m-1}}=0,$$

$$f'''(\zeta)-\frac{(\rho\nu)m}{\mu}f'^2(\zeta)+\frac{(\rho\nu)\frac{m+1}{2}}{\mu}f(\zeta)f''(\zeta)+\frac{h_1}{N_1}(T_\infty-T_w)f'''(\zeta)$$
$$-\frac{h_1}{N_1}(T_\infty-T_w)(f'''(\zeta)\theta(\zeta)+f''(\zeta)\theta'(\zeta))+\frac{3(n-1)\Gamma^2b^3x^{3m-1}}{2\nu}f''^2(\zeta)f'''(\zeta)$$
$$+\frac{\sigma J^2}{\rho b}x^{1-m}(1-f'(\zeta))+\frac{(\rho\nu)m}{\mu}=0,$$

$$f'''(\zeta)+m(1-f'^2(\zeta))+\frac{m+1}{2}f(\zeta)f''(\zeta)+\frac{h_1}{N_1}(T_\infty-T_w)f'''(\zeta)$$
$$-\frac{h_1}{N_1}(T_\infty-T_w)(f''(\zeta)\theta'(\zeta)+f'''(\zeta)\theta(\zeta))+\frac{3(n-1)\Gamma^2b^3x^{3m-1}}{2\nu}f''^2(\zeta)f'''(\zeta)$$
$$+\frac{\sigma J^2}{\rho b}x^{1-m}(1-f'(\zeta))=0,$$

$$f'''(\zeta)+m(1-f'^2(\zeta))+\frac{m+1}{2}f(\zeta)f''(\zeta)+\xi f'''(\zeta)$$
$$-\xi(f''(\zeta)\theta'(\zeta)+f'''(\zeta)\theta(\zeta))+\frac{3}{2}(n-1)We^2f''^2(\zeta)f'''(\zeta)$$
$$+M^2(1-f'(\zeta))=0. \quad (3.21)$$

The following derivatives will help to convert the equation (3.3) into the dimensionless form.

- $T=(T_\infty-T_w)\theta(\zeta)+T_w.$
- $\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}((T_\infty-T_w)\theta(\zeta)+T_w),$

$$=(T_\infty-T_w)\theta'(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}}\frac{m-1}{2}x^{\frac{m-3}{2}}y,$$
$$=(T_\infty-T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}}\frac{m-1}{2}x^{\frac{m-3}{2}}y\theta'(\zeta).$$

- $\dfrac{\partial T}{\partial y} = \dfrac{\partial}{\partial y}((T_\infty - T_w)\theta(\zeta) + T_w),$

$$= (T_\infty - T_w)\theta'(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}},$$

$$= (T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta).$$

- $u\dfrac{\partial T}{\partial x} = (bx^m f'(\zeta))((T_\infty - T_w)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}}\dfrac{m-1}{2}x^{\frac{m-3}{2}}y\theta'(\zeta)),$

$$= \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}(T_\infty - T_w)x^{\frac{3m-3}{2}}yf'(\zeta)\theta'(\zeta). \tag{3.22}$$

- $v\dfrac{\partial T}{\partial y} = -(b\nu)^{\frac{1}{2}}x^{\frac{m-1}{2}}\left[\dfrac{m-1}{2}(\zeta)f'(\zeta) + \dfrac{m+1}{2}f(\zeta)\right]$

$$((T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)),$$

$$= -b\frac{m+1}{2}(T_\infty - T_w)x^{m-1}f(\zeta)\theta'(\zeta)$$

$$- b\frac{m-1}{2}\left(\frac{b}{\nu}\right)^{\frac{1}{2}}(T_\infty - T_w)x^{m-1}yf'(\zeta)\theta'(\zeta),$$

$$= -b\frac{m+1}{2}(T_\infty - T_w)x^{m-1}f(\zeta)\theta'(\zeta)$$

$$- \frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)(T_\infty - T_w)x^{\frac{3m-3}{2}}yf'(\zeta)\theta'(\zeta). \tag{3.23}$$

- $k(T) = k^*[N_2 + h_2(T - T_w)],$

$$= k^*[N_2 + h_2((T_\infty - T_w)\theta(\zeta) + T_m - T_w)],$$

$$= k^*[N_2 + h_2(T_\infty - T_w)\theta(\zeta)].$$

- $k(T)\dfrac{\partial T}{\partial y} = (k^*N_2 + k^*h_2(T_\infty - T_w)\theta(\zeta))((T_\infty - T_w)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)),$

$$= k^*N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta) + k^*h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta(\zeta)\theta'(\zeta).$$

- $\dfrac{\partial}{\partial y}(k(T)\dfrac{\partial T}{\partial y}) = \dfrac{\partial}{\partial y}\left(k^*N_2(T\infty - T_w)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)\right)$

$$+ \frac{\partial}{\partial y}\left(k^*h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta(\zeta)\theta'(\zeta)\right),$$

$$= k^*N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right)x^{m-1}\theta''(\zeta)$$

$$+ k^*h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right)x^{m-1}\left(\theta'^2(\zeta) + \theta(\zeta)\theta''(\zeta)\right).$$

$$\frac{1}{\rho C_p}\frac{\partial}{\partial y}(k(T)\frac{\partial T}{\partial y}) = k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu\rho C_p}\right)x^{m-1}\theta''(\zeta)$$
$$+ k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu\rho C_p}\right)x^{m-1}\left(\theta'^2(\zeta) + \theta(\zeta)\theta''(\zeta)\right). \tag{3.24}$$

Using (3.22)-(3.24), equation (3.3) becomes:

$$\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}(T_\infty - T_w)x^{\frac{3m-3}{2}}yf'(\zeta)\theta'(\zeta)$$
$$- b\frac{m+1}{2}(T_\infty - T_w)x^{m-1}f(\zeta)\theta'(\zeta)$$
$$- \frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)(T_\infty - T_w)x^{\frac{3m-3}{2}}yf'(\zeta)\theta'(\zeta)$$
$$= k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu\rho C_p}\right)x^{m-1}\theta''(\zeta)$$
$$+ k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu\rho C_p}\right)x^{m-1}\left(\theta'^2(\zeta) + \theta(\zeta)\theta''(\zeta)\right),$$
$$- b\frac{m+1}{2}(T_\infty - T_w)x^{m-1}f(\zeta)\theta'(\zeta)$$
$$= \frac{1}{\rho C_p}k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right)x^{m-1}\theta''(\zeta)$$
$$+ \frac{1}{\rho C_p}k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right)x^{m-1}\left(\theta'^2(\zeta) + \theta(\zeta)\theta''(\zeta)\right),$$
$$\theta''(\zeta) + \frac{b(T_\infty - T_w)x^{m-1}\frac{m+1}{2}}{\frac{1}{\rho C_p}k^* N_2(T_\infty - T_w)\frac{b}{\nu}x^{m-1}}f(\zeta)\theta'(\zeta)$$
$$+ \frac{\frac{1}{\rho C_p}k^* h_2(T_\infty - T_w)^2\frac{b}{\nu}x^{m-1}}{\frac{1}{\rho C_p}k^* N_2(T_\infty - T_w)\frac{b}{\nu}x^{m-1}}(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)) = 0$$
$$\theta''(\zeta) + \frac{\mu C_p}{k^* N_2}\frac{m+1}{2}f(\zeta)\theta'(\zeta) + \frac{h_2}{N_2}(T_\infty - T_w)\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right) = 0,$$
$$\theta''(\zeta) + Pr\frac{m+1}{2}f(\zeta)\theta'(\zeta) + \epsilon\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right) = 0. \tag{3.25}$$

To convert the equation (3.4) into ordinary differential form, the following procedure has been carried out.

- $\phi(\zeta) = \dfrac{C - C_\infty}{C_w - C_\infty},$

  $C = (C_w - C_\infty)\phi(\zeta) + C_\infty.$
- $\dfrac{\partial C}{\partial x} = \dfrac{\partial}{\partial x}\left((C_w - C_\infty)\phi(\zeta) + C_\infty\right),$

$$= (C_w - C_\infty)\phi'(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} y\frac{m-1}{2}x^{\frac{m-3}{2}},$$

$$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} \frac{m-1}{2}x^{\frac{m-3}{2}}y\phi'(\zeta).$$

- $$u\frac{\partial C}{\partial x} = (bx^m f'(\zeta))(C_w - C_\infty)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} \frac{m-1}{2}x^{\frac{m-3}{2}}y\phi'(\zeta),$$
$$= (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right) x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta). \tag{3.26}$$
- $$\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}((C_w - C_\infty)\phi(\zeta) + C_\infty),$$
$$= (C_w - C_\infty)\phi''(\zeta)\left(\frac{b}{\nu}\right)x^{m-1},$$
$$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta).$$
- $$v\frac{\partial C}{\partial y} = -b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}f(\zeta)\phi'(\zeta)$$
$$- (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right) x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta). \tag{3.27}$$
- $$\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)\phi''(\zeta)\left(\frac{b}{\nu}\right)x^{m-1},$$
$$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta).$$
- $$D\frac{\partial^2 C}{\partial y^2} = D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta). \tag{3.28}$$

Using equations (3.26)-(3.28) in equation (3.4), we get

$$\begin{aligned}
&((C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right) x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta)) \\
&\quad - b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}f(\zeta)\phi'(\zeta) \\
&\quad - (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right) x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta) \\
&= D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta), \\
&\phi''(\zeta) + \frac{b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}}{D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}}f(\zeta)\phi'(\zeta) = 0, \\
&\phi''(\zeta) + \frac{m+1}{2}\frac{\nu}{D}f(\zeta)\phi'(\zeta) = 0,
\end{aligned}$$

$$\phi''(\zeta) + \frac{m+1}{2} L_e f(\zeta)\phi'(\zeta) = 0. \tag{3.29}$$

Now for converting the associated boundary conditions into the dimensionless form, the following steps have been taken:

$$u = u_w(x). \tag{3.30}$$

$$u = bx^m f'(\zeta).$$

$$u_w(x) = ax^m.$$

From equation (3.30), we get

$$bx^m f'(\zeta) = ax^m,$$

$$bf'(\zeta) = a, \qquad at \;\; \zeta \to 0.$$

$$f'(0) = \frac{a}{b},$$

$$f'(0) = B. \tag{3.31}$$

$$v = v_w. \tag{3.32}$$

$$v = -(bv)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left[\frac{m+1}{2} f(\zeta) + \zeta \frac{m-1}{2} f'(\zeta)\right]$$

From equation (3.32), we get

$$-(bv)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left[\frac{m+1}{2} f(\zeta) + \zeta \frac{m-1}{2} f'(\zeta)\right] = v_w \qquad at \;\; y \to 0$$

$$-(bv)^{\frac{1}{2}} \frac{m+1}{2} x^{\frac{m-1}{2}} f(\zeta) - \zeta \frac{m-1}{2} f'(\zeta) = v_w \qquad at \;\; \zeta \to 0$$

$$f(0) = \frac{-2v_w}{(b\nu)^{\frac{1}{2}}(m+1)x^{\frac{m-1}{2}}},$$

$$f(0) = S. \tag{3.33}$$

$$C = C_w. \tag{3.34}$$

$$C = (C_w - C\infty)\phi(\zeta) + C_\infty.$$

$$(C_w - C_\infty)\phi(\zeta) + C_\infty = C_w,$$

$$(C_w - C_\infty)\phi(\zeta) = C_w - C_\infty.$$

$$\phi(\zeta) = \frac{C_w - C_\infty}{C_w - C_\infty},$$

$$\phi(\zeta) = 1, \qquad at \ \ \zeta \to 0$$

$$\phi(0) = 1.$$

$$\theta'(0) = 1.$$

$$u \to u_e(x) = bx^m.$$

$$bx^m f'(\zeta) \to bx^m \qquad at \ \ \zeta \to \infty$$

$$f'(\zeta) \to 1. \qquad at \ \zeta \to \infty$$

$$C \to C_\infty,$$

$$\phi(\zeta)(C_w - C_\infty) + C_\infty \to C_\infty,$$

$$\phi(\zeta)(C_w - C_\infty) \to 0, \qquad at \ \ \zeta \to \infty$$

$$\phi(\zeta) \to 0 \qquad at \ \ \zeta \to \infty$$

$$\theta(\zeta) \to 0 \qquad at \ \ \zeta \to \infty$$

The final dimensionless form of the governing model, is

$$f'''(\zeta) + m(1 - f'^2(\zeta)) + \frac{m+1}{2} f(\zeta) f''(\zeta) + \xi f'''(\zeta) - \xi (f''(\zeta)\theta'(\zeta)$$
$$+ f'''(\zeta)\theta(\zeta)) + \frac{3}{2}(n-1) We^2 f''^2(\zeta) f'''(\zeta) M^2 (1 - f'(\zeta)) = 0. \tag{3.35}$$

$$\theta''(\zeta) + Pr \frac{m+1}{2} f(\zeta)\theta'(\zeta) + \epsilon \left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right) = 0. \tag{3.36}$$

$$\phi''(\zeta) + \frac{m+1}{2} Le f(\zeta)\phi'(\zeta) = 0. \tag{3.37}$$

The associated boundary conditions (3.6) shown as :

$$\zeta \to 0 : f(0) = S, \ f'(0) = B, \ \theta'(0) = -1, \ \phi(0) = 1.$$

$$\zeta \to \infty : f'(\zeta) = 1, \ \theta(\zeta) = 0, \ \phi(\zeta) = 0.$$

Different parameters used in equations (3.35)-(3.37) are explained as follows:

$$We^2 = \frac{\Gamma^2 b^3 x^{3m-1}}{\nu}, \ M^2 = \frac{\sigma J^2}{\rho b}, \ \xi = h_1(T_\infty - T_w), \ \epsilon = h_2(T_\infty - T_w),$$

$$S = \frac{-2v_w}{(b\nu)^{\frac{1}{2}}(m+1)x^{\frac{m-1}{2}}}, \ B = \frac{a}{b}, \ Pr = \frac{\mu C_p}{k^*}, \ L_e = \frac{\nu}{D}.$$

3.3 Method of Solution

The shooting technique has been used to compute the numerical solution of the ordinary differential equations (3.35)-(3.37). Equations (3.35) and (3.36) are solved numerically and then f is used in equation (3.37). The following notations have been used:

$$
\begin{aligned}
&f = Y_1,\ f' = Y_1' = Y_2,\\
&f'' = Y_2' = Y_3,\ f''' = Y_3'.\\
&\theta = Y_4,\ \theta' = Y_4' = Y_5,\ \theta'' = Y_5'.
\end{aligned}
$$

By using the above notations in equations (3.35) and (3.36), the following system of ODEs is obtained:

$$
\begin{aligned}
&Y_1' = Y_2, && Y_1(0) = S.\\
&Y_2' = Y_3, && Y_2(0) = B.\\
&Y_3' = \left(\frac{-m(1-Y_2^2) - \frac{m+1}{2}Y_1Y_3 + \xi Y_3Y_5 - M^2(1-Y_2)}{1+\xi-\xi Y_4 + \frac{3}{2}(n-1)We^2Y_3^2}\right), && Y_3(0) = r.\\
&Y_4' = Y_5, && Y_4(0) = q.\\
&Y_5' = \left(\frac{-Pr\frac{m+1}{2}Y_1Y_5 - \epsilon Y_5^2}{1+\epsilon Y_4}\right), && Y_5(0) = 1.
\end{aligned}
$$

Missing conditions r and q assumed to satisfy the following relation:

$$Y_2(\zeta_\infty, r, q) = 1, \quad Y_4(\zeta_\infty, r, q) = 0. \tag{3.38}$$

The above set of equations can be solved by using Newtons method with the following iterative formula:

$$\begin{bmatrix} r^{(n+1)} \\ q^{n+1} \end{bmatrix} = \begin{bmatrix} r^{(n)} \\ q^{(n)} \end{bmatrix} - \begin{bmatrix} \frac{\partial Y_2}{\partial r} & \frac{\partial Y_2}{\partial q} \\ \frac{\partial Y_4}{\partial r} & \frac{\partial Y_4}{\partial q} \end{bmatrix}^{-1} = \begin{bmatrix} Y_2 - 1 \\ Y_4 \end{bmatrix} \tag{3.39}$$

To execute the Newton's method, we further apply the following notations:

$$\frac{\partial Y_1}{\partial r} = Y_6,\ \frac{\partial Y_2}{\partial r} = Y_7,\ \frac{\partial Y_3}{\partial r} = Y_8,\ \frac{\partial Y_4}{\partial r} = Y_9,\ \frac{\partial Y_5}{\partial r} = Y_{10}.$$
$$\frac{\partial Y_1}{\partial q} = Y_{11},\ \frac{\partial Y_2}{\partial q} = Y_{12},\ \frac{\partial Y_3}{\partial q} = Y_{13},\ \frac{\partial Y_4}{\partial q} = Y_{14},\ \frac{\partial Y_5}{\partial q} = Y_{15}.$$

As a result of these new notations, the Newton's iterative scheme gets the form:

$$\begin{bmatrix} r^{(n+1)} \\ q^{n+1} \end{bmatrix} = \begin{bmatrix} r^{(n)} \\ q^{(n)} \end{bmatrix} - \begin{bmatrix} Y_7 & Y_{12} \\ Y_9 & Y_{14} \end{bmatrix}^{-1} = \begin{bmatrix} Y_2 - 1 \\ Y_4 \end{bmatrix} \tag{3.40}$$

Now differentiate the above system of five first order ODEs with respect to r and q, we get ten more ODEs. Writing all these fifteen ODEs together, we have the following IVP.

$$Y_6' = Y_7, \qquad Y_6(0) = 0.$$

$$Y_7' = Y_8, \qquad Y_7(0) = 0.$$

$$\begin{aligned} Y_8' = &\left(\frac{1}{(1+\xi-\xi Y_4+\frac{3}{2}(n-1)We^2Y_3^2)^2}\right)((1+\xi-\xi Y_4+\frac{3}{2}(n-1)We^2Y_3^2) \\ &(2mY_2Y_7 - \frac{m+1}{2}(Y_1Y_8+Y_3Y_6)+\xi(Y_3Y_{10}+Y_5Y_8)+M^2Y_7) \\ &-(-m(1-Y_2^2)-\frac{m+1}{2}Y_1Y_3+\xi Y_3Y_5-M^2(1-Y_2))(-\xi Y_9 \\ &+3(n-1)We^2Y_3Y_8)), \qquad Y_8(0) = 1. \end{aligned}$$

$$Y_9' = Y_{10}, \qquad Y_9(0) = 0.$$

$$\begin{aligned} Y_{10}' = &\frac{1}{(1+\epsilon Y_4)^2}((1+\epsilon Y_4)(-Pr\frac{m+1}{2}(Y_1Y_{10}+Y_5Y_6)-2\epsilon Y_5Y_{10}) \\ &-(-Pr\frac{m+1}{2}Y_1Y_5-\epsilon Y_5^2)(\epsilon Y_9)), \qquad Y_{10}(0) = 0. \end{aligned}$$

$$Y_{11}' = Y_{12}, \qquad Y_{11}(0) = 0.$$

$$Y_{12}' = Y_{13}, \qquad Y_{12}(0) = 0.$$

$$\begin{aligned} Y_{13}' = &\left(\frac{1}{(1+\xi-\xi Y_4+\frac{3}{2}(n-1)We^2Y_3^2)^2}\right)((1+\xi-\xi Y_4+\frac{3}{2}(n-1) \\ &We^2Y_3^2)(2mY_2Y_{12}-\frac{m+1}{2}(Y_1Y_{13}+Y_3Y_{11})+\xi(Y_3Y_{15}+Y_5Y_{13}) \\ &+M^2Y_{12})-(-m(1-Y_2^2)-\frac{m+1}{2}Y_1Y_3+\xi Y_3Y_5-M^2(1-Y_2)) \end{aligned}$$

$$(-\xi Y_{14} + 3(n-1)We^2 Y_3 Y_{13})), \qquad Y_{13}(0) = 0.$$

$$Y_{14}' = Y_{15}, \qquad Y_{14}(0) = 1.$$

$$Y_{15}' = \left(\frac{1}{1+\epsilon Y_4}\right)((1+\epsilon Y_4)(-Pr\frac{m+1}{2}(Y_1 Y_{15} + Y_5 Y_{11}) - 2\epsilon Y_5 Y_{15})$$
$$-(-Pr\frac{m+1}{2}Y_1 Y_5 - \epsilon Y_5^2)(\epsilon Y_{14})), \qquad Y_{15}(0) = 0.$$

The stopping criteria for the Newton's method is set as:

$$max\,\{|\ Y_2(\zeta_\infty) - 1\ |,\ \ |\ Y_4(\zeta_\infty)\ |\} < \varepsilon,$$

where ε is a small positive number. For all the calculations in this chapter, we have set ε=10^{-10}.

$$\phi = Z_1,\ \phi' = Z_1' = Z_2,\ \phi'' = Z_2'.$$

By using the above notations in equation (3.37), the following system of ODEs is obtained:

$$Z_1' = Z_2, \qquad Z_1(0) = 1.$$

$$Z_2' = -\frac{m+1}{2}LefZ_2, \qquad Z_2(0) = Q.$$

The above IVP will be numerically solved by RK technique of order four. In the above initial value problem, the missing condition Q satisfies the following relation:

$$Z_1(\zeta_\infty, Q) = 0.$$

To solve the above equation for Q, Newton's method which has the following iterative scheme will be implemented.

$$Q^{(n+1)} = Q^{(n)} - \frac{Z_1(\zeta_\infty, Q)}{Z_1'(\zeta_\infty, Q)}.$$

To incorporate Newton's method, we further utilize the following notions:

$$\frac{\partial Z_1}{\partial Q} = Z_3, \qquad \frac{\partial Z_1}{\partial Q} = Z_4.$$

As a result, the following IVP is obtained:

$$\begin{aligned} Z_3' &= Z_4, & Z_3(0) &= 0.\\ Z_4' &= -\frac{m+1}{2} Lef Z_4, & Z_4(0) &= 1. \end{aligned}$$

The stopping criteria for the Newton's method is set as:

$$\mid Z_1(\zeta_\infty, Q) \mid < \varepsilon.$$

3.4 Results and Discussion

The physical impact of significant parameters on the skin friction, Nusselt number and Sherwood number has been explained through graphs and tables. In the present survey, the shooting method has been opted for reproducing the values of $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$. The results presented in Tables 3.1-3.5 illustrate the impact of significant parameters on $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$.

In Table 3.1, for the rising values of suction parameter S and stretching parameter $B = 0$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are found to be increased. Furthermore, for accelerating value of S with $B = -3$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are increased.

In Table 3.2, for the increasing the value of the power law index n with $B = 2$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are increased. Furthermore, for accelerating value of power law index n with $B = -2$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are found to be decreased.

In Table 3.3, for the rising the value of nonlinearity parameter m with $B = 0$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are increased. Furthermore, for accelerating value of m with $B = -3$, $(Re_x)^{\frac{1}{2}} Cf_x$ and $(Re_x)^{\frac{-1}{2}} Nu_x$ are found to be increased.

In Table 3.4, for the increasing values of the Magnetic parameter M with $B = 2$,

$(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ are decreased. Furthermore, for accelerating value of M with $B = -2$, $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ are increased.

In Table 3.5, for the rising values of Weissenberg number We with $B = 2$, $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ are found to be increased. Furthermore, for accelerating value of We and $B = -2$, $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ are decreased.

Figure 3.2 represents the influence of power law index n on the dimensionless velocity profile $f'(\zeta)$ in the presence of B. It is clearly shown that $f'(\zeta)$ is increasing function by expanding the values of n. In the case of stretching, the opposite behavior has been observed.

Figure 3.3 illustrates the impact of suction parameter S on the dimensionless $f'(\zeta)$. It is observed that $f'(\zeta)$ is increasing by rising the values of S.

Figure 3.4 displays the influence of M on the $f'(\zeta)$. By increasing the values of M, $f'(\zeta)$ is increased in the case of shrinking. In the case of stretching, the opposite behavior has been observed.

Figure 3.5 delineates to show the impact of nonlinearity stretching parameter m on $f'(\zeta)$. By enhancing the values of m, $f'(\zeta)$ is increasing in the case of stretching.

Figure 3.6 shows the impact of We on $f'(\zeta)$. This graph indicates that with an increment in the values of We, $f'(\zeta)$ is increased in the case of stretching. In the event of stretching, the opposite behavior has been noted.

Figure 3.7 represents the impact of viscous parameter ξ on the dimensionless velocity profile $f'(\zeta)$. It can be noted that $f'(\zeta)$ is decreasing function by rising the values of ξ .

Figure 3.8 delineates the impact of nonlinearity stretching parameter m on the dimensionless temperature $\theta(\zeta)$. Due to an increment in m, $\theta(\zeta)$ is decreased.

Figure 3.9 delineates the impact of Pr on $\theta(\zeta)$. It is clearly shown that $\theta(\zeta)$ is increasing by enhancing the values of Pr.

Figure 3.10 illustrates the effect of ξ on $\theta(\zeta)$. It is clearly shown that $\theta(\zeta)$ is increasing by expanding the values of ξ.

Figure 3.11 shows the impact of Le on the dimensionless concentration $\phi(\zeta)$. It is observed that the concentration distribution is a decreasing function by increasing the values of Le.

Table 3.1: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7$.

B	S	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
0	5.0	2.9020	4.8292
0	5.5	3.0122	5.3449
0	6.0	3.1178	5.8622
0	6.5	3.2191	6.3808
0	7.0	3.3163	6.9004
0	7.5	3.4098	7.4208
-3	5.0	4.6210	4.2457
-3	5.5	4.8563	4.8175
-3	6.0	5.0709	5.3808
-3	6.5	5.2688	5.9377
-3	7.0	5.4531	6.4897
-3	7.5	5.6258	7.0380

Table 3.2: Numerical results for $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ s = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7$.

B	n	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	5	-3.0451	5.0573
2	6	-2.8827	5.0595
2	7	-2.7542	5.0613
2	8	-2.6485	5.0629
2	9	-2.5594	5.0643
-2	5	4.4331	4.4792
-2	6	4.1437	4.4692
-2	7	3.9193	4.4610
-2	8	3.7378	4.4542
-2	9	3.5863	4.4483

TABLE 3.3: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $s = 5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ M = 0.5,\ Pr = 0.7.$

B	m	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
0	7.0	4.4003	13.5537
0	7.5	4.5076	14.4272
0	8.0	4.6105	15.3008
0	8.5	4.7105	16.3008
0	9.0	4.8047	17.0482
0	10	4.9856	18.7959
-3	7.0	6.8340	12.9560
-3	7.5	6.9964	13.8293
-3	8.0	7.1523	14.7027
-3	9.0	7.4474	16.4500
-3	10	7.7229	18.1976

TABLE 3.4: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $s = 5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7.$

B	M	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	5.0	-3.6571	5.0618
2	6.5	-3.9449	5.0582
2	7.5	-4.1149	5.0882
2	8.5	-4.3294	5.1355
2	9.0	-4.4245	5.2195
2	10	-4.6130	5.3786
-2	5.0	5.9985	4.5607
-2	6.5	6.5953	4.5741
-2	7.5	6.8953	4.5841
-2	8.5	7.3509	4.5902
-2	9.0	7.5346	4.6573
-2	10	7.8846	4.6633

TABLE 3.5: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ n = 5,\ s = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7.$

B	We	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	0.6	-2.1328	5.0715
2	1.0	-1.6140	5.0817
2	1.4	-1.3383	5.0877
2	1.8	-1.1625	5.0917
2	2.2	-1.0386	5.0946
2	2.2	-1.0056	5.0970
-2	0.6	2.8860	4.4186
-2	1.0	2.1173	4.3812
-2	1.4	1.8282	4.3660
-2	1.8	1.7016	4.3593
-2	2.2	1.6360	4.3558
-2	2.4	1.5560	4.3468

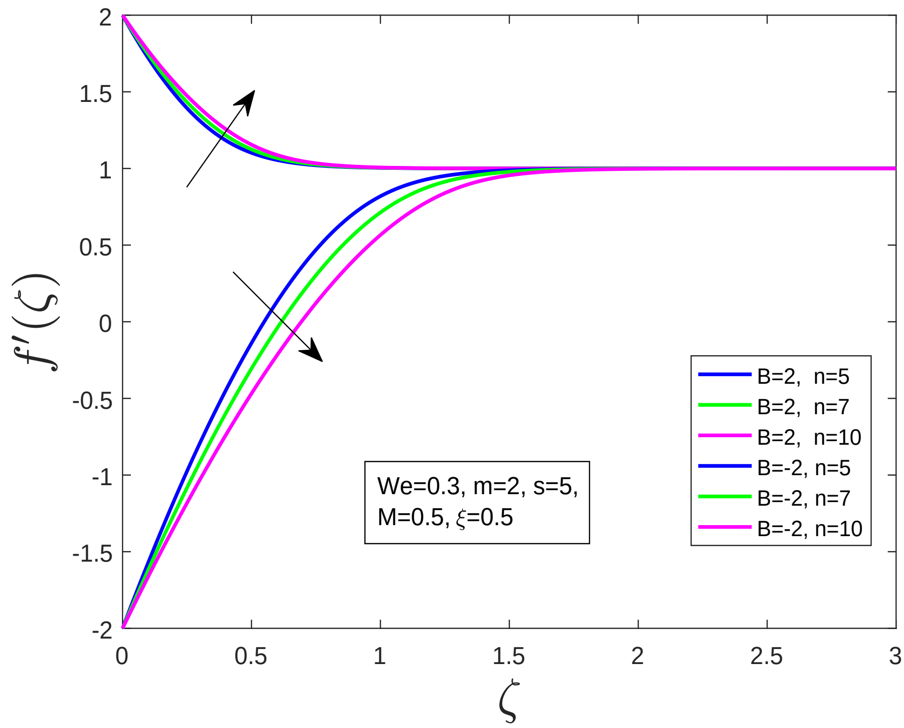

FIGURE 3.2: Impact of stretching parameter with power law index on the dimensionless velocity profile.

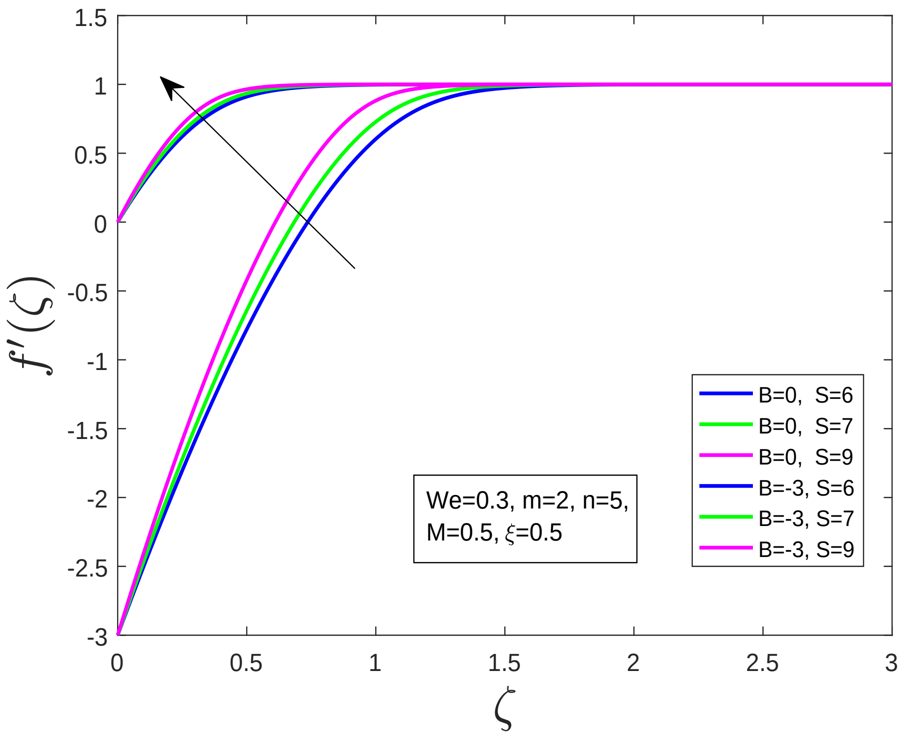

FIGURE 3.3: Impact of stretching parameter with suction parameter on the dimensionless velocity profile.

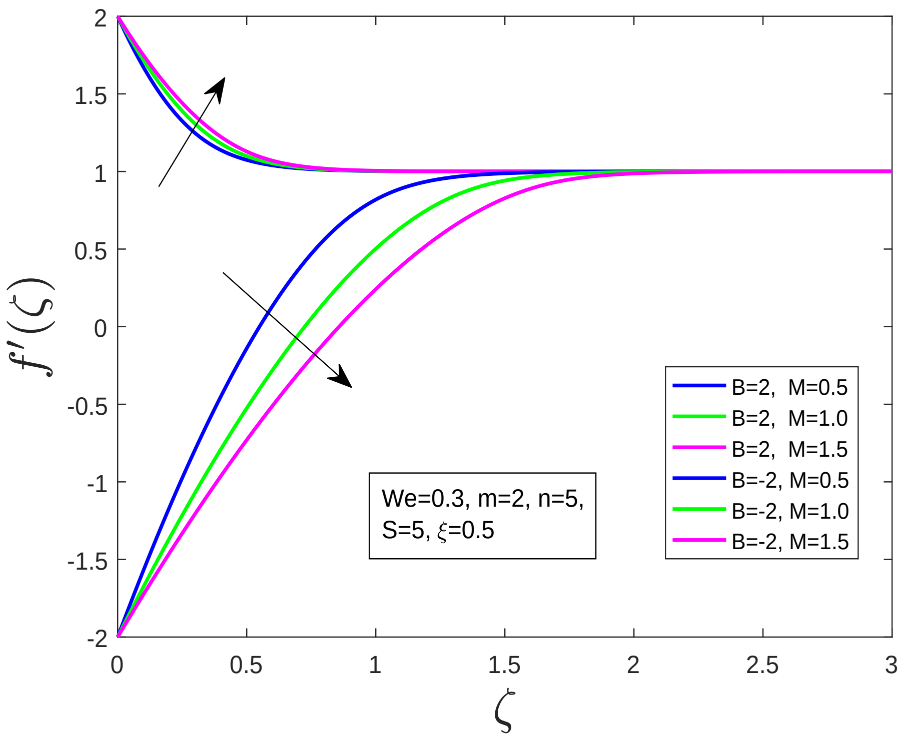

FIGURE 3.4: Impact of stretching parameter with magnetic parameter on the dimensionless velocity profile.

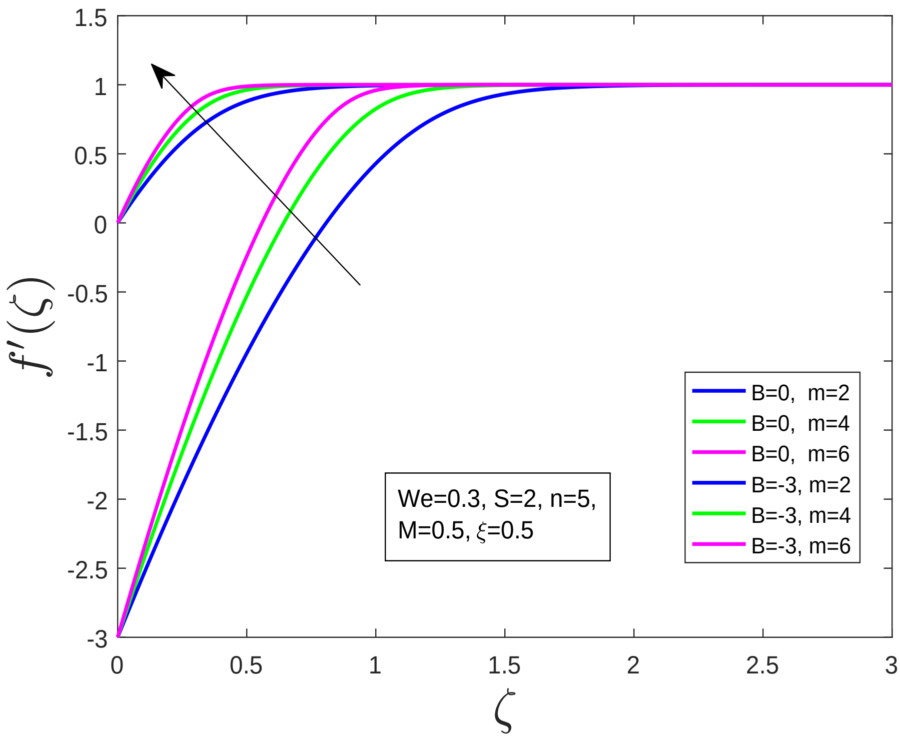

FIGURE 3.5: Impact of stretching parameter with nonlinearity stretching parameter on the dimensionless velocity profile.

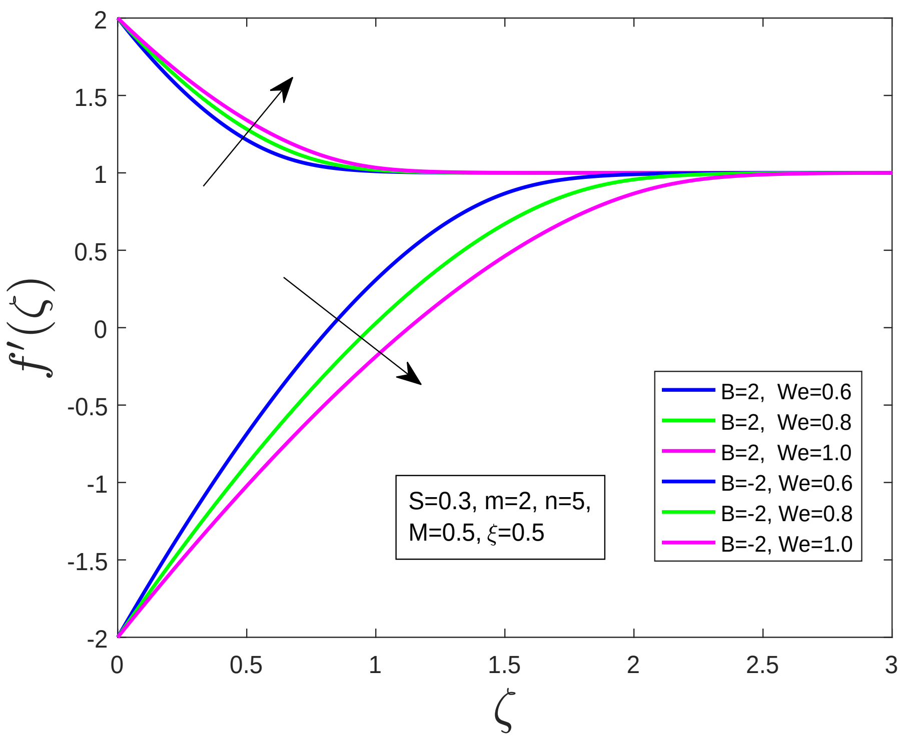

FIGURE 3.6: Impact of stretching parameter with Weissenberg number on the dimensionless velocity profile.

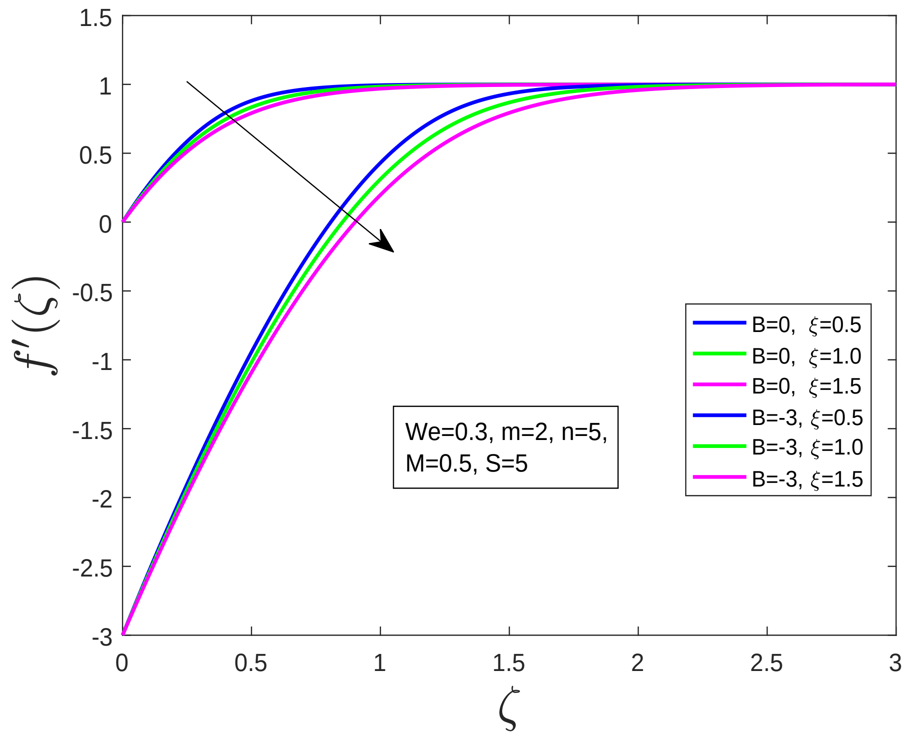

FIGURE 3.7: Influence of viscous parameter factor on dimensionless velocity profile.

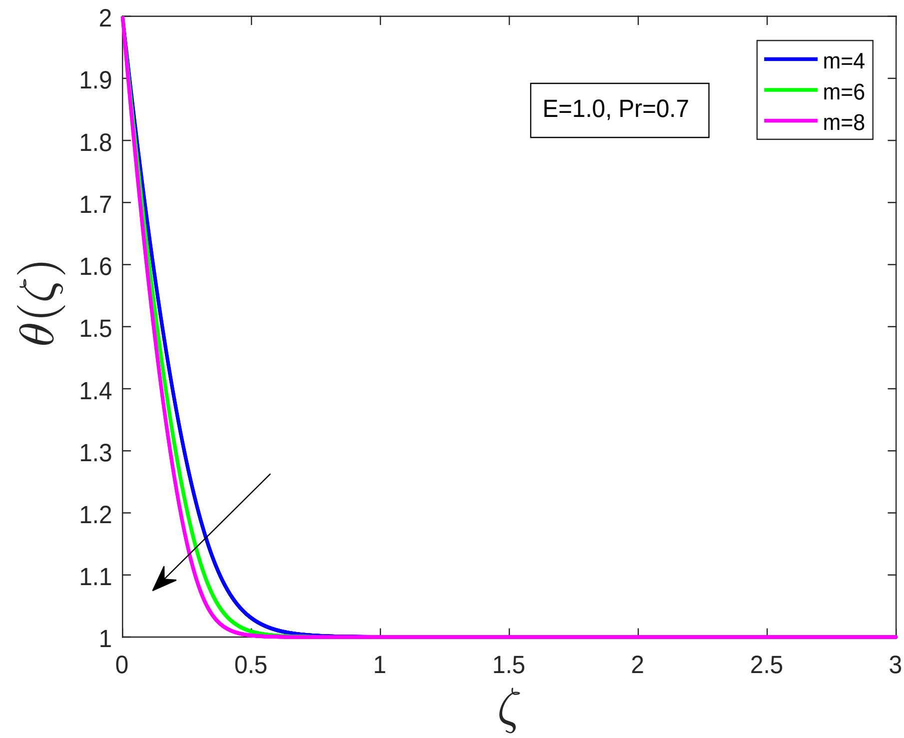

FIGURE 3.8: Influence of nonlinearity stretching parameter factor on dimensionless temperature profile.

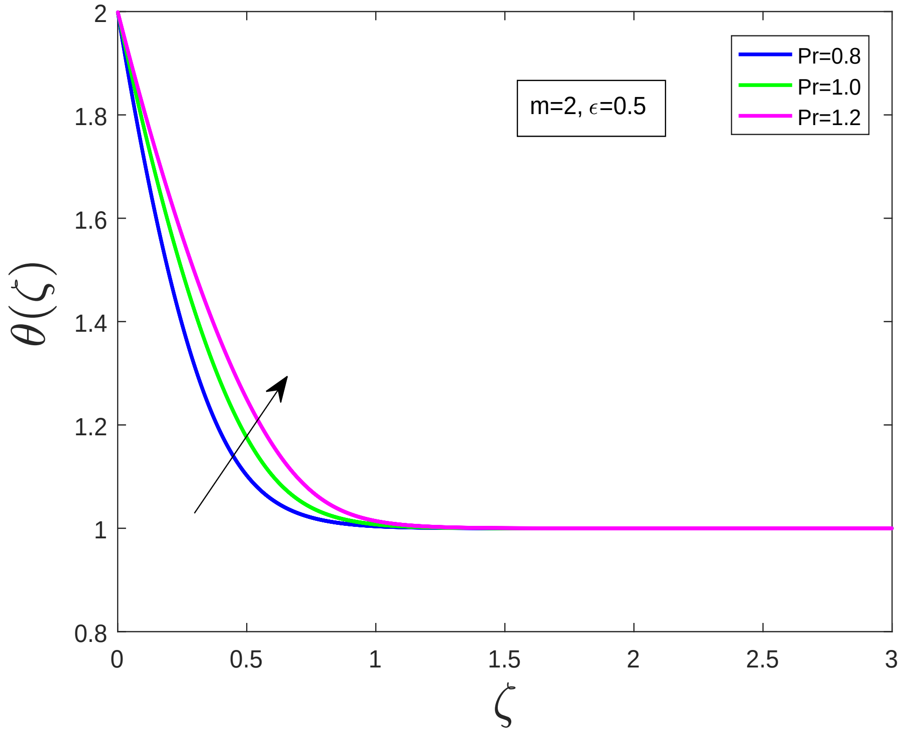

Figure 3.9: Influence of Prandtl number factor on dimensionless temperature profile.

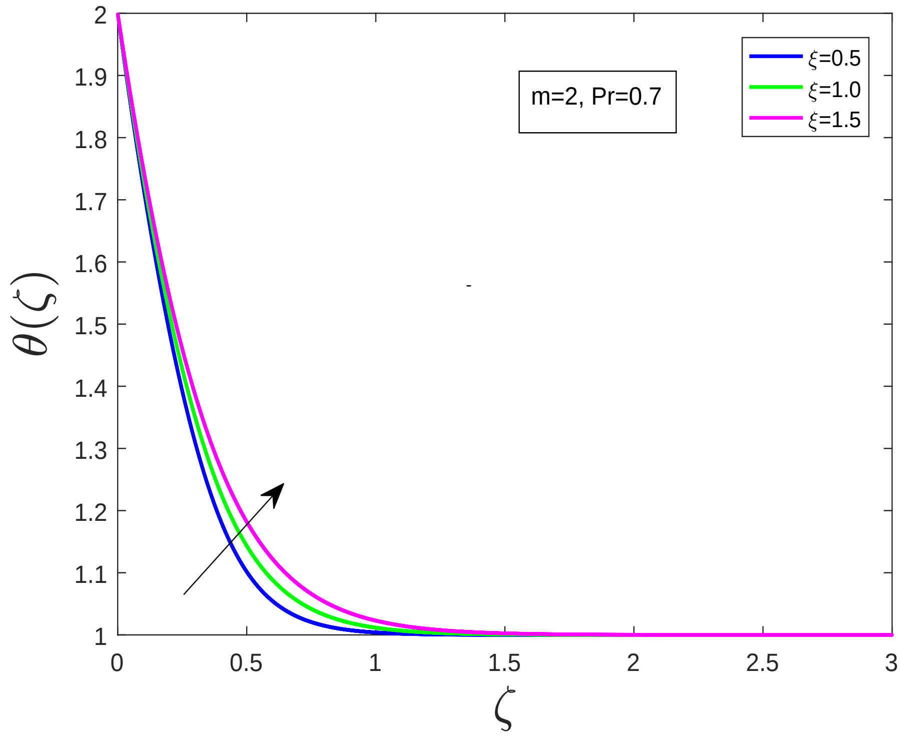

Figure 3.10: : Influence of viscous parameter factor on dimensionless temperature profile.

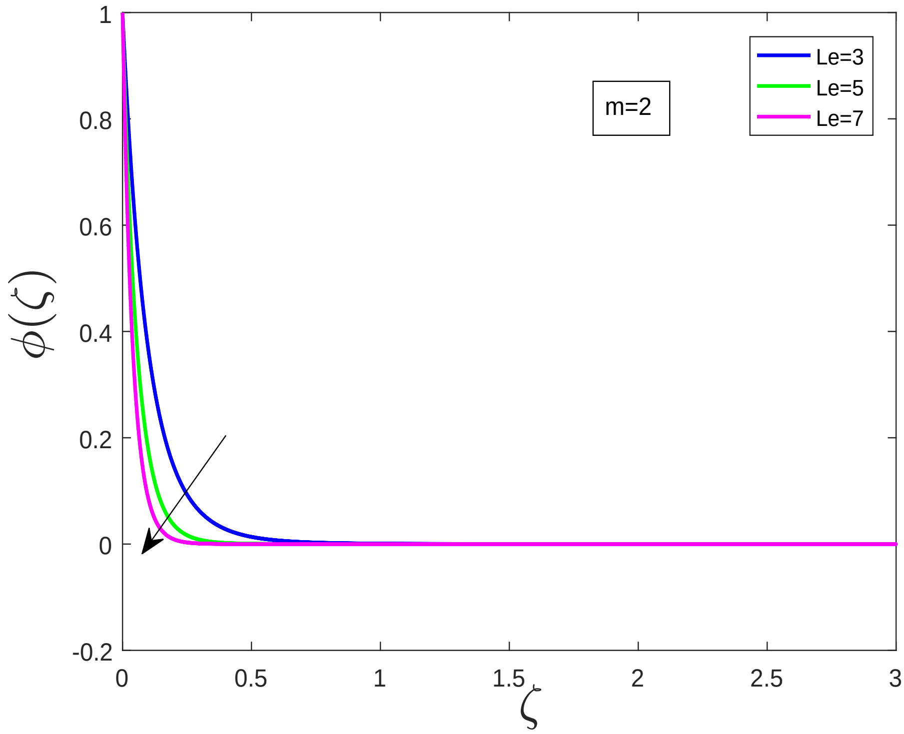

FIGURE 3.11: Influence of Lewis number factor on dimensionless concentration profile.

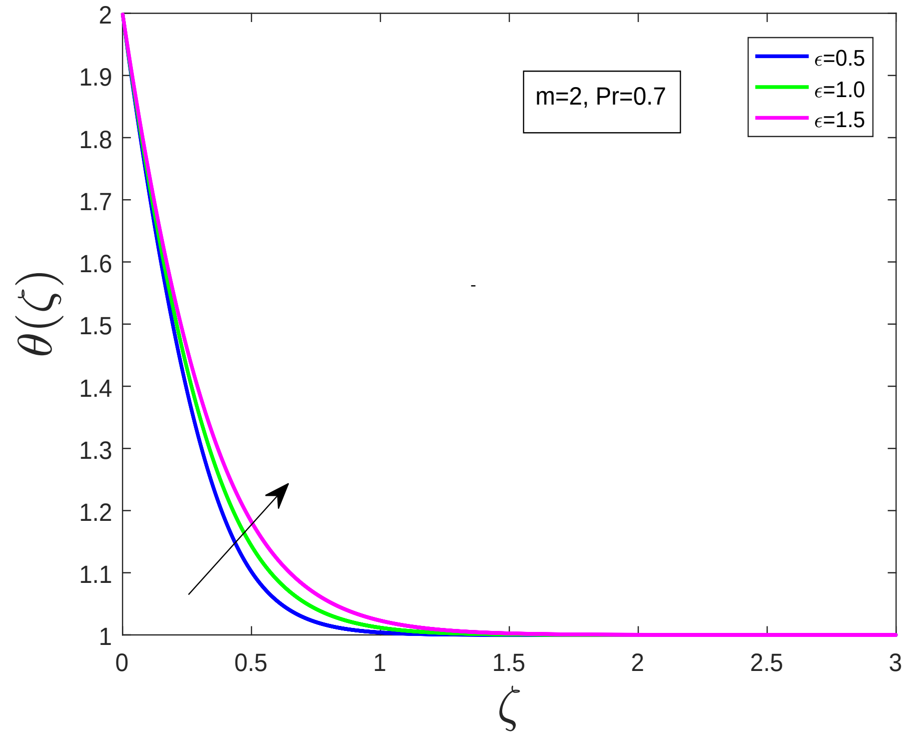

FIGURE 3.12: Influence of thermal conductivity parameter factor on dimensionless temperatureprofile.

Chapter 4

Effect of Thermal Radiation and Chemical Reaction on MHD Carreau Fluid Flow over a Stretching Sheet

4.1 Introduction

This chapter contains the extension of the model [42] by considering thermal radiation in energy equation. The chemical reaction are also included in the concentration equation. The governing coupled nonlinear PDEs are transformed into ODEs by using the appropriate transformations. In order to solve the ODEs the shooting method is implemented in MATLAB. At the end of this chapter is numerical solution for various parameters is discussed for the dimensionless velocity, temperature and concentration distributions. Investigation of obtained numerical results are given through tables and graphs.
The set of equations describing the flow as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{4.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+3\nu\frac{n-1}{2}\Gamma^2\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}$$
$$+\frac{\sigma J^2}{\rho}(u_e-u)+u_e\frac{\partial u_e}{\partial x}. \tag{4.2}$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}. \tag{4.3}$$
$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+Kr(C-C_\infty). \tag{4.4}$$

The following similarity transformation.

$$\left.\begin{aligned}\theta(\zeta)&=\frac{T-T_w}{T_\infty-T_w},\quad \phi(\zeta)=\frac{C-C_\infty}{T_w-T_\infty},\\ \psi&=(bv)^{\frac{1}{2}}x^{\frac{m+1}{2}}f(\zeta),\quad \zeta=\frac{b}{v}^{\frac{1}{2}}yx^{\frac{m-1}{2}}.\end{aligned}\right\} \tag{4.5}$$

- $u=bx^m f'(\zeta).$ (4.6)
- $v=-(bv)^{\frac{1}{2}}x^{\frac{m-1}{2}}\left[\frac{m+1}{2}f(\zeta)+\zeta\frac{m-1}{2}f'(\zeta)\right].$ (4.7)

The detailed procedure for the conversion of (4.1)-(4.4) into the dimensionless form has been described in the upcoming discussion.

- $\frac{\partial u}{\partial x}=\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta)+bmx^{m-1}f'(\zeta).$ (4.8)
- $\frac{\partial v}{\partial y}=-bmx^{m-1}f'(\zeta)-\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}x^{\frac{3m-3}{2}}yf''(\zeta).$ (4.9)

The detailed procedure for the conversion of (4.1) has been discussed in chapter 3.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

Now, for the momentum equation (4.2), dimensionless form we have discussed complete procedure in chapter 3.

$$f'''(\zeta)+m(1-f'^2(\zeta))+\frac{m+1}{2}f(\zeta)f''(\zeta)+\xi f'''(\zeta)$$

$$- \xi(f''(\zeta)\theta'(\zeta) + f'''(\zeta)\theta(\zeta)) + \frac{3}{2}(n-1)We^2 f''^2(\zeta)f'''(\zeta)$$
$$M^2(1 - f'(\zeta)) = 0. \tag{4.10}$$

To convert the equation (4.3) into ordinary differential form, the following procedure has been carried out.

- $\dfrac{\partial T}{\partial x} = (T_\infty - T_w)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}} \dfrac{m-1}{2} x^{\frac{m-3}{2}} y\theta'(\zeta).$
- $\dfrac{\partial T}{\partial y} = (T_\infty - T_w)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}} \theta'(\zeta).$
- $$u\frac{\partial T}{\partial x} = (bx^m f'(\zeta))((T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} \frac{m-1}{2} x^{\frac{m-3}{2}} y\theta'(\zeta)),$$
$$= \left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}(T_\infty - T_w)x^{\frac{3m-3}{2}} y f'(\zeta)\theta'(\zeta). \tag{4.11}$$
- $$v\frac{\partial T}{\partial y} = -(b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left[\frac{m-1}{2}(\zeta)f'(\zeta) + \frac{m+1}{2}f(\zeta)\right]$$
$$(T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta),$$
$$= -b\frac{m+1}{2}(T_\infty - T_w)x^{m-1} f(\zeta)\theta'(\zeta)$$
$$- b\frac{m-1}{2}\left(\frac{b}{\nu}\right)^{\frac{1}{2}}(T_\infty - T_w)x^{m-1} y f'(\zeta)\theta'(\zeta),$$
$$= -b\frac{m+1}{2}(T_\infty - T_w)x^{m-1} f(\zeta)\theta'(\zeta)$$
$$- \frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)(T_\infty - T_w)x^{\frac{3m-3}{2}} y f'(\zeta)\theta'(\zeta). \tag{4.12}$$
- $$k(T) = k^*[N_2 + h_2(T - T_w)],$$
$$= k^*[N_2 + h_2((T_\infty - T_w)\theta(\zeta) + T_m - T_w)],$$
$$= k^*[N_2 + h_2(T_\infty - T_w)\theta(\zeta)].$$
- $$k(T)\frac{\partial T}{\partial y} = (k^* N_2 + k^* h_2(T_\infty - T_w)\theta(\zeta))((T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)),$$

$$= k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)$$
$$+ k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta(\zeta)\theta'(\zeta).$$

- $$\frac{\partial}{\partial y}(k(T)\frac{\partial T}{\partial y}) = \frac{\partial}{\partial y}\left(k^* N_2(T\infty - T_w)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta'(\zeta)\right)$$
$$+ \frac{\partial}{\partial y}\left(k^* h_2(T_\infty - T_w)^2 \left(\frac{b}{\nu}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\theta(\zeta)\theta'(\zeta)\right),$$
$$= k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right) x^{m-1}\theta''(\zeta) + k^* h_2(T_\infty - T_w)^2$$
$$\left(\frac{b}{\nu}\right) x^{m-1}\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right),$$
$$\frac{1}{\rho C_p}\frac{\partial}{\partial y}(k(T)\frac{\partial T}{\partial y}) = k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right) x^{m-1}\theta''(\zeta) + k^* h_2(T_\infty - T_w)^2$$
$$\left(\frac{b}{\nu}\right) x^{m-1}\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right). \tag{4.13}$$
- $$\frac{\partial q_r}{\partial y} = \frac{16\sigma^* T_0^3}{3K^*}\frac{\partial^2 T}{\partial y^2}.$$
- $$\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2}{\partial y^2}((T_\infty - T_w)\theta(\zeta) + T_w),$$
$$= (T_\infty - T_w)\theta''(\zeta)\left(\frac{b}{\nu}\right) x^{m-1},$$
$$= (T_\infty - T_w)\left(\frac{b}{\nu}\right) x^{m-1}\theta''(\zeta).$$
- $$\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} = \frac{1}{\rho C_p}\frac{16\sigma^* T_0^3}{3k_1^*}(T_\infty - T_w)\left(\frac{b}{\nu}\right) x^{m-1}\theta''(\zeta). \tag{4.14}$$

Using equations (4.11)-(4.14),

$$\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)\frac{m-1}{2}(T_\infty - T_w)x^{\frac{3m-3}{2}} y f'(\zeta)\theta'(\zeta) - b\frac{m+1}{2}(T_\infty - T_w)x^{m-1} f(\zeta)\theta'(\zeta)$$
$$-\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)(T_\infty - T_w)x^{\frac{3m-3}{2}} y f'(\zeta)\theta'(\zeta) = \frac{1}{\rho C_p}k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right)$$
$$x^{m-1}\theta''(\zeta) + \frac{1}{\rho C_p}k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right) x^{m-1}\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right)$$
$$-\frac{1}{\rho C_p}\frac{16\sigma^* T_0^3}{3k_1^*}(T_\infty - T_w)\frac{b}{\nu}x^{m-1}\theta''(\zeta),$$
$$-b\frac{m+1}{2}(T_\infty - T_w)x^{m-1} f(\eta)\theta'(\zeta) = \frac{1}{\rho C_p}k^* N_2(T_\infty - T_w)\left(\frac{b}{\nu}\right) x^{m-1}\theta''(\zeta)$$
$$+\frac{1}{\rho C_p}k^* h_2(T_\infty - T_w)^2\left(\frac{b}{\nu}\right) x^{m-1}\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right),$$
$$-\frac{1}{\rho C_p}\frac{16\sigma^* T_0^3}{3k_1^*}(T_\infty - T_w)\frac{b}{\nu}x^{m-1}\theta''(\zeta),$$

$$\theta''(\zeta) + \frac{m+1}{2}\frac{\rho C_p \nu}{k_1 - \frac{16\sigma^* T_0^3}{3k^*}} f(\zeta)\theta'(\zeta) + \frac{h_2}{k_1 - \frac{16\sigma^* T_0^3}{3k^*}}$$

$$(T_\infty - T_w)\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right) = 0,$$

$$\theta''(\zeta) + \frac{\mu C_p}{k^* - \frac{16\sigma^* T_0^3}{3k_1^*}}\frac{m+1}{2} f(\zeta)\theta'(\zeta) + \frac{h_2}{k^* - \frac{16\sigma^* T_0^3}{3k_1^*}}$$

$$(T_\infty - T_w)\left(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)\right) = 0,$$

$$\theta''(\zeta) + \frac{m+1}{2}\frac{C_p \mu}{k_1(1 - \frac{16\sigma^* T_0^3}{3k^* k_1})} f(\zeta)\theta'(\zeta)$$

$$+ \frac{\epsilon}{1 - \frac{16\sigma^* T_0^3}{3k^* k_1}}(\theta(\zeta)\theta''^{(\zeta)} + \theta'^2(\zeta)) = 0,$$

$$\theta''(\zeta) + \frac{Pr\frac{m+1}{2}}{1 - \frac{4}{3}R} f(\zeta)\theta'(\zeta) + \frac{\epsilon}{1 - \frac{4}{3}R}(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)) = 0. \tag{4.15}$$

The dimensionless form of (4.4), the complete derivations has been discussed below:

- $\phi(\zeta) = \dfrac{C - C_\infty}{C_w - C_\infty}.$
- $C = (C_w - C_\infty)\phi(\zeta) + C_\infty.$
- $\dfrac{\partial C}{\partial x} = \dfrac{\partial}{\partial x}\left((C_w - C_\infty)\phi(\zeta) + C_\infty\right),$

$$= (C_w - C_\infty)\phi'(\zeta)\left(\frac{b}{\nu}\right)^{\frac{1}{2}} y\frac{m-1}{2}x^{\frac{m-3}{2}},$$

$$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)^{\frac{1}{2}}\frac{m-1}{2}x^{\frac{m-3}{2}} y\phi'(\zeta).$$

- $u\dfrac{\partial C}{\partial x} = (bx^m f'(\zeta))(C_w - C_\infty)\left(\dfrac{b}{\nu}\right)^{\frac{1}{2}}\dfrac{m-1}{2}x^{\frac{m-3}{2}} y\phi'(\zeta),$

$$= (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-3}{2}} y f'(\zeta)\phi'(\zeta). \tag{4.16}$$

- $\dfrac{\partial C}{\partial y} = \dfrac{\partial}{\partial y}((C_w - C_\infty)\phi(\zeta) + C_\infty),$

$$= (C_w - C_\infty)\phi''(\zeta)\left(\frac{b}{\nu}\right)x^{m-1},$$

$$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta).$$

- $v\dfrac{\partial C}{\partial y} = -(b\nu)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left[\dfrac{m-1}{2}(\zeta)f'(\zeta) + \dfrac{m+1}{2}f(\zeta)\right]$

$$(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta),$$

$$= -b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}f(\zeta)\phi'(\zeta)$$

$$- (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta). \tag{4.17}$$

- $$\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)\phi''(\zeta)\left(\frac{b}{\nu}\right)x^{m-1},$$

  $$= (C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta).$$

- $$D\frac{\partial^2 C}{\partial y^2} = D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta). \tag{4.18}$$

- $$Kr(C - C_\infty) = Kr(C_w - C_\infty)\phi(\zeta) + C_\infty - C_\infty,$$

  $$= Kr(C_w - C_\infty)\phi(\zeta). \tag{4.19}$$

Using equations (4.16)-(4.19),

$$(C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta) - b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}f(\zeta)\phi'(\zeta)$$

$$- (C_w - C_\infty)\frac{m-1}{2}\left(\frac{b^{\frac{3}{2}}}{\nu^{\frac{1}{2}}}\right)x^{\frac{3m-3}{2}}yf'(\zeta)\phi'(\zeta)$$

$$= D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta) + Kr(C_w - C_\infty)\phi(\zeta)$$

$$- b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}f(\zeta)\phi'(\zeta) = D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}\phi''(\zeta)$$

$$+ Kr(C_w - C_\infty)\phi(\zeta)$$

$$\phi''(\zeta) + \frac{b(C_w - C_\infty)\frac{m+1}{2}x^{m-1}}{D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}}f(\zeta)\phi'(\zeta) + \frac{Kr(C_w - C_\infty)}{D(C_w - C_\infty)\left(\frac{b}{\nu}\right)x^{m-1}}\phi(\zeta) = 0,$$

$$\phi''(\zeta) + \frac{m+1}{2}\frac{\nu}{D}f(\zeta)\phi'(\zeta) + \frac{\nu}{D}\frac{Kr}{bx^{m-1}}\phi(\zeta) = 0,$$

$$\phi''(\zeta) + \frac{m+1}{2}L_e f(\zeta)\phi'(\zeta) + \frac{2Kr(m+1)}{2b(m+1)x^{m-1}}Le\phi(\zeta) = 0,$$

$$\phi''(\zeta) + \frac{m+1}{2}L_e f(\zeta)\phi'(\zeta) + \frac{m+1}{2}\gamma Le\phi(\zeta) = 0. \tag{4.20}$$

- $$u = u_w(x). \tag{4.21}$$

  $$u = bx^m f'(\zeta).$$

  $$u_w(x) = ax^m.$$

Consider the equation (4.21)

$$bx^m f'(\zeta) = ax^m,$$
$$bf'(\zeta) = a,$$
$$f'(0) = \frac{a}{b}, \qquad as \;\; \zeta \to 0.$$
$$f'(0) = B. \tag{4.22}$$

- $v = v_w. \tag{4.23}$

$$v = -(bv)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left[\frac{m+1}{2} f(\eta) + \zeta \frac{m-1}{2} f'(\zeta) \right].$$

From equation (4.23), we get

$$-(bv)^{\frac{1}{2}} x^{\frac{m-1}{2}} \left[\frac{m+1}{2} f(\zeta) + \zeta \frac{m-1}{2} f'(\zeta) \right] = v_w,$$
$$-(bv)^{\frac{1}{2}} \frac{m+1}{2} x^{\frac{m-1}{2}} f(\zeta) + \zeta \frac{m-1}{2} f'(\zeta) = v_w, \qquad as \;\; y \to 0.$$
$$f(0) = \frac{-2v_w}{(b\nu)^{\frac{1}{2}} (m+1) x^{\frac{m-1}{2}}}, \qquad as \;\; \zeta \to 0.$$
$$f(0) = S. \tag{4.24}$$

- $C = C_w. \tag{4.25}$

$$C = (C_w - C\infty) + C_\infty,$$
$$(C_w - C_\infty)\phi(\zeta) + C_\infty = C_w,$$
$$(C_w - C_\infty)\phi(\zeta) = C_w - C_\infty,$$
$$\phi(\zeta) = \frac{C_w - C_\infty}{C_w - C_\infty},$$
$$\phi(\zeta) = 1, \qquad as \;\; \zeta \to 0.$$
$$\phi(0) = 1. \tag{4.26}$$
$$\theta'(0) = 1. \tag{4.27}$$

- $u \to u_e(x) = bx^m.$

$$bx^m f'(\zeta) \to bx^m, \qquad as \;\; \zeta \to \infty.$$
$$f'(\infty) \to 1.$$

- $C \to C_\infty$

$$\phi(\zeta)(C_w - C_\infty) + C_\infty \to C_\infty.$$
$$\phi(\zeta)(C_w - C_\infty) \to 0, \qquad as \quad \zeta \to \infty.$$
$$\phi(\infty) \to 0, \qquad as \quad \zeta \to \infty.$$
$$\theta(\infty) \to 0, \qquad as \quad \zeta \to \infty.$$

The final dimensionless form of the governing model, is

$$f'''(\zeta) + m(1 - f'^2(\zeta)) + \frac{m+1}{2} f(\zeta) f''(\zeta) + \xi f'''(\zeta)$$
$$- \xi(f''(\zeta)\theta'(\zeta) + f'''(\zeta)\theta(\zeta)) + \frac{3}{2}(n-1)We^2 f''^2(\zeta) f'''(\zeta)$$
$$M^2(1 - f') = 0. \tag{4.28}$$
$$\theta''(\zeta) + \frac{Pr\frac{m+1}{2}}{1 - \frac{4}{3}R} f(\zeta)\theta'(\zeta) + \frac{\epsilon}{1 - \frac{4}{3}R}(\theta(\zeta)\theta''(\zeta) + \theta'^2(\zeta)) = 0. \tag{4.29}$$
$$\phi''(\zeta) + \frac{m+1}{2} L_e f(\zeta)\phi'(\zeta) + \frac{m+1}{2}\gamma Le\phi(\zeta) = 0. \tag{4.30}$$

Different parameter used in equations (4.28)-(4.30) are formulated as follows:

$$We^2 = \frac{\Gamma^2 b^3 x^{3m-1}}{\nu}, \ M^2 = \frac{\sigma J^2}{\rho b}, \ \xi = h_1(T_\infty - T_w), \ \epsilon = h_2(T_\infty - T_w),$$
$$S = \frac{-2v_w}{(b\nu)^{\frac{1}{2}}(m+1)x^{\frac{m-1}{2}}}, \ B = \frac{a}{b}, \ Pr = \frac{\mu C_p}{k^*}, \ Le = \frac{\nu}{D},$$
$$R = \frac{4\sigma^* T_0^3}{k^* k_1}, \ \gamma = \frac{2Kr}{b(m+1)x^{m-1}}.$$

4.2 Method of Solution

The shooting method has been used to solve the ordinary differential equations (4.28)-(4.30). Equations (4.28) and (4.29) are solved numerically and then f is used in equation (4.30).The following notations have been considered:

$$f = Z_1, \ f' = Z_1' = Z_2,$$
$$f'' = Z_2' = Z_3, \ f''' = Z_3'.$$
$$\theta = Z_4, \ \theta' = Z_4' = Z_5, \ \theta'' = Z_5'.$$

By using the above notations in equation (4.28)-(4.29), the following scheme of ODEs is attained:

$$
\begin{aligned}
Z_1' &= Z_2, && Z_1(0) = S.\\
Z_2' &= Z_3, && Z_2(0) = B.\\
Z_3' &= \frac{-m(1-Z_2^2) - \frac{m+1}{2}Z_1Z_3 + \xi Z_3 Z_5 - M^2(1-Z_2)}{1+\xi-\xi Z_4 + \frac{3}{2}(n-1)We^2 Z_3^2}, && Z_3(0) = r.\\
Z_4' &= Z_5, && Z_4(0) = q.\\
Z_5' &= \frac{(1-\frac{4}{3}R)\left(-Pr\frac{m+1}{2}Z_1Z_5 - \epsilon Z_5^2\right)}{1+\epsilon Z_4}, && Z_5(0) = 1.
\end{aligned}
$$

The above IVP will be numerically determined by RK technique of order four. In the above initial value problem, the missing conditions r and q satisfy the following relation:

$$Z_2(\zeta_\infty, r, q) = 1, \quad Z_4(\zeta_\infty, r, q) = 0.$$

By using Newton's method which has the following iterative scheme:

$$\begin{bmatrix} r^{(n+1)} \\ q^{n+1} \end{bmatrix} = \begin{bmatrix} r^{(n)} \\ q^{(n)} \end{bmatrix} - \begin{bmatrix} \frac{\partial Z_2}{\partial r} & \frac{\partial Z_2}{\partial q} \\ \frac{\partial Z_4}{\partial r} & \frac{\partial Z_4}{\partial q} \end{bmatrix}^{-1} = \begin{bmatrix} Z_2 - 1 \\ Z_4 \end{bmatrix} \tag{4.31}$$

Now, introduce the following notations:

$$
\begin{aligned}
&\frac{\partial Z_1}{\partial r} = Z_6,\ \frac{\partial Z_2}{\partial r} = Z_7,\ \frac{\partial Z_3}{\partial r} = Z_8,\ \frac{\partial Z_4}{\partial r} = Z_9,\ \frac{\partial Z_5}{\partial r} = Z_{10}.\\
&\frac{\partial Z_1}{\partial q} = Z_{11},\ \frac{\partial Z_2}{\partial q} = Z_{12},\ \frac{\partial Z_3}{\partial q} = Z_{13},\ \frac{\partial Z_4}{\partial q} = Z_{14},\ \frac{\partial Z_5}{\partial q} = Z_{15}.
\end{aligned}
$$

As a result of these new notations, the Newton's iterative scheme gets the form:

$$\begin{bmatrix} r^{(n+1)} \\ q^{n+1} \end{bmatrix} = \begin{bmatrix} r^{(n)} \\ q^{(n)} \end{bmatrix} - \begin{bmatrix} Z_7 & Z_{12} \\ Z_9 & Z_{14} \end{bmatrix}^{-1} = \begin{bmatrix} Z_2 - 1 \\ Z_4 \end{bmatrix} \tag{4.32}$$

Now differentiate the above system of five first order ODEs with respect to r and q. Writing all these ten ODEs together, we have the following IVP.

$$Z_6' = Z_7, \qquad Z_6(0) = 0.$$

$$Z_7' = Z_8, \qquad Z_7(0) = 0.$$

$$Z_8' = \frac{1}{(1+\xi-\xi Z_4+\frac{3}{2}(n-1)We^2Z_3^2)^2}((1+\xi-\xi Z_4+\frac{3}{2}(n-1)We^2Z_3^2)$$
$$(2mZ_2Z_7-\frac{m+1}{2}(Z_1Z_8+Z_3Z_6)\xi(Z_3Z_{10}+Z_5Z_8)+M^2Z_7)-(-m(1-Z_2^2)$$
$$-\frac{m+1}{2}Z_1Z_3+\xi Z_3Z_5-M^2(1-Z_2))(-\xi Z_9+3(n-1)We^2Z_3Z_8)),$$
$$Z_8(0) = 1.$$

$$Z_9' = Z_{10}, \qquad Z_9(0) = 0.$$

$$Z_{10}' = \frac{1}{1+\epsilon Z_4}(1+\epsilon Z_4)(1-\frac{4}{3}R)\Bigg(-Pr\frac{m+1}{2}(Z_1Z_{10}+Z_5Z_6)-2\epsilon Z_5Z_{10}\Bigg)$$
$$-(1-\frac{4}{3}R)\Bigg(-Pr\frac{m+1}{2}Z_1Z_5-\epsilon Z_5^2\Bigg)(\epsilon Z_9), \qquad Z_{10}(0) = 0.$$

$$Z_{11}' = Z_{12}, \qquad Z_{11}(0) = 0.$$

$$Z_{12}' = Z_{13}, \qquad Z_{12}(0) = 0.$$

$$Z_{13}' = \Bigg(\frac{1}{1+\xi-\xi Z_4+\frac{3}{2}(n-1)We^2Z_3^2}\Bigg)((1+\xi-\xi Z_4+\frac{3}{2}(n-1)We^2Z_3^2)$$
$$(2mZ_2Z_{12}-\frac{m+1}{2}(Z_1Z_{13}+Z_3Z_{11})+\xi(Z_3Z_{15}+Z_5Z_{13})+M^2Z_{12})$$
$$-(-m(1-Z_2^2)-\frac{m+1}{2}Z_1Z_3+\xi Z_3Z_5-M^2(1-Z_2))(-\xi Z_{14}$$
$$+3(n-1)We^2Z_3Z_{13})), \qquad Z_{13}(0) = 0.$$

$$Z_{14}' = Z_{15}, \qquad Z_{14}(0) = 1.$$

$$Z_{15}' = \frac{1}{1+\epsilon Z_4}(1+\epsilon Z_4)(1-\frac{4}{3}R)\Bigg(-Pr\frac{m+1}{2}(Z_1Z_{15}+Z_5Z_{11})-2\epsilon Z_5Z_{15}\Bigg)$$
$$-(1-\frac{4}{3}R)\Bigg(-Pr\frac{m+1}{2}Z_1Z_5-\epsilon Z_5^2\Bigg)(\epsilon Z_{14}), \qquad Z_{15}(0) = 0.$$

The threshold for the Newton's method is set as,

$$max\,\{|\ Z_2(\zeta_\infty)-1\ |,\ \ |\ Z_4(\zeta_\infty)\ |\} < \varepsilon$$

The equation (4.30) will be numerically solved by using shooting method. The following notions has been used:

$$\phi = Y_1, \ \phi' = Y_1' = Y_2, \ \phi'' = Y_2'.$$

By using the above notations in equations (4.30), the following system of ODEs is obtained:

$$\begin{aligned} &Y_1' = Y_2, && Y_1(0) = 1. \\ &Y_2' = -\frac{m+1}{2} Lef Y_2 - \frac{m+1}{2}\gamma Le Y_1, && Y_2(0) = Q. \end{aligned}$$

The above IVP will be numerically determined by RK technique of order four. In the above initial value problem, the missing condition Q satisfy the following relation:

$$Z_1(\zeta_\infty, Q) = 0. \tag{4.33}$$

Newton method which has the following iterative scheme:

$$Q^{(n+1)} = Q^{(n)} - \frac{G(\zeta)}{G'(\zeta)}, \quad where \quad G(\zeta) = Z_1(\zeta_\infty, Q) = 0.$$

To incorporate Newton's method, we further utilize the following notions:

$$\frac{\partial Y_1}{\partial Q} = Y_3, \quad \frac{\partial Y_1}{\partial Q} = Y_4.$$

Now differentiate above system of two first order ODEs with respect to Q,

$$\begin{aligned} &Y_3' = Y_4, && Y_3(0) = 0. \\ &Y_4' = -\frac{m+1}{2} LeC1 Y_4 - \frac{m+1}{2}\gamma Le Y_3, && Y_4(0) = 1. \end{aligned}$$

The stopping criteria for the Newton's method is set as:

$$| Z_1(\zeta_\infty, Q) | < \varepsilon.$$

4.3 Results and Discussion

The principle object is about to examine the impact of some different parameters against the velocity, temperature and concentration distribution. The impact of different factors like nonlinearity parameter m, magnetic parameter M, Weissenberg number We, thermal radiation R and chemical reaction Kr is observed graphically. Numerical outcomes of the Nusselt number $(Re_x)^{\frac{-1}{2}} Nu_x$ and skin friction $(Re_x)^{\frac{1}{2}} Cf_x$ for the distinct values of some fixed parameters are shown in Tables 4.1-4.5.

Figure 4.1 represents the influence of n on the dimensionless $f'(\zeta)$ in the presence of B. It is clearly shown that the $f'(\zeta)$ is increased by enhancing the value of n. In the case of stretching, the opposite behavior has been observed.

Figure 4.2 illustrate the impact of suction parameter S on the dimensionless $f'(\zeta)$. It is clearly shown that the $f'(\zeta)$ is increased by enhancing the value of S.

Figure 4.3 delineated to show the impact of nonlinearity stretching parameter m on $f'(\zeta)$. By enhancing the value of m, the $f'(\zeta)$ is increased in the case of stretching.

Figure 4.4 shows the impact of We on $f'(\zeta)$. This graph indicates that an increment in the value of We, the $f'(\zeta)$ is increased in the case of stretching. In the event of stretching, the opposite behavior has been noted.

Figure 4.5 represents the impact of viscous parameter ξ on $f'(\zeta)$. It can be noted that, the $f'(\zeta)$ is decreased by increasing the value of ξ in the case of stretching.

Figure 4.6 delineated to show the impact of m on $\theta(\zeta)$. The increment of m, the $\theta(\zeta)$ is decreased.

Figure 4.7 represents the influence of Pr on the dimensionless $\theta(\zeta)$. It is clearly observed that the $\theta(\zeta)$ is increased by enhancing the value of Pr, the thermal boundary layer thickness increases for both shear thickening and shear thinning fluids.

Figure 4.8 illustrate the effect of viscous parameter ξ on $\theta(\zeta)$. It is clearly shown that the $\theta(\zeta)$ is increased by enhancing the value of ξ.

Figure 4.9 displays the impact of Le on $\phi(\zeta)$. Concentration profile decelerate for the boosting values of Le thus we have get a small molecular diffusivity and thermal boundary layer.

Figure 4.10 delineated to show the impact of chemical reaction parameter γ on the dimensionless $\phi(\zeta)$. The increment of γ, $\phi(\zeta)$ is decreased.

TABLE 4.1: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7,\ R = -0.2$.

B	S	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
0	5.0	2.9150	3.7910
0	5.5	3.0239	4.1949
0	6.0	3.1283	4.6006
0	6.5	3.2286	5.0078
0	7.0	3.3250	5.4160
0	7.5	3.4283	5.8006
0	8.0	3.5286	6.2078
0	8.5	3.6250	6.6160
0	9	3.7240	7.1160
-3	5.0	4.6336	3.2377
-3	5.5	4.8673	3.6922
-3	6.0	5.0804	4.1397
-3	6.5	5.2772	4.5819
-3	7.0	5.4604	5.0202
-3	7.5	5.6150	5.5910
-3	8.0	5.8336	6.0377
-3	8.5	6.0673	6.5922

TABLE 4.2: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ s = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7,\ R = -0.2$.

B	n	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	5	-3.0564	3.9932
2	6	-2.8924	3.9954
2	7	-2.7628	3.9973
2	8	-2.6564	3.9989
2	9	-2.5666	4.0004
-2	5	4.4452	3.4641
-2	6	4.1541	3.4529
-2	7	3.9285	3.4436
-2	8	3.7460	3.4358
-2	9	3.5939	3.4290

TABLE 4.3: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $s = 5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ M = 0.5,\ Pr = 0.7,\ R = -0.2$.

B	m	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
0	7.0	4.4039	10.6706
0	7.5	4.5109	11.3597
0	8.0	4.6136	12.0488
0	8.5	4.7236	13.0388
0	9.0	4.8074	13.4275
0	10	4.9880	14.8064
-3	7.0	6.8370	10.0829
-3	7.5	6.9991	10.7710
-3	8.0	7.1548	11.4594
-3	8.5	7.2048	12.4194
-3	9.0	7.4495	12.8367
-3	10	7.7248	14.2147

TABLE 4.4: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $s = 5,\ n = 5,\ We = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7,\ R = -0.2$.

B	M	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	5.0	-3.6655	4.0224
2	6.5	-3.9524	4.0190
2	7.5	-4.0324	4.0590
2	8.5	-4.3354	4.1607
2	9.0	-4.4299	4.2827
2	10	-4.6175	4.4957
-2	5.0	6.0054	3.5956
-2	6.5	6.6013	3.6081
-2	7.5	6.9013	3.6181
-2	8.5	7.3560	3.6231
-2	9.0	7.5392	3.7326
-2	10	7.8889	3.7378

TABLE 4.5: Numerical outcomes of $(Re_x)^{\frac{1}{2}}Cf_x$ and $(Re_x)^{\frac{-1}{2}}Nu_x$ for some fixed parameters $M = 0.5,\ n = 5,\ s = 0.3,\ \xi = 0.5,\ \epsilon = 0.5,\ m = 2,\ Pr = 0.7,\ R = -0.2$.

B	We	$(Re_x)^{\frac{1}{2}}Cf_x$	$(Re_x)^{\frac{-1}{2}}Nu_x$
2	0.6	-2.1373	4.0081
2	1.0	-1.6163	4.0192
2	1.4	-1.3397	4.0260
2	1.8	-1.1635	4.0305
2	2.2	-1.0394	4.0339
-2	0.6	2.8912	3.3964
-2	1.0	2.1198	3.3520
-2	1.4	1.8296	3.3339
-2	1.8	1.7025	3.3259
-2	2.2	1.6366	3.3217

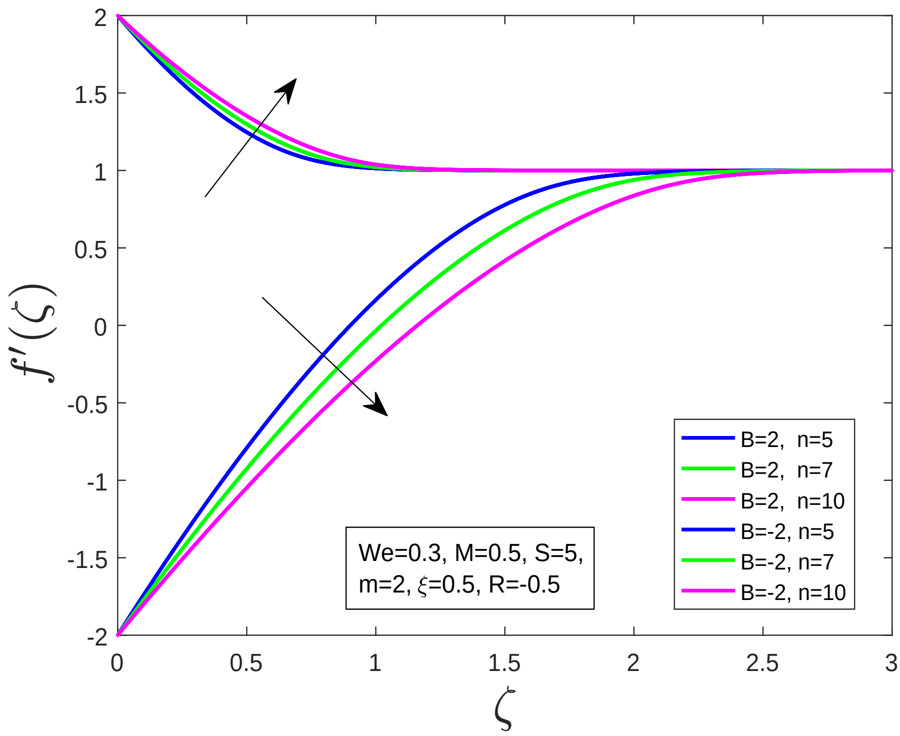

FIGURE 4.1: Impact of stretching parameter with power law index on the dimensionless velocity profile.

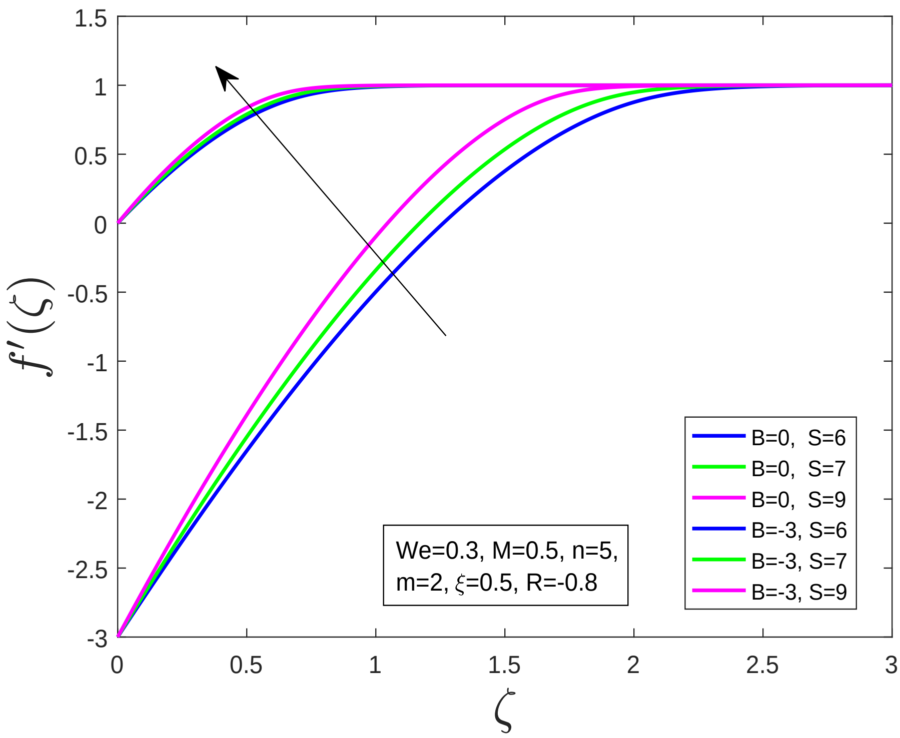

FIGURE 4.2: Impact of stretching parameter with suction parameter on the dimensionless velocity profile.

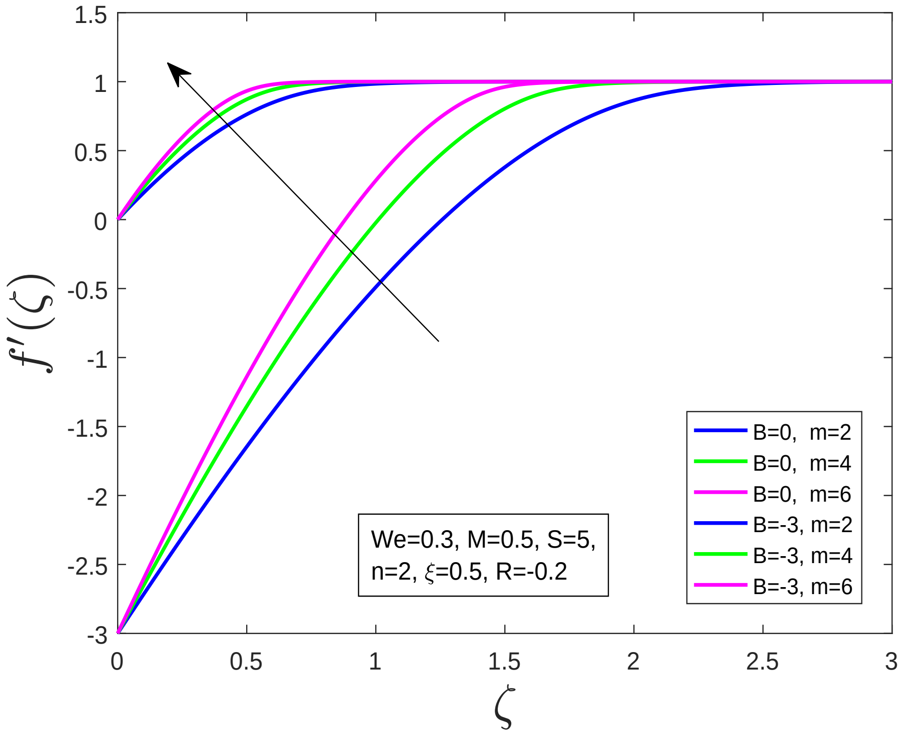

FIGURE 4.3: Impact of stretching parameter with nonlinearity stretching parameter on the dimensionless velocity profile.

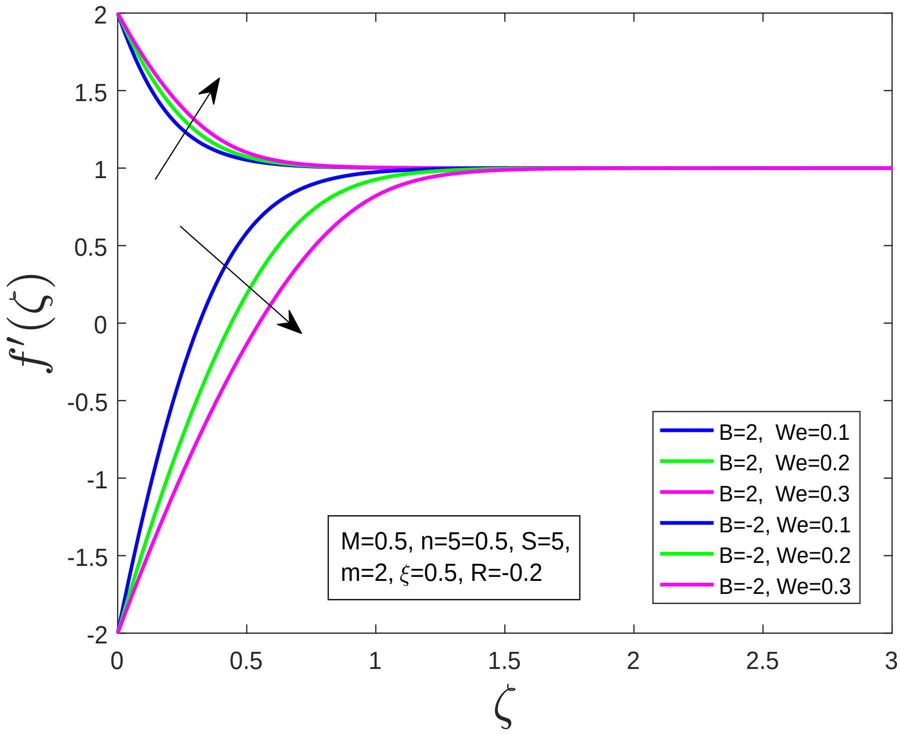

FIGURE 4.4: Impact of stretching parameter with Weissenberg number on the dimensionless velocity profile.

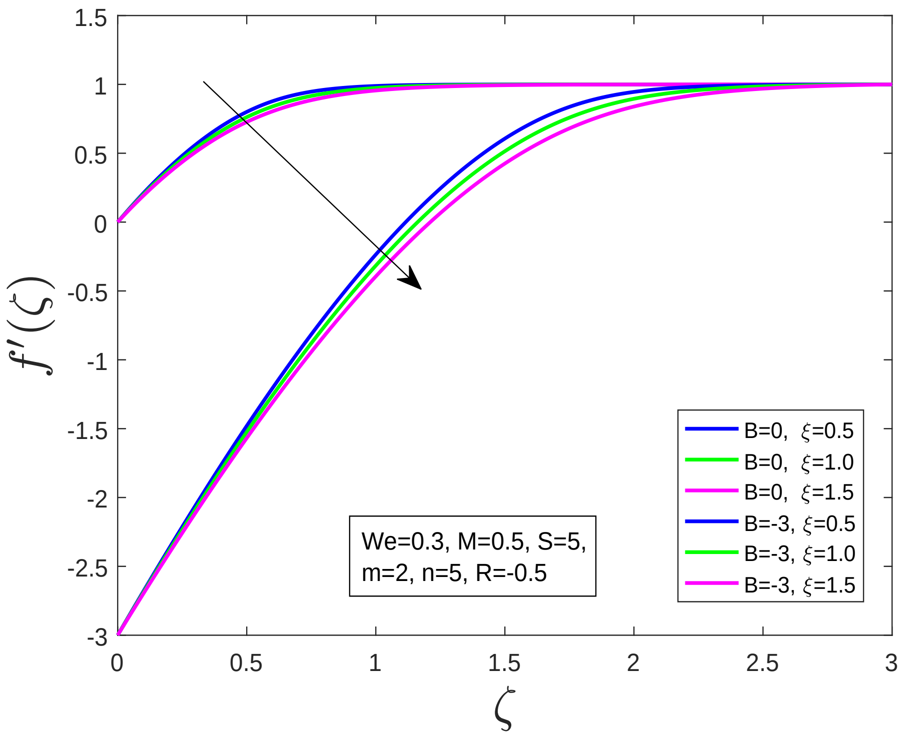

FIGURE 4.5: Influence of viscous parameter factor on dimensionless velocity profile.

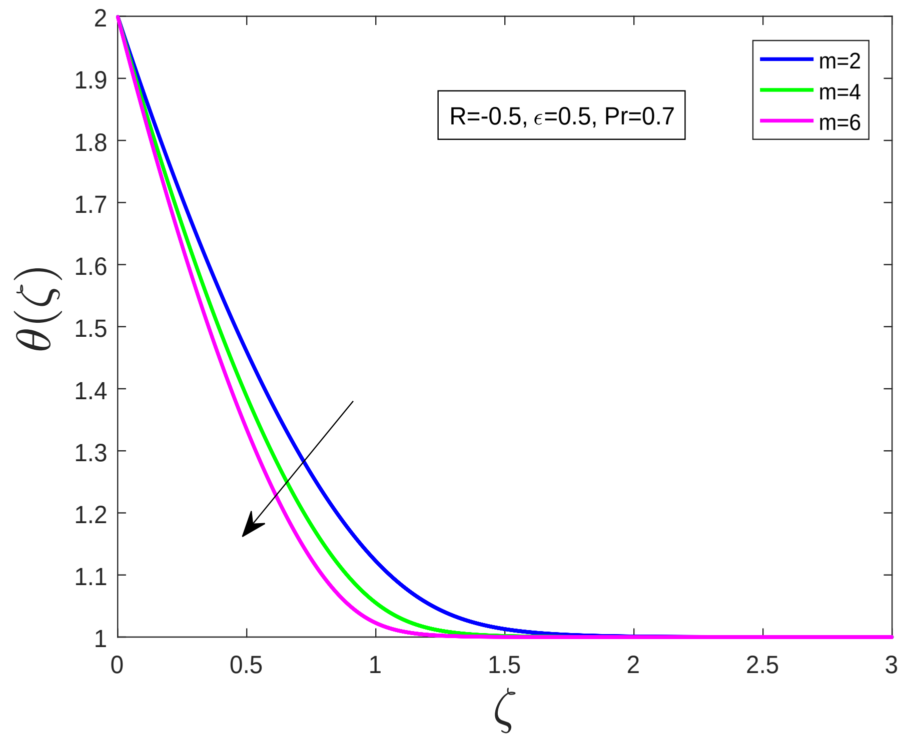

FIGURE 4.6: Influence of nonlinearity stretching parameter factor on dimensionless temperature profile.

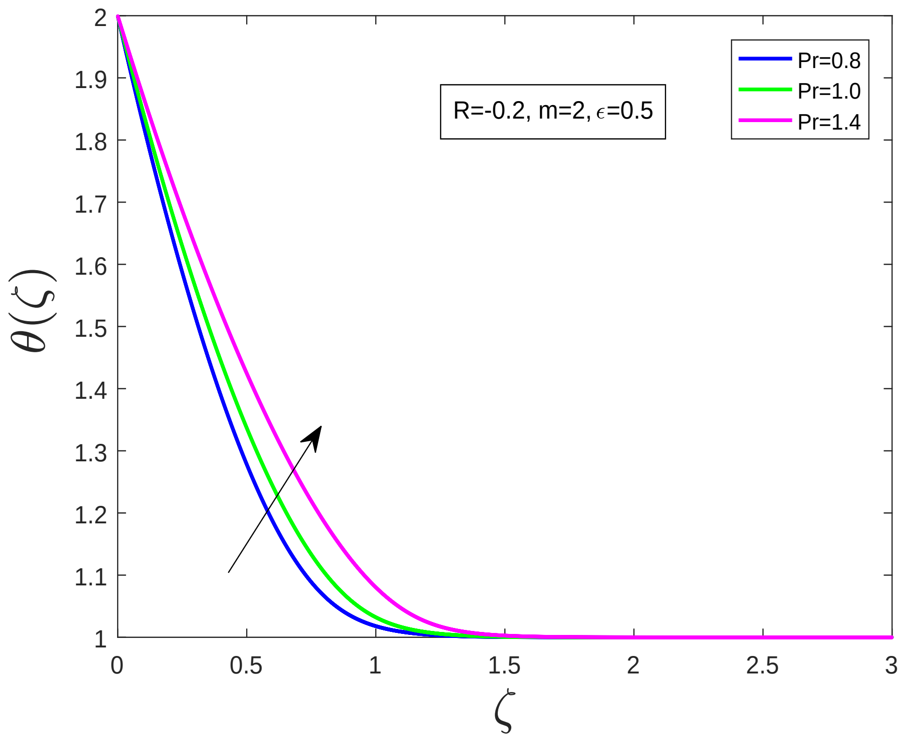

FIGURE 4.7: Influence of Prandtl number factor on dimensionless temperature profile

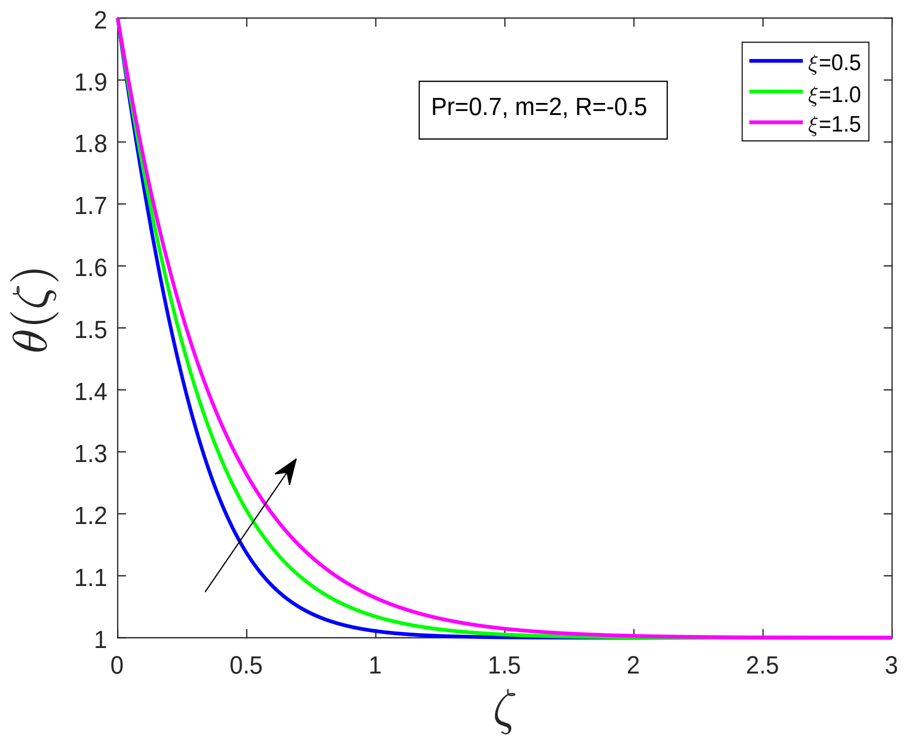

FIGURE 4.8: Influence of viscous parameter factor on dimensionless temperature profile.

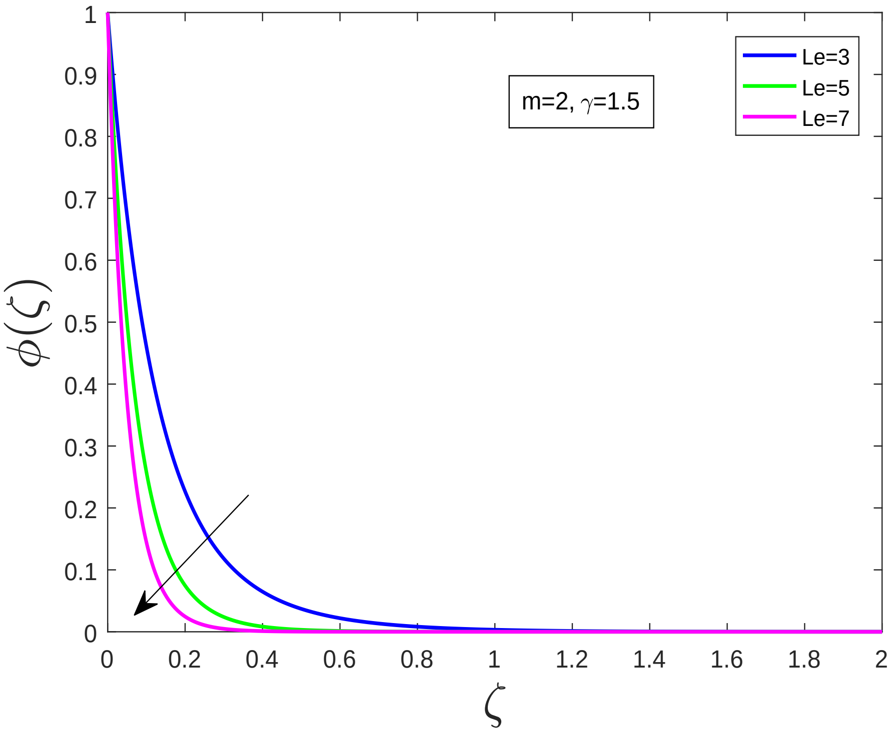

FIGURE 4.9: Influence of Lewis number factor on dimensionless concentration profile.

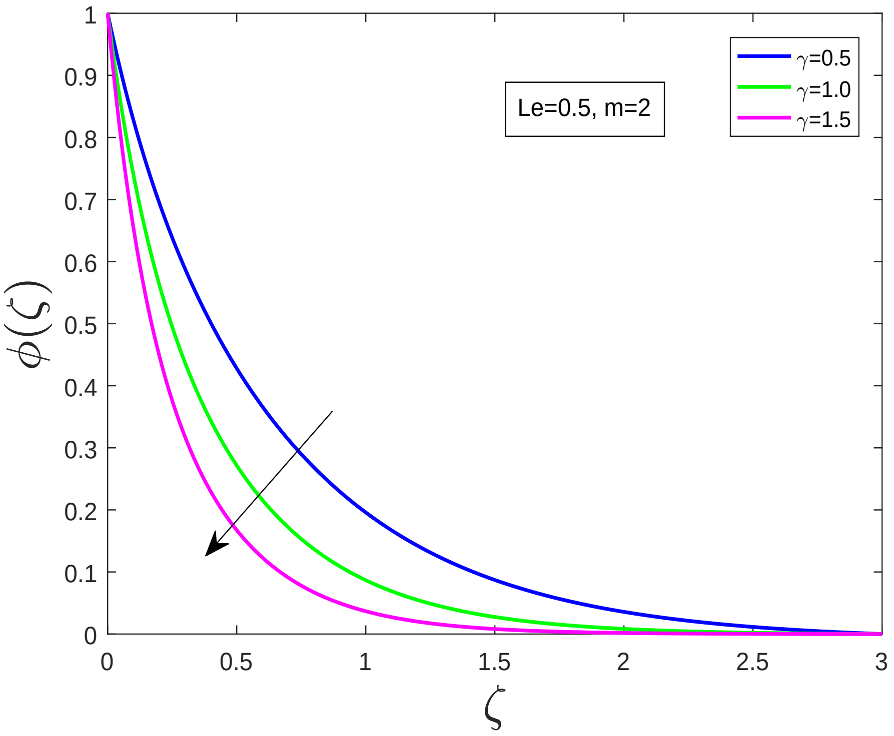

FIGURE 4.10: Influence of chemical reaction factor on dimensionless concentration profile.

Chapter 5

Conclusion

This research work represents the 2-D boundary layer flow of MHD Carreau fluid flow over a stretching sheet with the variable of viscosity, heat transfer and thermal conductivity. Furthermore, the impacts of magnetic parameter, suction parameter, nonlinearity parameter and Weissenberg number are discussed. The obtained mathematical model contains the nonlinear PDEs of continuity equation, momentum equation, energy equation and concentration equation. Furthermore these PDEs are converted into a system of nonlinear ODEs by applying the similarity transformation. For the numerical results of ODEs, shooting technique is utilized. The dimensionless velocity behavior, temperature distribution, skin friction, Nusselt number and sherwood number have been analyzed for different values of various parameters. The numerical results are explained through different figures and tables.

- For the enhancing values of We and n, the velocity distribution is increased. In case of stretching, the opposite behavior has been obsered.
- The temperature profile is decreased due to the increasing values of nonlinearity stretching parameter.
- A decrement is noticed in concentration profile due to the accelerating values of Lewis number.
- A increment is noticed in the temperature profile by enhancing values of Prandtl number.

- The increasing values of viscous parameter cause an enhanement in temperature distribution.
- A decrement is noticed in concentration profile due to the accelerating values of chemical reaction parameter.

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