CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

Motion Control of Robotic Arm Manipulator Using PID and Sliding Mode Technique

by

Zaheer Abbas

A thesis submitted in partial fulfillment for the degree of Master of Science

in the

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To my parents

CAPITAL UNIVERSITY OF SCIENCE & TECHNOLOGY
ISLAMABAD

CERTIFICATE OF APPROVAL

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Abstract

In the last decades, many researches have proposed concerning the motion control and position regulation for manipulators. Motion control has important applications in many areas, for example industrial robotics, autonomous systems, and virtual prototyping, etc.

In this work, an exhaustive research of the existing literature related to the thesis has been carried out, which has served to create a comprehensive database used to perform a detailed historical review of developments since its starts to the current state of the art and the latest trends.

In this thesis two control methodologies for the position control of robotic arm manipulator are used, i.e. PID and Sliding Mode control. Both techniques have good performance but, the Sliding mode control being insensitive to parametric variations, disturbance rejection and good performance against uncertainties has much better results than PID control.

The simulation results are also presented which shows SMC has better performance as compared to PID control technique.

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Abbreviations

DOBC	Disturbance Observer Based Control
2DOF	Two Degrees of Freedom
FOPID	Fractional Order Proportional Integral Derivative
NN	Neural Network
PID	Proportional Integral Derivative
SMC	Sliding Mode Control
SM	Sliding Mode

Symbols

e	Tracking error
K_p, K_i, K_d	Proportional, integral and derivative gain
L_i	Length of each link
$\mathcal{L}$	Lagrangian
M_i	Mass of each link
q	Joint angle/position
q^d	Desired joint angle
S	Sliding surface
τ	Torque acting at joint
x_i	State variables
θ	Joint angle
ξ	State variable
ϕ	Boundary layer thickness

Chapter 1

Introduction

1.1 Background

The Robot technology is widely used in many areas of life like, in industrial applications, space, sensing and monitoring technologies, computer design, control and automation, mechanical designing, practical math, man machine interface. In this chapter a brief introduction describing the various parts that are related to this work, i.e., design and motion control of robotic manipulator is presented.

A robot manipulator is a programmable, multipurpose manipulator that is used to move about different types of objects, parts, items or special devices through changeable programmed movements to perform different types of tasks [1].

Robots were introduced in Industries in early 50s. The basic idea behind the introduction of robotics technology in industry was, the replacement of human in the tasks, having high risk factor, repetitiveness, also to increase the production and for the improvement of product quality. In comparison to autonomous systems the robot manipulators have established their importance and are used at larger level to take over humans in cyclic tasks. They were initially designed for handling with radioactive and other hazardous materials by using manipulator arms.

By recent advancements in the field of control, they are used in various applications like;

- Automatic welding
- Laser cutting
- Assembly lines
- Guided Missile
- Robotically assisted surgery
- In space like, NASA developed robots (mars rovers, curiosity, space and opportunity etc.)

1.2 Parts of Manipulator

It is a mechanism of rigid arms which consists on a series of segments, mostly sliding or jointing, called cross-slides, which grip and move objects by different degrees of freedom. Also, it is an electronically controlled mechanism, where each segment has to perform different functions by interacting with its workspace. They are also commonly called as robotic arms. The study and the research of robot manipulators mostly deals with orientations and positions of the different sections that build up the robotic manipulators [2].

They are mainly consists on an assembly of links, joints and end-effector. The brief description of these parts is given below.

1.2.1 Links

These are the rigid parts of the manipulator.

1.2.2 Joints

Part of the mechanism that connects the two links with each other. Joints permit controlled or limited relative motion between the links. The table mentioned below provides brief information related to different types of joints.

Table 1.1: Joints types of robotic manipulator.

S. No.	Type	Description
1	Revolute	These types of joints permits rotation motion about an axis.
2	Cylindrical	The types of joints which permits rotation and translation about an axis.
3	Prismatic	Permits relative translation motion around an axis.
4	Spherical	Gives three degrees of rotational motion freedom around the center of the joint. They are also refereed as a ball socket joint.
5	Planar	Provides translational motion on a plane and rotation motion around an axis perpendicular to plane.

1.2.3 End Effector

It is the part of the manipulator, which interacts with its workspace to perform various tasks.

1.3 Classification of Manipulators

There are many criteria's according to which the robot manipulator can be classified. Here we will classify them on the basis of motion characteristics and Kinematic structure.

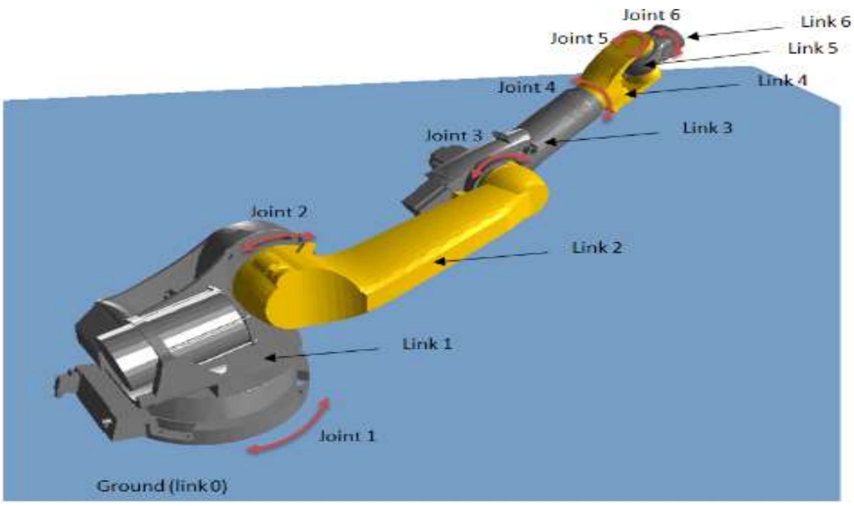

FIGURE 1.1: Different parts of robotic manipulator.

1.3.1 Motion Characteristics

On the basis of motion characteristics the robot manipulator can be classified into Planar, Spherical and Spatial manipulator [1].

1.3.1.1 Planar Type

In Planar type manipulators all the links move in planes parallel to each other.

1.3.1.2 Spherical Type

In these types of manipulators all the links perform spherical movements around a common motionless point.

1.3.1.3 Spatial Type

These are the manipulators for which, at minimum one link of the manipulator must have a spatial movement.

1.3.2 Kinematic Structure

On the basis of Kinematic structure the robot manipulator can be classified into open loop or serial, parallel and hybrid manipulator.

1.3.2.1 Open loop Robot

These are the robots for which, links are connected in such a way that they form a open loop. They are also called as serial manipulators.

1.3.2.2 Parallel Robot

A parallel robot is a manipulator, for which arms or links form a close loop.

1.3.2.3 Hybrid Robot

These are the combination of both, the serial and parallel type of manipulators.

1.4 Degrees of Freedom

It is described as the number of Independent parameters that are needed to fully describe a mechanism in its configuration space [3]. The equation used to determine the number of degrees of freedom of a robot is as follows

$$DOF = \lambda(l-1) - \sum_{j=1}^{m} (\lambda - f_j) \qquad (1.1)$$

Here l indicates the number of links (including the ground link), m shows the number of total joints, f_j is number of degrees of freedom of the j^{th} joint and λ has value depending upon type of mechanisms.

1.5 Motivation and Problem Statement

The Motion control of the robot manipulators is a demanding job, it has been researched at both the industrial as well as academic level by the researchers for more or less than couple of decades. Basically, the robots used in industries are the type of machines which are applied for the automation purposes, although the specifications of a particular application can be exactly formulated, but there is no limit regarding what a user requires about desired functionality and similarly the desired performance for the motion or trajectory control of the robot [3]. The performance of the motion control is application dependent.

Few examples of performance requirements are:

- For application like laser cutting and laser welding the accuracy of the path is essential performance requirement.
- In case of painting and dispensing the accuracy of the speed is the requirement.
- For material handling sort of applications the repetitive cycles with minimum times are needed.
- There are processes for which minimum overshoots and a small settling times are the necessary performance requirement parameters, like spot welding.
- There are also applications for which tight control is needed like machining.

The applications stated above requires the clear planning of motion paths, generation of trajectories and also the perfect design of the control. Do to the nonlinear and varying dynamics , the robot motion control becomes a challenging task [4]. Also, the uncertainties of the parameters in mechanical part and the actuation part of the manipulator makes this problem even more complex.

Motion control of the robot manipulator is always a matter of high concern for the designers because manipulators have different number of inputs and outputs, nonlinear and time varying dynamics. Various control techniques have been adapted to overcome this issue over the years. Each of them has its own advantages and disadvantages.

The extensive use of robotic manipulator for various applications is the real source of motivation to present a control methodology that can better cater the problems associated with manipulators to achieve good performance.

Here, we will implement two different control techniques for position or movement control of a robotic arm manipulator. That is proportional integral derivative (PID) and sliding mode technique.

PID control is the simplest of all control techniques used for the motion control of robotic manipulators. It has K_p, K_i, and K_d, parameters which are selected manually to achieve desire control performance. It has good trajectory tracking or error tracking response, better convergence time to achieve stability. In contrary to this, the sliding mode being robust nonlinear control methodology provides more better performance than the PID control.

A comparison between both the techniques on the basis of simulation results will be presented in the later chapter. The graphs for joints error tracking and for the force applied at joints are provided for the comparison between the two types of controllers.

Chapter 2

Literature Review

2.1 Introduction

There is a long past history for the motion control of robotic manipulators, it always presents a broad research area for the researchers and engineers of control system design because of the the new developments in intelligent control methodologies. A vast number of model oriented control techniques have been used for controlling the positions of robots like [5], Disturbance observer based controller [6], Artificial Neural Networks (ANNs) [7, 8], Fuzzy logic control [9].

These model oriented techniques requires precise mathematical or in other words ideal model for the control of manipulator, which makes them highly complex and computational wise more time taking, more importantly for the manipulator with high degrees of freedom. In contrary to this, the control techniques which are not model oriented, do not requires the precise information of parameters, neither mechanical nor for the actuation part of the manipulator. These model less techniques makes the design process easy [10].

Irrespective of the advancement in the area of control system design, the PID control technique is the most widely used strategy in industries due to the ease of implementation and simpleness of its design [11, 12].

PID control is a usual model less feedback control methodology, which is been used widely for industrial application due to its performance reliability, simple design, and implementation both in hardware and softwares. More importantly, it does not need the ideal mathematical model for the implementation [13].

Similarly, because of the order reduction feature, insensitive behavior for parametric changes and disturbances, the sliding mode approach is a proficient method for the control of higher order complex dynamics systems. This control approach relies on a law refereed as reaching law. Which has the capability to tailored the dynamic properties of the plant in reaching phase, and to control the chattering associated with the control input [14].

2.2 Overview of Various Control Techniques

The brief overview of different techniques use for motion control of robotic arm manipulator and proposed methodologies is as follows.

2.2.1 Computed Torque Control

Computed Torque Control is a eminent motion control method for manipulators that is in fact often used in combination with the PID-controller. The principle objective of a computed torque controller is to linearize and decouple the dynamical equations of motion, so that, each joint can be considered independently. It can transform the inherently nonlinear dynamics of the robotic arm into a seemingly linear system [5].

It is a model based control strategy. For a model oriented control strategy a plausibly precise mathematical model of the physical plant under the control consideration is required. So, the designing of the physical plant as reasonably as achievable is the most prior step in the use of model oriented control. A mathematical design derived under the assumptions of rigid body dynamics may not represent the real attributes of the physical system, as every mechanical system exhibits structural flexibility to some extent. So, the flexibility effects of the physical system must have to be modeled as well, to mathematically capture the real behaviour of the plant [2].

After designing the mathematical model, the subsequent challenge is to execute the control strategy based on the system model. There are two main problems linked with the use of absolute flexible joint mathematical design for synchronized control.

1. Getting real time, joint configuration parameters and their derivatives, is one of the toughest task, if not impossible, because of the hardware restrictions.
2. The real time computation of inverse dynamics is to be made in computed torque control approach.

With increase in the mathematical complications of a given physical system, the computational load also grows. Although, the fast processing of the computers have surmount many high speed computational problems, but for real time computations it is burdensome to apply the absolute higher order flexible joint mathematical model. The real time inverse dynamics calculations using parallel computation for a complex system like a robot manipulator with different degree of freedom is not unattainable. However, simplifying a higher order model to recover a reduced order model, which would represent the actual system with reasonable accuracy, is quite acceptable [15].

Computed torque control (CTC) allows the design of considerably more precise, energy efficient, lower feedback gains and complaint controls for robots [5]. Comparative study between PID and CTC results that, CTC is better as it works for non-linear system. But it is more complex than PID tecnique in implementation.

2.2.2 Disturbance Observer Based Control

These types of controllers have a long history for various industry related applications. Also denoted as DOBC. It is basically the disturbance and uncertainty estimation technique. Few of the approaches which are used for the aforesaid estimation are as follows;

- The first approach for this purpose is DOBC
- Another method is disturbance adjustment strategy
- Disturbance rejection active control
- Anti disturbance control strategy

For the above mentioned techniques, generally both the uncertainties and disturbances are considered as interlinked with each other. An estimation method is used for the evaluation of total disturbance, which comprised of uncertainties and disturbances in the system.

Usually, disturbances and uncertainties are present in every physical system, whether industrial or non industrial. They impose negative impacts on the functionality and performance of the system, and most of the times renders the system unstable [16]. The disturbance rejection is always a matter of high concern for the designer. It can be rejected or suppressed by feed forward method provided that computable. But, most of the times, it is not quantifiable directly or very expensive to compute. So, the better method to cater this problem is the estimation of disturbances through different computable parameters. Finally, a controller based on these estimation is applied for the rejection of disturbances.

In a similar way, this concept is also used to handle uncertainties in dynamical model of the system, which takes place due to intentional or unintentional negligence of the control system designer in modeling phase. The ultimate objective of this control methodology is to achieve robust performance [3]. Which is the main and important boosting reasons for the designers, for using these controllers at broader ranges for various applications. There are different types of such control algorithms but each of them has the same core idea, that is, observer for the estimation of disturbance or uncertainties and a compensator on the basis of these estimations.

In DOBC technique the accumulative effect of disturbance torques is estimated, which is based on the difference of output of nominal and actual physical model of the system [17]. In case of robotic manipulators with many links, the DOBC takes the coupling torque from one link to other as an external unknown torque, providing an opportunity for the independent control of the joint. This leads to the designing of a simple controller.

It is the most effective and optimal result oriented control strategy for robotic manipulators. Without using any extra sensor it can eliminate the external unknown disturbances. We can use it for the estimation of frictions, motion control or for independent control of joints.

2.2.3 Neural Network Control

It uses the mental computational or processing power for the generation of strategies or algorithms which are used for the designing of complicated patterns and predications related issues.

It has the capability of analyzing and modeling the highly complicated nonlinear relationships. This aspect makes ANN as one of the prominent control strategy because practically, most of the relations between inputs and outputs are nonlinear and complicated.

The main features of the artificial neural network control strategy, which distinguishes it from others are as follows; [8]

- Generalization of given problem by learning.
- Can be used for real time applications.
- Not much prior information required.
- Very easy to implement.
- Can be used for challenging problems.

One of its major drawback is, the requirement for the high level of computational power, as robotic manipulators have complex dynamics, therefore needs good computational strength for the designing.

2.2.4 Proportional Integral Derivative Control

It is one of most prominent and most widely used control methodology for industrial applications world over. The importance and wide acceptance of this technique is due to its good performance for various applications, and also the simplicity of its design [18]. One can use it in most simple and straight forward way.

In this research PID control technique is implemented for the motion control of robotic manipulator and quite satisfactory performance is observed through simulation results.

The PID algorithm is mainly comprised of three important parameters, which are called as proportional, integral and derivative gains. These parameters are tuned to meet the desired specifications of the system [11]. Its methodology is, to monitor the input from the sensor, on the basis of which output for the actuator is evaluated by using the afore said three gains.

Other than these three parameters, it also have process variables. Which are described, as the parameters of the system which are to be controlled [12]. In real life, we have many examples of process parameters like temperature (°C), pressure (psi), and flow (cmh). The process variable is measured as an input from the sensor, which then used in the feedback loop for the control of system. One another important parameter is the set point, actually it is the value which we desired, that process variable should have for optimal performance of the system. The process variable keeps on updating through control feedback loop until it attains the set point. The feedback control loop uses the difference between the process and set point parameters for the computation of output to the actuator. As an example, suppose flow is the process parameter with value 200 cmh and set

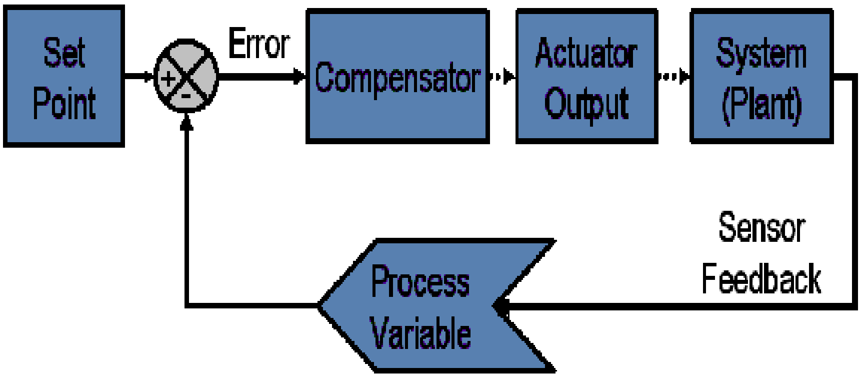

Figure 2.1: Block diagram of typical feedback system.

point is 600 cmh, in this case the compensator will take the relevant action based on the difference between the set point and process value, to meet the desired flow requirements, by computing the output for the actuator. A typical example of feedback system is illustrated in Fig. 2.1.

It is not only the actuator output that have impact on the performance of system but also, there are some external inputs that can influence its behavior. As in case of flow, the process parameter might not reach its set point may be due to the some leakage in any system component, this leakage acts as an a disturbance input to the system.

The proportional parameter of the PID control methodology relies on the difference between the set point and the process parameter, which is also called as error [19]. It is the ratio of the output value for the actuator to error between process and set point value. For example the error is 20 and proportional gain is 10 then output for the actuator will be 200. By increasing the proportional parameter, the speed of control system response can be increased. But, if it is increased to a very high value, it can cause oscillations in the process parameter response. Which may also render the system unstable [16].

Integral parameter sum up the error with time and rises up slowly. It keeps on increasing until the error becomes zero. The basic purpose of this gain is to make the system steady state error zero. The steady state error is the difference between final values for the process and set point parameters [12]. If the integral gain can not drive the system steady state error to zero and takes controller into saturation point, it is called integral gain wrap up.

If the process variable is rising rapidly, in such case derivative parameter decreases the output to the actuator to overcome the situation [16] as the derivative parameter respond very strongly to changes in error or process parameter. The derivative parameter depends upon variation rate of process variable. The speed of the control system response can be increased by rising the value of the derivative parameter. As the derivative behavior is sensitive to noise in process parameter so, its value is mostly kept low.

The hit and trial method is used for the adjustment of PID controller gains [19]. Which can be made easy, by making clear understanding of its three gains. Initially the derivative and integral parameter are kept zero, then the proportional parameter is gradually increased unless the output of feedback loop starts oscillating. Increasing the proportional term, above this point make the system response quick. However, its large value may turn the system unstable. After adjusting the proportional term, now increase the integral parameter keeping the derivative gain at zero, the oscillating response of system will get stop and the steady state error will reach its desired value, it also adds overshoot to the system response, which is quite essential to some extent for the quick response against sudden variations [16].

After having set, both proportional and integral variables, now adjust the derivative parameter. Gradually increase its value unless, system attains its desired set point. The increase in the value of derivative gain result in minimizing the system overshoot response and also increase the stability. However, too much increase in its value makes the system behaviour noise sensitive. So, derivative gain should be kept low.

2.2.5 Sliding Mode Control

Also, called variable structure control (VSC). It is a nonlinear control approach with distinguishing features, like insensitive behavior towards parametric changes, rejection of disturbances, good accuracy and ease of implementation [3].

The basic idea behind the Sliding mode technique is to bring the system states to a specific surface in state space, called as sliding surface. Once the system states attains or approaches the sliding surface, the controller job is to keep them as closer to the surface as possible. The sliding mode control design is based on two steps, which are as follows; [14]

- In the first step sliding surfaces are designed which meets the system requirements.
- In the second step a control law is established which can drives the system states towards the surfaces.

This technique have two remarkable advantages. First the dynamic response of the plant can be customized by selecting a specific sliding function [20]. Secondly, feedback loop behavior is insensitive to parametric uncertainties. These uncertainties can be disturbances, model uncertainties and bounded nonlinearities.

In real life, it used for nonlinear control applications where disturbances and uncertainties are the matter of high concern. This technique does not requires exact knowledge about the dynamics of the system under control consideration [21].

Chattering is the biggest problem associated with SM control strategy. Which is basically the oscillations of input signal with different frequency and amplitude. These oscillations of the input signal can damage, disrupt or wear the controller as well as physical system [2].

Sliding mode is the good nonlinear control methodology for the best performance of robotic arm manipulator as they have complicated nonlinear dynamics.

Chapter 3

Position Control of Robotic Manipulator using PID Control

3.1 Introduction

This chapter mainly focus on robot manipulator motion control problem, that is the position regulation of its arms. A basic method that can be used to solve this problem is called joint space technique, by using which first given problem is converted into a path which is desirable for the joints to follow, then a control strategy or law is suggested, which is used for computing the torques needed to be apply at the joints to get desired motion response from the manipulator.

Inspite of the new development in the area of control, proportional-integral-derivative (PID) technique is the most extensively use methodology in industries due to its simplicity of design, implementation and clear physical meanings of its three gains. PID control is used for the position tracking of the joints of robotic arm manipulator as its offers good tracking with small overshoot and minimum tracking error.

Electrical, pneumatic or hydraulic actuation mechanisms are mostly used for driving the joints of a robot. These actuation devices impose torques or forces at the joints, which result in movements of robotic arms. A precise control strategy is necessary for the control of robotic arms to meet the desired motion specifications.

Generally used methods for the motion control of the robot, depends at large extent on its mathematical model. The modeling and control of robot manipulator is an open field of research for the control designers as well as for researchers [11].

Proportional integral derivative control has very straight forward formation and comprehensive physical interpretation of its gain parameters. Its performance is adequate and widely accepted in industries. Using PID control algorithms every single joint of manipulator is mostly independently controlled [12].

The error and trial method for the tuning of PID controllers is quite easy provided that the robots have reduction type transmission mechanisms. Because for such robots torques are more smoother and can be attain with less effort. This really makes easier to move robotic arms with large masses.

3.2 Dynamic Model

The dynamic model of the robot manipulator is its response in terms of the movements of its arms to the forces applied by electrical, pneumatic or any other type actuator at its joints. [1].

A set of nonlinear second order differential equations are used for describing the dynamics of a manipulator. These differential equations mainly rely on inertial and kinematics characteristics of the manipulator. The dynamic model of the manipulator can be derived by accumulating all the forces acting on it. But in this research we will use lagrangian technique for the dynamics.

First, the equations of motion are to be determined for the manipulator. Actually, for control of a manipulator the torques acting at joints, resulting in its movements are very essential for desired motion control.

For the precise computation of torques from the actuators, we need a good mathematical model of the manipulator. In other words a model that can help us to directly calculate the torques from it. After finding the forces or torques at joints, the next phase is to design a control feedback law by keeping in view the system requirements. This control law will continuously update the forces to overcome the deviations from the desired trajectory [18].

As the dynamic model of manipulator is, the relation between the joint forces and its movements. So, for dynamic modeling we need to create a relationship between joints position parameters, their derivative like velocity, acceleration and the torques acting at the joints [2].

This model relies heavily on the established balance of forces in Newton's second law (Eq. 3.1) or rotational motion law of Euler (Eq. 3.2).

$$\sum F = \frac{d}{dt}(mv) \tag{3.1}$$

$$\sum T = \frac{d}{dt}(I\omega) = I\dot{\omega} + \omega X(I\omega) \tag{3.2}$$

For single link robot manipulator as shown in Fig. 3.1, the balance of forces-torques would result in the following equation [22]:

$$\tau - MgL\cos\theta = I\frac{d^2\theta}{dt^2} \tag{3.3}$$

$$\tau = ML^2\ddot{\theta} + MgL\cos\theta \tag{3.4}$$

3.2.1 Approaches for Dynamic Model

3.2.1.1 Direct Dynamic Model

The direct dynamic model describes the joint coordinates or configuration parameters as a function of forces or torques acting at the joints. i.e.

$$\theta(t) = f(\tau(t))$$

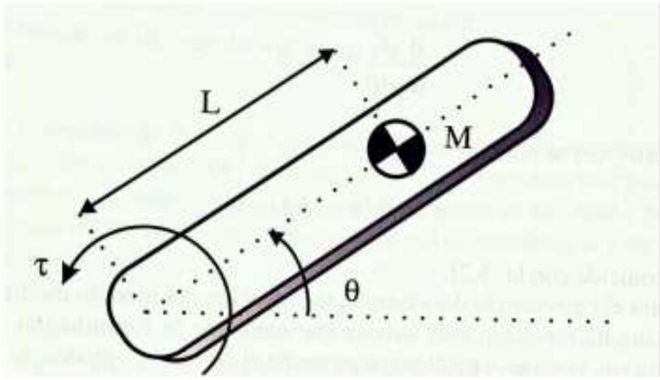

FIGURE 3.1: Model of the one link manipulator.

3.2.1.2 Inverse Dynamic Model

This model represents forces or torques acting at joints as function of its coordinates or configuration parameters.i.e.

$$\tau(t) = g(\theta(t))$$

3.2.2 Lagrangian Formulation

The best technique that can be used for obtaining the dynamic model is, using the Lagrangian approach based on energetic considerations, that is the difference between kinetic energy (KE) and potential energy (PE). This formulation greatly facilitates in deriving the complicated mathematical model of a manipulator [2]. The lagrangian equation is

$$\mathcal{L}(q(t), \dot{q}(t)) = KE(q(t), \dot{q}(t)) - PE(q(t)) \tag{3.5}$$

For the lagrangian approach, only kinetic and potential energies of the system are needed to calculate. This approach is less vulnerable to errors than accumulating all the forces acting on the robot like inertial and actuator forces etc.

3.2.3 Euler-Lagrange

The Euler Lagrange equation is second order partial differential equation. It is also refereed as Lagrange equation. The Euler lagrange equation uses the lagrangian equation in its computations. [2].

Actually, to get the Euler Lagrange equation solution, first the kinetic and potential energies of the system are evaluated. Then these equations are used in Lagrangian equation, which is basically the difference of kinetic and potential energies.

Finally, the Lagrangian $\mathcal{L}$, is presented to Euler Lagrange equation which apply partial derivatives on kinetic and potential energies of the system under consideration. Here we use Euler Lagrange for the evaluation of forces acting at the joints. It is represented as follow [2]

$$F = \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right] - \frac{\partial \mathcal{L}}{\partial \theta} \tag{3.6}$$

Where, F is the force acting on the joints of the manipulator and $\mathcal{L}$, represents the Lagrangian equation.

3.2.4 Kinetic Energy

The equation for the kinetic energy of a system is

$$KE = \frac{1}{2}mv^2 \tag{3.7}$$

Here, m is for the mass of robotic arm and v for the velocity of the joint. Further, the mass of each arm is considered at the end of the link .

For the robot manipulator shown in Fig.3.2 we obtain the following dynamic model

$$K_E = \frac{1}{2}I\dot{\theta}^2 \tag{3.8}$$

Here, $I = ML^2$ as inertia tensor and $\dot{\theta}$ is the angular velocity of robotic arm.

3.2.5 Potential Energy

The potential energy equation is defined as

$$P_E = mgl \tag{3.9}$$

Where g is the gravitational force and l is the length of projection of the link. We can write it as

$$P_E = Mgh$$

$$P_E = MgL \sin\theta \tag{3.10}$$

Putting the Kinetic and Potential energy equations in Lagrangian relation (3.5) for the robotic manipulator

$$\mathcal{L}(q(t), \dot{q}(t)) = \frac{1}{2} ML^2\dot{\theta}^2 - MgL \sin\theta \tag{3.11}$$

Now the force applied at the joint can be computed by using Euler Lagrange equation;

$$\frac{\partial \mathcal{L}}{\partial \theta} = -MgL \cos\theta$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = ML^2\dot{\theta}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = ML^2\ddot{\theta} \tag{3.12}$$

From the Euler-Lagrange equation finally we have the force

$$F = ML^2\ddot{\theta} + MgL \cos\theta \tag{3.13}$$

Irrespective of the method used for driving the dynamic model, the outcome is always a nonlinear equation as shown;

$$F = M(q)\ddot{q} + C(q, \dot{q}) + G(q) \tag{3.14}$$

Where $q \in R^n$ represents the link position, n is the joint or degree of freedom and $F \in R^n$ represents control input. $M \in R^{n\times n}$ is the inertia matrix, C, the Coriolis/centripetal matrix, and $G(q)$ represents the gravity vector.

3.3 PID Control

Considering the nonlinear equation that are derive from Euler Lagrange equation, the control input parameter F is torque applied by the actuator at joints. Which is unknown, the purpose of this torque is to make the joint to follow the suggested motion.

For this purpose following PID control method will be used [11];

$$F = K_p e + K_i \int_0^t e(\tau) d\tau + k_d \dot{e} \tag{3.15}$$

Here,

e, represents tracking error, K_p, K_i and K_d are gains of the PID controller.

The above equation for the PID control, can also be written as

$$F = K_p e + K_d \dot{e} + \xi \tag{3.16}$$

Where,

$$\dot{\xi} = K_i e$$

and initial value of ξ

$$\xi(0) = \xi_0$$

Due to the integral in the equation for the PID control (3.16) an extra state is added to substitute it, which is represented as ξ and the derivative as $\dot{\xi} = K_i e$. The equation obtained by substituting the F from Eq.(3.16) in the equation (3.14).

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = K_p e + K_d \dot{e} + \xi \tag{3.17}$$

we can write it in form of state vector $[\xi \; e \; \dot{e}]^T$ like;

$$\frac{d}{dt}\begin{bmatrix} \xi \\ e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} e \\ \dot{e} \\ qd - M(q)^{-1}[K_p e + K_d \dot{e} + K_i \xi - C\left(q,\dot{q}\right)\dot{q} - G\left(q\right)] \end{bmatrix} \tag{3.18}$$

3.3.1 Model for PID Control of Robot Manipulator

Here, we will implement the PID control technique to control the torque acting at the joints of manipulator, for driving them to follow the desired movements. For the mathematical model, we will concentrate on a two link robot manipulator as represented in Fig. 3.2.

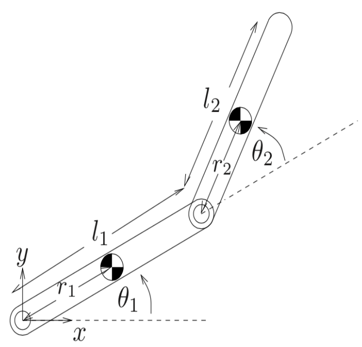

FIGURE 3.2: Two Link Robot Manipulator Model.

The energy based Lagrangian equation (3.5) will be solved by calculating the kinetic and potential energy. However, we do not know the velocity but, we know the derivative of position with respect to time is velocity. So, we done the position in the end of the link by the use of the parameters known to us, that is, we can use

$$\begin{aligned} x_1 &= l_1 \sin\theta_1 \\ y_1 &= l_1 \cos\theta_1 \\ x_2 &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \\ y_2 &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \end{aligned} \tag{3.19}$$

K.E

By putting in kinetic energy equation we can have expression as below;

$$K_E = \frac{1}{2}M_1\dot{x}_1^2 + \frac{1}{2}M_1\dot{y}_1^2 + \frac{1}{2}M_2\dot{x}_2^2 + \frac{1}{2}M_2\dot{y}_2^2 \tag{3.20}$$

By simplification

$$\begin{aligned} K_E = \frac{1}{2}(M_1 + M_2)L_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2L_2^2\dot{\theta}_1^2 + M_2L_2^2\dot{\theta}_1\dot{\theta}_2 \\ + \frac{1}{2}M_2L_2^2\dot{\theta}_2^2 + M_2L_1L_2\cos\theta_2(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1^2) \end{aligned} \tag{3.21}$$

P.E

The Potential energy is

$$P_E = M_1gL_1\cos\theta_1 + M_2g(L_1cos\theta_1 + L_2\cos(\theta_1 + \theta_2)) \tag{3.22}$$

Using the Lagrangian equation we have [2]

$$\mathcal{L} = K_E - P_E$$

$$\mathcal{L} = \frac{1}{2}(M_1 + M_2)L_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2L_2^2\dot{\theta}_1^2 + M_2L_2^2\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}M_2L_2^2\dot{\theta}_2^2$$

$$+ M_2L_1L_2\cos\theta_2(\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1^2) - (M_1gL_1\cos\theta_1 \tag{3.23}$$

$$+ M_2g(L_1\cos\theta_1 + L_2\cos(\theta_1 + \theta_2))$$

Now, we will use the Lagrange Euler Eq. (3.6) in order to compute force applied at the joints of the robot.

$$F_{\theta_{1,2}} = \frac{d}{dt}\left[\frac{\partial\mathcal{L}}{\partial\dot{\theta}_{1,2}}\right] - \frac{\partial\mathcal{L}}{\partial\theta_{1,2}} \tag{3.24}$$

So, by simplification of dynamic equations

$$F_{\theta_1} = ((M_1 + M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 + (M_2L_2^2$$

$$+ M_2L_1L_2\cos\theta_2)\ddot{\theta}_2 - M_2L_1L_2\sin\theta_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) \tag{3.25}$$

$$- (M_1 + M_2)gL_1\sin\theta_1 - M_2gL_2\sin(\theta_1 + \theta_2)$$

Where F_{θ_1} is the force/torque acting at 1^{st} joint of robotic manipulator.

and

$$F_{\theta_2} = \left(M_2L_2^2 + M_2L_1L_2\cos\theta_2\right)\ddot{\theta}_1 + M_2L_2^2\ddot{\theta}_2$$

$$\tag{3.26}$$

$$- M_2L_1L_2\sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 - M_2gL_2\sin(\theta_1 + \theta_2)$$

Where F_{θ_2} is the force/torque acting at joint 2 of robotic manipulator. Now, the system motion can be described by

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

and

$$M(q) = \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix}$$

where

$$D_1 = (M_1 + M_2)L_1^2 + M_2 L_2^2 + 2M_2 L_1 L_2 \cos\theta_2$$

$$D_2 = M_2 L_2^2 + M_2 L_1 L_2 \cos\theta_2$$

$$D_3 = M_2 L_2^2 + M_2 L_1 L_2 \cos\theta_2$$

$$D_4 = M_2 L_2^2$$

and

$$C(q, \dot{q}) = \begin{bmatrix} -M_2 L_1 L_2 \sin\theta_2 (2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \\ -M_2 L_1 L_2 \sin(\theta_2) \dot{\theta}_1 \dot{\theta}_2 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} -(M_1 + M_2) g L_1 \sin\theta_1 - M_2 g L_2 \sin(\theta_1 + \theta_2) \\ -M_2 g L_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$

$$F = \begin{bmatrix} F_{\theta_1} \\ F_{\theta_2} \end{bmatrix}$$

Having the system equation,

$$\ddot{q} = M(q)^{-1}[-C(q, \dot{q}) - g(q)] + \hat{F} \tag{3.27}$$

$$\hat{F} = M(q)^{-1} F \Leftrightarrow F = M(q)\hat{F} \tag{3.28}$$

By, decoupling the system to have non physical input

$$\hat{F} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{3.29}$$

But, the physical inputs to the system

$$\begin{bmatrix} f_{\theta_1} \\ f_{\theta_2} \end{bmatrix} = M(q) \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{3.30}$$

System error signals can be written as

$$\begin{aligned} e(\theta_1) &= \theta_{1f} - \theta_1 \\ e(\theta_2) &= \theta_{2f} - \theta_2 \end{aligned} \tag{3.31}$$

These equations represents the difference of final and initial positions of manipulator. Here we will use the following final positions values;

$$\begin{bmatrix} \theta_{1f} \\ \theta_{2f} \end{bmatrix} = \begin{bmatrix} \frac{\pi}{2} \\ -\frac{\pi}{2} \end{bmatrix} \tag{3.32}$$

the initial positions values are

$$\theta_0 = \begin{bmatrix} -\frac{\pi}{2} \\ \frac{\pi}{2} \end{bmatrix} \tag{3.33}$$

With this knowledge, now we are able to formulate the PID based control structure by applying classical linear law (3.15). So, system equation will become

$$\ddot{q} = M(q)^{-1}[-C(q, \dot{q}) - g(q)] + \hat{F} \tag{3.34}$$

With

$$\hat{F} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} K_{p1}\left(\theta_{1f} - \theta_1\right) - K_{d1}\dot{\theta}_1 + K_{i1}\int e\left(\theta_1\right) dt \\ K_{p2}\left(\theta_{2f} - \theta_2\right) - K_{d2}\dot{\theta}_2 + K_{i2}\int e\left(\theta_2\right) dt \end{bmatrix} \tag{3.35}$$

We will consider the actual inputs acting at joints (3.30). We have added an extra state for each joint position as a replacement for integral in PID law for simulation purposes:

$$\begin{aligned} z_1 &= \int e(\theta_1)\,dt \\ \dot{z}_1 &= \theta_{1f} - \theta_1 \\ z_2 &= \int e(\theta_2)\,dt \\ \dot{z}_2 &= \theta_{2f} - \theta_2 \end{aligned} \tag{3.36}$$

So, the system equations are

$$\left\{ \begin{array}{c} \dot{z}_1 = \theta_{1f} - \theta_1 \\ \dot{z}_2 = \theta_{2f} - \theta_2 \\ \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = M(q)^{-1}[-C(q,\dot{q}) - G(q)] + \hat{F} \end{array} \right\} \tag{3.37}$$

3.4 Simulation Results

The Proportional Integral Derivative algorithm is used for the control of a two link robot arm manipulator. PID gain variables are tuned manually for attaining the satisfactory performance of the manipulator through control action.

The control operation is evaluated against various values of aforesaid parameters. Here simulation results with list of parameters used for good performance are presented.

TABLE 3.1: Parameters for the Simulation

Sr#	**Symbol**	**Definition**	**Value**
1	l_1	Length for first link	1m
2	l_2	Length for second link	1m
3	m_1	Mass for first link	1kg
4	m_2	Mass for second link	1kg
5	g	Gravitational constant	9.8m/s^2
6	K_{p1}	Proportional gain for 1st link	15
7	K_{d1}	Derivative gain for 1st link	7
8	K_{i1}	Integral gain for 1st link	10
9	K_{p2}	Proportional gain for 2nd link	15
10	K_{d2}	Derivative gain for 2nd link	10
11	K_{i2}	Integral gain for 2nd link	10

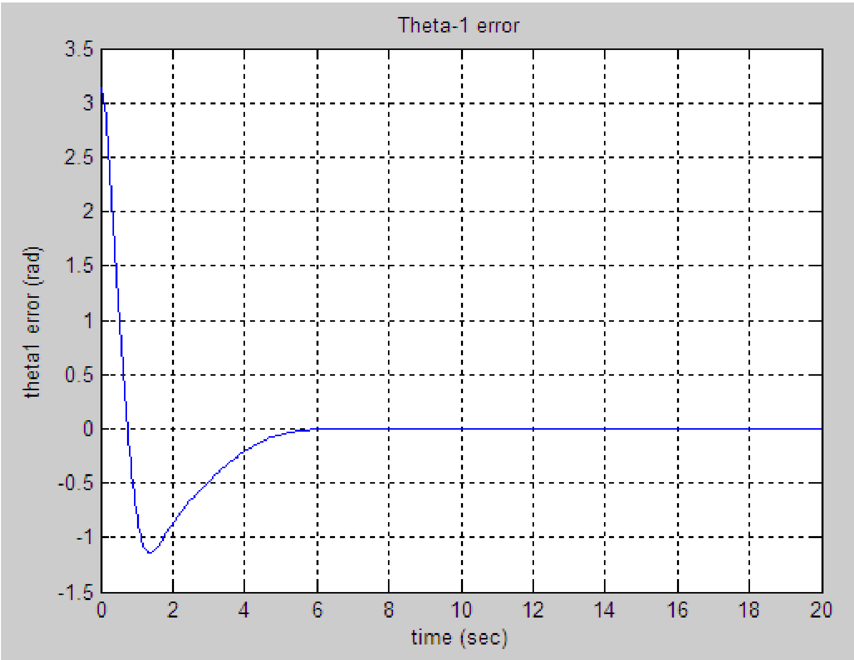

FIGURE 3.3: Joint1 Tracking Error.

Figure 3.3 depicts the position error for the link one. It shows, link one reaches the desired stable position in considerably fast time and remains stable thereafter using the K_p, K_i and K_d mentioned in Table 3.1.

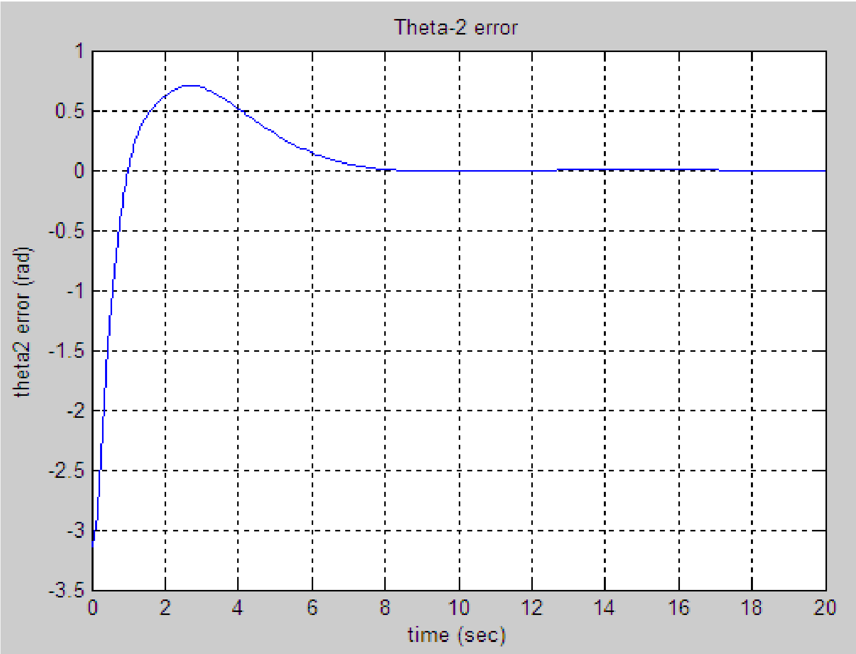

FIGURE 3.4: Joint2 Tracking Error.

Similarly, Fig. 3.4 shows the quick response of the control strategy in making the robotic arm to reach its desired position for the second link of the robot manipulator.

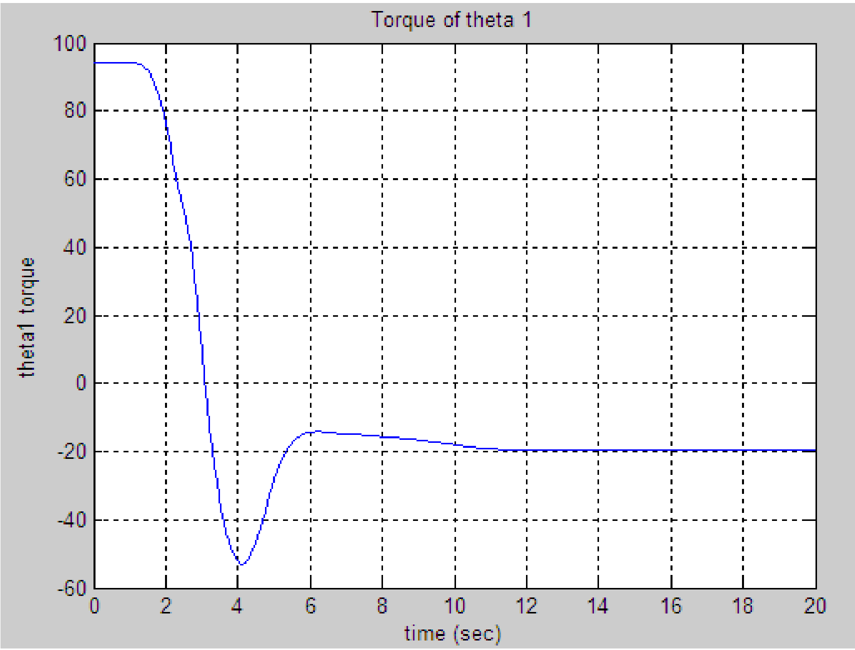

FIGURE 3.5: Force/Torque applied at joint1.

Figure 3.5 indicates the force or torque applied for position tracking of link 1, initially the force applied on the link one has random behavior, but it gets stabilize very quickly.

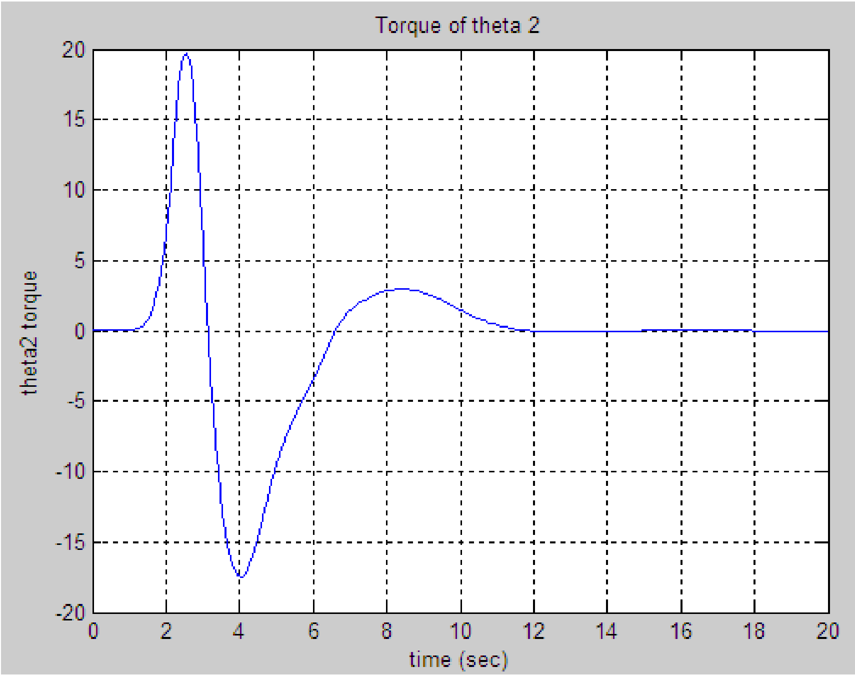

FIGURE 3.6: Force/Torque applied at joint2.

It can be seen in Fig. 3.6, the Force or Torque applied at second link for tracking the position has slight overshoot or peaks initially. Similar to the force applied at link one, it's also get stabilize quickly.

From simulation results, we have experienced that the proportional gain K_p depends on error term, also, by increasing the value of K_p the speed of the system response can be increased, where as K_d the derivative gain respond to changes in error, and K_i the integral gain which has to drive the system to a zero steady state error.

3.5 Conclusion

The PID is by far the most prevalent controller in the process industry due to ease of its implementation and satisfactory performance as compared to other control techniques. From the Simulation results presented above, we conclude that robotic manipulator perform well under the action of PID controller. The tracking error for both joint is minimum also force applied at both joints gets stable very quickly with very small over shoot. Hence, the PID control is good technique for motion control of robot arm manipulator provided that PID parameters are adjusted precisely.

Chapter 4

Position Control of Robotic Manipulator using Sliding Mode Control (VSC) Technique

4.1 Introduction

In this chapter, we will concentrate on a nonlinear control method refereed as the Sliding Mode or variable structure control (VSC) for the motion control of a robotic arm manipulator.

The most of the work done in seventies and eighties was mainly focused on the robust behavior of the controllers. The controllers that can withstand their performance and stability even in the presence of external disturbances, discrepancies in physical system and model are called as robust controllers. The robust control technique that emerge during this era is sliding mode control [20].

This control technique has been used for various applications over the years for optimal and robust performance, like in flight control, robotic manipulators, in tracking systems, electrical and mechanical field, adaptive methodologies and many more [16, 20, 23].

The sliding mode control terminology was initially introduced in the perspective of variable structure control. SM control has proven its importance by robustness behavior for control problems having different discrepancies in their model. It is one of the best control technique for systems with complicated dynamics operating under the influence of different uncertainties and disturbances. The order reduction property of SMC provides an opportunity to control the higher order systems having complex dynamics [10].

Apart from the advantages, it also has one major disadvantage, known as chattering. This phenomenon is basically the oscillations of input control signal, due to the discontinuous behavior of control method. The presence of these unwanted oscillations is highly dangerous because they can completely destroy the system and the controller functionality.

The robustness behavior of sliding mode control methodology is firmly linked to oscillations of control signal at high frequency. But practically these oscillations are of finite frequency and their amplitude is also finite resulting in degradation of system performance. They are two major reasons for this behavior. Firstly, during the design phase some of the system dynamics are ignored, either intentionally or unintentionally considering them as fast dynamics, these dynamics get excited by the fast switching of the controller. Secondly, the use of microcontrollers for implementation also result in chattering [24].

Robot manipulator being highly complicated system with nonlinear dynamics, requires a nonlinear controller technique for their best functionality and optimal performance [2].

4.2 Dynamics of Robot Manipulator

In control system design, there are numerous problems which requires position tracking. Depending upon the application, mostly force is used as an input parameter to achieve the objective. Here, we will focus on the position control of robot manipulator using sliding mode control approach. The robotic arm manipulator has different number of links which are connected with each other through joints. Actuators imposed force or torque at joints to drive them according to the requirement. These actuators also have dynamics which are mostly ignored in the design phase, by considering them stable and quicker than the inertial dynamics. Such negligence in the design phase ultimately leads to chattering issue.

The dynamic model of robot is

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \tag{4.1}$$

Here, $q \in R^n$ represents joint configuration variables, and $\tau \in R^n$ represents control input. $M \in R^{n\times n}$ is inertia matrix, $C(q,\dot{q})\dot{q}$, the Coriolis vector, and $G(q)$ represents the gravity vector [1].

4.3 Sliding Mode Control Design

The sliding mode control methodology is the collection of decision strategies and feedback laws. To determine which control law is acting on the system under the sliding mode control at any moment of time, its depends upon the system status and corresponding decision strategy. This particular control strategy has two phase. First one is called reaching mode while the second is sliding mode. For the sliding mode control technique, first a appropriate switching function is designed by keeping in view the required sliding mode dynamics and than a control law is designed in such a way that the reaching condition can be easily met [2]. Usually, the required sliding mode dynamics are stable, very fast and error less responses. The general form of switching manifold is;

$$S\left(x\right) = C^T x = 0 \tag{4.2}$$

For a system with m inputs.

$$S_j = \left\{ x | s_j = C_j^T x = 0 \right\}, \quad j = 1, \cdots, m \tag{4.3}$$

The dynamic response of a system after reaching the sliding surface and remain restricted to it, is refereed as ideal behavior or ideal motion. There are two main positive aspects of such sliding motion

- First, its capability of reducing the order of the system, this really makes it a suitable choice for the control of higher order complex systems.
- The second remarkable advantage is, insensitive nature for parametric variations and rejection of external disturbances, making it a robust control technique [20].

The reaching law must be precisely designed because, not only the attractiveness of the sliding surface for the system state parameters but also the dynamic performance of the system in both reaching and sliding mode depends on it. Further, it also provides means for chattering intensity control in the input signal. So, it has complete capability to influence the dynamics of system.

Reaching law has mathematical formation in the form of differential equation which describes the dynamics for the switching function. The reaching condition is also a differential equation of an asymptotically stable switching function. In reaching phase the dynamic properties of system can be altered as per requirement by making suitable choice of variables in differential equation. The constant rate reaching law can be defined as [24]:

$$\dot{s} = -ksign(s) \tag{4.4}$$

Dynamic model of robot manipulator from Eq. (4.1) is

$$M(q)\ddot{q} + C(q, \dot{q}) + g(q) = U \tag{4.5}$$

Here u represent input force/torque acting at the joint. By simplification of robot dynamic, Lagrangian and Euler Lagrange equations, as illustrated in previous chapter we obtained

$$M(q) = \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix}$$

Where

$$\ddot{q} = M(q)^{-1}[-C(q,\dot{q}) - G(q) + U] \tag{4.6}$$

The principle of designing SMC law for arbitrary order plants is to force the error and derivative of error of a variable to zero. The robot arm is to track a desired motion $q^d(t)$. Define an error vector [2]:

$$x = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} q^d - q \\ \dot{q}^d - \dot{q} \end{bmatrix} \tag{4.7}$$

Now, the sliding surface for position and velocity control of robotic arm manipulator with 2DOF can be defined as

$$S = \left(\frac{d}{dt} + \lambda\right) x \tag{4.8}$$

$$\dot{S} = \ddot{q}^d - \ddot{q} + \lambda \dot{e}_1 \tag{4.9}$$

Here, We will use error and its time derivative as system states $(e, \dot{e})$. The control law for the reaching condition can be defined as

$$\frac{1}{2}\frac{d}{dt}S^2 = -\eta|S| \tag{4.10}$$

Where, $\eta > 0$ From equation (4.6) and (4.9), we have

$$\dot{S} = \ddot{q}^d + M(q)^{-1}[C(q,\dot{q}) + G(q) - U] + \lambda \dot{e}_1 \tag{4.11}$$

By applying the invariance condition to switching surface S, the corresponding equivalent control is given by putting $\dot{S} = 0$.

$$U_{eq} = [C(q,\dot{q}) + G(q)] + M(q)[\ddot{q}^d + \lambda \dot{e}_1] \tag{4.12}$$

Now, introducing switching law

$$U_{SW} = -ksign\,(s)$$

$$sign\,(s) = \begin{cases} -1, s < 0 \\ 0, s = 0 \\ 1, s > 0 \end{cases} \tag{4.13}$$

So, the overall controller becomes

$$U = U_{eq} + U_{Sw}$$

$$U = [C(q,\dot{q}) + G(q)] + M(q)[\ddot{q}^d + \lambda \dot{e}_1] - ksign\,(s) \tag{4.14}$$

This will make equation (4.11) as

$$\dot{S} = -M(q)^{-1} ksign\,(s) \tag{4.15}$$

Now, we will apply Lyapunov function candidate

$$V = \frac{1}{2} S^2 \tag{4.16}$$

Taking derivative and further simplification

$$\begin{aligned} \dot{V} &= S\dot{S} \\ \dot{V} &= -sM(q)^{-1} ksign\,(s) \\ \dot{V} &= -M(q)^{-1} k|s| \end{aligned} \tag{4.17}$$

Which, ultimately becomes

$$\dot{V} = -M(q)^{-1}k|s| \tag{4.18}$$

The above equation shows that the sliding condition defined in equation (4.10) is met.

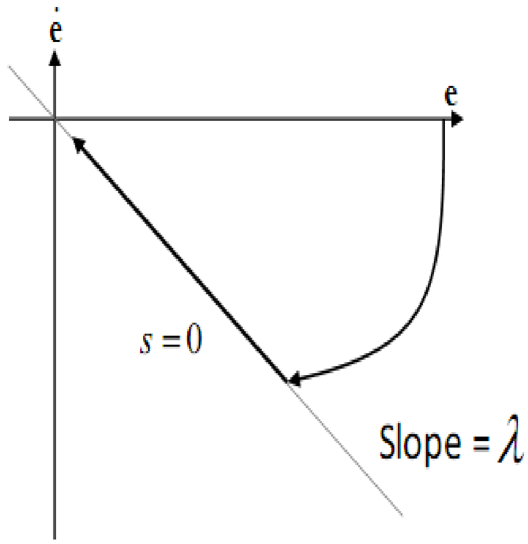

Figure 4.1: Sliding surface.

Now, the variable structure control of a robotic manipulator is designed from its dynamics presented in chapter 3. The equations (4.5) and (4.6) can be written as [16]

$$\begin{aligned} \ddot{x} &= f + u \\ or\ddot{q} &= f + u \end{aligned} \tag{4.19}$$

Where, f is the dynamics of robot manipulator and u is the control input.

From equation (4.8) error $(e, \dot{e})$ as state variable the sliding surface of the system can be defined

$$S = \left(\frac{d}{dt} + \lambda\right) e$$

$$S = \dot{e} + \lambda e \tag{4.20}$$

$$\dot{S} = \ddot{e} + \lambda\dot{e} \tag{4.21}$$

By putting $\ddot{e}$=$\ddot{q}$−$\ddot{q}^d$ and relation (4.19) in (4.21)

$$\dot{S} = f + u - \ddot{q}^d + \lambda\dot{e} \tag{4.22}$$

By applying invariance Condition i.e $\dot{S} = 0$

$$\begin{aligned} 0 &= f + u - \ddot{q}^d + \lambda\dot{e} \\ u_{eq} &= -f + \ddot{q}^d - \lambda\dot{e} \end{aligned} \tag{4.23}$$

Since our dynamic model f is an approximation, the approximated law that can attain $\dot{S} = 0$ can be expressed as

$$u_{eq} = -\hat{f} + \ddot{q}^d - \lambda\dot{e} \tag{4.24}$$

For Joint 1

$$\begin{aligned} \hat{f}_1 = ((M_1 + M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 + (M_2L_2^2 + M_2L_1L_2\cos\theta_2) \\ \ddot{\theta}_2 - M_2L_1L_2\sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 - M_2L_1L_2\sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ \dot{\theta}_2 - (M_1 + M2)gL_1\sin\theta_1 - M_2gL_2\sin(\theta_1 + \theta_2) \end{aligned} \tag{4.25}$$

and

$$\begin{aligned} u_{eq1} = -((M_1 + M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 - (M_2L_2^2 + M_2L_1L_2\cos\theta_2) \\ \ddot{\theta}_2 + M_2L_1L_2\sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 + M_2L_1L_2\sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ \dot{\theta}_2 + (M_1 + M2)gL_1\sin\theta_1 + M_2gL_2\sin(\theta_1 + \theta_2) + \ddot{q}^q - \lambda_1\dot{e}_1 \end{aligned} \tag{4.26}$$

For Joint 2

$$\hat{f}_2 = (M_2L_2^2 + M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 + (M_2L_2^2)\ddot{\theta}_2$$
$$+ M_2L_1L_2\sin(\theta_2)\dot{\theta}_1^2 - M_2gL_2\sin(\theta_1+\theta_2) \tag{4.27}$$

$$u_{eq2} = -(M_2L_2^2 + M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 - (M_2L_2^2)\ddot{\theta}_2$$
$$- M_2L_1L_2\sin(\theta_2)\dot{\theta}_1^2 + M_2gL_2\sin(\theta_1+\theta_2) + \ddot{q}^q - \lambda_2\dot{e}_2 \tag{4.28}$$

Introducing the switching surface and applying the overall control law

$$u = u_{eq} + u_{Sw} \tag{4.29}$$

$$u_1 = -((M_1+M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 - (M_2L_2^2 + M_2L_1L_2\cos\theta_2)\ddot{\theta}_2$$
$$+ M_2L_1L_2\sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 + M_2L_1L_2\sin(\theta_2)(\dot{\theta}_1+\dot{\theta}_2)\dot{\theta}_2$$
$$+ (M_1+M_2)gL_1\sin\theta_1 + M_2gL_2\sin(\theta_1+\theta_2) + \ddot{q}^d - \lambda_1\dot{e}_1 - k_1sign(s) \tag{4.30}$$

and

$$u_2 = -(M_2L_2^2 + M_2L_1L_2\cos\theta_2)\ddot{\theta}_1 - (M_2L_2^2)\ddot{\theta}_2 - M_2L_1L_2\sin(\theta_2)\theta_1^2$$
$$+ M_2gL_2\sin(\theta_1+\theta_2) + \ddot{q}^d - \lambda_2\dot{e}_2 - k_2sign(s) \tag{4.31}$$

Putting the values of u (u_1, u_2) for joint 1 and 2, respectively, in equation (4.22), we have

$$\dot{S} = -ksign(s) \tag{4.32}$$

where, k is (k_1, k_2). Using Lyapunov function candidate

$$\begin{aligned} V &= \tfrac{1}{2}s^2 \\ \dot{V} &= s\dot{s} \\ \dot{V} &= -sksign(s) \\ \dot{V} &= -k|s| \end{aligned} \tag{4.33}$$

$$\dot{V} \leq -k|s| \tag{4.34}$$

Hence, sliding condition mentioned in Eq.(4.10) is satisfied for both joints.

4.4 Chattering

It can be described as the undesirable oscillations, that appears in various sliding mode control implementations. These oscillation have finite frequency and amplitude. They are actually associated with the input control signal. The main reason behind these oscillations is the discontinuous nature and high frequency switching of the controller, which causes the exciting of the dynamics of system, which were actually neglected in the design phase also called unmodeled dynamics. These dynamics can be of actuators or sensors, they are overlooked during design process with assumptions of being generally faster than the other system dynamics.

For the elimination of this unwanted phenomenon, there is no need to must have the detailed or precise model of all the system constituents. The elimination of chattering is too much important to make this technique as a viable control solution. Because this problem imposed a major hurdle for utilizing the extraordinary and remarkable features of SMC in different control applications.

The switching strategy, which is the backbone of the sliding mode control is not considered as source of chattering because in ideal case its frequency is almost infinity. Here we will consider this phenomenon that is Chattering, as highly unwanted finite frequency oscillatory response due to the unmodeled dynamics of the system.

4.4.1 Boundary Layer Method

When system states attains the sliding surface, the control signal start oscillating at high frequency. These oscillations are refereed as chattering as discussed in previous section. Chattering has many adverse effects on the system performance and even, can damage the system and controller.

There are different techniques used for the reduction or the elimination of this problem. One out of these is called boundary layer method. It is the most extensively used approach for the elimination of chattering. In which, a region around the sliding surface is defined as boundary layer, for the approximation of discontinuous sign function in this region a smooth continuous function is defined. In control strategy, it is achieved by the replacement of sign(s) in boundary layer region by sat (s). The boundary layer function is [16, 20];

$$sat\left(s\right) = \begin{cases} -1, \frac{s}{\phi} < 0 \\ 0, \left|\frac{s}{\phi}\right| < 1 \\ 1, \frac{s}{\phi} < -1 \end{cases} \tag{4.35}$$

and, the switching law becomes

$$U_{SW} = -ksat\left(s\right) \tag{4.36}$$

Here, ϕ represents the thickness of smooth boundary region [20]. If system states moves out of this region, the sign(s) function will replace boundary layer function which will influence the controller to drive the states towards the sliding surface. On reaching the surface the sign(s) will be replaced again with sat(s).

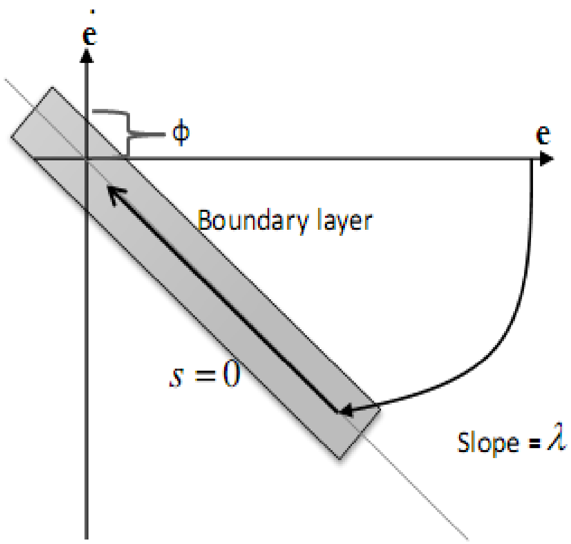

FIGURE 4.2: Smooth region.

One reason, that can be considered for the generation of the chattering is the discontinuous nature of control methodology. This leads to think about the replacement of discontinuous controller with a continuous one, so that the chattering can be eliminated. But as discussed previously, chattering is the high frequency oscillations which may persist even by the use of continuous controller. Therefore, we need apply a technique that can better tackle this problem.

Apart form this, the replacement of the discontinuous controller with a continuous function in boundary layer region can reduce or eliminate chattering, has been a widely accepted opinion. For the effectiveness of this method the thickness of the boundary layer should be precisely selected [24].

We can conclude, that the better way for the elimination of chattering is, one must design the system precisely during the modeling phase. All the system dynamics must be consider in the mathematical model of the system. In this way, the phenomenon of chattering can be eliminated right at the initial stage [2, 24].

4.5 Simulation Result

The following simulink model is used in simulation for the motion control of robotic arm manipulator.

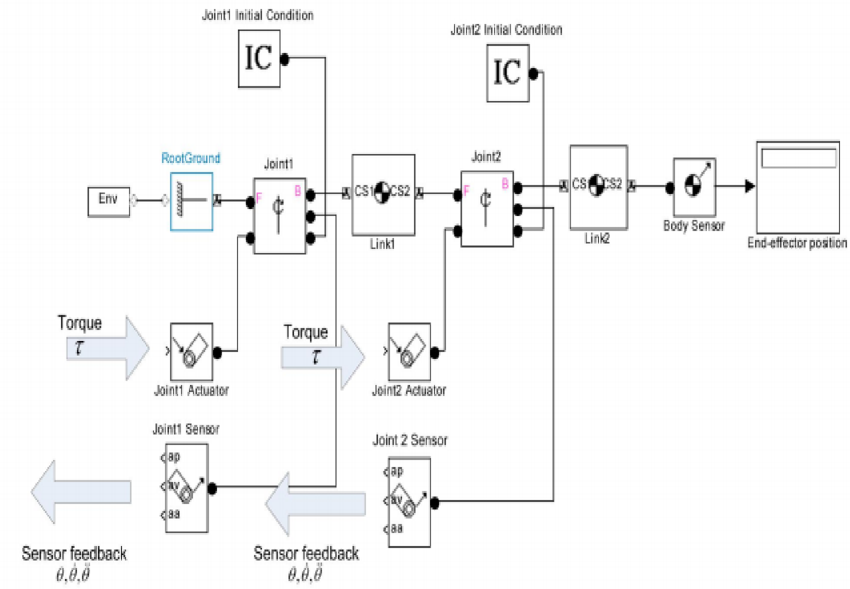

FIGURE 4.3: Simulink Model of Robot Manipulator.

The Sliding mode control (SMC) algorithm is applied for the motion control of robot manipulator. Here result are presented on two basis i.e.

- SMC without boundary layer
- SMC with boundary layer for the elimination of chattering

4.5.1 SMC Without Boundary Layer

The of list parameters used for the simulation along with result for error tracking and control force applied at both joint are given below.

and

$$\theta_1\,(0) = -90^0, \theta_2\,(0) = 90^0$$

TABLE 4.1: Parameters list for SMC simulation.

S. No.	**Symbol**	**Definition**	**Value**
1	l_1	Length for first link	1m
2	l_2	Length for second link	1m
3	m_1	Mass for first link	1kg
4	m_2	Mass for second link	1kg
5	g	Gravitational constant	9 .8 m/s^2
6	λ_1	Sliding surface constant for 1^{st} link	4
7	λ_2	Sliding surface constant for 2^{nd} link	4
8	K_1	For 1^{st} link	10
9	K_2	For 2^{nd} link	4

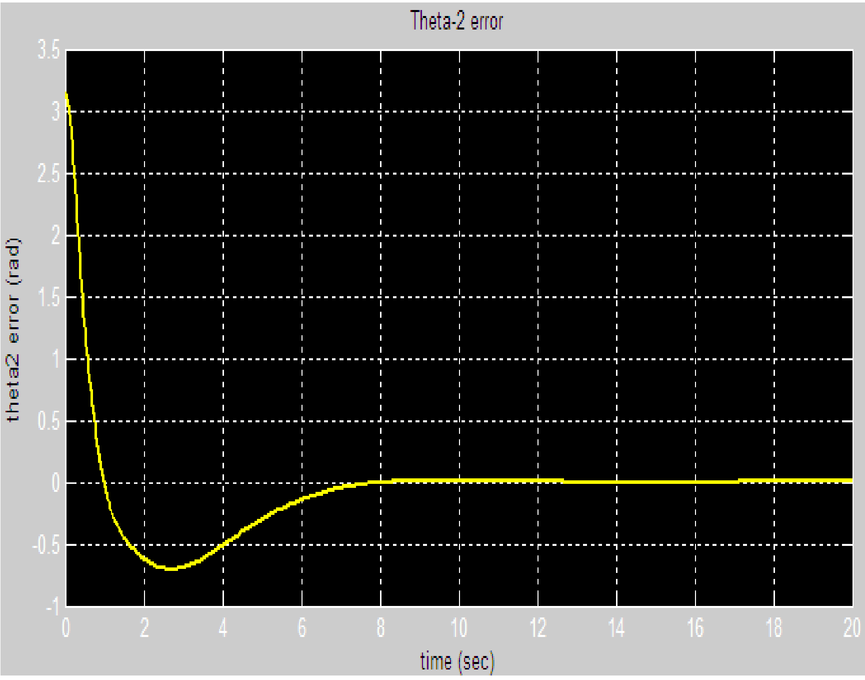

FIGURE 4.4: Joint 1 tracking error without boundary layer.

From Fig. 4.4, joint "1" reaches its desired position in one and half second that is much better as compared to its performance under the PID control.

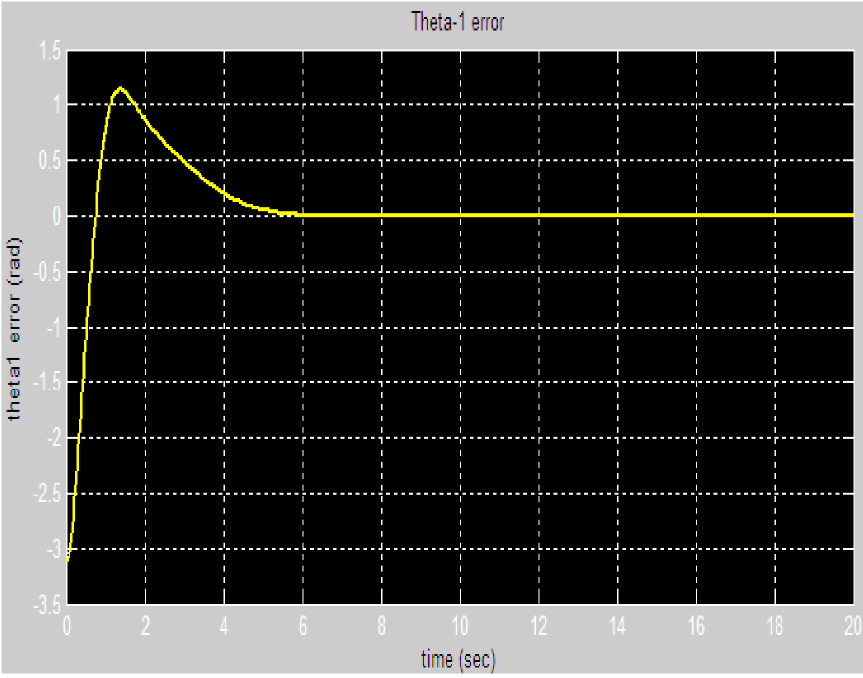

FIGURE 4.5: Joint 2 tracking error without boundary layer.

The response of Joint two in Fig.4.5, is very similar to that of joint one. Infact, it reaches the stable position even quicker and with minimum error than first one.

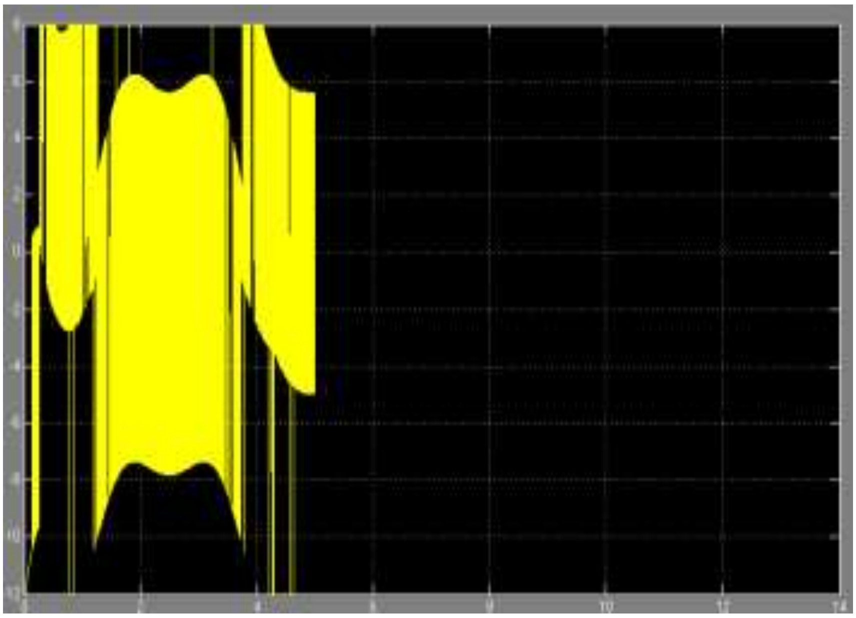

FIGURE 4.6: Joint one force/torque without boundary layer.

Figure 4.6 shows the main problem with sliding mode control called chattering. It is highly undesirable for both controller as well as for the system because it can badly effect the performance and may cause damage to physical system.

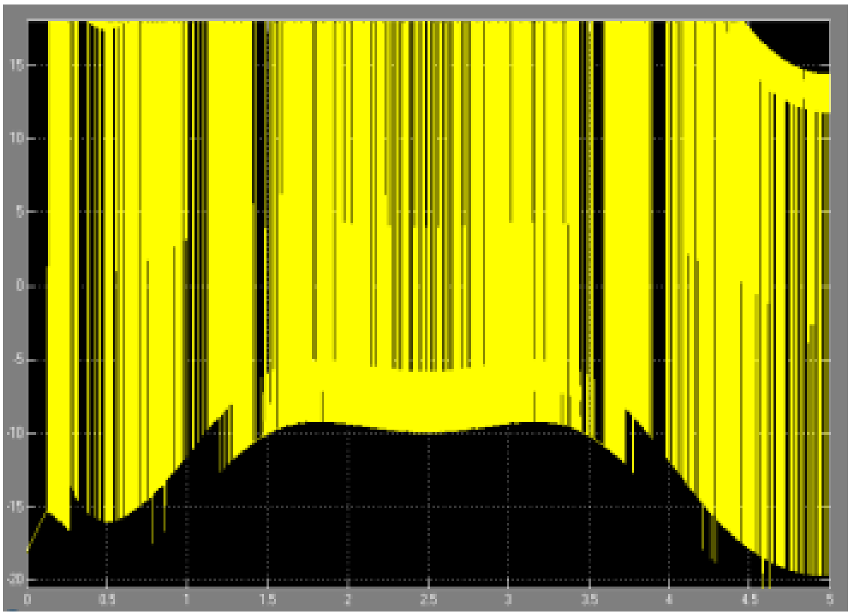

FIGURE 4.7: Joint two force/torque without boundary layer.

Figure 4.7 shows that chattering problem also exist with joint two.

4.5.2 SMC with Boundary Layer

To cater the chattering problem, we will use the boundary layer approach by replacing the sign(s) in the boundary layer region with a smooth continuous function. These functions will replace each other according to the situation. In side the boundary layer region the sat(s) will take over while outside the boundary sign(s). The thickness of boundary layer is denoted by ϕ and its value for joint one and two are as follow.

$$\phi_1 = 0.01 \quad \text{and} \quad \phi_2 = 0.01$$

The values of all other parameters are same as listed in Table 4.1. The simulation results are as follows

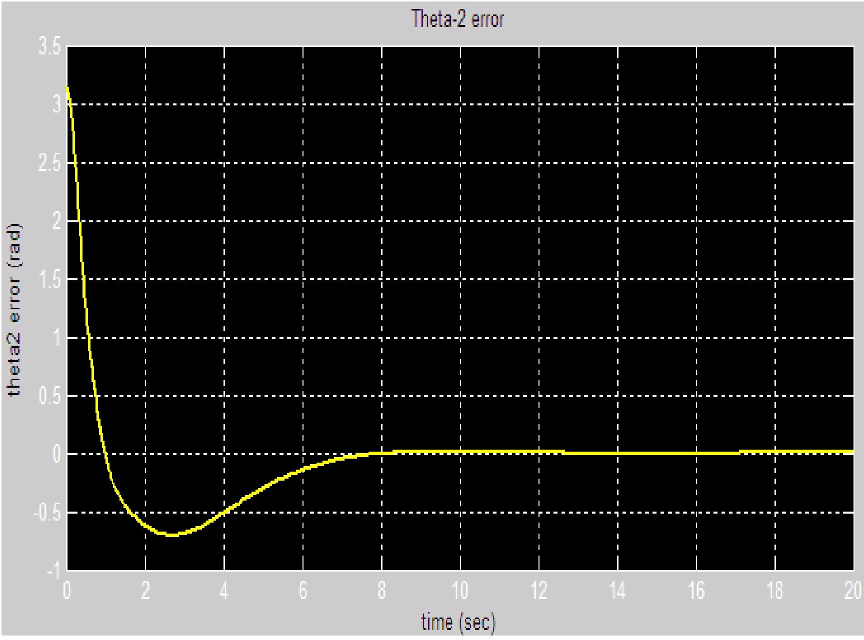

FIGURE 4.8: Joint 1 tracking error with boundary layer.

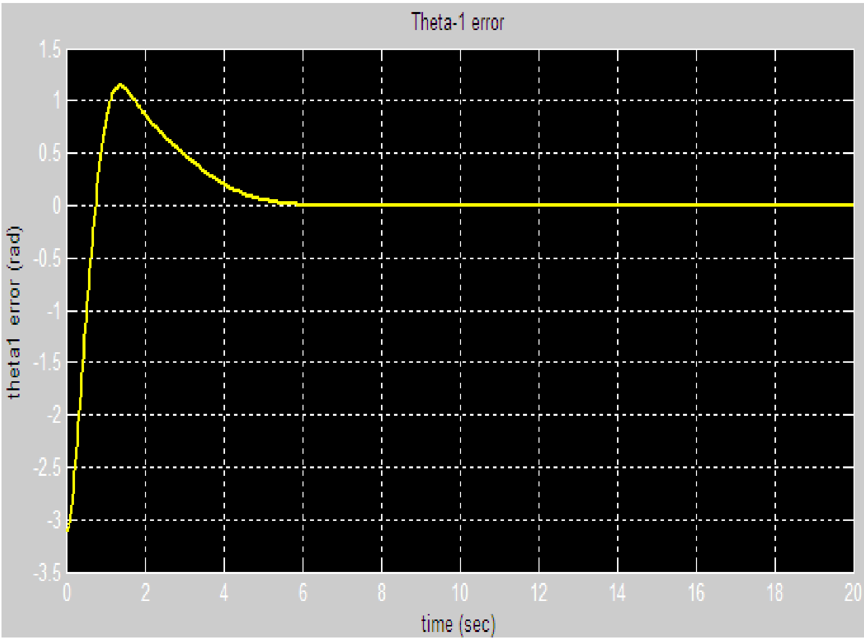

FIGURE 4.9: Joint 2 tracking error with boundary layer.

The tracking error response shown in Fig. 4.8 and Fig. 4.9 for SMC with introduction of boundary layer method is same as shown in Fig. 4.4 and Fig. 4.5 with SMC without boundary layer method. So, the tracking error behavior of both controllers is same.

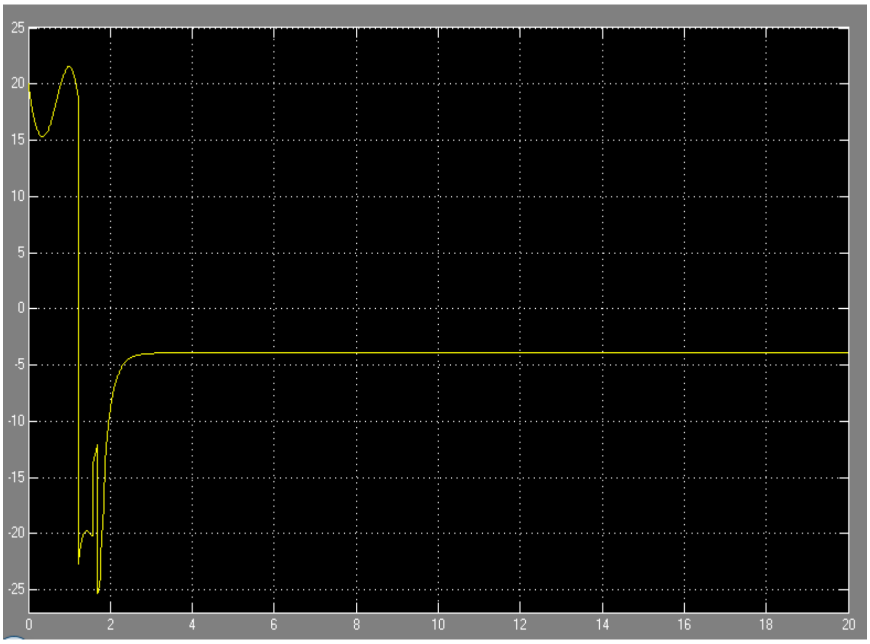

Figure 4.10: Joint one force/torque with boundary layer.

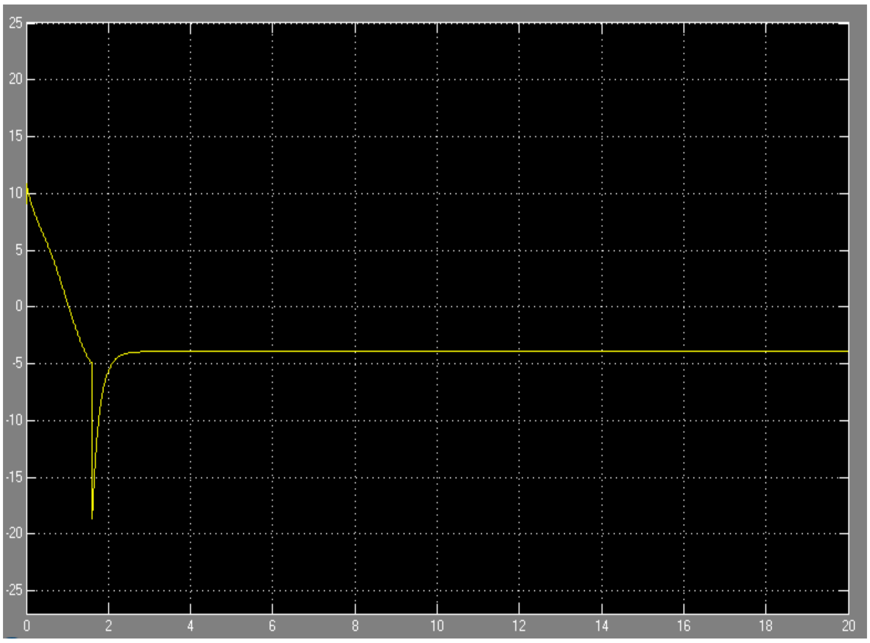

Figure 4.11: Joint two force/torque with boundary layer.

The introduction of boundary layer method completely eliminates the chattering from both joints of robotic arm manipulator. This is quite evident from Fig. 4.10 and Fig. 4.11.

4.6 Comparison

In this section comparison between the simulation results of SMC and PID control techniques is presented.

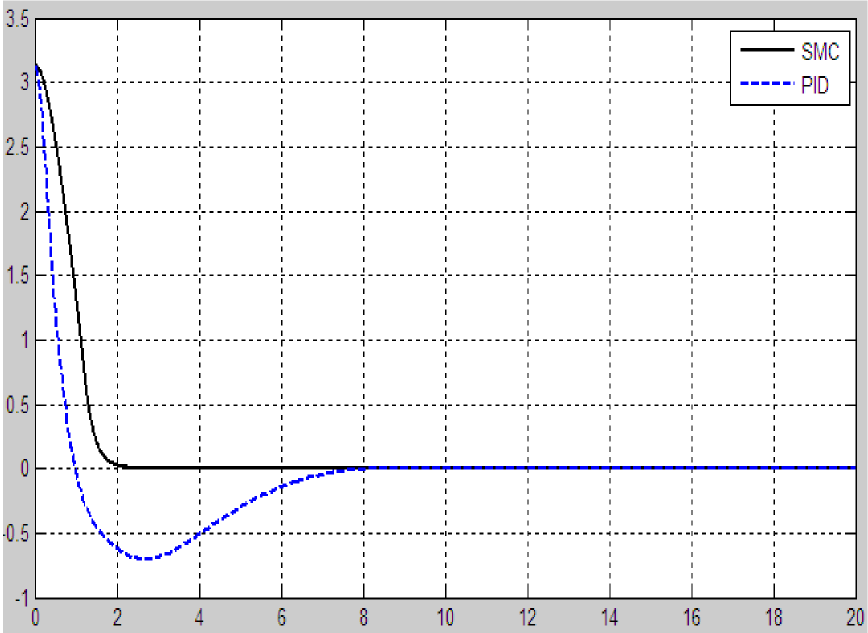

FIGURE 4.12: Joint One Tracking Error Comparison.

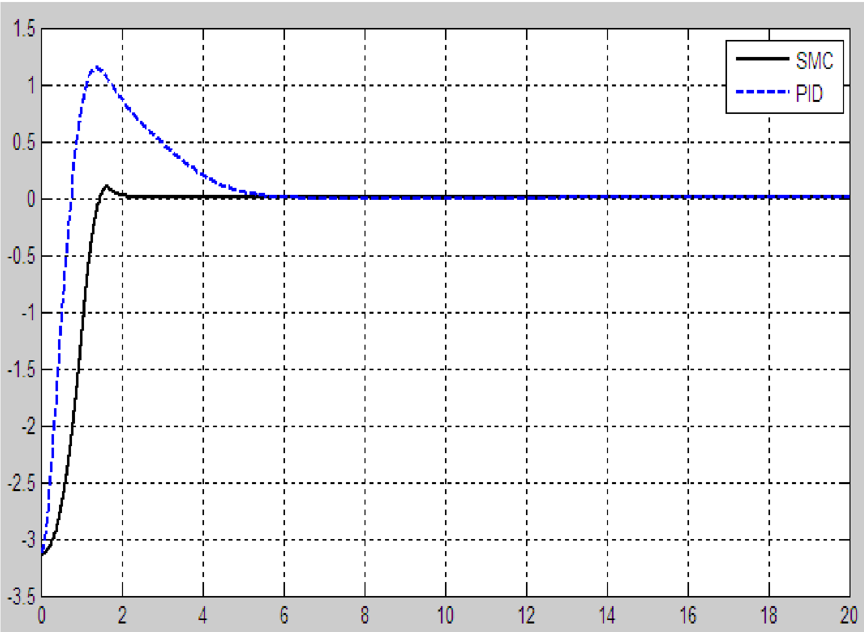

FIGURE 4.13: Joint Two Tracking Error Comparison.

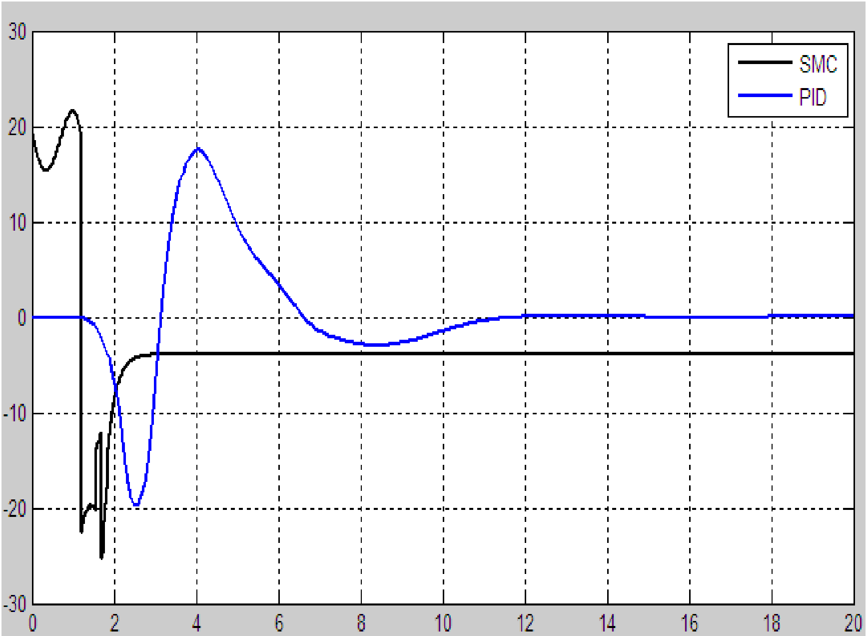

FIGURE 4.14: Joint One force Comparison

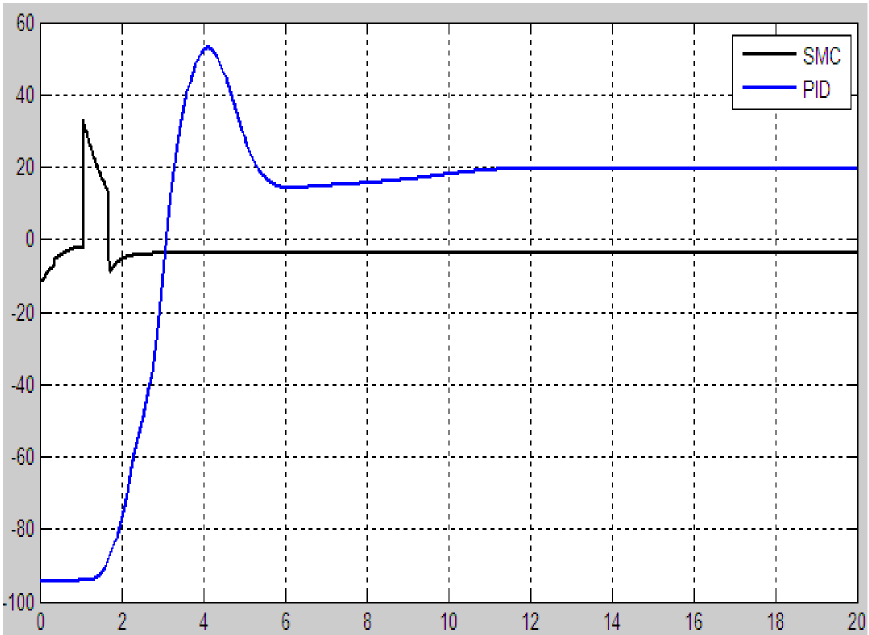

FIGURE 4.15: Joint Two force Comparison

The above simulation results clearly shows that SMC has better performance than PID control technique.

4.6.1 Parametric Variation

Keeping all the parameters same as listed in table 3.1 for PID control and in table 4.1 for SMC simulations, except m1 and m2 which are changed to 10Kg each.The simulation results are given below.

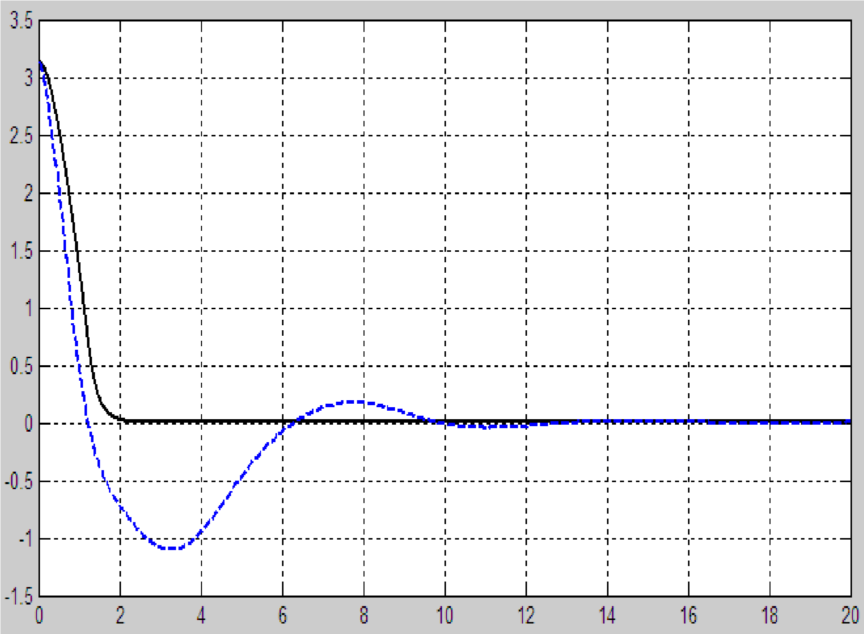

FIGURE 4.16: Joint One Tracking Error Comparison.

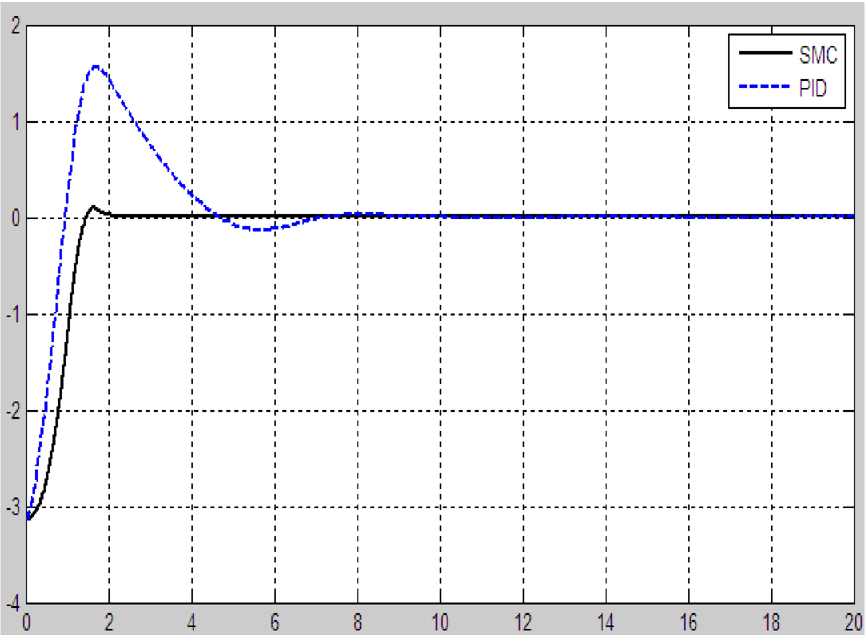

FIGURE 4.17: Joint Two Tracking Error Comparison.

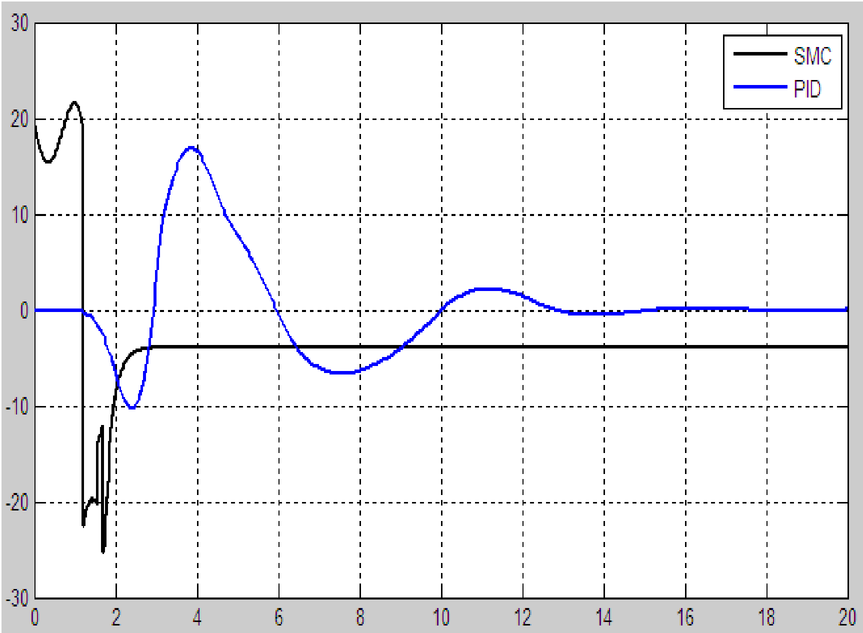

FIGURE 4.18: Joint One force Comparison

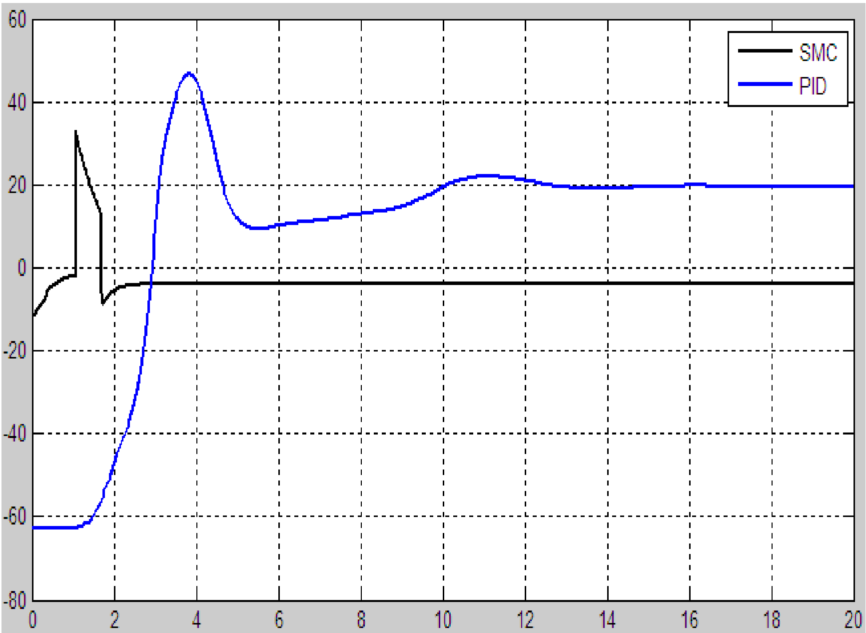

FIGURE 4.19: Joint Two force Comparison

The above figures indicates, that the Sliding Mode Control being insensitive to parametric variations has good performance with variation in mass parameters as compared to PID control.

4.6.2 Robustness Test

A sinusoidal signal as external disturbance is applied at both the joints to verify the robustness of the control techniques. Simulation results are presented below.

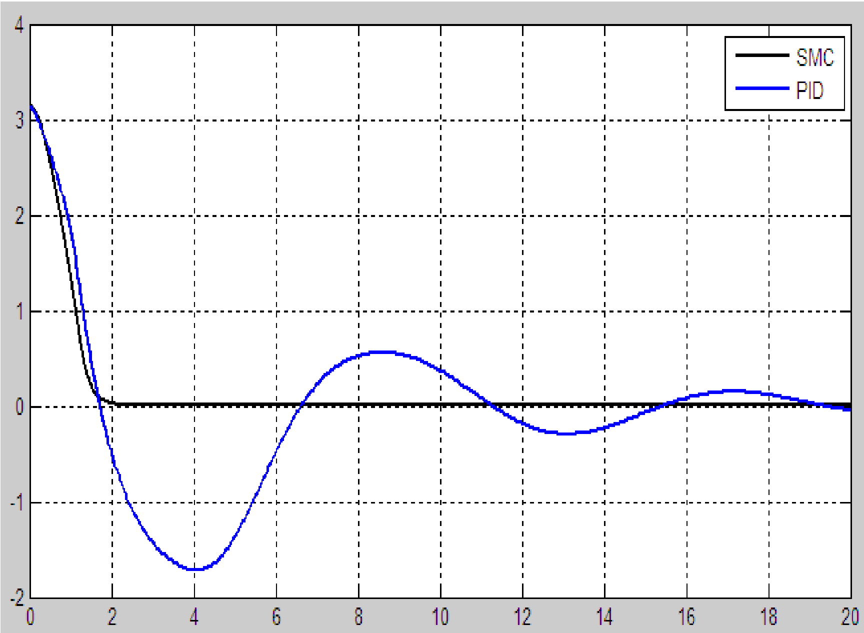

FIGURE 4.20: Joint One Tracking Error Comparison.

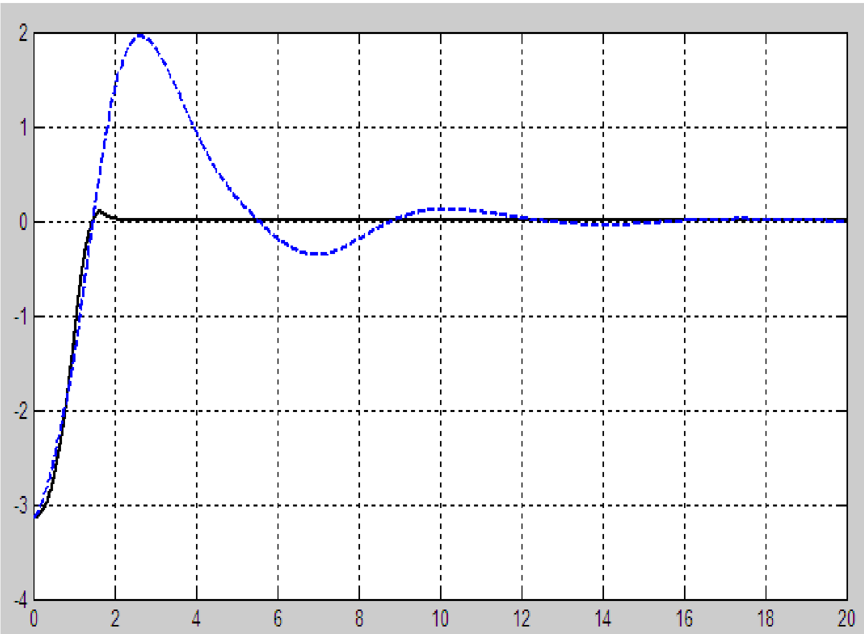

FIGURE 4.21: Joint Two Tracking Error Comparison.

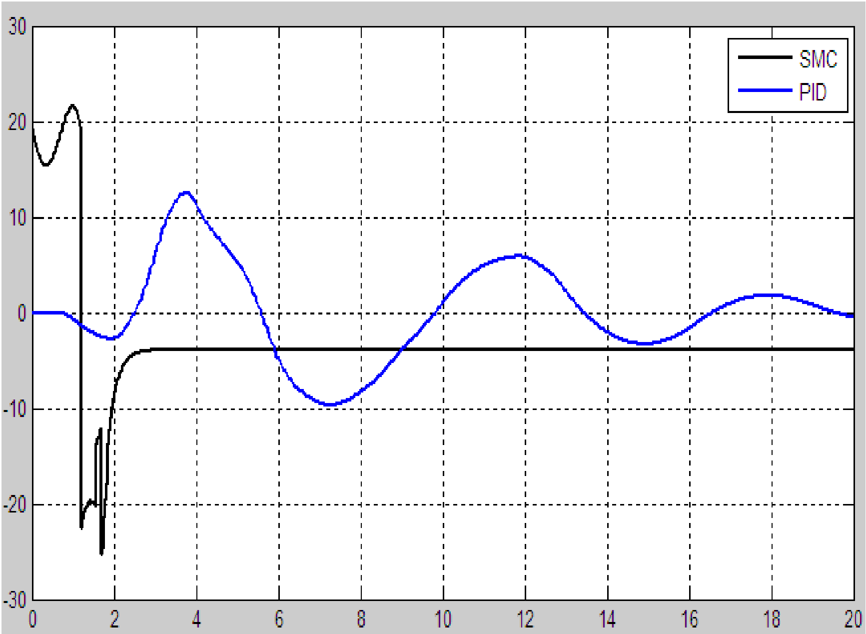

FIGURE 4.22: Joint One force Comparison

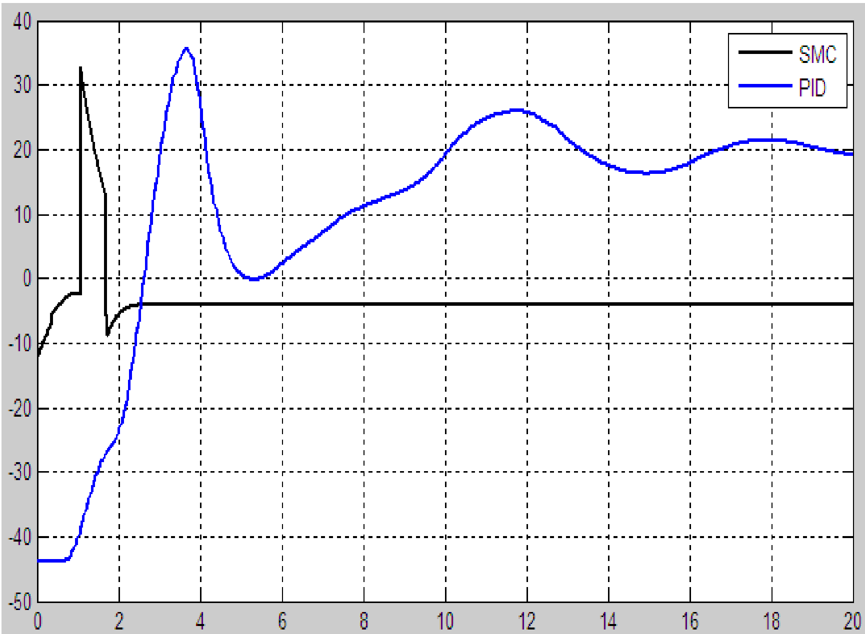

FIGURE 4.23: Joint Two force Comparison

It is clear from above simulation results that the sliding mode control technique being robust perform well even in the presence of external disturbances as compared to PID Control.

4.7 Conclusion

Sliding mode control shows good performances for nonlinear system with parameteric uncertainties and unmodeled disturbances. From above simulation results, the sliding mode control methodology has better dynamic response and minimum tracking error performance. The biggest drawback of this technique is chattering with control input which is handled by introducing boundary layer method. The overall result is satisfactory.

Chapter 5

Conclusion and Future Work

5.1 Introduction

This research presents a absolute study for controlling robot manipulator. The whole process depends on two things;

- The first one is modeling the manipulator
- The second one is controlling the manipulator

The modeling process includes complete kinematics “forward and inverse kinematics” analysis of the robot. A complete mathematical model of two link robot was developed.

Controlling process requires the designing of all constituents of controllers this means identifying the PID and Sliding Mode controller input and output, choosing the recommended control rule base.

5.2 Conclusion

The objective of this thesis was the motion control of the robot manipulator to trace desired trajectory by two different control methodologies, i.e. PID Control and Sliding Mode Control.

The proposed controllers were implemented for the motion control of a two link manipulator, in order to evaluate the effectiveness of the control strategies. By analyzing the simulation results, we can conclude that the proposed control methodologies are capable of attaining the desired motion control effectively. Further, they can be used for the system which required very small tracking error.

Also the results proved that the both suggested PID and SMC controller has good performances such as fast response and small errors for different desired tracking functions and it can be used, for more degrees of freedom robotic arm systems.

Further, Sliding Mode control algorithm has much better performance in comparison to PID technique, as sliding mode has very fast convergence time to attain the desired stable position with minimum tracking error and also the robust one. The only problem with SMC approach is the chattering which can be eliminated by introducing boundary layer.

5.3 Future Work

On the basis of the experience gained from the research work described in this study, many research oriented questions comes to mind. So, the few suggestions for the future work are as follows;

1. A future work can focuses on different types of controllers like passivity based control, Integral Sliding mode control etc.
2. Extend the system to more degree of freedom and apply same control techniques addressed in this research work or any other control methodology.

3. Add some disturbance to the system.

4. Artificial intelligence method can be used for the elimination of chattering in SMC.

Bibliography

[1] R. Kelly, V. S. Davila, and J. A. L. Perez, *Control of robot manipulators in joint space.* Springer Science & Business Media, 2006.

[2] M. W. Spong and M. Vidyasagar, *Robot dynamics and control.* John Wiley & Sons, 2008.

[3] T. R. Kurfess, *Robotics and automation handbook.* CRC press, 2004.

[4] A. Green and J. Z. Sasiadek, "Dynamics and trajectory tracking control of a two-link robot manipulator," *Modal Analysis*, vol. 10, no. 10, pp. 1415–1440, 2004.

[5] M. A. Llama, R. Kelly, and V. Santibanez, "Stable computed-torque control of robot manipulators via fuzzy self-tuning," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 30, no. 1, pp. 143–150, 2000.

[6] "Some facts about time-optimal control of a class of rigid manipulators."

[7] O. Barambones and V. Etxebarria, "Robust neural control for robotic manipulators," *Automatica*, vol. 38, no. 2, pp. 235–242, 2002.

[8] F. L. Lewis, A. Yesildirek, and K. Liu, "Multilayer neural-net robot controller with guaranteed tracking performance," *IEEE Transactions on Neural Networks*, vol. 7, no. 2, pp. 388–399, 1996.

[9] J. Yen and R. Langari, *Fuzzy logic: intelligence, control, and information.* Prentice Hall Upper Saddle River, NJ, 1999, vol. 1.

[10] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, *Robotics: modelling, planning and control.* Springer Science & Business Media, 2010.

[11] P. Tomei, "A simple PD controller for robots with elastic joints," *IEEE Transactions on Automatic Control*, vol. 36, no. 10, pp. 1208–1213, 1991.

[12] R. Ortega, A. Loria, and R. Kelly, "A semiglobally stable output feedback PID regulator for robot manipulator, automatic control," *IEEE Transaction August*, vol. 40, 1995.

[13] I. Cervantes and J. Alvarez-Ramirez, "On the PID tracking control of robot manipulators," *Systems & Control Letters*, vol. 42, no. 1, pp. 37–46, 2001.

[14] V. Utkin, "Sliding modes in optimization and control problems," 1992.

[15] J. Y. Luh, M. W. Walker, and R. P. Paul, "On-line computational scheme for mechanical manipulators," *Journal of Dynamic Systems, Measurement, and Control*, vol. 102, no. 2, pp. 69–76, 1980.

[16] J.-J. E. Slotine, W. Li *et al.*, *Applied nonlinear control.* Prentice Hall Englewood Cliffs, NJ, 1991, vol. 199, no. 1.

[17] C. Zhongyi, S. Fuchun, and C. Jing, "Disturbance observer-based robust control of free-floating space manipulators," *IEEE Systems Journal*, vol. 2, no. 1, pp. 114–119, 2008.

[18] J.-J. E. Slotine, "The robust control of robot manipulators," *The International Journal of Robotics Research*, vol. 4, no. 2, pp. 49–64, 1985.

[19] S. Arimoto, "Stability and robustness of PID feedback control for robot manipulators of sensory capability," in *Robotics Research, 1st Int. Symp.* MIT Press, 1984, pp. 783–799.

[20] J.-J. Slotine and S. S. Sastry, "Tracking control of non-linear systems using sliding surfaces, with application to robot manipulators," *International Journal of Control*, vol. 38, no. 2, pp. 465–492, 1983.

[21] W. Gao and J. C. Hung, "Variable structure control of nonlinear systems: A new approach," *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 45–55, 1993.

[22] D. Valério and J. S. da Costa, "Digital implementation of non-integer control and its application to a two-link robotic arm," in *European Control Conference (ECC), 2003.* IEEE, 2003, pp. 2267–2272.

[23] V. Utkin, J. Guldner, and J. Shi, "Sliding mode control in electromechanical systems,(1999)," *Taylar & Francis Ltd*, pp. 115–129.

[24] C. Edwards and S. Spurgeon, *Sliding mode control: theory and applications.* CRC Press, 1998.