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Non-Linear Control Techniques for Stabilization of Planar Vertical Take-off and Landing Aircraft

by

Yaseen

A thesis submitted in partial fulfillment for the degree of Master of Science

in the

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This thesis is dedicated to my caring parents and family.

CERTIFICATE OF APPROVAL

Non-Linear Control Techniques for Stabilization of Planar Vertical Take-off and Landing Aircraft

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(Yaseen)

Abstract

This work presents non-linear control algorithms for stabilizing a planar vertical take-off and landing aircraft (PVTOL), That belongs to family of underactuated mechanical systems having three degrees of freedom (3DOF) with two control inputs. The suggested methodologies for PVTOL are based on first order sliding mode control, Robust adaptive sliding mode control and the backstepping control. In adaptive sliding mode control, the system is first transformed into a particular structure through input transformation, which contain some unknown terms. The dimension of system increases. The unknown term is computed adaptively and then system is stabilized via adaptive sliding mode control. Also the sliding mode control which contain discontinuous term and adapted laws are derived in such a way that the time derivative of a Lyapunov function becomes strictly negative. In second approach transformation is applied and new control inputs injected in dynamical model of PVTOL. Linear sliding manifold is defined and system is forced toward sliding manifold via first order sliding mode control. In backstepping approach system is transformed into specific structure and system is stabilized. Matlab simulation results reveal the effectiveness of the suggested control algorithms on PVTOL system.

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Abbreviations

ASMC	Adaptive sliding mode control
DOF	Degree of freedom
PVTOL	Planar vertical take-off and landing
RP	Reaching Phase
SS	Sliding surface
TORA	Translational oscillatory and rotational actuator
UAV	Unmanned aerial vehicles
UMS	Underactuated mechanical system
VTOL	vertical take-off and landing

Symbols

ϵ	epsilon
F_d	spring damping force
g	gravitational acceleration
s	sliding surface
τ	torque
θ	angle
ω	angular velocity
u	control input

Chapter 1

Introduction

1.1 Background

This chapter introduces the research work carried out in this thesis. First, building upon the background, the author explains how motivation for this work was developed. This chapter concludes with an overview of this thesis.

Mechanical Systems are among the oldest systems invented by humans to be used as in our daily life. Application of mechanical systems are used in almost every aspect of practical life. Examples of such systems includes pulley, wheel, sewing machines, steam engines, automobiles, robots, aerospace and marine systems. When use of these systems became more common, their manual operation become less productive, so humans started thinking to automate these systems for better qualitative and quantitative output. The need of automation leads to the application of Control theory in mechanical systems.

From control point of view, mechanical control systems have been studied under the following subclasses:

1. **Fully Actuated Mechanical Systems:** Those Systems which have same number of actuators compared to the number of degrees of freedom.

2. **Underactuated Mechanical Systems:** Those Systems which have less number of actuators compared to the number of degrees of freedom.

3. **Nonholonomic Systems:** Those Systems which have non-integrable 1st order constraints on their velocities.

Control of fully actuated mechanical systems is not a challenging problem because linear control methods like pole placement or frequency domain analysis can be used to design control framework for such systems, nonlinear techniques of feedback linearization [1] is also applicable.

Planar vertical take-off and landing aircraft (PVTOL) is 3DOF underactuated mechanical system. Research in control of underactuated system including PVTOL sysetm started in 90's [2–5]. In past decades, level of interest has been increased in underactuated systems. The underactuated mechanical systems have been of great importance in research and are considered as prototype systems for nonlinear complex systems. When the usefulness of underactuated mechanical systems (which have less number of actuators than configuration variables) was realized in applications of science and engineering, its research shifted to practical nature. Underactuated systems have numerous applications in science and technology including robotics, aerospace and marine systems. The feature of underactuation make their control distinct from other non-linear systems. This property of "underactuation" is due to the following reasons.

1. Natural dynamics of the systems like surface and water vehicles.

2. Also this property can be chosen to reduce cost and for more practical advantages like reduction of weight in space and underwater vehicles.

3. Underactuation phenomenon can be applied to design low order nonlinear systems for obtaining acknowledgment about control of higher order underactuated systems, (e.g. the Beam-and-Ball system, the TORA system, Rotating pendulum, quadrotor system, PVTOL etc.).

4. Underactuation may raise in case of actuator's failure, (e.g. in a aircraft or surface vessel).

Considering application of underactuated systems, starting from group of robotics includes flexible-link joints, mobile robots and many others kinds of manipulators. Underactuated class also includes surface and underwater vehicles such as surface vessels, twin rotor system etc. Due to highly nonlinear and complex behavior of underactuated mechanical systems, control design for such type of systems is observed as a challenging task, due to less actuators compared to the degrees of freedom to be controlled. Also it is an active field of research due to benefits of "underactuation" property.

Many control methods are applied on fully actuated systems (where number of actuators are equal to the degrees of freedom to be controlled) such as partial feedback linearization collocated and non-collocated, passivity, adaptive and fuzzy control. These techniques can not be applied on underactuated mechanical systems, due to non-holonomic nature of such systems.

In this research work, we focus on stabilization of planar vertical take-off and landing aircraft system (PVTOL). First order sliding mode control, adaptive sliding mode control and backstepping techniques are applied to stabilize the PVTOL system. Efficasy of the suggested algorithms is verified by the simulation studies via MATLAB software.

1.2 Motivation

Practical importance and theoretically challenging nature of underactuated mechanical systems motivates us for investigating a control design framework for realization of aforementioned benefits in practical applications.

Non-linear behaviour and the reduced dimension of the input-space are the basic reasons which makes the use of UMS complicated. Many control methods were designed to make it suitable to non linearities and to minimize the system order to lower dimensional model. Nevertheless, the aforementioned methods are not practically relevant due to system constraints (e.g. limitation of actuator power), which is motivational aspects regarding PVTOL underactuated system. Control of PVTOL system becomes challenging problem. The aforementined discussion clarifies to intend a control design approach that may able to provide required robustness with minimal chattering and improved performance.

Need and importance of underactuated mechanical systems are already established in engineering and military applications due to its applicability toward surface and underwater vehicles, and for getting more practical advantages like weight and cost reduction. The importance of research in this domain is the ability of backup control capability in order of the control of failure of fully-actuated system. The aspect of stabilization of PVTOL underactuated system will always appear likely or unlikely in aforementioned scenarios, which is discussed in this thesis and becomes the notion of this work.

1.3 Application of Research

In this modren era, we live in a world of machines. These machines can be underactuated or fully actuated. In case of failure of fully actuated systems due to constrution constraints and actuator faliure, the degree of freedom become more than the number of acutators, so the only choice to deal with it is as underactuated system. Due to application perspective of UMS, world is quite rich while considering larger scale, every industry is equipped with the machinery based on underactuation phenomenon up to some extent.

Due to its broad range of applications in science and engineering, control of underactuated systems is extremely important and so it is now an active field for researchers. Further, applications of underactauted systems includes marine, space robot, spacecraft, PVTOL, areal vehicles, under water vehicles, mechatronics and hybrid machines. In several applications, simplification in actuation system can minimize weight, system structure design and energy consumption while keeping its proper functionality. In few other applications, UMS are designed to tolerate actuators failure. In wider sense, all non-linear control systems are underactuated upto some extent, because to apply available control laws, it is convenient to neglect several high order dynamics and some non-linearities.

1.4 Problem Statement and Research Objectives

Underactuated mechanical systems have practical importance along with the added benefits of underactuation. But the advantages associated with uncertainties come at higher cost of difficult control design due to complicated non-linearity and control coupling. Lack of direct actuation for some degree of freedoms of planar vertical and planar landing aircraft, the PVTOL becomes more sensitive to external disturbances.

The research goal in this work is to investigate, using backstepping, sliding mode control and adaptive sliding mode control theory is applied to get a improved performance and robust control of PVTOL. Finally the suggested framework will be numerically validated in MATLAB.

1.5 Thesis Organization

The outline of the thesis look like:

Chapter 2 – Literature Review: In this chapter, we will get re-view about literature already published about PVTOL underactuated system.Then the suggested work is established by analyzing of this literature for PVTOL.

Chapter 3 – Control algorithms for stabilization of PVTOL UMS with 3-DOF: In this chapter, first order sliding mode control, adaptive sliding mode and backstepping control are suggested for PVTOL underactuated systems.

Chapter 4 – Application to algorithms: In this chapter, the suggested algorithms is applied to PVTOL underactuated mechanical systems and simulations are verified using MATLAB.

Chapter 5 – Conclusion and Future Work: This specific chapter conclude the applied algorithms results and also cover the upcoming work on these systems, which can be achieved.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, Literature about PVTOL underactuated mechanical system and sliding mode control are reviewed and presented. Different control design techniques developed over the past years are reviewed.

For fully-actuated systems, a wide range of design algorithms exists for the sake to improve performance and robustness which consist of optimal control, feedback linearization, passivity based control strategies etc. These algorithms are not suitable to apply for the whole class of underactuated systems because most often such system are not linearizable using smooth feedback [6] , and sometime because of unstable hidden modes of these systems.

Many traditional and recent strategies of non-linear control design including backstepping [6] [7] , forwarding [7] [8] [9], low-gain/high-gain designs [10] and sliding mode control (SMC) [11] are not openly applicable to UMS leaving some of the few special exceptions (e.g. the cart-pole system and beam-and-ball system). This is due to the fact that a method for transforming underactuated systems into cascade nonlinear systems with upper/lower triangular or nontriangular structural properties has not yet been discovered.

2.2 Underactuated Mechanical Systems

A system , which has less numbers of actuators when compared to its degree of freedom is known as underactuated mechanical system. UMS are used for the purpose to have a cost reduction, weight reduction and energy usage reduction while keeping important features of the underactuation. Due to its broad range of applications in science and engineering, control of underactuated systems is extremely important and so it is now an active field for researchers.

During the past two decade, many researchers having interests in non linear control theory, automation and robotics, autonumous vehicles control and particularly control of PVTOL underactuated system. Control of general PVTOL and other underactuated mechanical systems is presently considered a big open problem based on surveys [12–14].

Examples of these systems are mobile robot, helicopter, undreractuated manipulator, space robot, spacecraft, surface vessels, PVTOL and under water vehicles. Fully actuated system doesn't have such challenges as in underactuated mechanical systems. Many control techniques have been presented [6] [7], which includes backstepping, energy and passive-built regulator, intelligent and fuzzy control and hybrid and switched control. It's difficult to pinpoint the general concept that permits to conduct a regular investigation of PVTOL underactuated system because the variety and broad research on this topic.

Spong [15] did first generalization of underactuated systems, where it was shown that UMS could be partially linearized by feedback locally. According to variable of actuation, he proposed changes in the input that convert non-linear models into partially linear models. But, the new control comes in both converted subsystems. However, first classification of underactuated mechanical systems according to equivalent Control Flow Diagram was given by Seto [16], He showed the method of generalized forces to be transmitted by degrees of freedom.

Later on, Olfati-Saber [17] gave second classification of underactuated mechanical system, which is based upon several system's structural properties like integrable normalized generalized momentums, kinetic symmetry, actuation mode and interacting inputs [17]. Some of the most recognized work with respect to energy point of view include Spong [15] ,Astrom [18] and Bloch [19] . In the same way, passivity-built procedures also includes in swinging and routing the former systems, but for the purpose to guide them to the homoclinic path. Jankovic [20] and Sepulchre [21] also possess his work on passivity-based control and introduces the system transformation in a cascade form. Kolesnichenko also posed such work for TORA and pendubot. Hauser [22] posed approximate linearization methodology for ball and beam balancer. In [23–25] stabilization of PVTOL is discussed. Inertia wheel pendulums and cranes have been investigated widely because of their extensive use in industry. Also reviewing models, applications and control techniques are studies and conserved.

2.3 VTOL

VTOL are aerial crafts which can take-off and lands vertically, with no run on a runway. It theoretically means that it can take-off and land almost anywhere. The VTOL technology is mainly classified based on the nature of requirement such as transport and air strike. The transport class aircraft with VTOL technology are mostly rotorcraft like Helicopters, Gyrodynes, tilt rotors and tilt wings, which use advanced turbo shaft engines. On the other hand, the attack jets which use VTOL technology primarily contain light weight, efficient turbofans augmented with power lift fans to initiate take off.

VTOL aircrafts requires less physical space and infrastructure to get into the flight as compared to other planes, which means more fighters on single craft carrier making it dream for military purposes. Some areal vehicles are really VTOL aircraft. The first operational VTOL jet aircraft was the British Royal Air Force Hawker Harrier, established in 1969. It was one of several successes among many failed efforts to develop VTOL that were underway in the 60s. The motive behind establishing a VTOL is to manufacture an aircraft that is able of vertical take-off like helicopter or quad-rotor having the preferable characteristics of fixed-wing aircraft like high cruise speeds.

Aviation control is an important problem of control that seems in several applications like space-craft, helicopter and aircraft. Whole dynamics of aircraft and helicopters are very complex and some how unmanageable for control purpose. It's also interesting to keep the main and important features, which must be examined while designing control law for practical aircraft.

2.4 Modeling of PVTOL

Unmanned aerial vehicles (UAVs) is huge part of the electronics industry, because of its adaptability and mainly because of the dropping costs of their electronic parts. Unmanned aerial vehicles is a type of areal vehicles which is capable of

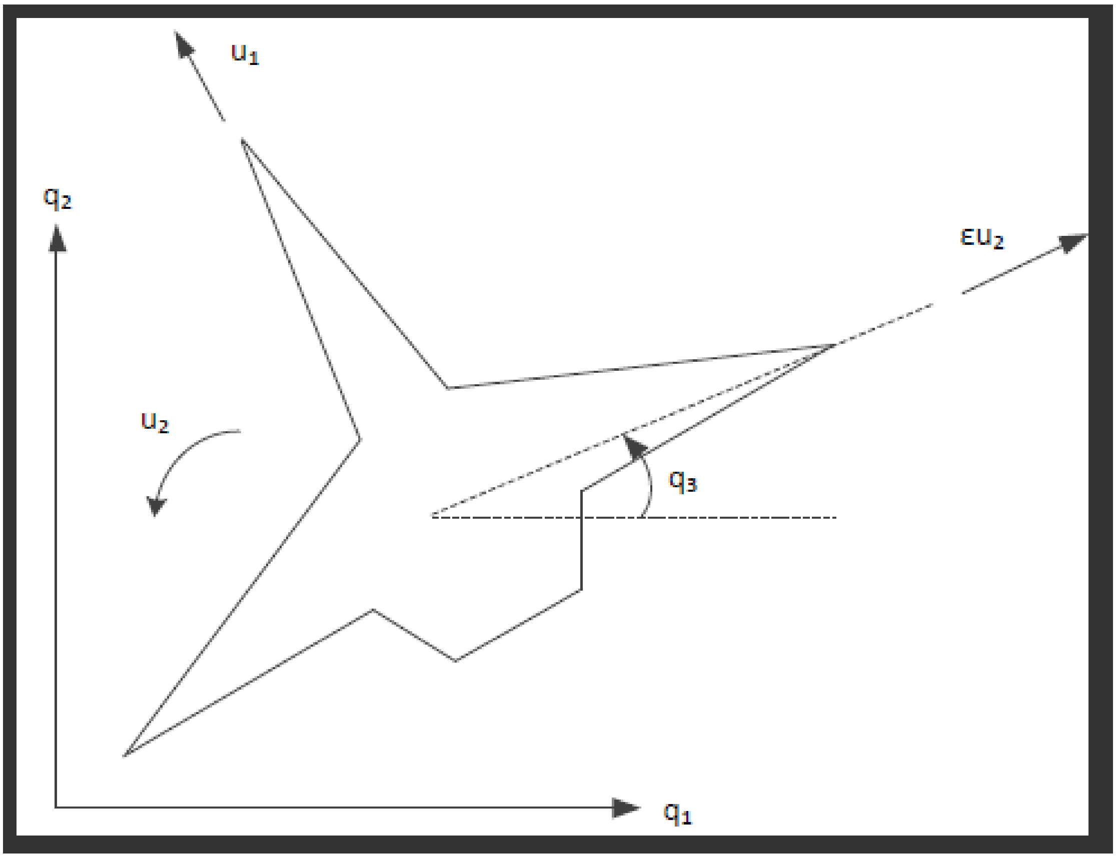


FIGURE 2.1: The VTOL

taking-off vertically like helicopters and it is presented by planer vertical take-off and landing (PVTOL) craft model. PVTOL is a testing non-linear system for control designers and researchers. This system also presents a specific case of what is known as "motion control". Reliability requirements of areal vehicles needs fault detection and isolation. So non-linear procedure for observation and protection of faults are used. Suggestion is given by Lagrangian model for PVTOL aircraft for the purpose of developing an algorithm for the detection, identification and isolation of the faults [26, 27].

2.4.1 Lagrangian Model

There is an immense quantity of literature of Lagrange's equations of motions. Structure of Planar vertical take off and landing (PVTOL) aircraft is presented in the Fig (2.2).

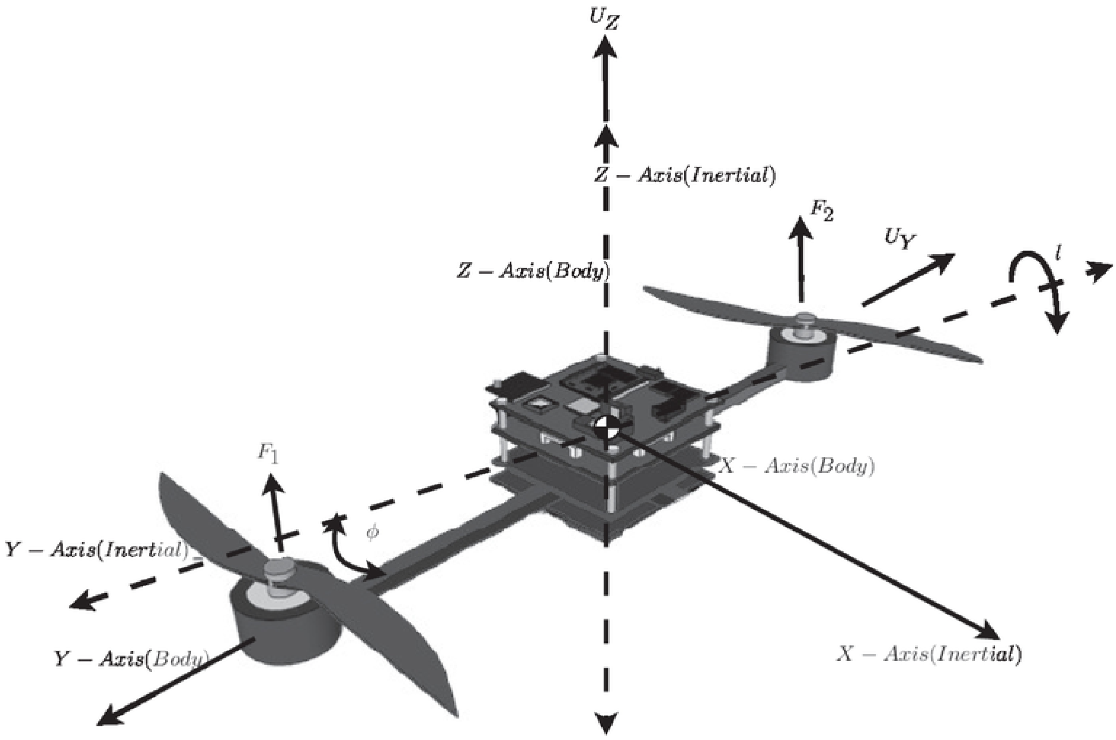


FIGURE 2.2: Structure of PVTOL

The throughout linear position of the PVTOL is described in the inertial frames x-y-z axis with two generic coordinates, an additional generic coordinate $\xi \triangleq [x, y]^T$, while the angular position is described in the inertial frame. Consider pitch angle as θ,i.e.the rotation angle about the y-axis, and the yaw angle as ψ, i.e. the rotation of the PVTOL around the z-axis is zero. Roll angle is the only angular movement θ i.e, about the x-axis, its rotation.

$$\xi = \begin{bmatrix} y \\ z \end{bmatrix}, \ \eta = \phi, \ q = \begin{bmatrix} y \\ z \\ \phi \end{bmatrix} \tag{2.1}$$

Body frame's origin (also the origin of the inertial frame) is center of mass of the Planar vertical take-off and landing (PVTOL) systems. The PVTOL have uniform structure with aligned two arms with the body's x-axis. J_x represent the inertia.

Lagrangian is described as the sum of kinetic energy subtracted the potential energy E_{pot}. In Planar vertical take off and landing (PVTOL) case, the kinetic energy includes 2 parts, first is composed of translational energy E_{tran} and the other one is the rotational energy E_{rot}.

$$\mathcal{L}(q, \dot{q}) = E_{rot} + E_{tran} - E_{pot} \tag{2.2}$$

Equation of lagrange is considered by the equation

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q} = \begin{bmatrix} f_y \\ f_z \\ \ell_f \end{bmatrix} \tag{2.3}$$

In which f_y represents generic forces acting on the y-axis, f_z represents generic forces acting on z-axis and ℓ_f is the moment. For results of PVTOL

$$E_{tran} = \frac{1}{2} m [\dot{y} \ \ \dot{z}] \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} \tag{2.4}$$

$$E_{rot} = \frac{1}{2} J_x \omega^2 = \frac{1}{2} J_x \dot{\phi}^2 \tag{2.5}$$

$$E_{pot} = mgz \tag{2.6}$$

through which the Lagrangian's motion eqs gives

$$\mathcal{L}(q,\dot{q}) = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 + \frac{1}{2}m\dot{J}_x\dot{\phi}^2 - mgz \tag{2.7}$$

and the terms

$$\frac{\partial \mathcal{L}}{\partial \dot{q}} = \begin{bmatrix} m\dot{y} \\ m\dot{z} \\ J_x\dot{\phi} \end{bmatrix} \tag{2.8}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \begin{bmatrix} m\ddot{y} \\ m\ddot{z} \\ J_x\dot{\omega} \end{bmatrix} = \begin{bmatrix} m\ddot{y} \\ m\ddot{z} \\ J_x\ddot{\phi} \end{bmatrix} \tag{2.9}$$

$$\frac{\partial \mathcal{L}}{\partial q} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix} \tag{2.10}$$

The generic forces are given by (in inertial-frame)

$$f_y = \cos(\phi)U_y - sin(\phi)U_z \tag{2.11}$$

$$f_z = \sin(\phi)U_y + cos(\phi)U_z \tag{2.12}$$

Here

- U_z represents (the force's sum for the every single rotor) total thrust forces, which are acting on body frame's z-axis.
- U_y refer to the side forces on the body frame's y-axis.
- While ℓ_f is the moment, which is acting on the rolling angle.

Motions equations given as;

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_x \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ m \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\phi) & 0 \ \sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1_x \end{bmatrix} \begin{bmatrix} U_X \\ U_z \\ \ell_f \end{bmatrix} \tag{2.13}$$

2.5 Examples of Underactuated Mechanical System

The underactuated mechanical systems consist of the ball and beam system, translational oscillator with rotational actuator (TORA) system, the acrobat, the cart-pole system, the pendubot, inertia-wheel pendulum,the double inverted pendulum and the rotating pendulum.

All examples are selected because of the complexity of their control design and fact that for control and analysis purposes that they are of high interest in the literature. We briefly introduce some examples with its related control design task.

2.5.1 Pendubot and Acrobot

Acrobot and Pendubot are two-linked manipulators with one actuator at shoulder and at the elbow respectively. Both manipulators have identical equation of motion and alike graphically too. The stabilization of to link manipulator of its upright equilibrium point ($q_1 = \pi/2$ *and* $q_2 = 0$) is the control task from any initial condition.

One famous approach of control is the Energy based control used to swing up the system from its steady downward spot upto some precarious up right spot, and to swap it for stabilization to a linear controller.

Pendubot and the Acrobot graphically appears to be really similar (i.e. the share the exact same inertia matrix). The contrast in position of their one actuator make big difference in their normal characterization (i.e, standard form) and control design.

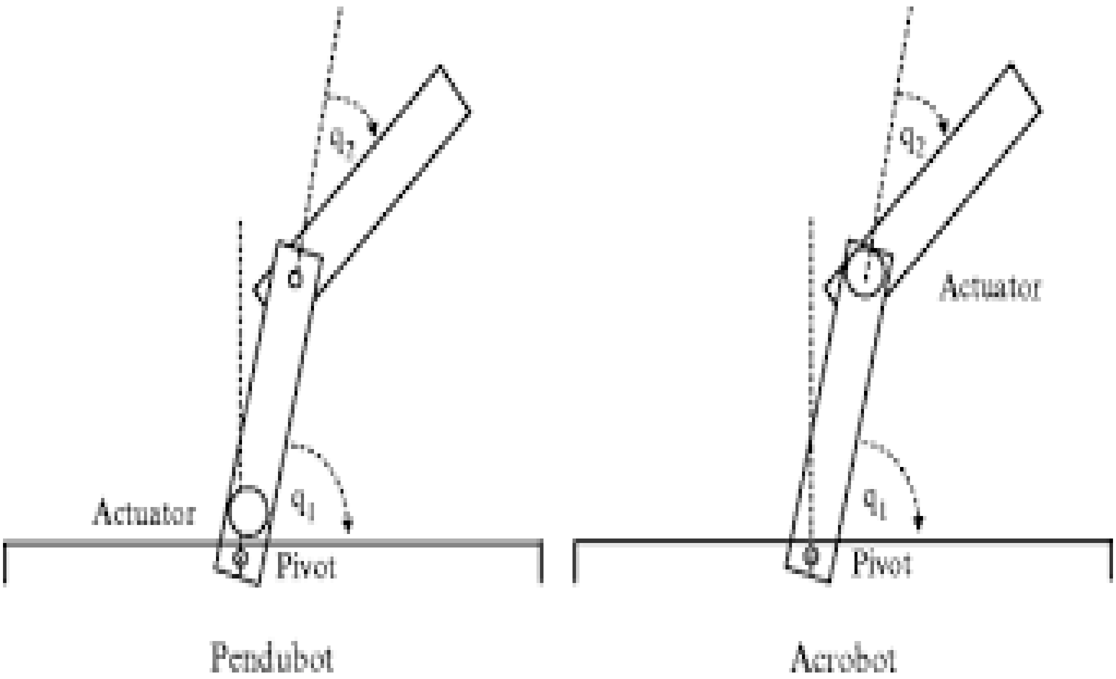


FIGURE 2.3: Pendubot And Acrobot

Swinging up these double inverted pendulum system from their downward initial positions and bringing them near to their up right euilibrium position, then changing to linear controller around upright equilibrium point as proposed in literature [28–30] .

2.5.2 The Cart-Pole System and Rotating Pendulum

Cart and pole system is a benchmark nonlinear underactuated system. For nonlinear control study, It can be utilized as a test for several control algorithms varification. Control job is the swinging up the pendulum out of steady downward zero state. ($q_1 = 0$ *and* $q_2 = \pi$) vertical unbalanced equilibrium point ($q_2 = 0$), having retaining cart at its original point ($q_1 = 0$).

Resemblance between the Cart-Pole system and the rotating pendulum is that both have the similar model of the potential energy. Swinging up control design for inverted pendulum in the Cart and Pole system [15, 31, 32] and the Rotating pendulum [18, 33, 34] has been done by several researchers to stabilize the Cart-Pole system and Rotating Pendulum respectively.

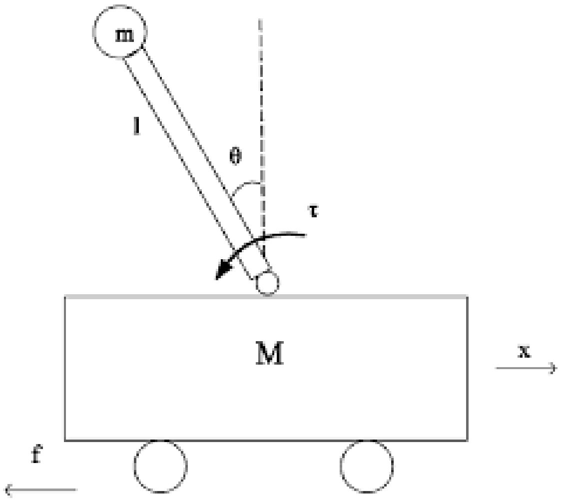


FIGURE 2.4: Cart Pole System

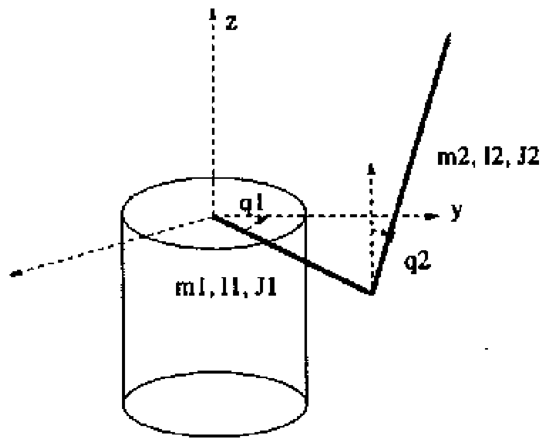


FIGURE 2.5: Rotating pendulum

2.5.3 The Ball and Beam System

Ball and beam system consists of beam able to move up and downward with motor connected at its one end whereas the another end of beam is fixed. As this beam is made of metal and iron ball is allowed to move freely on it, bringing the ball to middle of the beam by applying some moment starting from any initial condition on the beam is the control task.

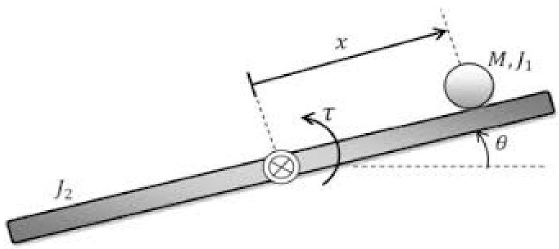


FIGURE 2.6: The Ball-and-Beam System

Due to complexity of this system, tracking and stabilization for the ball and beam system using output or state feedback has been observed by many researchers and designer [10, 35–37]. In [22], tracking of the ball-and-beam system was considered utilizing approximate input-output linearization. Similarly, Using output feedback , Teel and Praly [36] presented the stabilization of this system.

In [38], global stabilization of the ball-and-beam systems was attained with friction using a numeric category of the method of building of Lyapunov functions having some cross terms, that was basically because of Praly and Mazenc [39].

2.5.4 TORA System

The TORA system is a non-linear benchmark example for different control techniques and was first introduced in [40]. This system contains an oscillating translational stage with mass m_1, which is controlled with rotational eccentric mass m_2 to make sure the horizontal displacement q_1.

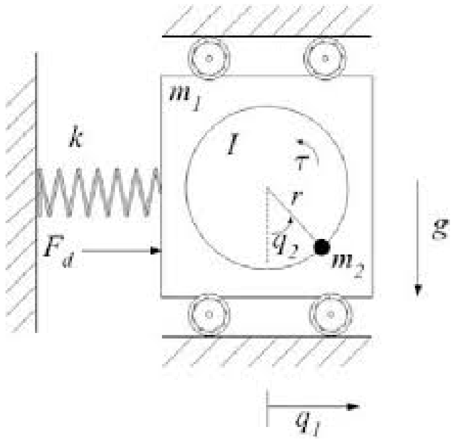


FIGURE 2.7: The Ball-and-Beam System

TORA isn't fully feedback linearizable, because of the fact, an easy swap of coordinates was available convert TORA to the cascade nonlinear system, which experience some great notice by many research scholars [20, 41, 42] . In almost all of these tasks, the TORA system was consider as Standard sample for passivity-based techniques and tracking/stabilization utilizing output feed-back in zero gravity, i.e. g = 0.

2.5.5 The Inertia-Wheel Pendulum

Spongy was the first to introduce inertia-wheel pendulum in [43]. It consist of a pendulum which has at its end a revolving uniform inertia wheel. Pendulum is unactuated and with help of rotating wheels the system is going to be controlled. While the wheel stops rotating , to make the pendulum stable in its up-right equilibrium point is the control task. it is not important to consider the specific angle of revolution of the wheel.

The first example of a flat UMS is inertia-wheel pendulum with 2 degrees of freedom (DOF) and single control input. Which is because of its constant inertial matrices.

In [44], to swing up the pendulum, an energy based methodology is used and then a supervisory based switching strategy is deployed to change to a stabilizing local nonlinear controller via comparatively big region of attraction.

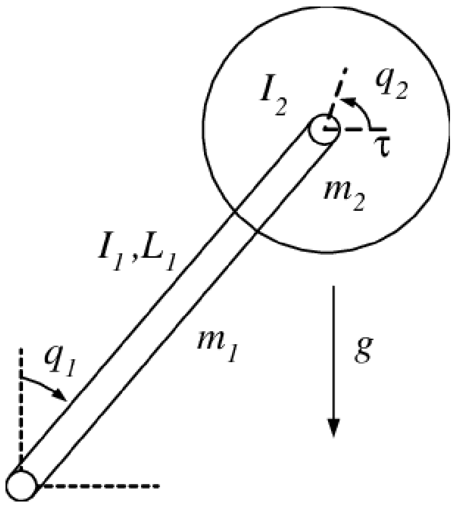


FIGURE 2.8: Inertia Wheel System

2.6 Sliding Mode Control

For the control of underactuated mechanical systems, Uncertainties is a big issue. As a gap exist between practical system and the model, which yields these model uncertainties, unmodeled dynamics and parameter variations.
For overcoming such issues of uncertainties, some techniques are used like robust control etc. but these techniques also have some limitations like, it is some time applicable to small uncertainties only or sometime sensitive to unstructured uncertainties [45, 46].

A variable structured controller, such as sliding mode controller gets valuable attention from researchers in recent years. It provides a very good system performance against disturbance rejection, model imperfection and to aforementioned uncertainties.

Sliding mode control strategy is found to be very successful with respect to underactuated systems, their applications can be found in inverted pendulum [47] , surface vessel [48, 49] , helicopter [50], ball and beam [51] , satellite [52] , overhead crane [53] and TORA [54] . Some researchers also worked to develop a universal sliding mode control for the underactuated systems [49, 55] .

SMC grew very rapidly from last two decades, but it is effected from a chattering phenomena discussed in next section. SMC happens in two stages named sliding phase and reaching phase.

2.6.1 Sliding Surface

For applying SMC on a system, switching surface design is the first step. The switching-surface is also known as sliding-surface. When we define the sliding surface, the aforementioned two stages comes in to place. Reaching phase is achieved first, which is accountable for the attraction of system states from any initial condition to the sliding surface. While the reaching phased is attained, and the system placed on the sliding surface, then sliding phase comes into place, and the states of the system slides towards the equilibrium point using some discontinuous control action (that too shows some robustness). Fig. 2.9 shows the reaching phase (RP), sliding mode (SM) and sliding surface (SS).

2.6.2 Chattering Phenomenon

Because of the discontinuous nature of sliding mode control, high frequency oscillations will be produced in the system, which is named as chattering. This occurrence leads to undesirable oscillations that degrade the performance of the system. In real applications, it is quite dangerous for the mechanically moving parts along with actuators health, and it also increases the use of the actuators, it may also lead towards total system failure. To overcome chattering effect, numerous solutions of chattering problem have been suggested in [56].

Based on the estimation of sliding variable, A design scheme was presented [57]. The describing function approach was evolved for chattering investigation of structure in the occurrence of the un-modeled dynamics. Another way to minimize chattering effect on the system is using some Higher Order SMC (HOSM) control approach in [58].

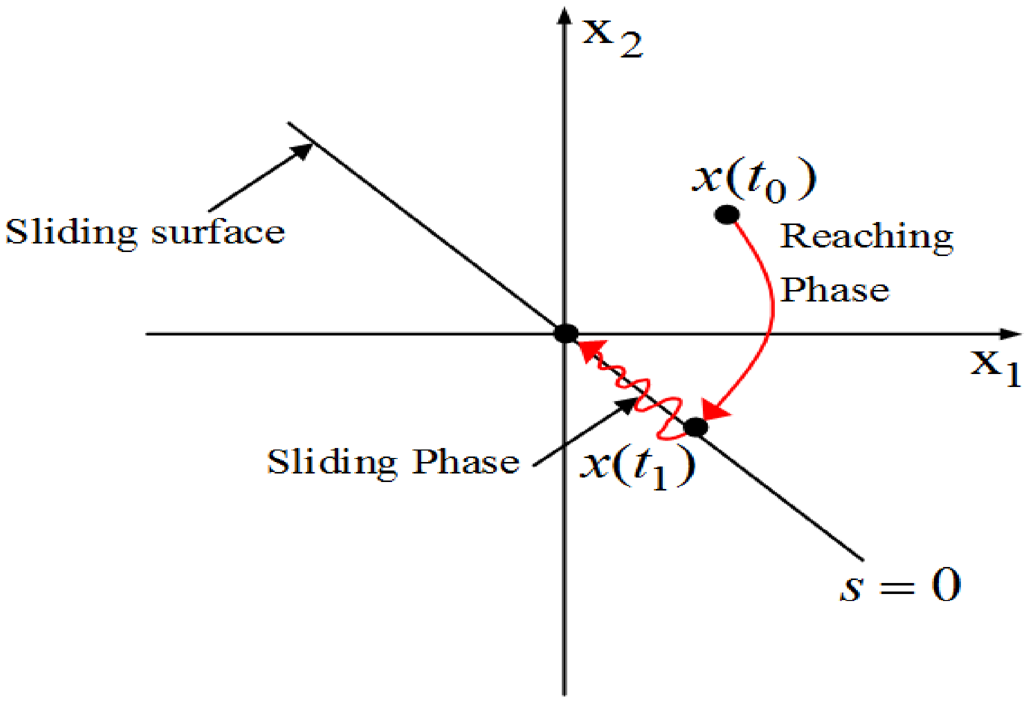


FIGURE 2.9: The Sliding Phase, Reaching Phase and Sliding Surface

Chapter 3

Control Algorithms for Stabilization of PVTOL UMS with 3 DOF

The planar vertical take-off and landing aircraft problem (PVTOL) falls under actuated systems UMS with 3 degrees of freedom (3DOF) and two control inputs. In the last decades, it has attracted a lot of attention of control researchers as presented in [13, 59]. The PVTOL is a particular case of the so called motion control problem and is considered as a benchmark non linear control system problem.

3.1 Adaptive Sliding Mode Control

In this sections control technique is presented for stabilization of PVTOL based on adaptive sliding mode technique. First of all, Input transformation is used to transform the system into a special structure containing some unknown term. The dimension of the system increases. The unknown term is then computed adaptively and the system is stabilized using adaptive sliding mode control. Computer simulation results show the effectiveness of the suggested control algorithm on these systems.

Given Fig (3.1) is a simplified planer model of a real vertical take off and landing plane (PVTOL). Dynamics of PVTOL is given in [17].

$$\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= -u_1 sin\theta + \epsilon u_2 cos\theta \\
\dot{y_1} &= y_2 \\
\dot{y_2} &= u_1 cos\theta + \epsilon u_2 sin\theta - g \\
\dot{\theta} &= \omega \\
\dot{\omega} &= u_2
\end{aligned} \tag{3.1}$$

Where,

$x_1 = q_1$ (Horizontal displacement)

$y_1 = q_2$ (Vertical displacement)

$\theta = q_3$ (Roll angle, which PVTOL make with horizontal line).

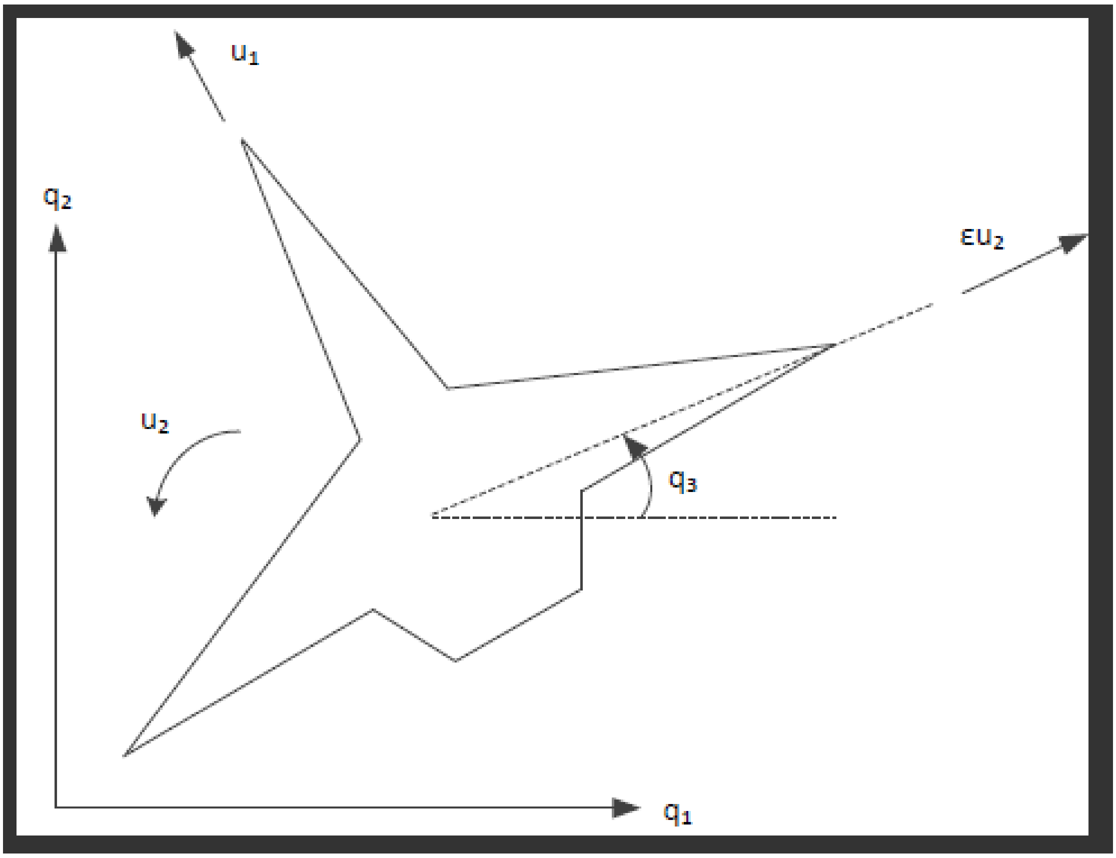


FIGURE 3.1: The VTOL

u_1 is collective input and u_2 is the couple as given in the Fig. (3.1). A small coefficient parameter ϵ is defined, that presents the aircrafs coupling between the pitching moment and the lateral acceleration. Gravitational acceleration is given by the term g. In case with $\epsilon << 1$ results in a weak input coupling, as given in the literature [17] [60] [61] . When $|\epsilon|$ is relatively small, the system is non minimum phase, and if $|\epsilon| >> 1$, the system is slightly strongly non-minimum phase [17] . The stabilization of non-minimum phase system is considered to be difficult, we are interested in a strong input coupling case that is the non-minimum phase. By defining the state variables as:

$$\begin{aligned} x_1 &= x \quad & x_2 &= \dot{x_1} \quad & x_3 &= y = y_1 \\ x_4 &= \dot{x_3} \quad & x_5 &= \theta \quad & x_6 &= \dot{x_5} \end{aligned} \tag{3.2}$$

the dynamics can be written as:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -u_1 sinx_5 + \epsilon u_2 cosx_5 \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= u_1 cosx_5 + \epsilon u_2 sinx_5 - g \\ \dot{x_5} &= x_6 \\ \dot{x_6} &= u_2 \end{aligned} \tag{3.3}$$

These equations represents the state space model for PVTOL system.

3.1.1 The Control Problem

A desired set point is given $x_{des} \in R^6$, A feedback strategy is build in control's terms $u_i : R^6 \rightarrow R, i = 1,2$ in such a way that the desired set point x_{des} is an attractive set for (3.3), so that there exists an $\epsilon > 0$, in such a way $x(t, t_0, x_0) \rightarrow x_{des}$ as $t \rightarrow \infty$ for either initial condition $(t_0, x_0) \in R$. With out losing the generality, we supposed that $x_{des} = 0$, which could be attained by some appropriate translation of coordinate systems.

3.1.2 Applied Algorithm

Step 1:

By choosing

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\sin x_5 & \cos x_5 \\ \frac{1}{\epsilon}\cos x_5 & \frac{1}{\epsilon}\sin x_5 \end{bmatrix} \begin{bmatrix} x_3 \\ x_5 + g + v \end{bmatrix} = \begin{bmatrix} -x_3 \sin x_5 + \cos x_5 (x_5 + g + v) \\ \frac{1}{\epsilon} x_3 \cos x_5 + \frac{1}{\epsilon} \sin x_5 (x_5 + g + v) \end{bmatrix} \tag{3.4}$$

where v is designed such that the system expressed in (3.3) becomes,

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3 \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= x_5 + v \\ \dot{x_5} &= x_6 \\ \dot{x_6} &= u_2 = \frac{1}{\epsilon} x_3 \cos x_5 + \frac{1}{\epsilon} \sin x_5 (x_5 + g + v) \end{aligned} \tag{3.5}$$

Step2:

Now define new state variables as:

$z_1 = x_1,\ z_2 = x_2,\ z_3 = x_3,\ z_4 = x_4,\ z_5 = x_5 + v,\ z_6 = x_6 + \dot{v},\ z_7 = v,\ z_8 = \dot{v}$

then system (3.5) becomes as:

$$\begin{aligned} \dot{z_1} &= z_2 \\ \dot{z_2} &= z_3 \\ \dot{z_3} &= z_4 \\ \dot{z_4} &= z_5 \\ \dot{z_5} &= z_6 \\ \dot{z_6} &= u_2 + \ddot{v} = \frac{1}{\epsilon} z_3 \cos(z_5 - z_7) + \frac{1}{\epsilon} \sin(z_5 - z_7)(z_5 + g) + \ddot{v} \\ \dot{z_7} &= z_8 \\ \dot{z_8} &= \ddot{v} \end{aligned} \tag{3.6}$$

Step 3:

By adding and subtracting ω in last equation $\dot{z}_8 = \ddot{v}$ we have $\dot{z}_8 = \ddot{v} + \omega - \omega$. Assume that second ω is unknown and can be computed adaptively. Let $\hat{\omega}$ be the estimate value of ω and $\tilde{\omega}$=$\omega - \hat{\omega}$ be the error in the estimation of ω. Therefore the system (3.6) can be written as:

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= z_5 \\
\dot{z}_5 &= z_6 \\
\dot{z}_6 &= u_2 + \ddot{v} = \frac{1}{\epsilon} z_3 \cos(z_5 - z_7) + \frac{1}{\epsilon} \sin(z_5 - z_7)(z_5 + g) + \ddot{v} \\
\dot{z}_7 &= z_8 \\
\dot{z}_8 &= \ddot{v} + \omega - \hat{\omega} - \tilde{\omega}
\end{aligned} \tag{3.7}$$

Eq. (3.7) can be decompose into two subsystems.

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= z_4 \\
\dot{z}_4 &= z_5 \\
\dot{z}_5 &= z_6 \\
\dot{z}_6 &= u_2 + \ddot{v} = \frac{1}{\epsilon} z_3 \cos(z_5 - z_7) + \frac{1}{\epsilon} \sin(z_5 - z_7)(z_5 + g) + \ddot{v}
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
\dot{z}_7 &= z_8 \\
\dot{z}_8 &= \ddot{v} + \omega - \hat{\omega} - \tilde{\omega}
\end{aligned} \tag{3.9}$$

Eq. (3.8) represents the first subsystem and (3.9) represent these second subsystem.

Step 4:

Defining the Hurwitz sliding surface for (3.8) and (3.9) using formula
$s = (1 + \frac{d}{dt})^{n-1} z_i$
which gives for subsystem (3.8)

$$s_1 = z_1 + 5z_2 + 10z_3 + 10z_4 + 5z_5 + z_6 \tag{3.10}$$

and for (3.9)

$$s_2 = z_7 + z_8 \tag{3.11}$$

then

$$\dot{s_1} = z_2 + 5z_3 + 10z_4 + 10z_5 + 5z_6 + u_2 + \ddot{v} \tag{3.12}$$

by choosing

$$\ddot{v} = -z_2 - 5z_3 - 10z_4 - 10z_5 - 5z_6 - u_2 - ksign(s_1) - ks_1 \tag{3.13}$$

we have

$$\dot{s_1} = -ksign(s_1) - ks_1 \tag{3.14}$$

and

$$\dot{s_2} = z_8 + \ddot{v} + \omega - \hat{\omega} - \tilde{\omega} \tag{3.15}$$

Step 5:

By choosing a Lyapunov function $V = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 + \frac{1}{2}\tilde{\omega}^2$, design ω and the adaptive laws for $\tilde{\omega}$ and $\hat{\omega}$ such that $\dot{V} < 0$.

Theorem 1:

Consider a Lyapunov function $V = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 + \frac{1}{2}\tilde{\omega}^2$, then $\dot{V} < 0$ if ω and the adaptive laws for $\tilde{\omega}$ and $\hat{\omega}$ are choosen as:

$\omega = -z_8 - \ddot{v} + \hat{\omega} - k_1 sign(s_2) - k_1 s_2$

$\dot{\tilde{\omega}} = s_2 - k_2\tilde{\omega}$

$\dot{\hat{\omega}} = -s_2 + k_2\tilde{\omega},$

here, $k_1, k_2 > 0$.

since,

$$\begin{aligned}\dot{V} &= s_1\dot{s_1} + s_2\dot{s_2} + \tilde{\omega}\dot{\tilde{\omega}} \\ &= s_1(-ksign(s_1) - ks_1) + s_2(z_8 + \ddot{v} + \omega - \hat{\omega} - \tilde{\omega}) + \tilde{\omega}\dot{\tilde{\omega}} \\ &= -k|s_1| - ks_1^2 + s_2(z_8 + \ddot{v} + \omega - \hat{\omega}) + \tilde{\omega}(\dot{\tilde{\omega}} - s_2)\end{aligned} \tag{3.16}$$

By using:

$\omega = -z_8 - \ddot{v} + \hat{\omega} - k_1 sign(s_2) - k_1 s_2$

$\dot{\tilde{\omega}} = s_2 - k_2\tilde{\omega}$

$\dot{\hat{\omega}} = -s_2 + k_2\tilde{\omega}$, $k_1, k_2 > 0$.

We have,

$\dot{V} = -k|s_1| - ks_1^2 - k_1|s_2| - ks_2^2 - k_2\tilde{\omega}^2 < 0$

From this we conclude that $s_1, s_2, \tilde{\omega} \to 0$. Since $s_1, s_2 \to 0$, therefore $z_i \to 0$, $i = 1, 2, ..., 8$. thus $x_i \to 0$, $i = 1, 2, ..., 6$.

3.2 First Order Sliding Mode Control

Here, we present control algorithm for the stabilization of PVTOL based on first order sliding mode control technique.

3.2.1 Applied Algorithm

The dynamic model of PVTOL is given as in [13]

$$\begin{aligned}\dot{q_1} &= p_1 \\ \dot{p_1} &= -sin(\theta_1)u_1 + \epsilon cos(\theta_1)u_2 \\ \dot{q_2} &= p_2\end{aligned}$$

$$\dot{p_2} = cos(\theta_1)u_1 + \epsilon sin(\theta_1)u_2 - g$$
$$\dot{\theta}_1 = \theta_2 \tag{3.17}$$
$$\dot{\theta}_2 = u_2.$$

After introducing the following transformation proposed in [62]:

$$\begin{aligned} x_1 &= q_1 - \epsilon sin(\theta_1) \\ x_2 &= p_1 - \epsilon\theta_2 cos(\theta_1) \\ y_1 &= q_2 + \epsilon(cos(\theta_1) - g) \\ y_2 &= p_2 - \epsilon\theta_2 sin(\theta_1) \end{aligned} \tag{3.18}$$

The system of Eq. 3.17 can be expressed as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -sin(\theta_1)\bar{u_1} \\ \dot{y_1} &= y_2 \\ \dot{y_2} &= cos(\theta_1)\bar{u_1} - g \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= u_2. \end{aligned} \tag{3.19}$$

In Eq. 3.19 $\bar{u_1} = u_1 - \epsilon\theta_2^2$. Let

$$v_1 = -\sin\theta_1\bar{u}_1 \tag{3.20}$$

$$v_2 = \cos\theta_1\bar{u}_1 - g \tag{3.21}$$

So the dynamics 3.19 becomes

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= v_1 \\ \dot{y_1} &= y_2 \\ \dot{y_2} &= v_2 \end{aligned}$$

$$\begin{aligned} \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= u_2. \end{aligned} \tag{3.22}$$

Multiply $-sin\theta_1$ and $cos\theta_1$ with Eq. (3.20) and (3.21) respectively to get

$$-\sin\theta_1 v_1 = \sin^2\theta_1 \bar{u}_1 \tag{3.23}$$

$$\cos\theta_1 v_2 = \cos^2\theta_1 \bar{u}_1 - \cos\theta_1 \tag{3.24}$$

Add Eq. (3.23) and (3.24) to get

$$\begin{aligned} \bar{u}_1 &= -\sin\theta_1 v_1 + \cos\theta_1 v_2 + \cos\theta_1 \\ u_1 &= -\sin\theta_1 v_1 + \cos\theta_1 v_2 + \cos\theta_1 + \epsilon\theta_2^2 \end{aligned} \tag{3.25}$$

Define $v_1 = y_1$ and $v_2 = \theta_1$, system (3.22) becomes

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= y_1 \\ \dot{y_1} &= y_2 \\ \dot{y_2} &= \theta_1 \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= u_2. \end{aligned} \tag{3.26}$$

Define state variables $x_3 = y_1$, $x_4 = y_2$, $x_5 = \theta_1$, $x_6 = \theta_2$. So Eq. 3.26 becomes

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3 \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= x_5 \\ \dot{x_5} &= x_6 \\ \dot{x_6} &= u_2. \end{aligned} \tag{3.27}$$

Defining the Hurwitz sliding surface for (3.26) by

$s = (1 + \frac{d}{dt})^5 x_1$

which gives,

$s = x_1 + 5x_2 + 10x_3 + 10x_4 + 5x_5 + x_6.$

then

$\dot{s} = x_2 + 5x_3 + 10x_4 + 10x_5 + 5x_6 + u_2.$

by choosing

$u_2 = -x_2 - 5x_3 - 10x_4 - 10x_5 - 5x_6 - ksign(s) - ks,\ k > 0,$

we have

$\dot{s} = -ksign(s) - ks.$

Consider a Lyapunov function

$V = \frac{1}{2}s^2,$

by taking derivative,

$\dot{V} = s\dot{s}$

$\dot{V} = |s|(-ksign(s) - ks)$

then $\dot{V} < 0$ if here, $k > 0$.

3.3 Backstepping Control

3.3.1 Applied Algorithm

The dynamic model of PVTOL is given as in [13]

$$
\begin{aligned}
\dot{q}_1 &= p_1 \\
\dot{p}_1 &= -sin(\theta_1)u_1 + \epsilon cos(\theta_1)u_2 \\
\dot{q}_2 &= p_2 \\
\dot{p}_2 &= cos(\theta_1)u_1 + \epsilon sin(\theta_1)u_2 - g \\
\dot{\theta}_1 &= \theta_2 \\
\dot{\theta}_2 &= u_2.
\end{aligned}
\tag{3.28}
$$

After introducing the following transformation proposed in [62]:

$$\begin{aligned} x_1 &= q_1 - \epsilon sin(\theta_1) \\ x_2 &= p_1 - \epsilon\theta_2 cos(\theta_1) \\ y_1 &= q_2 + \epsilon(cos(\theta_1) - g) \\ y_2 &= p_2 - \epsilon\theta_2 sin(\theta_1) \end{aligned} \tag{3.29}$$

The system of Eq. 3.28 can be expressed as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -sin(\theta_1)\bar{u_1} \\ \dot{y_1} &= y_2 \\ \dot{y_2} &= cos(\theta_1)\bar{u_1} - g \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= u_2. \end{aligned} \tag{3.30}$$

In Eq. 3.19 $\bar{u_1} = u_1 - \epsilon\theta_2^2$. Let

$$v_1 = -\sin\theta_1\bar{u}_1 \tag{3.31}$$

$$v_2 = \cos\theta_1\bar{u}_1 - g \tag{3.32}$$

So the dynamics 3.30 becomes

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= v_1 \\ \dot{y_1} &= y_2 \\ \dot{y_2} &= v_2 \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= u_2. \end{aligned} \tag{3.33}$$

By $v_1 = y_1$ and $v_2 = \theta_1$, System in 3.33 becomes

$$\dot{x_1} = x_2$$

$$\begin{aligned}\dot{x_2} &= y_1\\ \dot{y_1} &= y_2\\ \dot{y_2} &= \theta_1\\ \dot{\theta}_1 &= \theta_2\\ \dot{\theta}_2 &= u_2.\end{aligned} \tag{3.34}$$

Define state variables $x_3 = y_1$, $x_4 = y_2$, $x_5 = \theta_1$, $x_6 = \theta_2$. So Eq. 3.34 becomes

$$\begin{aligned}\dot{x_1} &= x_2\\ \dot{x_2} &= x_3\\ \dot{x_3} &= x_4\\ \dot{x_4} &= x_5\\ \dot{x_5} &= x_6\\ \dot{x_6} &= u_2.\end{aligned} \tag{3.35}$$

Let x_2 be the virtual input, z_1 be the error and α_1 be the stabilizing control

$$\begin{aligned}z_1 &= x_2 - \alpha_1\\ x_2 &= z_1 + \alpha_1\end{aligned} \tag{3.36}$$

so;

$$\dot{x_1} = z_1 + \alpha_1 \tag{3.37}$$

take lyapunov function

$$V_1 = \frac{1}{2}x_1^2 \tag{3.38}$$

So its derivative

$$\dot{V_1} = x_1\dot{x_1} = x_1(z_1 + \alpha_1) \tag{3.39}$$

Choose $\alpha_1 = -x_1$, so that

$$\dot{V_1} = -x_1^2 + x_1 z_1 \tag{3.40}$$

Hence

$$\dot{x_1} = -x_1 + z_1 \tag{3.41}$$

and

$$\begin{aligned} \dot{z_1} &= \dot{x_2} - \dot{\alpha_1} = x_3 - (-\dot{x_1}) \\ &= x_3 + x_2 = x_3 + z_1 + \alpha_1 \\ \dot{z_1} &= x_3 + z_1 - x_1 \end{aligned} \tag{3.42}$$

Let x_2 be the virtual input, z_2 be the error and α_2 be the stabilizing control

$$\begin{aligned} z_2 &= x_3 - \alpha_2 \\ x_3 &= z_2 + \alpha_2 \end{aligned} \tag{3.43}$$

so

$$\dot{z_1} = -x_1 + z_1 + z_2 + \alpha_2 \tag{3.44}$$

Chose Lyapunov

$$V_2 = \frac{1}{2} z_1^2 + V_1 \tag{3.45}$$

So its derivative

$$\begin{aligned} \dot{V_2} &= z_1 \dot{z_1} + \dot{V_1} \\ &= -x_1^2 + z_1 x_1 + z_1 \dot{z_1} \\ &= -x_1^2 + z_1(\dot{z_1} + x_1) \\ \dot{V_2} &= -x_1^2 + z_1(-x_1 + z_1 + z_2 + \alpha_2 + x_1) \end{aligned} \tag{3.46}$$

Chose $\alpha_2 = -2z_1$

$$\dot{V_2} = -x_1^2 - z_1^2 + z_1 z_2 \tag{3.47}$$

Hence

$$\dot{z}_1 = -x_1 - z_1 + z_2 \tag{3.48}$$

and

$$\begin{aligned}
\dot{z}_2 &= \dot{x}_3 - \dot{\alpha}_2 \\
&= x_4 - (2\dot{z}_1) = x_4 + 2\dot{z}_1 \\
&= x_4 + 2(-x_1 - z_1 + z_2) \\
\dot{z}_2 &= x_4 - 2x_1 - 2z_1 + 2z_2
\end{aligned} \tag{3.49}$$

Let x_4 be the virtual input, z_3 be the error and α_3 be the stabilizing control

$$\begin{aligned}
z_3 &= x_4 - \alpha_3 \\
x_3 &= z_3 + \alpha_3
\end{aligned} \tag{3.50}$$

so

$$\dot{z}_2 = -2x_1 - 2z_1 + 2z_2 + z_3 + \alpha_3 \tag{3.51}$$

Chose Lyapunov

$$V_3 = \frac{1}{2} z_2^2 + V_2 \tag{3.52}$$

So its derivative

$$\begin{aligned}
\dot{V}_3 &= z_2 \dot{z}_2 + \dot{V}_2 \\
&= -x_1^2 + z_1^2 + z_1 z_2 + z_2 \dot{z}_2 \\
&= -x_1^2 - z_1^2 + z_2(z_1 + \dot{z}_2) \\
&= -x_1^2 - z_1^2 + z_2(z_1 - 2x_1 \\
&- 2z_1 + 2z_2 + z_3 + \alpha_3 \\
\dot{V}_2 &= -x_1^2 - z_1^2 + z_2(z_1 - 2x_1 - 2z_1 + 2z_2 + z_3 + \alpha_3)
\end{aligned} \tag{3.53}$$

Chose $\alpha_3 = -2x_1 + z_1 - 3z_2$

$$\dot{V}_3 = -x_1^2 - z_1^2 + z_2^2 + z_2 z_3 \tag{3.54}$$

Hence

$$\dot{z}_2 = -z_1 - z_2 + z_3 \tag{3.55}$$

and

$$\begin{aligned}
\dot{z}_3 &= \dot{x}_4 - \dot{\alpha}_3 \\
&= x_5 - (2\dot{x}_1 + \dot{z}_1 - 3\dot{z}_2) \\
&= x_5 - [2(x_1 + z_1) + (-x_1 - z_1 + z_2) - 3(-z_1 - z_2 + z_3)] \\
&= x_5 - [-2x_1 + 2z_1 - x_1 - z_1 - z_2 + 3z_1 + 3z_2 - 3z_3] \\
&= x_5 - [-3x_1 + 4z_1 + 2z_2 - 3z_3] \\
\dot{z}_3 &= x_5 + 3x_1 - 4z_1 - 2z_2 + 3z_3
\end{aligned} \tag{3.56}$$

Let x_5 be the virtual input, z_4 be the error and α_4 be the stabilizing control

$$\begin{aligned}
z_4 &= x_5 - \alpha_4 \\
x_5 &= z_4 + \alpha_4
\end{aligned} \tag{3.57}$$

so

$$\dot{z}_3 = -3x_1 - 4z_1 - 2z_2 + 3z_3 + z_4 + \alpha_4 \tag{3.58}$$

Chose Lyapunov

$$V_4 = \frac{1}{2}z_3^2 + V_3 \tag{3.59}$$

So its derivative

$$\begin{aligned}
\dot{V}_4 &= z_3\dot{z}_3 + \dot{V}_3 \\
&= -x_1^2 - z_1^2 - z_2^2 + z_2 z_3 + z_3\dot{z}_3
\end{aligned}$$

$$\begin{aligned}
&= -x_1^2 - z_1^2 - z_2^2 + z_2 z_3 + z_3 \dot{z}_3 \\
&= -x_1^2 - z_1^2 - z_2^2 + z_3(z_2 + \dot{z}_3) \\
&= -x_1^2 - z_1^2 - z_2^2 + z_3(z_2 + 3x_1 - 4z_1 - 2z_2 + 3z_3 + z_4 + \alpha_4) \\
\dot{V}_3 &= -x_1^2 - z_1^2 - z_2^2 + z_3(3x_1 - 4z_1 - z_2 + 3z_3 + z_4 + \alpha_4)
\end{aligned} \tag{3.60}$$

Chose $\alpha_4 = -3x_1 + 4z_1 + z_2 - 4z_3$

$$\dot{V}_4 = -x_1^2 - z_1^2 - z_2^2 - z_3^2 + z_3 z_4 \tag{3.61}$$

Hence

$$\dot{z}_3 = -z_2 - z_3 + z_4 \tag{3.62}$$

and

$$\begin{aligned}
\dot{z}_4 &= \dot{x}_5 - \dot{\alpha}_4 \\
&= x_6 - (-3\dot{x}_1 + 4\dot{z}_1 + \dot{z}_2 - 4\dot{z}_3) \\
&= x_6 - [-3(-x_1 + z_1) + 4(-x_1 - z_1 + z_2) + (-z_1 - z_2 + z_3) - 4(-z_2 - z_3 + z_4)] \\
&= x_6 - [3x_1 - 3z_1 - 4x_1 - 4z_1 + 4z_2 - z_1 - z_2 + z_3 + 4z_2 + 4z_3 - 4z_4] \\
&= x_6 - [-x_1 - 8z_1 + 7z_2 + 5z_3 - 4z_4] \\
\dot{z}_4 &= x_6 + x_1 + 8z_1 - 7z_2 - 5z_3 + 4z_4
\end{aligned} \tag{3.63}$$

Let x_6 be the virtual input, z_5 be the error and α_5 be the stabilizing control

$$\begin{aligned}
z_5 &= x_6 - \alpha_5 \\
x_6 &= z_5 + \alpha_5
\end{aligned} \tag{3.64}$$

so

$$\dot{z}_4 = x_1 + 8z_1 - 7z_2 - 5z_3 + 4z_4 + z_5 + \alpha_5 \tag{3.65}$$

Hence

Chose Lyapunov

$$V_5 = \frac{1}{2}z_4^2 + V_4 \tag{3.66}$$

So its derivative

$$\begin{aligned}
\dot{V}_5 &= z_4\dot{z}_4 + \dot{V}_4 \\
&= -x_1^2 - z_1^2 - z_2^2 - z_3^2 + z_3 z_4 + z_4\dot{z}_4 \\
&= -x_1^2 - z_1^2 - z_2^2 - z_3^2 + z_4(z_3 + \dot{z}_4) \\
\dot{V}_5 &= -x_1^2 - z_1^2 - z_2^2 - z_3^2 + z_4(x_1 + 8z_1 - 7z_2 - 4z_3 + 4z_4 + z_5 + \alpha_5)
\end{aligned} \tag{3.67}$$

Chose $\alpha_5 = -x_1 - 8z_1 + 7z_2 + 4z_3 - 5z_4$

$$\dot{V}_5 = -x_1^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 z_5 \tag{3.68}$$

Hence

$$\dot{z}_4 = -z_3 - z_4 + z_5 \tag{3.69}$$

and

$$\begin{aligned}
\dot{z}_5 &= \dot{x}_6 - \dot{\alpha}_5 \\
&= u_2 - (-\dot{x}_1 - 8\dot{z}_1 + 7\dot{z}_2 + 4\dot{z}_3 - 5\dot{z}_4) \\
&= u_2 - [-(-x_1 + z_1) - 8(-x_1 - z_1 + z_2) + 7(-z_1 - z_2 + z_3) \\
&+ 4(-z_2 - z_3 + z_4) - 5(-z_3 - z_4 + z_5)] \\
&= u_2 - [9x_1 - 19z_2 + 8z_3 + 9z_4 - 5z_5] \\
\dot{z}_4 &= u_2 - 9x_1 + 19z_2 - 8z_3 - 9z_4 + 5z_5
\end{aligned} \tag{3.70}$$

Chose Lyapunov

$$V_6 = \frac{1}{2}z_5^2 + V_5 \tag{3.71}$$

So its derivative

$$\begin{aligned}
\dot{V}_6 &= z_5\dot{z}_5 + \dot{V}_5 \\
&= -x_1^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 z_5 + z_5\dot{z}_5
\end{aligned}$$

$$\begin{aligned} &= -x_1^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_5(z_4 + \dot{z}_5) \\ \dot{V}_6 &= -x_1^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_5(-9x_1 + 19z_2 - 8z_3 - 8z_4 + 5z_5 + u_2) \end{aligned} \tag{3.72}$$

Chose $u_2 = 9x_1 - 19z_2 + 8z_3 + 8z_4 - 6z_5$

so

$$\dot{V}_6 = -x_1^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 - z_5^2 < 0 \tag{3.73}$$

and

$$\dot{z}_5 = -z_4 - z_5 \tag{3.74}$$

System in z_i becomes;

$$\begin{aligned} \dot{x}_1 &= -x_1 + z_1 \\ \dot{z}_1 &= -x_1 - z_1 + z_2 \\ \dot{z}_2 &= -z_1 - z_2 + z_3 \\ \dot{z}_3 &= -z_2 - z_3 + z_4 \\ \dot{z}_4 &= -z_3 - z_4 + z_5 \\ \dot{z}_5 &= -z_4 - z_5 \end{aligned} \tag{3.75}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} \tag{3.76}$$

So the system states x_i in the form of z_i can be written as;

$$\begin{aligned} x_1 &= z_1 \\ x_2 &= -x_1 + z_1 \\ x_3 &= -z_1 + z_2 \end{aligned}$$

$$
\begin{aligned}
x_4 &= 2x_1 + z_1 - 3z_2 + z_3 \\
x_5 &= -3x_1 + 4z_1 + z_2 - 4z_3 + z_4 \\
x_6 &= -x_1 - 8z_1 + 7z_2 + 4z_3 - 5z_4 + z_5
\end{aligned}
\tag{3.77}
$$

Chapter 4

Applications of Algorithms

Introduction

In this chapters, Suggested control strategies are applied to PVTOL systems. These system have three degree of freedom (3DOF) and two control inputs. MATLAB simulation is executed to indicate the effectiveness of the suggested control strategies.

4.1 PVTOL Aircraft

PVTOL is a nonlinear benchmark underactuated system with 3DOF and 2 control inputs. In non-linear systems, researchers are struggling to stabilize the dynamics of PVTOL. In literature many researchers proposed different techniques for stabilization of PVTOL. Sliding mode control and higher order sliding mode control are proposed in literature in the presense of model uncertainities and external disturbances. A. Popov and C. Aguilar-Ibanez proposed a sliding mode control in [13, 63]. Simple real-time control strategy is proposed in [64]. Gupta and Aguilar-Ibanez presents a model predective control and backstepping techniqe respectivelly [65, 66]. Inspired by the prevailing works, we are going to acquire robust stabilization of PVTOL.

Given Fig (3.1) is a simplified planer model of a real vertical take off and landing plane (PVTOL). Dynamics of PVTOL is given in [17].

$$\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= -u_1 sin\theta + \epsilon u_2 cos\theta \\
\dot{y_1} &= y_2 \\
\dot{y_2} &= u_1 cos\theta + \epsilon u_2 sin\theta - g \\
\dot{\theta} &= \omega \\
\dot{\omega} &= u_2
\end{aligned} \tag{4.1}$$

By defining the state variables as: $x_1 = x$, $x_2 = \dot{x_1}$, $x_3 = y = y_1$, $x_4 = \dot{x_3}$, $x_5 = \theta$, $x_6 = \dot{x_5}$. the dynamics can be written as:

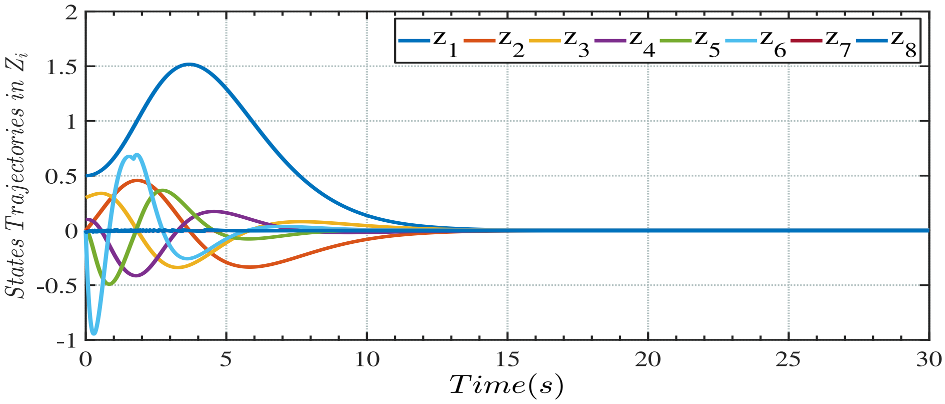
$$\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= -u_1 sinx_5 + \epsilon u_2 cosx_5 \\
\dot{x_3} &= x_4 \\
\dot{x_4} &= u_1 cosx_5 + \epsilon u_2 sinx_5 - g \\
\dot{x_5} &= x_6 \\
\dot{x_6} &= u_2
\end{aligned} \tag{4.2}$$

These equations represents the state space model for PVTOL system.

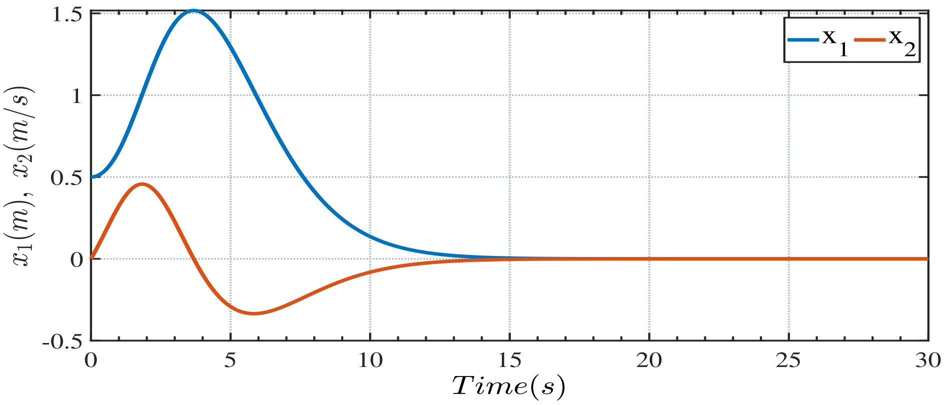
4.2 Simulation Results

In this work we suggested non linear algorithms for stabilization of PVTOL fr different initial conditions. Suggested control schemes are now used to stabilize a PVTOL system as considered in previous chapter. It is an underactuated mechanical system with 3 degree of freedom and control inputs.

4.2.1 Adaptive Sliding Mode Control



(a)



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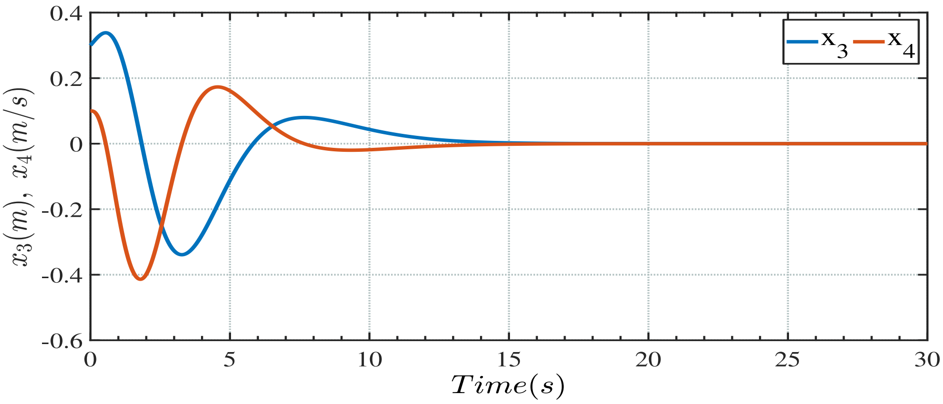
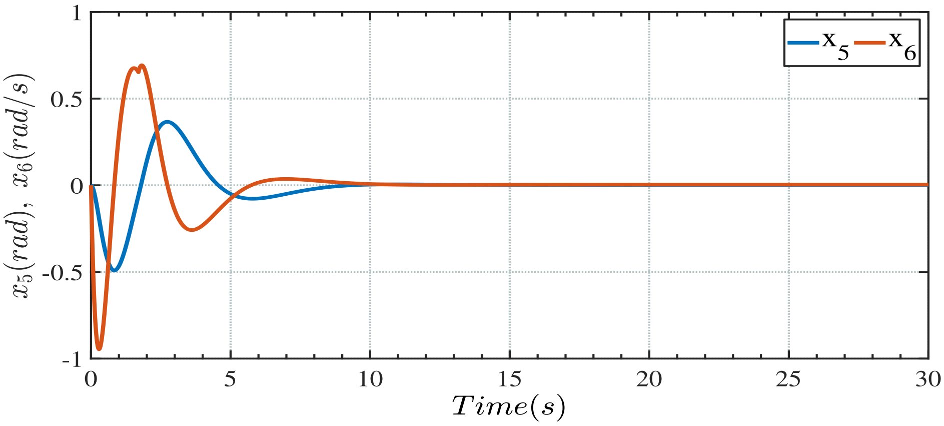
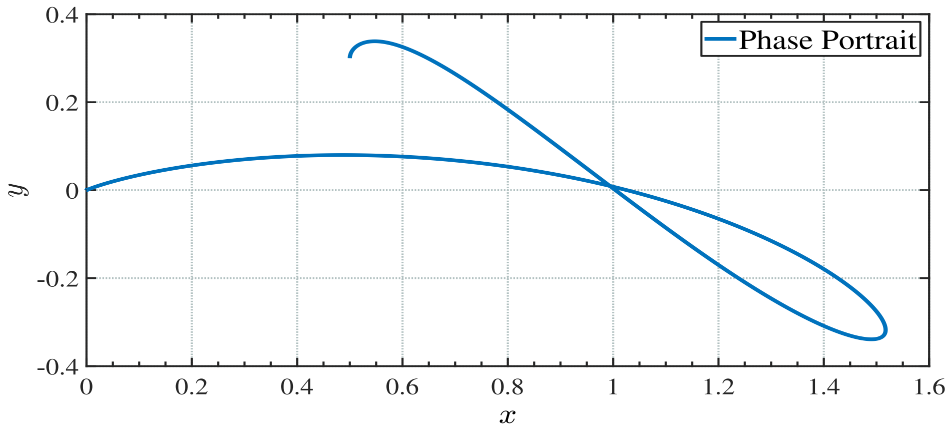


(c)

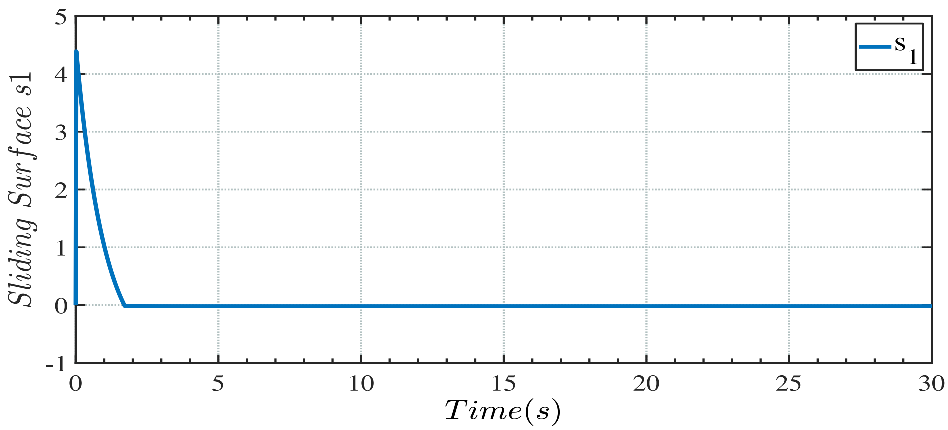
FIGURE 4.1: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0, 0.3, 0.1)$, (a) Represents system states in z_i, (b) Represents time history of horizontal displacement and velocity, (c) Represents time history of vertical displacement and velocity



(a)

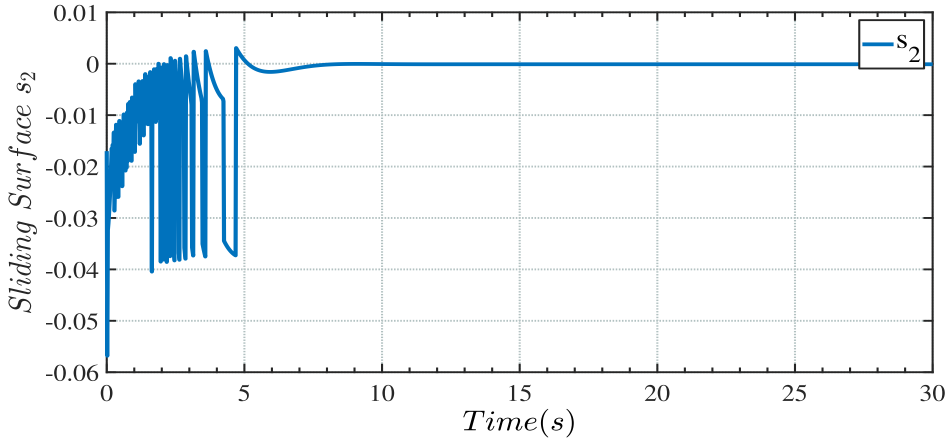


(b)

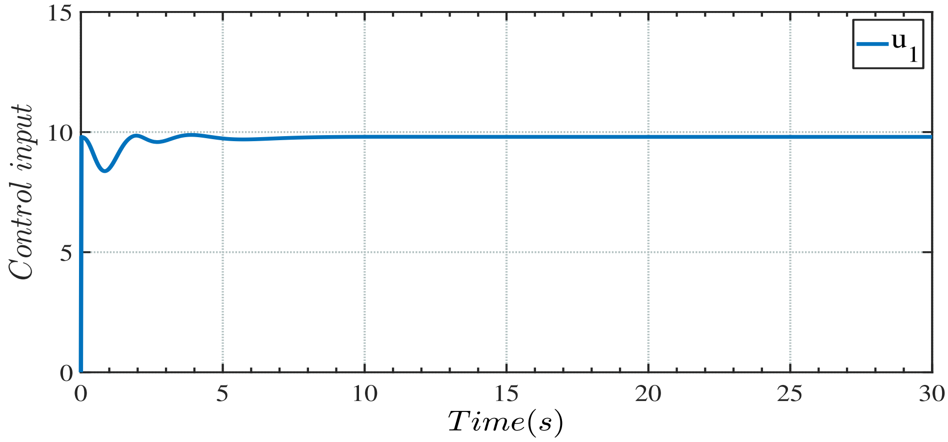


(c)

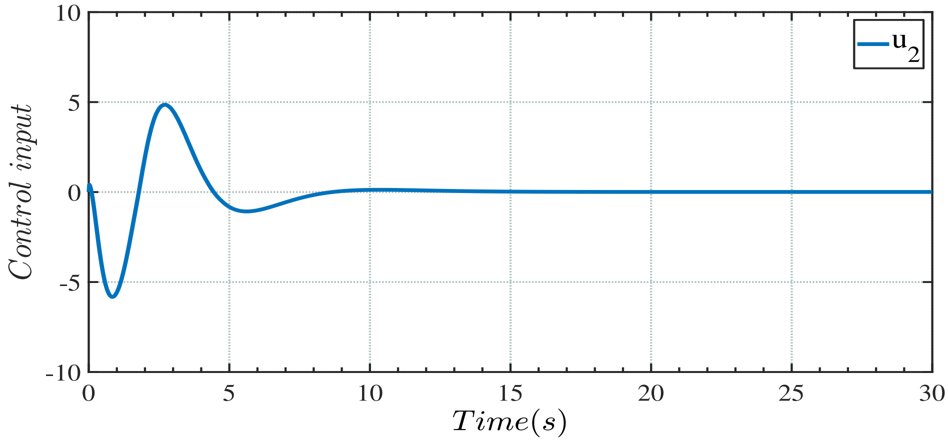
FIGURE 4.2: Closed loop response of PVTOL system corresponds to initial condition $(x_5(0), x_6(0)) = (0, 0)$, (a) Represents time history of roll angle and angular velocity, (b) Represents the phase portrait, (c) Represents time history of sliding surface s_1



(a)



(b)



(c)

FIGURE 4.3: Closed loop response of PVTOL system, (a) Time history of sliding surfaces s_2, (b),(c) Time history of control inputs u_1, ,(c) Time history of control inputs u_2

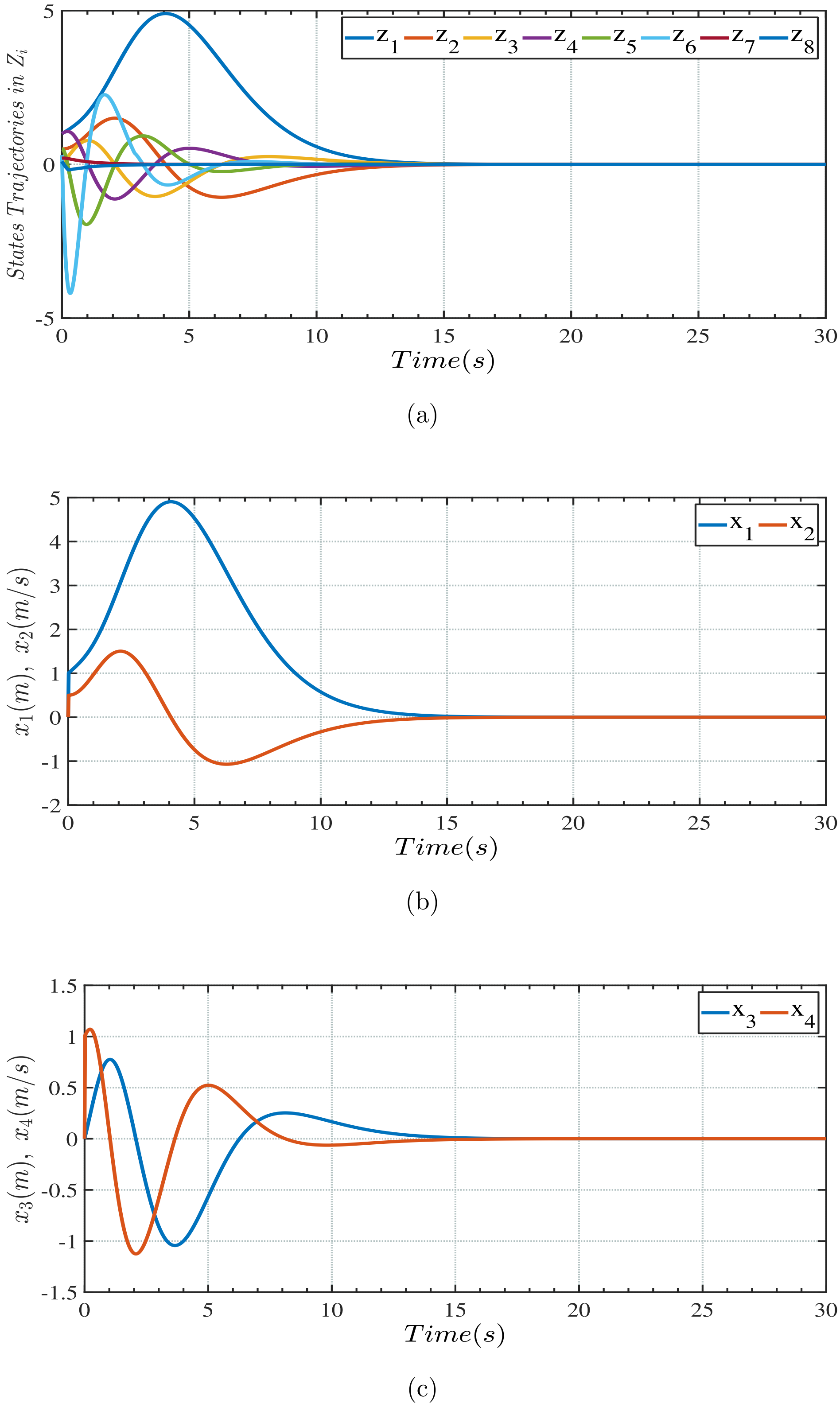
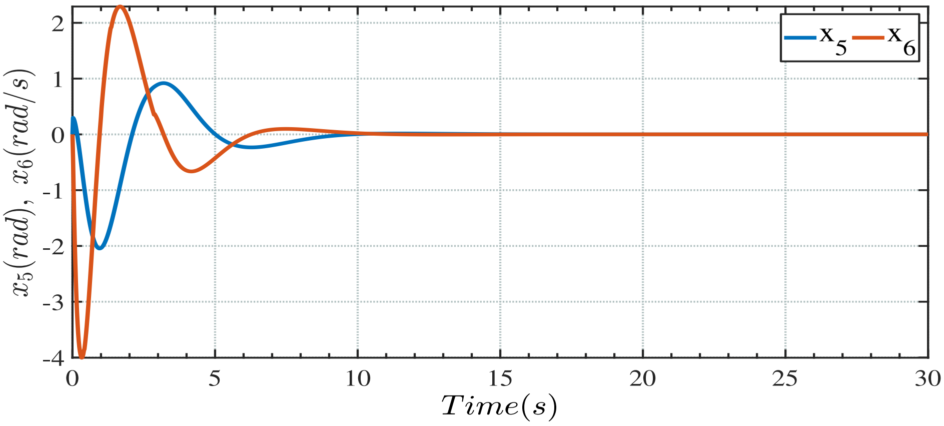
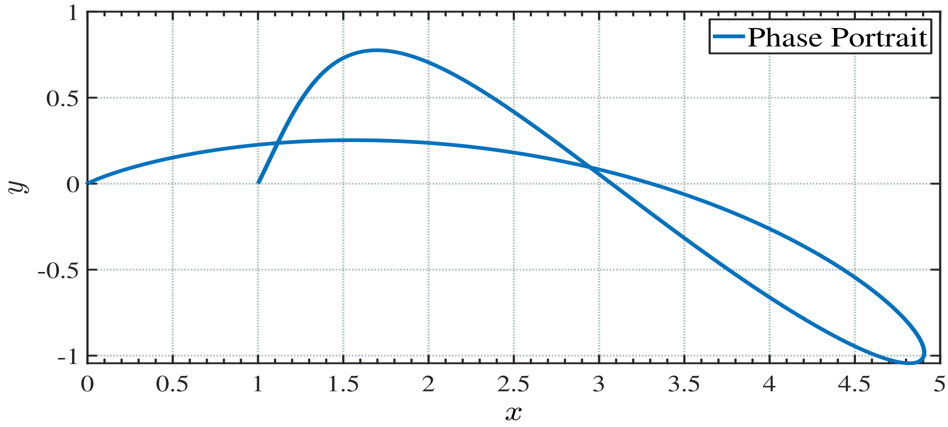


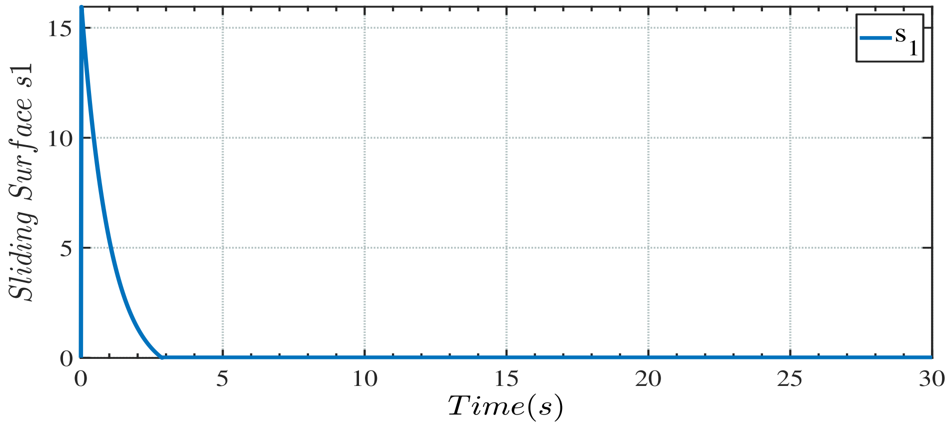
FIGURE 4.4: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0.5, 0, 1)$, (a) Represents system states in z_i, (b) Represents time history of horizontal displacement and velocity, (c) Represents time history of vertical displacement and velocity



(a)

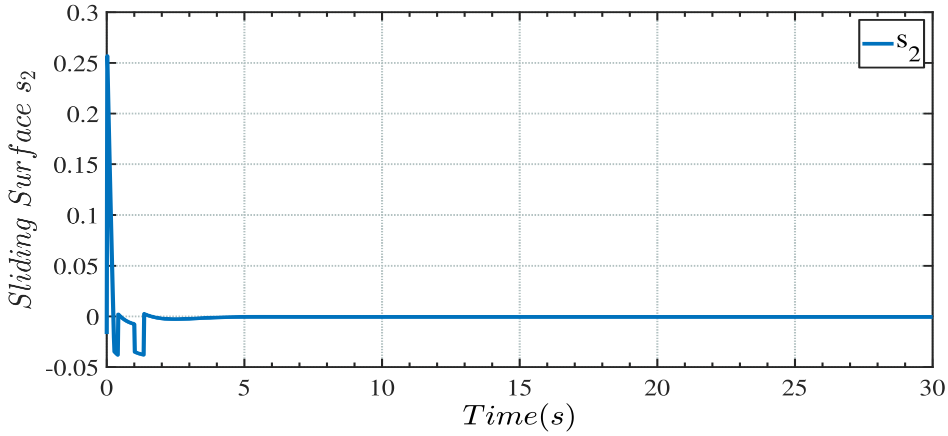


(b)

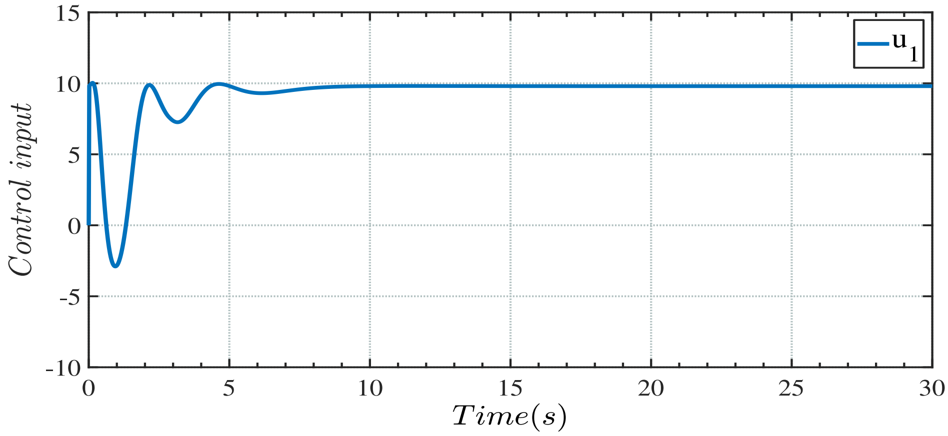


(c)

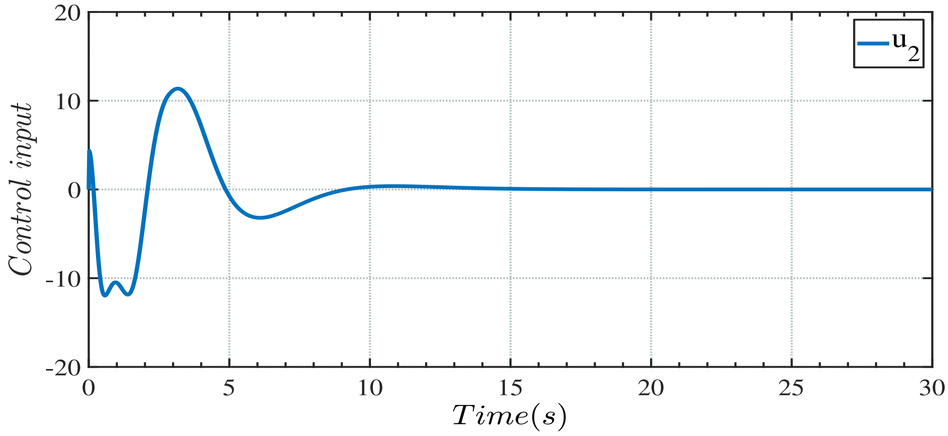
FIGURE 4.5: Closed loop response of PVTOL system corresponds to initial condition $(x_5(0), x_6(0)) = (0.3, 0.2)$, (a) Represents time history of roll angle and angular velocity, (b) Represents the phase portrait, (c) Represents time history of sliding surface s_1



(a)



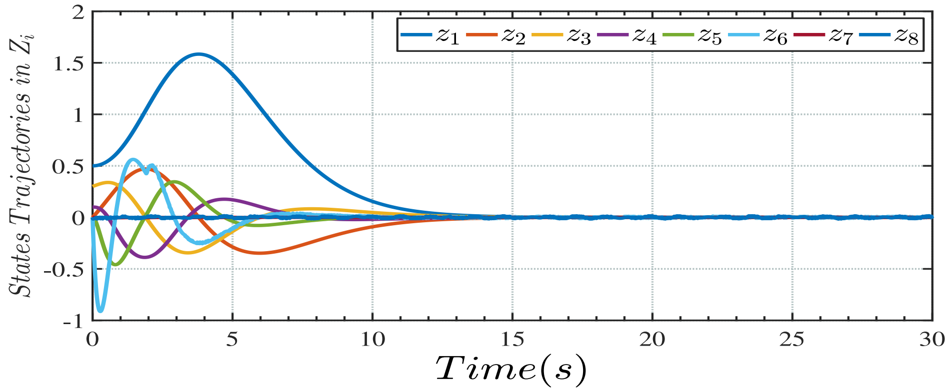
(b)



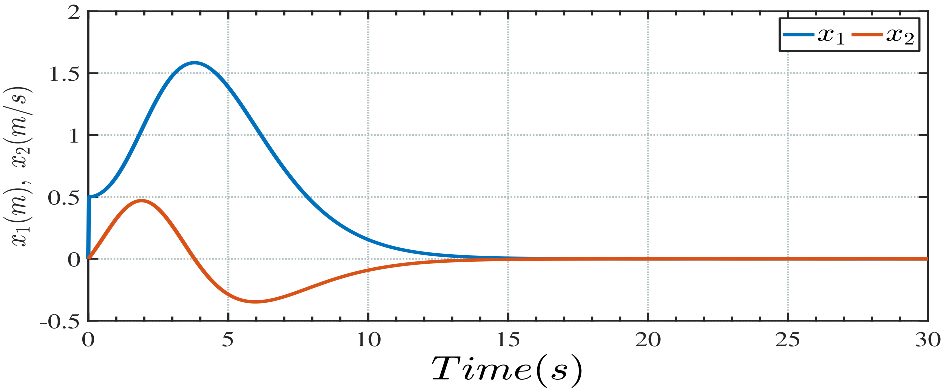
(c)

FIGURE 4.6: Closed loop response of PVTOL system, (a) Time history of sliding surfaces s_2, (b),(c) Time history of control inputs u_1, ,(c) Time history of control inputs u_2

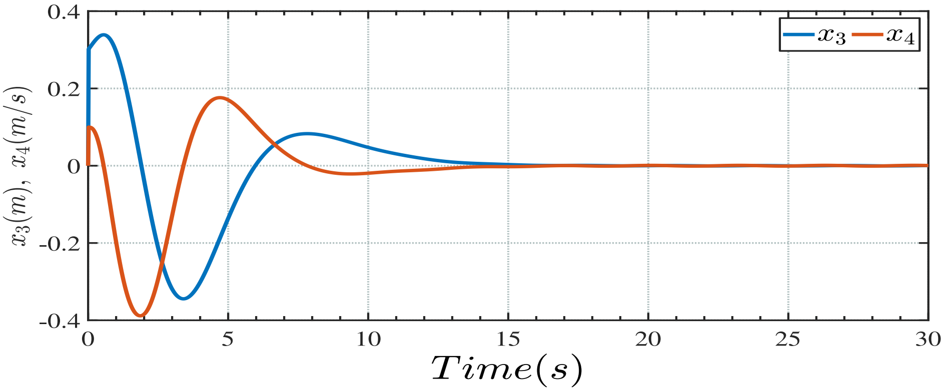
4.2.2 Adaptive Sliding Mode Control With Disturbances



(a)

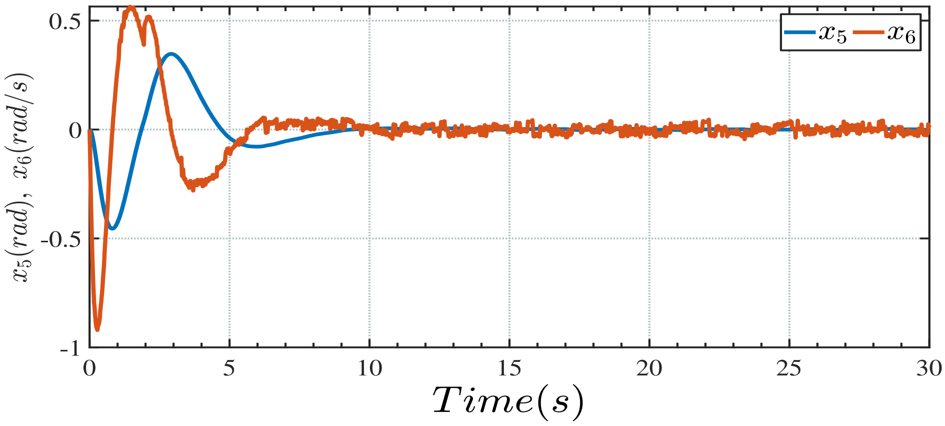


(b)

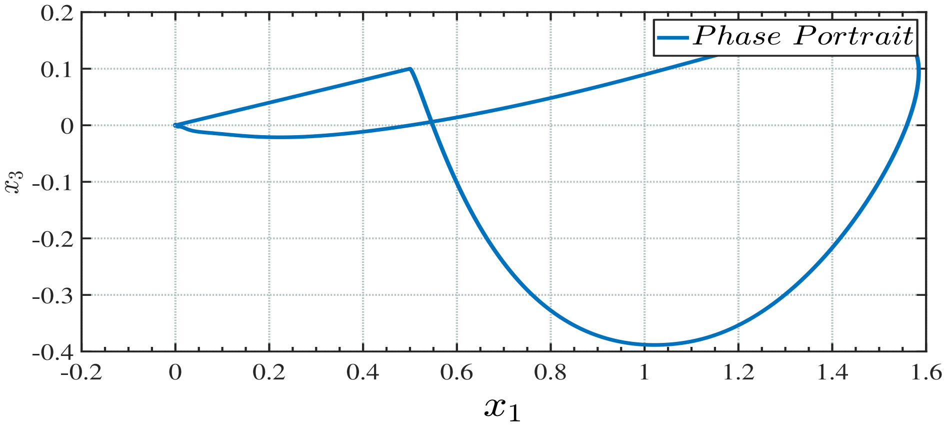


(c)

FIGURE 4.7: Closed loop response of PVTOL system in the presence of disturbances corresponds to initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10.5, 0, 0.3, 0.1)$, (a) Represents system states in z_i, (b) Represents time history of horizontal displacement and velocity, (c) Represents time history of vertical displacement and velocity



(a)



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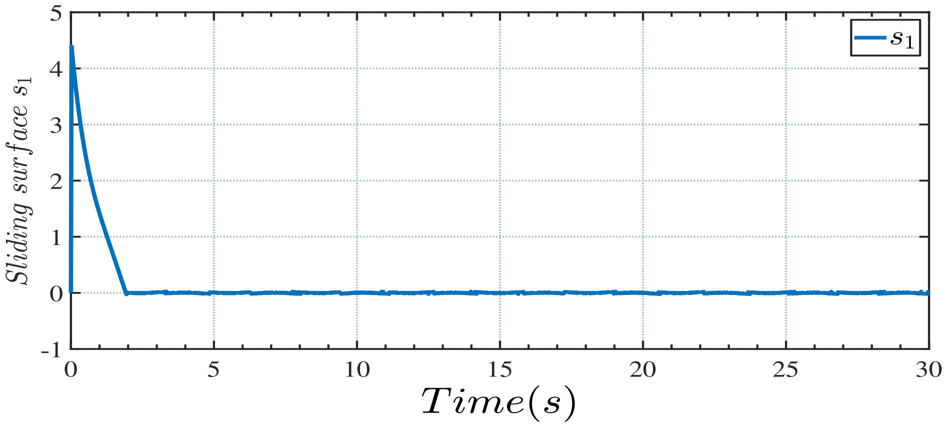
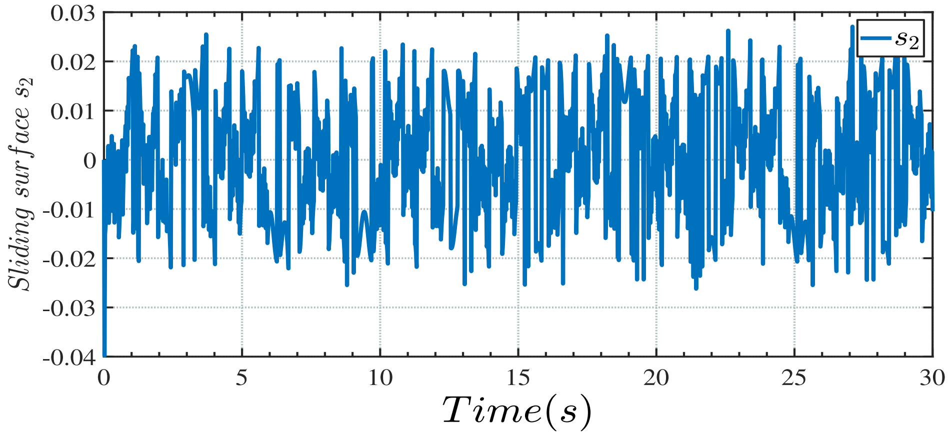
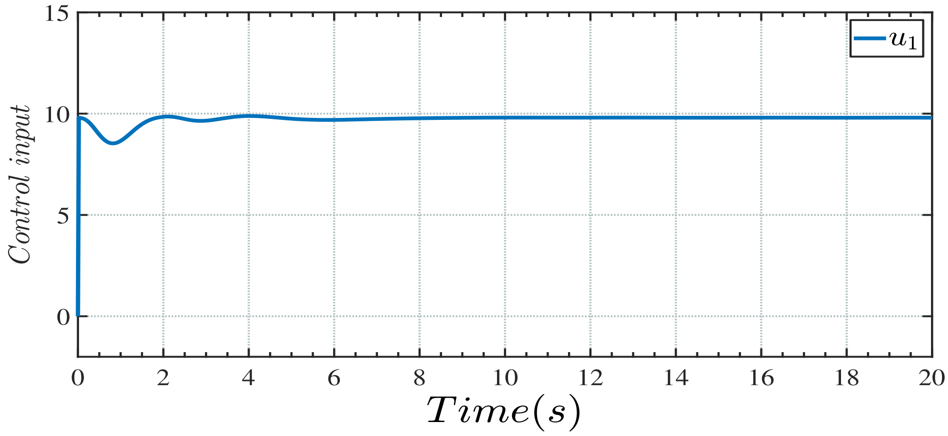


(c)

FIGURE 4.8: Closed loop response of PVTOL system in the presence of disturbances corresponds to initial condition $(x_5(0), x_6(0)) = (0, 0)$, (a) Represents time history of roll angle and angular velocity, (b) Represents the phase portrait, (c) Represents time history of sliding surface s_1



(a)



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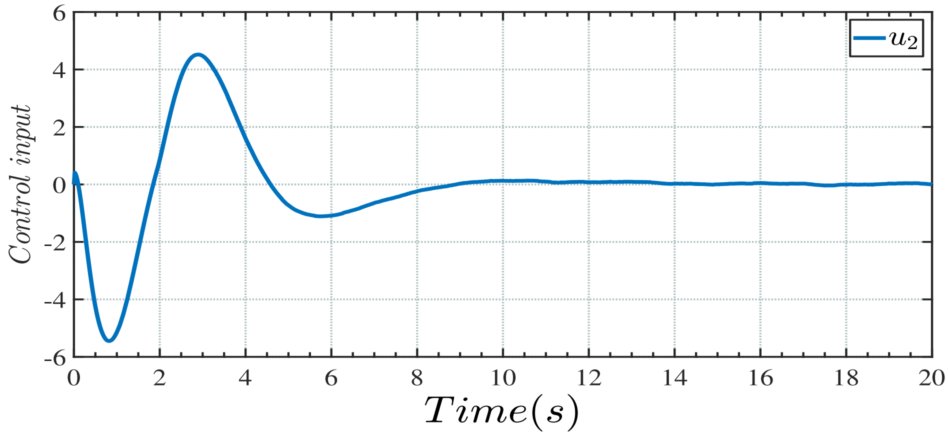
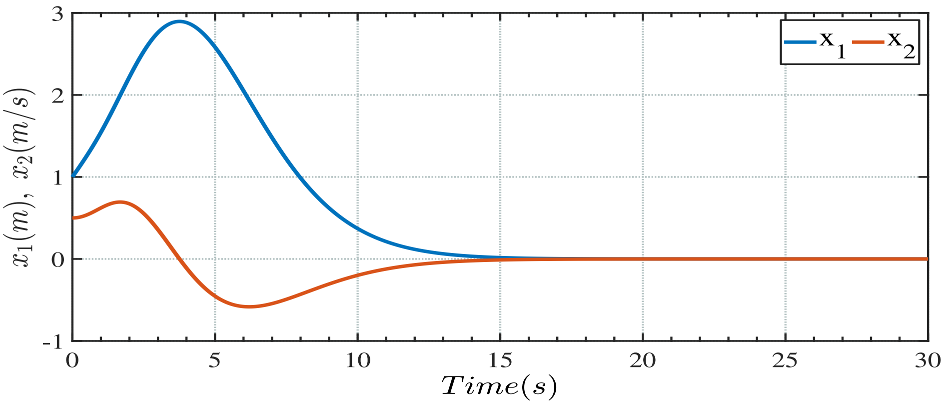


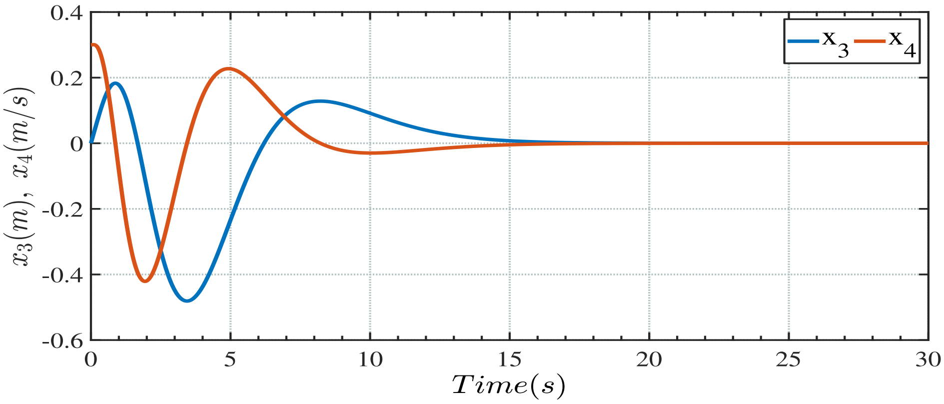
(c)

FIGURE 4.9: Closed loop response of PVTOL system in the presence of disturbances (a) Time history of sliding surfaces s_2, (b),(c) Time history of control inputs u_1, ,(c) Time history of control inputs u_2

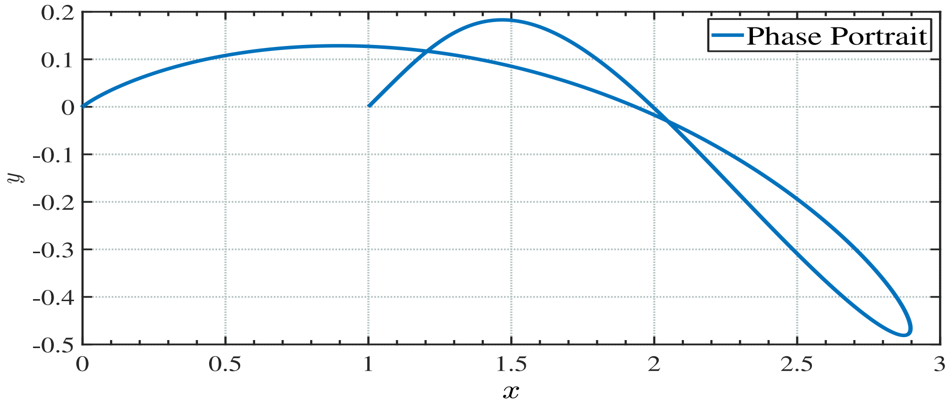
4.2.3 First Order Sliding Mode Control



(a)

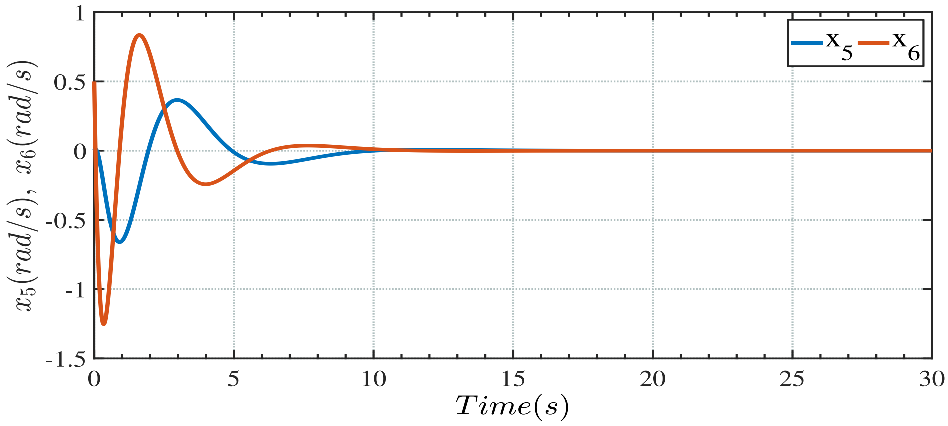


(b)

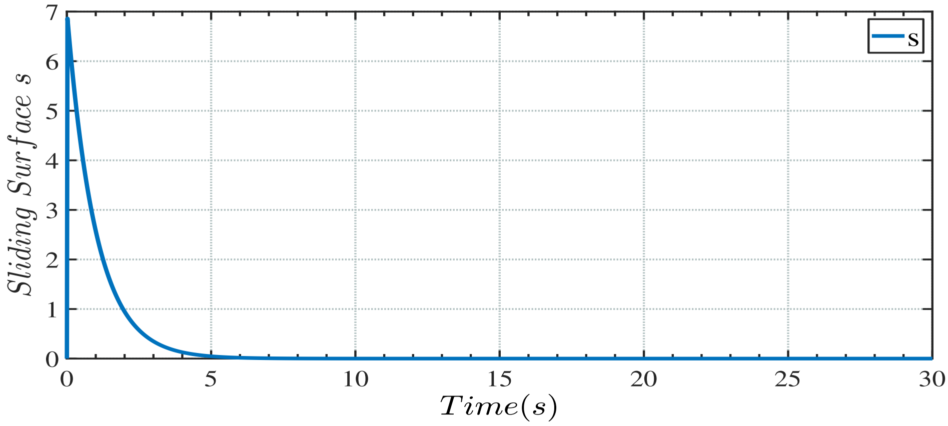


(c)

FIGURE 4.10: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0.5, 0, 0.3)$, (a) Represents time history of horizontal displacement and velocity, (b) Represents time history of vertical displacement and (c) Represents the phase portrait,



(a)



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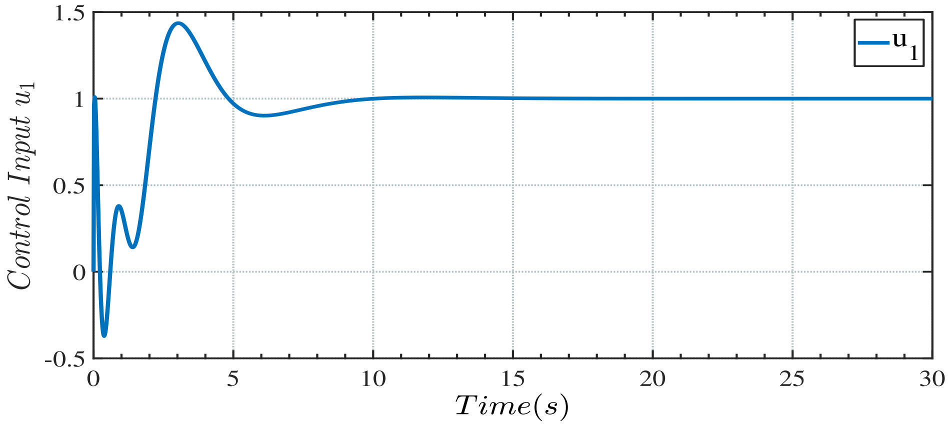
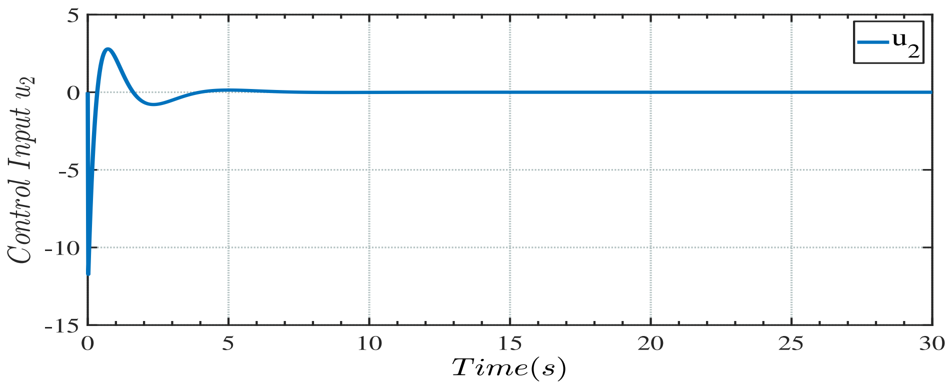


(c)

FIGURE 4.11: Closed loop response of PVTOL system corresponds to initial condition $(x_5(0), x_6(0)) = (0, 0.5)$, (a) Represents time history of roll angle and angular velocity, (b) Time history of sliding surface s (c) Time history of control input u_1

.



(d)

FIGURE 4.12: Closed loop response of PVTOL system (d) Time history of control input u_2

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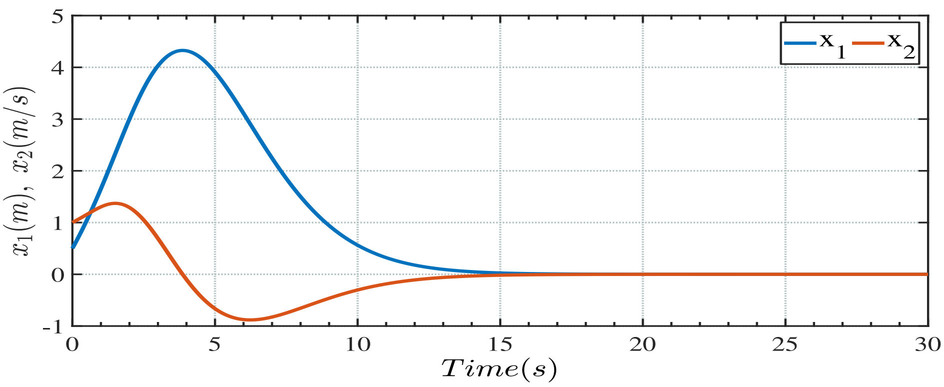
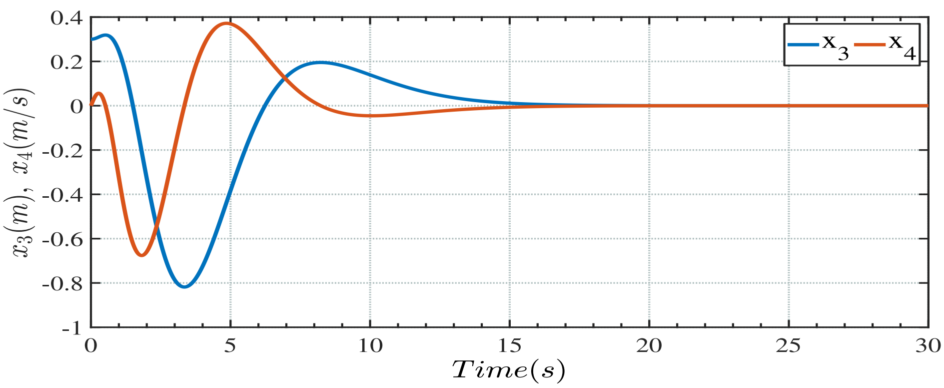
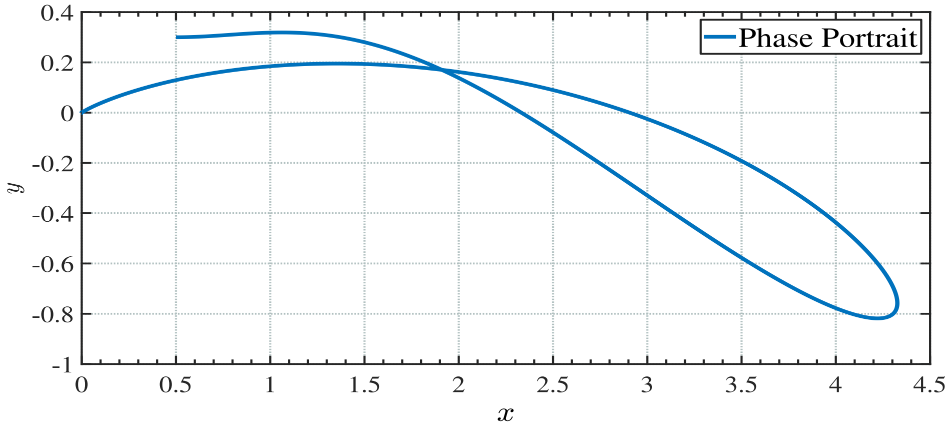


(a)

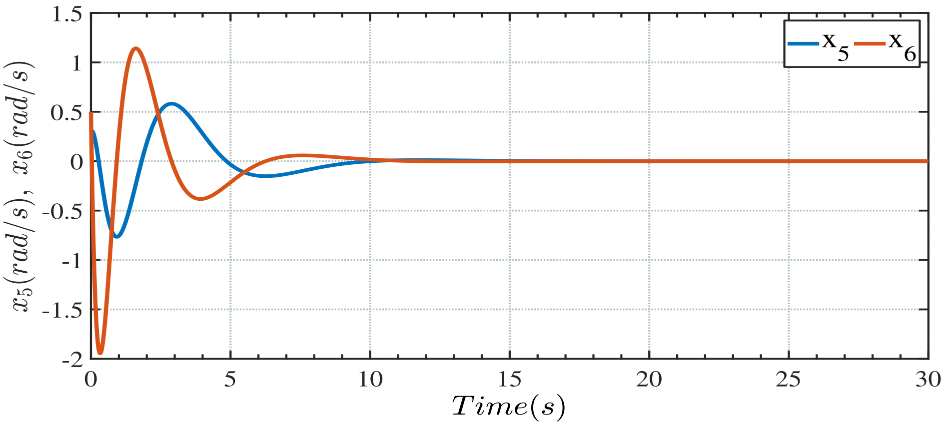


(b)

FIGURE 4.13: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0.5, 0, 0.3)$, (a) Represents time history of horizontal displacement and velocity, (b) Represents time history of vertical displacement and velocity



(a)



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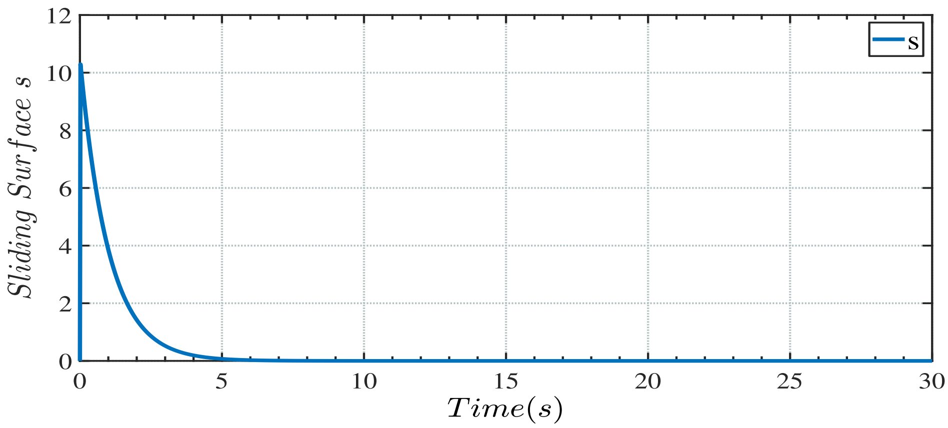
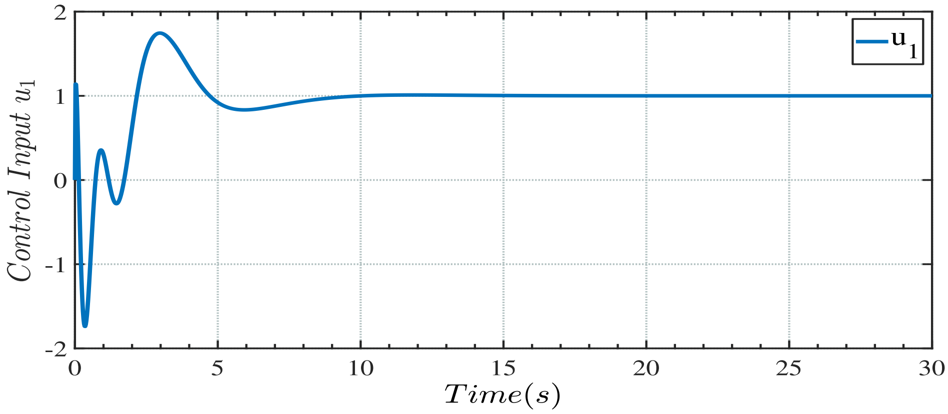


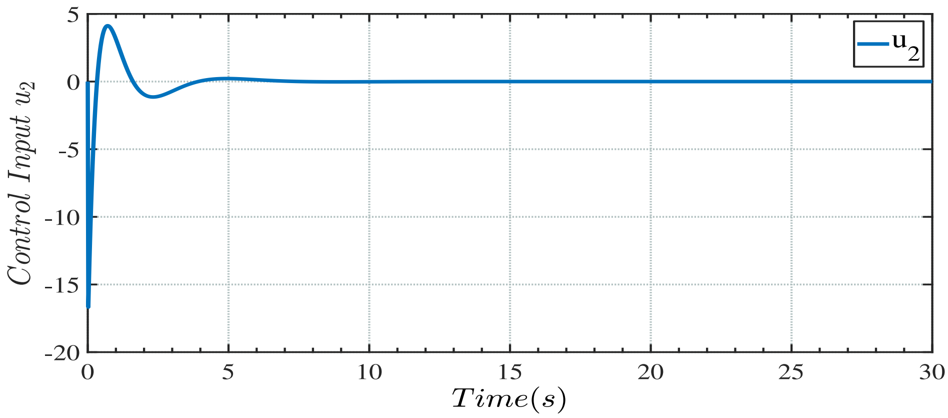
(c)

Figure 4.14: Closed loop response of PVTOL system corresponds to initial condition $(x_5(0), x_6(0)) = (0, 0.5)$, (a) Represents the phase portrait, (b) Represents time history of roll angle and angular velocity, (c) Time history of sliding surface s

.



(a)



(b)

FIGURE 4.15: Closed loop response of PVTOL system (a) Time history of control inputs u_1 (b) Time history of control inputs u_2

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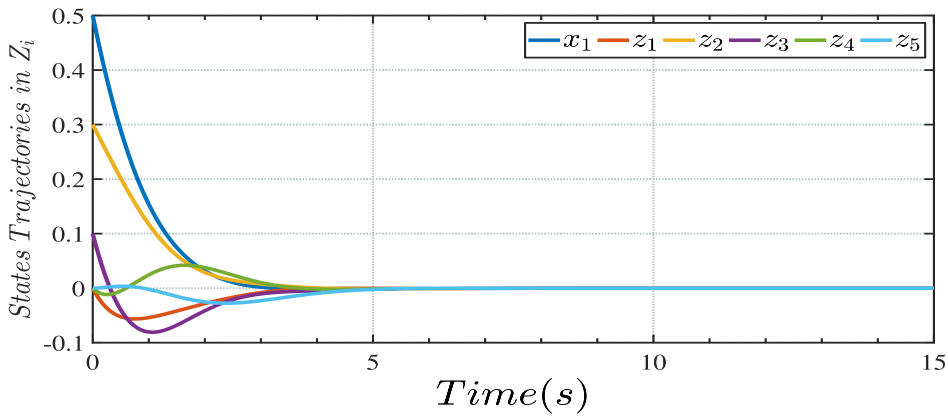
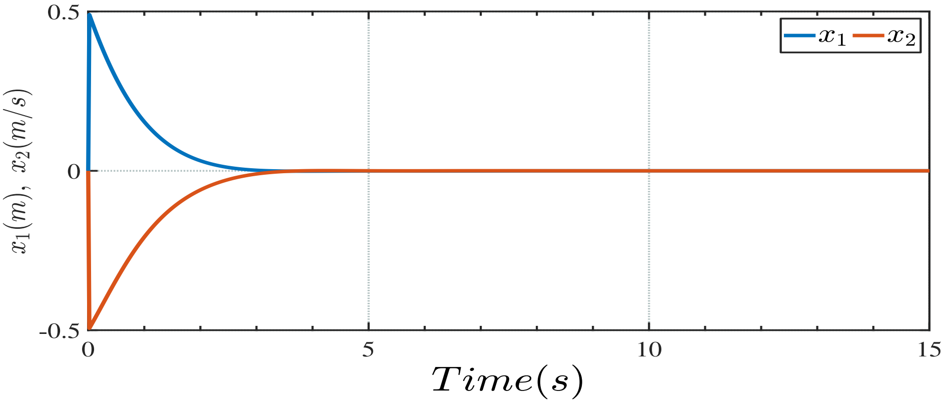
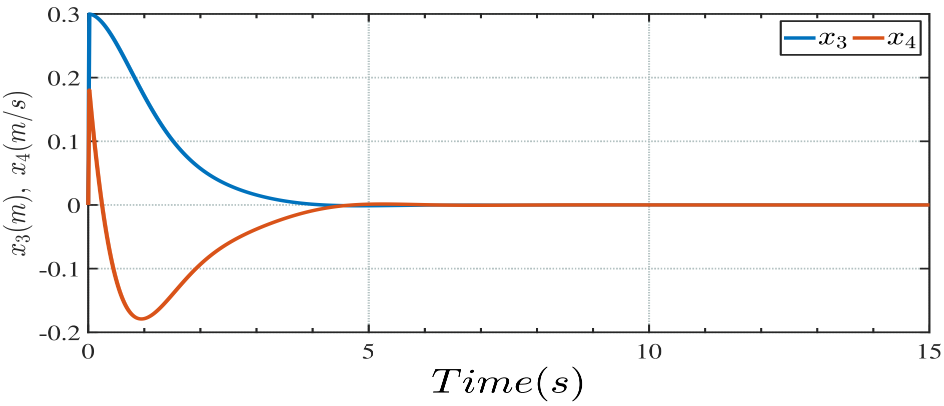
4.2.4 Backstepping Control



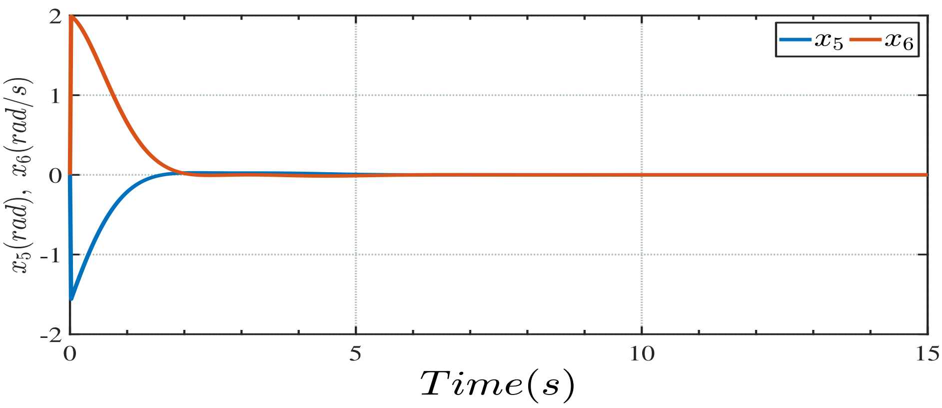
(a)



(b)

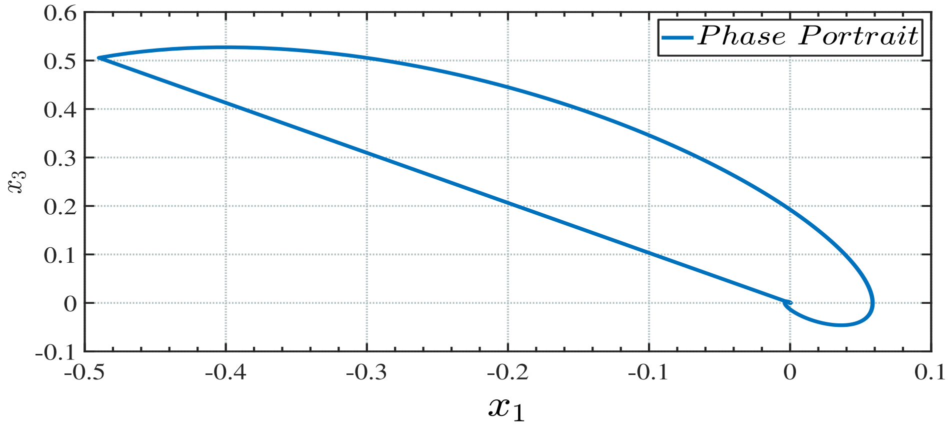


(c)

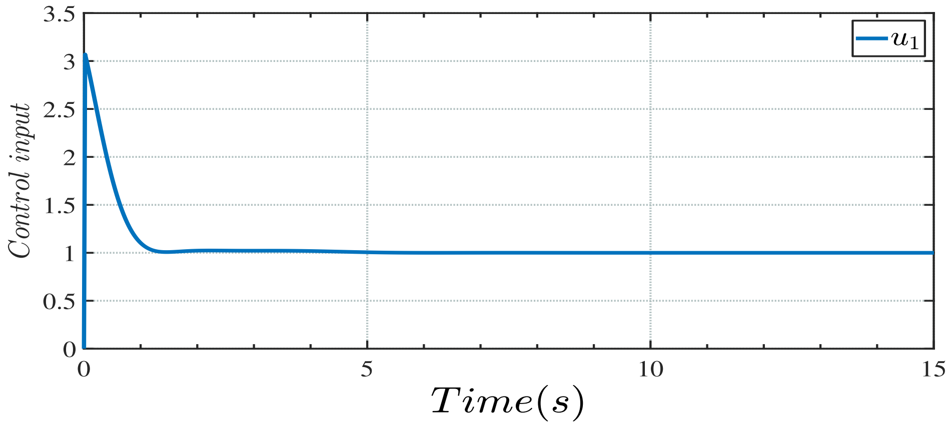


(d)

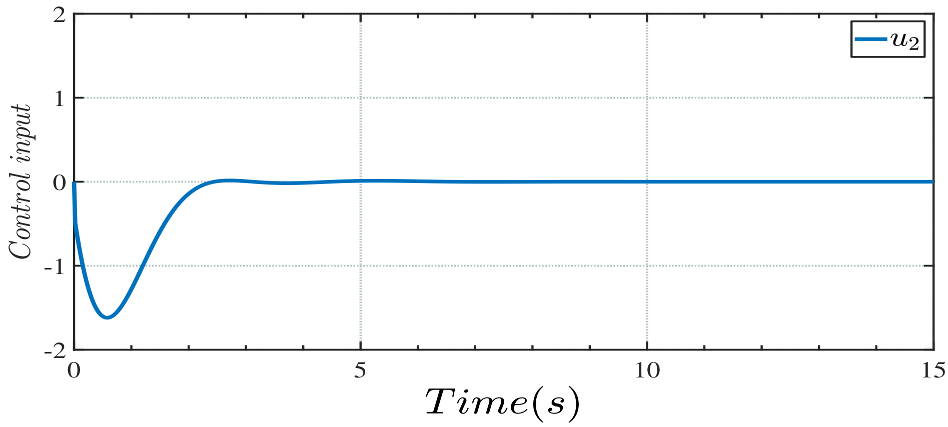
FIGURE 4.16: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), ..., x_6(0)) = (0, 0, 0.18, 0, 0, 0)$, (a) Represents time history of states in z_i (b) Represents time history of horizontal displacement and horizontal velocity x_1, x_2 (c) Represents time history of vertical displacement and vertical velocity x_3, x_4 (d) Represents time history of roll displacement and roll velocity x_5, x_6



(a)



(b)



(c)

FIGURE 4.17: Closed loop response of PVTOL system corresponds to initial condition $(x_1(0), ..., x_6(0)) = (0, 0, 0.18, 0, 0, 0)$, (a) Represents the phase portrait, (b)Time history of control input u_1 (c) Time history of control input u_2
.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

In previous decades, the interest in research of planar vertical take-off and landing aircraft (PVTOL) has been increased. There are numerous applications of this system in the field of aerospace, mechatronics, robotics, industry etc. This research work presents a stabilization of PVTOL system.The suggested methodologies is based on Robust adaptive sliding mode control, first order SMC and backstepping control. In adaptive SMC system is transformed through input transformation, which contains some unknown terms, the unknown term is adaptively computed. In first order sliding mode control, transformation is applied and new control inputs injected in system. In backstepping approach, system is transformed into specific structure and system is stabilized. The suggested techniques is applied to PVTOL systems with 3 DOF.

5.2 Future Research Directions

Based on this research work, certain directions are suggested for future research.

1. Extension of the suggested techniques to other UMS.

2. Apply Observers.

3. Apply sliding mode observer techniques.

4. Practical implementation of the suggested algorithms.

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